Student No.: ______________________

Qualifying/Placement Exam, Part-A
10:00 – 12:00, January 12, 2016, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

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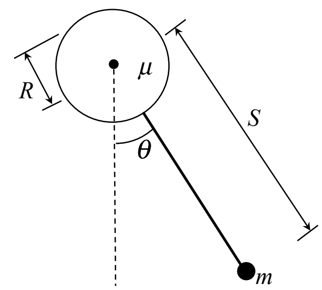
1. [10 pts] A pendulum is composed of a mass *m,* at the end of a massless rod, which places the mass at a distance *S,* from the (frictionless) axle of a disk with a uniformly distributed mass μ, and radius *R*, as shown in the figure.



   a) Using Lagrange Mechanics find the angular acceleration ($\ddot{\theta}$) of the system.

   b) Determine the period of motion of the pendulum in a small angle approximation.

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2. A particle with mass M, moves along the x-axis in a one-dimensional potential of the form, $V(x) = -Kxe^{-ax}$, where K and a are positive constants.

   a) [3 pts] Find an equilibrium position for the mass along the x-axis.

   b) [7 pts] For motion near the equilibrium position, find the period of small oscillations.

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3. [10 pts] Two identical particles, each with mass m and charge q, begin at rest and far apart. One of the particles is given an initial velocity v_0 toward the other. Determine the distance of closest approach, d, for the two particles, including any required constants to obtain that distance in meters.

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4. [10 pts] For the two vector fields given in (a) and (b) below, exhibit calculations that show which are possible electric fields. Here, k is a constant with the appropriate units.

   (a) $\mathbf{E} = k(xy\,\hat{\mathbf{x}} + 2yx\,\hat{\mathbf{y}} + 3xz\,\hat{\mathbf{z}})$

   (b) $\mathbf{E} = k(y^2\,\hat{\mathbf{x}} + (2xy + z^2)\,\hat{\mathbf{y}} + 2yz\,\hat{\mathbf{z}})$

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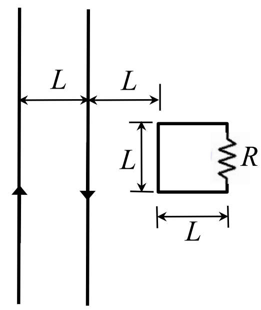
5. Two infinitely long parallel wires are separated by a distance L, and carry oppositely directed currents I with magnitudes that increase at the same fixed rate dI/dt. In the same plane as the wires is a square conducting loop with a side of length L, and located a distance L to the nearest wire, as shown in the figure.



a) [7 pts] Assuming that the loop includes a resistance, R, find the magnitude of the current, i, induced in the loop.

b) [3 pts] Explain why the current in the loop flows in a particular direction, clockwise or counter-clockwise.

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Qualifying/Placement Exam, Part-B
1:00 – 3:00, January 12, 2016, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
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$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

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1. [10 pts] The state vector for a two-level system evolves in time according to the Schrödinger equation:

$$i\hbar \frac{d\psi(t)}{dt} = H\psi(t)$$

where

$$H = \begin{pmatrix} 6E & -2E \\ -2E & 3E \end{pmatrix}, \quad \text{and} \quad \psi(0) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

and $E > 0$.

a) [2 pts] What are the possible energies of this system, i.e., the eigenvalues of *H*?

b) [3 pts] What are the (normalized) eigenvectors of *H?*

c) [3 pts] Express $\psi(t)$ in terms of the eigenvectors and energies of *H*.

d) [2 pts] What is the probability that a measurement of the energy of this state will yield the smaller of the two energies?

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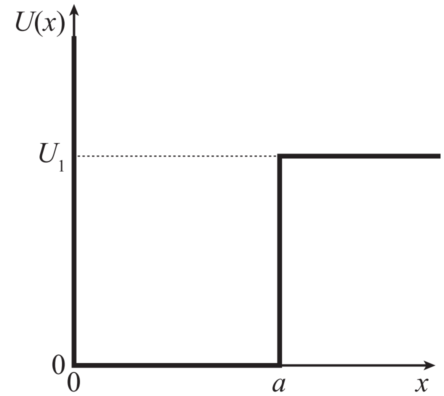

2. An electron is confined to a potential well given by

$$U(x) = \begin{cases} \infty \text{ for } x < 0 \\ 0 \text{ for } 0 \le x \le a \\ U_1 \text{ for } x > a \end{cases},$$

as shown in the figure.

The width of the well, $a = 1.00 \cdot 10^{-9}$ m, and $U_1 = 2.00$ eV.

[10 pts] This well has at least two bound states. Show whether or not it has a third bound state.

Note: $\hbar = 1.055 \cdot 10^{-34}$ Js, $m_e = 9.11 \cdot 10^{-31}$ kg and 1 eV = $1.602 \cdot 10^{-19}$ J.

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3. [10 pts] The energy eigenstates of the hydrogen atom are described by Schrodinger wavefunctions $\psi_{nlm}(\vec{r})$ where the energy of the ground state $[\psi_{100}(\vec{r})]$ is $E_0 = -13.6\,\mathrm{eV}$.

   Suppose the atom is in the state $0.8\psi_{100} + 0.6i\psi_{211}$.

   a) [4 pts] Find the expectation value of the energy.

   b) [3 pts] Find the expectation value of $\vec{L}^2$, the square of the angular momentum vector.

   c) [3 pts] Find the probability that a measurement of L_z, the *z*-component of angular momentum, would yield the result 0.

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4. [10 pts] An unstable particle of rest mass 1875.0 MeV/c^2 is initially at rest. It decays into two fragments, that fly off with velocities, $v_1 = 0.428c$ and $v_2 = -0.368c$. Find the rest masses of the fragments.
(Hint: for a particle with momentum pc and energy E, $\beta = pc/E$ and $\gamma = E/mc^2$)

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5. [10 pts] A 12.43-g fragment of charcoal is to be carbon dated. Measurements show that it has $1.01 \cdot 10^{-12}$ atoms of ^{14}C for each atom of ^{12}C. The tree from which the charcoal was produced died how many years ago? (Hint: The half-life of ^{14}C is 5730 yr., and the $^{14}C/^{12}C$ ratio in living organic matter is $1.20 \cdot 10^{-12}$.)

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6. [10 pts] Steel balls of mass M are dropped one after another from a height H by a high-precision device that attempts to hit a specific point on the ground. Estimate the typical distance away from that point that they will hit using the Uncertainty Principle.