

Student No.: \_\_\_\_\_

Qualifying/Placement Exam, Part-B  
3:30 – 5:30, January 9, 2017, 3239 BPS

Put your **Student Number** on every sheet of this  
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work!  
Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose!*

You may need the following constants:

$k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$	permittivity of free space
$\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$	Stefan-Boltzmann constant
$k = 1.4 \times 10^{-23} \text{ J/K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34} \text{ J}\cdot\text{s}$	Planck's constant
$= 6.58 \times 10^{-16} \text{ eV}\cdot\text{s}$	"
$c = 3.0 \times 10^8 \text{ m/s}$	speed of light
$e = 1.602 \times 10^{-19} \text{ C}$	charge of the electron

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1. [10 pts] A particle of mass  $m$  moving in the harmonic oscillator potential has the (normalized) wave function at time  $t = 0$  given by

$$\Psi(x, 0) = \frac{1}{3}\psi_0(x) - \frac{i}{\sqrt{3}}\psi_1(x) + \frac{(2+i)}{3}\psi_2(x)$$

where  $\psi_n(x)$  is the  $n^{\text{th}}$  eigenfunction, corresponding to energy  $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ .

- a) [4 pt] Write an expression for the wavefunction  $\Psi(x, t)$  at a nonzero time  $t$  later.
- b) [4 pt] What is the expectation value for the energy at the nonzero time  $t$ ?
- c) [2 pt] If the energy is measured at the time  $t$ , what is the probability that one would obtain the value of the energy found in part b)?

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2. [10 pts] A particle of mass  $m$  is contained in a one-dimensional box of width  $a$ . The potential energy  $U(x)$  is infinite at the walls of the box ( $x = 0$  and  $x = a$ ) and zero in between ( $0 < x < a$ ).

- (a) [6 pts] Solve the Schrodinger equation for the wave function and energy of this particle.
- (b) [2 pts] For the case  $n = 3$  (excited state), find the probability that the particle will be located in the region  $a/3 < x < 2a/3$ .
- (c) [2 pts] Consider the transition from the excited state  $n = 2$  to the ground state  $n = 1$ , calculate the wavelength of light emitted.

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3. [10 pts] The wave function of the ground state of a harmonic oscillator of force constant  $k$  and mass  $m$  is

$$\psi_0(x) = (\alpha/\pi)^{1/4} e^{-\alpha x^2/2}, \quad \alpha = m\omega_0/\hbar, \quad \omega_0^2 = k/m$$

Obtain an expression for the probability of finding the particle *outside* the classical region, where the classical region is defined as  $E \geq V(x)$ .

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4. [10 pts] Ultraviolet light is incident normally on the surface of a certain substance. The work function of the electrons in this substance is 3.44 eV. The incident light has an intensity of  $0.055 \text{ W/m}^2$ . The electrons are photoelectrically emitted with a maximum speed of  $4.2 \times 10^5 \text{ m/s}$ . What is the maximum number of electrons emitted from a square centimeter of the surface per second? Assume that none of the photons are reflected or heat the surface.

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5. [10 pts] The maximum energy  $E_{\max}$  of the electrons emitted in the decay of isotope  $^{14}\text{C}$  is 0.156 MeV. If the number of electrons with energy between  $E$  and  $E + dE$  is assumed to have the approximate form

$$N(E)dE \propto \sqrt{E}(E_{\max} - E)^2 dE$$

(a) [7 pts] Calculate the mean energy.

(b) [3 pts] Find the power generated assuming all the electrons are absorbed within a source of  $^{14}\text{C}$  emitting  $3.7 \times 10^7$  electrons per sec.

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6. [10 pts] The rest masses of the particles below are given in amu ( $1\text{amu} = 931.5\text{ MeV}$ ).

$^{12}\text{C}$  12.000000; p 1.007825; n 1.008665;  $\alpha$  4.002603

- (a) [3 pts] Calculate the energy needed to break up the  $^{12}\text{C}$  nucleus into its constituents.
- (b) [2 pts] If a  $^{12}\text{C}$  nucleus breaks up into 3 alphas, calculate the energy that is released.
- (c) [5 pts] If the alphas are to further break into neutrons and protons, then show that the overall energy needed is identical with the results in (a).