Student No.: ______________________

Qualifying/Placement Exam, Part-A
8:00 – 10:00, January 8, 2013, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99\times10^{9}\,\mathrm{Nm^2/C^2}$	Coulomb force constant
$\sigma = 5.7\times10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\,\mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\,\mathrm{J\cdot s}$	Planck's constant
$c = 3.0\times10^{8}\,\mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\,\mathrm{C}$	charge of the electron

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1. [10 pts] An isolated uniform conducting sphere with a radius of 10 cm is initially uncharged and then charged to a potential (with respect to ∞) of -10,000 V.

a. [5 pts] Find the energy stored in the system.
b. [5 pts] Find the number of electrons that must be brought to the sphere to obtain this potential.

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2. [10 pts] A long cylindrical capacitor of length L consists of an inner conducting wire of radius a, and a thin shell of radius b. There is a vacuum in the space between the two conductors.

a. [5 pts] If the capacitor is charged with a charge Q, find the electric field between the conductors as a function of the radial distance from the center of the wire. Assume that the length $L >> b$, so that end effects can be neglected.

b. [4 pts] In the space between the conductors, find the electric potential as a function of the radial distance from the center of the wire.

c. [1 pt] Find the capacitance of this arrangement of conductors.

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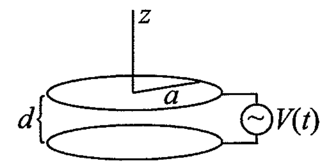

3. [10 pt] Two circular conducting plates with radius a, parallel to the xy-plane with $z = 0$ midway between the plates, are separated by a distance $d \ll a$, and connected to a time varying voltage source, $V(t)$. In your answers provide **magnitude** and **direction** for vectors.

a. [2 pts] Assuming the electric field is determined entirely by the potential $V(t)$, what is the time varying electric field **E** between the plates?
b. [3 pts] Using Maxwell's equations, determine the $\nabla \times \mathbf{B}$ between the plates; it is zero everywhere outside the plates.
c. [2 pts] Evaluate the $\oint \mathbf{B} \cdot d\ell$ around a coaxial loop at $z = 0$ with radius $r > a$, assuming that purely azimuthal magnetic field, $\mathbf{B} = B_\phi(r)\hat{\phi}$, dependent only on r, extends beyond the disks.
d. [3 pts] Stokes theorem states that $\oint \mathbf{B} \cdot d\ell = \int \nabla \times \mathbf{B} \cdot d\mathbf{A}$. Using this theorem and the results of parts b. and c., determine the radial dependence of **B** outside the plates.

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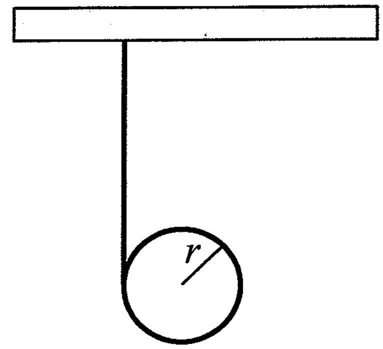

4. [10 pts] A string is wrapped around a uniform homogeneous cylinder whose radius is r and mass is m. The free end of the string is tied to the ceiling and the cylinder is allowed to fall (see the figure) starting from rest. As the string unwraps, the cylinder rotates.

a) [5 pts] Using Lagrangian mechanics, determine the linear acceleration of the center of mass of the cylinder.
b) [3 pts] What is the tension in the string?
c) [2 pts] What is the linear velocity of the cylinder after it has dropped down a distance h?.

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5. [10 pts] A comet, traveling in an elliptical orbit, has a distance r_1 and a speed v_1 at its closest approach to the sun. Using conservation laws, obtain a formula for r_2, the comet's farthest distance from the sun, in terms of (r_1, v_1, G, M), where G is Newton's constant and M is the mass of the sun, assumed to be much greater than the mass of the comet.

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6. [10 pts] Consider the Lagrangian, $L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)+k(x\dot{y}-y\dot{x})$, where m and k are constants.

a) [5 pts] Find Lagrange equations of motion.
b) [5 pts] Solve the equations, with these initial conditions:

$(x,y,z)=(0,0,0)$ and $(\dot{x},\dot{y},\dot{z})=(0,v_0,0)$.

Hint: For $\frac{d}{dt}\left[f(\dot{x},y)\right]=0$, then $f(\dot{x},y)=$ a constant.

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Qualifying/Placement Exam, Part-B
10:30 – 12:30, January 8, 2013, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

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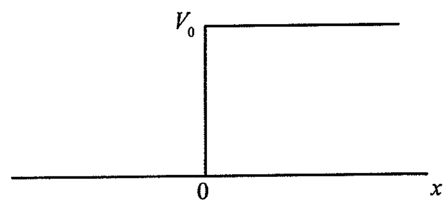
[10 pts] A particle of mass m travelling from $x = -\infty$ with amplitude A strikes the potential barrier illustrated in the figure. Assume that its energy E satisfies $0 < E < V_0$.



a. [2 pts] What is the form of the wave function for $x < 0$? Let the complex amplitude of the reflected wave be B.
b. [2 pts] What is the form of the wave function for $x > 0$? Let the amplitude of the transmitted wave be C.
c. [3 pts] By using the boundary conditions at $x = 0$, Determine B in terms of A.
d. [3 pts] Compute $|B|^2 / |A|^2$ and interpret the result.

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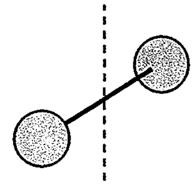

2. [10 pts] The Hamiltonian for a linear molecule (such as shown in the figure) free to rotate about its center of mass is

$$H = \frac{\vec{L}^2}{2I} + \alpha L_z ,$$

where I and α are constants. The eigenfunctions of $\vec{L}^2$ and L_z are the spherical harmonics $Y_\ell^m(\theta,\phi)$.

a. [5 pts] Derive a formula for the energy levels of the molecule.
b. [5 pts] The linear molecule is in the state $\psi(\theta,\phi)$ given by

$$\psi(\theta,\phi) = \frac{1}{\sqrt{6}}\left[Y_2^1(\theta,\phi) - iY_2^0(\theta,\phi) + 2Y_2^{-2}(\theta,\phi) \right]$$

What are the possible outcomes of a measurement of the energy of the molecule, and with what probability does each occur?

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3. [10 pts] A particle of mass m is in the ground state of a one-dimensional square well potential:

$$V(x,t<0)=\begin{cases} 0, & 0<x<L \\ \infty, & \text{other wise} \end{cases}.$$

a. [5 pts] Determine the ground state wave function for a particle in this potential.

b. [5 pts] At a time t = 0, the potential suddenly disappears. What is the probability density, dN/dp, for the particle to leave with the momentum p? Note that the probability density should be normalized so that $\int dp\, dN/dp = 1$.

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4. [10 pts] A 1.0 MeV γ-ray makes a head-on collision with a stationary, free electron ($mc^2 = 0.511$ MeV) and is scattered backwards through an angle of 180°. What kinetic energy does the electron acquire?

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5. [10 pt] A spaceship with a velocity of 0.83c travels from the Earth to Proxima Centauri (the nearest star to our Sun). Proxima Centauri is a distance of 4.24 light years from the Earth.

a) [4 pts] How long does the trip take as measured by the control center in Houston Texas?
b) [4 pts] How long does the trip take according to the passengers in the spaceship?
c) [2 pts] Messages from the spaceship are broadcast toward the Earth at a frequency of 90.5 MHz. At what frequency should the receivers on Earth be tuned to receive these messages?