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Qualifying/Placement Exam, Part-A
09:00 – 11:00, January 14, 2014, 4270 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99\times 10^9\ \text{Nm}^2/\text{C}^2$	Coulomb force constant
$\sigma = 5.7\times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$	Stefan-Boltzmann constant
$k = 1.4\times 10^{-23}\ \text{J/K}$	Boltzmann constant
$\hbar = 1.05\times 10^{-34}\ \text{J}\cdot\text{s}$	Planck's constant
$c = 3.0\times 10^8\ \text{m/s}$	speed of light
$e = 1.602\times 10^{-19}\text{C}$	charge of the electron

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1. [10 pts] At some time *t*, a proton travels with a velocity of $(1\times10^4\,\text{m/s})\hat{\imath}+(2\times10^4\,\text{m/s})\hat{\jmath}$ and is located at $x=3\text{m}, y=4\text{m}$. Find the magnetic field $\vec{B}$ at the following position: $x=2\text{m}, y=2\text{m}$.

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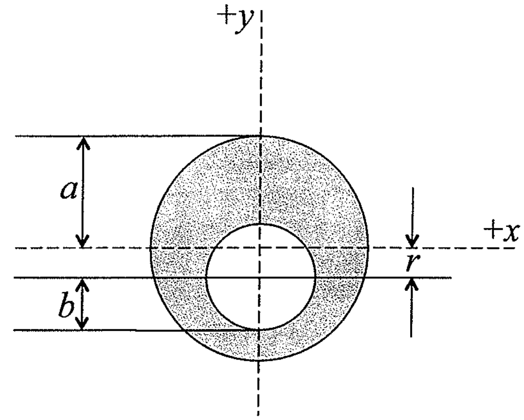
2. [10 pts] A uniformly charged sphere (radius a, and total charge Q) is centered at the origin. A spherical cavity (radius b) centered a distance r from the origin, is carved out from the sphere. Determine the electric potential a distance y from the origin along the +y-axis.



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3. [10 pts] An electromagnetic plane wave travels in vacuum at the speed of light c. The magnetic field of this wave is given by

$$\vec{B} = \vec{B}_0 \cos(\vec{a} \cdot \vec{r} + bt)$$

where $\vec{B}_0, \vec{a}$ and b are constants .

a. [5 pts] Give all conditions that must be satisfied by $\vec{B}_0, \vec{a}, b$, and c.
b. [5 pts] Suppose $\vec{B}_0 = B_0 \hat{z}$ and $\vec{a} = a\hat{y}$. Find the corresponding electric field $\vec{E}$.

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4. [10 pts] A uniform spherical ball of mass M and radius R is set rotating about a horizontal axis with an angular speed ω_0 and is placed gently on the floor. The initial center of mass velocity of the ball is zero. If the coefficient of sliding friction between the ball and the floor is μ, find the speed of the center of mass of the ball when it begins to roll without slipping. (The moment of inertia of the ball about an axis through its center of mass is $I = \frac{2}{5}MR^2$, and the linear and rotational velocities are independent until the ball rolls without slipping)

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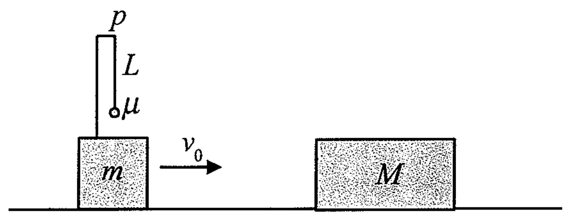
5. [10 pts] A small ball with mass μ is freely hung by a massless string of length L from a point p fixed above the center of a block with mass m, as shown in the figure. Initially moving with a uniform velocity v_0 on a frictionless surface, the mass m and the ball collide with and stick to another block with a larger mass M. Assume the mass of the ball is much less than either of the blocks, $\mu << (m, M)$.



a) [2 pts] What is the final velocity v of the blocks?

b) [8 pts] What is the minimum velocity v_0 required for the ball to circle around the point p.

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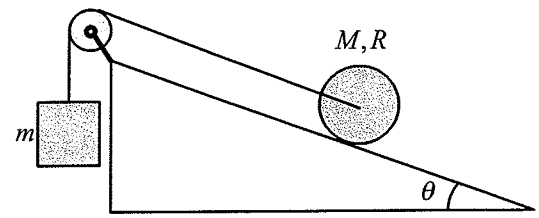
6. [10 pts] A cylinder of mass M and radius R rolls without slipping on an inclined plane, which is at an angle θ with respect to the horizontal. One end of the rope is attached to the axle of the cylinder and the other end is attached to a freely hanging mass m, with the rope passing over a frictionless pulley. (The masses of the rope and pulley are negligible.)



a) [4 pts] Write down the Lagrangian for this system, using the height y of the mass m as the dynamical variable.
b) [2 pts] Obtain a differential equation in y that describes the motion of the system.
c) [2 pts] Find the height $y(t)$ as a function of time if the system starts at rest at the height y_0.
d) [2 pts] What is the condition for the mass m to move upward?

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Qualifying/Placement Exam, Part-B
13:00 – 15:00, January 14, 2014, 4270 BPS

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1. [10 pts] A particle is confined to a one-dimensional box with $|x| < a$. Initially the spacial wavefunction is given by the form

$$\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}(a^2 - x^2)$$

a. [2 pts] If the wavefunction $\psi(x)$ is expanded in terms of the eigenfunctions of the box, will the expansion require
   i. Even parity eigenfunctions?
   ii. Odd parity eigenfunctions?
   iii. Both even and odd eigenfunctions?
   iv. Neither even nor odd eigenfunctions?
b. [2 pts] What is the expectation value of the momentum?.
c. [3 pts] What is the expectation value of the energy?
d. [3 pts] If an energy measurement is made, what is the most probable value for the energy?

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2. [10 pts] The energy eigenstates of the hydrogen atom are described by Schrodinger wavefunctions $\psi_{nlm}(\vec{r})$ where the energy of the ground state [$\psi_{100}(\vec{r})$] is $E_0 = -13.6\,\text{eV}$.

Suppose the atom is in the state $0.8\psi_{100} + 0.6i\psi_{100}$.

a. [4 pts] Find the expectation value of the energy.
b. [3 pts] Find the expectation value of $\vec{L}^2$, the square of the angular momentum vector.
c. [3 pts] Find the probability that a measurement of L_z, the z-component of angular momentum, would yield the result 0.

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3. [10 pts] A particle of mass m and momentum p moves in the positive x direction and encounters a potential "cliff" at the point $x = 0$,

$$V(x) = \begin{cases} 0, x < 0 \\ -V_0, x > 0 \end{cases}$$

where V_0 is a positive constant.

a) [8 pts] What is the probability that the particle is reflected from the "cliff" as a function of the momentum p.

b) [2 pt] Does this result surprise you and why?

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4. [10 pts] A monochromatic blue light with a wavelength of 434.2 nm is incident on a sample of cesium. Electrons emitted from the cesium surface are observed to have velocities ranging up to 5.491×10^5 m/s. ($m_e = 0.511$ MeV/c^2, $\hbar c = 197$ MeV·fm)

a) What is the work function for this sample of cesium.
b) Explain why there is a *range* of emitted electron velocities.
c) What is the wavelength of the fastest emitted electrons?

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5. [10 pt] The decay sequence for the isotope Uranium-235 begins with ${}^{235}_{92}\mathrm{U}$ and ends with ${}^{207}_{82}\mathrm{Pb}$.

a) [5 pts] What are the total number of α-particles and β-particles emitted per nucleus in the decay sequence.

b) [5 pts] If the atomic masses of the isotopes are $m_{\mathrm{U}} = 235.043923\mathrm{u}$ and $m_{\mathrm{Pb}} = 206.975881\mathrm{u}$ what is the total kinetic energy produced per nucleus in the decay sequence (MeV)? (The atomic mass of Helium is $m_{\mathrm{He}} = 4.002603\mathrm{u}$, and the atomic mass unit, $\mathrm{u} = 931.494\ \mathrm{MeV/c^2}$.)

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6. [10 pts] A neutral pion (π^0) has a rest energy of 135.0 MeV. A pion of momentum 200 MeV/c decays to two photons. If one of the photons travels in the same direction as the original pion, find the energies of the two photons by using energy and momentum conservation in special relativity.