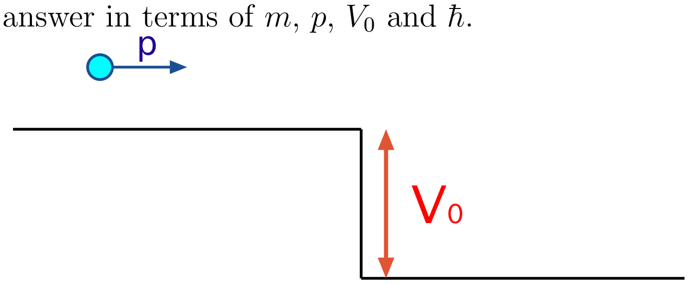
The problems listed below are meant to be representative of the material to be covered by the qualifier. The exam covers E&M, classical mechanics, thermodynamics, qauntum physics and relativity. The problems listed here focus more heavily on the material from upper-division undergraduate classes. Less attention was paid to representing material and concepts that would be covered in introductory level calculus-based physics courses, but it should be emphasized that roughly half the exam is based on such material. Of the five topics listed above, problems related to thermodynamics and relativity are definitely less numerous than those from the other three subjects.

Each section is devoted to its own topic and lists the specific chapters from texts from which one might study.

1 Quantum Physics

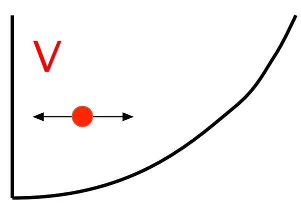
To study for the QM section of the qualifier, it is recommended that you review chapters 1-5 of *Introduction to Quantum Mechanics* by Giffiths. The problems below are representative of the material you might see on the exam.

1.1 Consider a beam of particles of mass m and momentum p that approaches the barrier of height V_0 from the left (the higher side of the barrier). What fraction of particles are reflected by the barrier? Give answer in terms of m, p, V_0 and $\hbar$.



1.2 Consider a particle of mass m in the potential well below,

$$V(x) = \begin{cases} \frac{1}{2}kx^2, & x > 0 \\ \infty, & x < 0 \end{cases}$$



In terms of k, m and $\hbar$, find the ground state energy of the well.

1.3 Light of wavelength λ is incident on a charged particle of mass m. For light scattered by an angle θ, DERIVE an expression for the wavelength of the scattered light, λ'. Express λ' in terms of θ, m, $\hbar$ and the speed of light c.

1.4 A particle of mass m is confined to an infinite square well,

$$V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$$

For $t = 0$, the particle is in the ground state of the above well. At time $t = 0$, the well is suddenly doubled in size to

$$V(x) = \begin{cases} 0, & -L < x < L \\ \infty, & \text{otherwise} \end{cases}$$

For $t > 0$, what is the probability of finding the state in the ground state of the new well?

1.5 The energy eigenstates for a particle of mass m confined to a square well of length L are described by the wave functions,

$$\psi_n(x) = \frac{\sin\left(n\frac{\pi x}{L}\right)}{\sqrt{L/2}}, \quad 0 < x < L.$$

Suppose that at a given instant the wave function is described by:

$$\Psi = (\cos\alpha)\psi_1 + i(\sin\alpha)\psi_3.$$

(a) Find the expectation of the energy $\langle H \rangle$. Express your answer in terms of α, $\hbar$, m and L.

(b) What is the probability density for finding the particle at $x = L/2$?

1.6 An electron's spin is initially pointed the x direction, i.e., $S_x\psi = (\hbar/2)\psi$. At time $t = 0$ the electron enters a region with magnetic field in the z direction that couples to the electron via the interation,

$$\mathcal{V} = -g\vec{B}\cdot\vec{S}, \quad \vec{S} = \frac{\hbar}{2}\vec{\sigma}.$$

Find the expectation $\langle S_x \rangle$ as a function of time. FYI:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

1.7 Neglecting interactions, consider a two-dimensional zero-temperature gas of neutrons of number density per area ρ. In terms of ρ, the neutron mass m and $\hbar$, derive the average kinetic energy of the neutrons. (Note: the neutron is a spin 1/2 particle)

2 Relativity and Modern Physics

In regards to relativity, you need to understand the material from Chapter 14 *Classical Dynamics* by Thornton and Marion. Another topic that is often explored concerns fundamental conservation laws. A standard text that describes conservation of baryon number, lepton number, etc., should be fine. The questions concerning conservation laws tend to be qualitative.

2.1 You are designing an accelerator to hurl relativistic particles of mass m_a at a stationary target of particles of mass m_b. The beam particles have energy $E_a = K + m_a c^2$, where K is the kinetic energy of the beam. The design should be such that collisions can create a resonance of mass $M > m_a + m_b$.

(a) In terms of m_a, E_a and c, what is the momentum of the beam particles p_a?

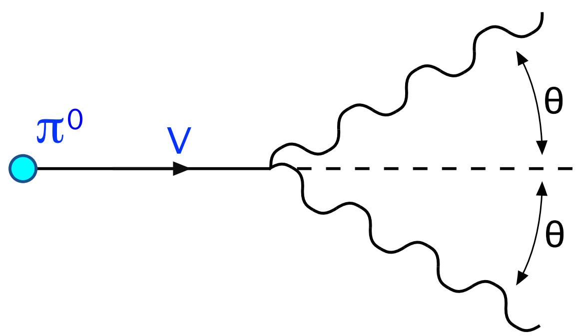
(b) Using energy and momentum conservation (or by exploiting relativistic invariants) find E_a in terms of m_a, m_b, M and c.

2.2 A muon (mass m_μ) has a lifetime $\tau_\mu = 2.2$ μs when decaying in the laboratory frame. A muon with relativistic energy E (includes rest-mass energy) travels down a beam pipe. What is the average distance one would expect the muon to travel before decaying? Give your answer in terms of m_μ, τ_μ, E and c.

2.3 A π^0 meson, moving at a relativistic speed v, decays in flight into two photons (which have zero mass). Both photons emerge at an angle θ relative to the original meson direction, as shown below.

(a) What is the total relativistic energy of the π^0 meson? Give answer in terms of v, c and m_π.

(b) Using conservation of energy and momentum, express the angle θ in terms of the same variables.



2.4 Consider the following interactions that are disallowed due to violations of fundamental conservation laws. Describe which conservation law(s) are violated (baryon number, lepton number, electric charge):

(a) $n \rightarrow p + e$

(b) $p + \bar{p} \rightarrow e + \bar{\nu}$

(c) $^5\text{He} \rightarrow p + p + p + n + n$

2.5 When describing radiation, one often encounters the terms α, β and γ. Which particles do these refer to?

3 Math Methods

From *Mathematical Methods in the Physical Sciences* by Mary L. Boas you should probably focus on chapters: 7,8,12,13,14,15. From *Essential Mathematical Methods for Physicists* by Weber and Arfken you should focus on chapters: 1,2,3,5,6,7,8.

3.1 Which of the following is a solution to Laplace's equation, $\nabla^2\phi(\vec{r}) = 0$, for $r > 0$? Circle all that work. Here, θ is the polar angle in spherical coordinates and ϕ is the azimuthal angle.

(a) $\phi = x^2 + y^2 + z^2$

(b) $\phi = r\cos\theta$

(c) $\phi = \frac{1}{r^2}\sin\theta\cos\phi$

(d) $\phi = \cos kx \sin ky \cos kz$

(e) $\phi = j_\ell(kr)P_\ell(\cos\theta)e^{im\phi}, \quad m \leq \ell, k > 0.$

3.2 Consider Schrödinger's equation in two dimensions ($\nabla^2 = \partial_x^2 + \partial_y^2$).

$$-\nabla^2\Phi(\vec{r}) = (k^2 - 2mV(r))\Phi(\vec{r}).$$

Using radial coordinates, $r \equiv \sqrt{x^2+y^2}$, $\phi \equiv \tan^{-1}(y/x)$, assume solutions of the form

$$\Phi(\vec{r}) = \phi_m(r)e^{im\phi}.$$

Derive the purely radial differential equation for $\phi_m(r)$.

3.3 Perform the following integral

$$\int_{-\infty}^{\infty} dx \frac{e^{ikx}}{x^2+a^2}.$$

3.4 Find the constant a such that the function,

$$f(x) = a\frac{\sin\Lambda x}{x}\Bigg|_{\Lambda\to\infty},$$

is a delta function.

3.5 Find the eigen-vectors and eigen-values of the matrix

$$\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$$

3.6 Consider the notation, $C(t) \equiv e^{iHt}Ce^{-iHt}$, where H and C are $n \times n$ matrices. Show that

$$\mathrm{Tr}\ e^{-\beta H}A(0)A(i\beta/2+z) \;=\; \mathrm{Tr}\ e^{-\beta H}A(0)A(i\beta/2-z). \tag{3.1}$$

3.7 For any n-dimensional state vectors $|\phi\rangle$ and $|\psi\rangle$, show that

$$\langle\phi|A|\psi\rangle^* = \langle\psi|A^\dagger|\phi\rangle,$$

where $A^\dagger_{ij} = (A_{ji})^*$. Here the bra and ket are defined so that $\langle\phi_i| \equiv |\phi\rangle^{T,*}$.

3.8 Consider the triangle wave form,

$$\begin{aligned} y(x) &= \alpha x, \quad -a < x < a, \\ y(x-2a) &= y(x). \end{aligned}$$

If $y(x)$ is expanded in a Fourier series,

$$y(x) = \sum_n a_n \cos(nkx) + b_n \sin(nkx),$$

(a) What is k?

(b) Which coefficients are zero?

3.9 Consider the periodic function, $y(x - 2\pi) = y(x)$

$$y(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

Find all the coefficients a_n and b_n for the Fourier expansion,

$$y(x) = \sum_n a_n \cos nx + b_n \sin nx.$$

3.10 Show that if a random number r is chosen with uniform probability with $0 < r < 1$, that the number

$$t = -\tau \ln(r)$$

will be generated with exponential probability proportional to $e^{-t/\tau}$.

4 Electrodynamics

From *Electromagnetism* by Pollack and Stump you should know chapters 3-11. Additionally, you should be able to solve all E&M problems from an elementary calculus-based textbook, such as Halliday and Resnick or Bauer and Westfall.

4.1 Beginning with Maxwell's equations, DERIVE the wave equation for an electromagnetic wave.

4.2 The upper half of a sphere ($z > 0$) has a uniform electric charge density of ρ, while the lower half has a uniform density $-\rho$. Find the electric potential $U(r, \theta)$ for large r, i.e., keep only the lowest power in $1/r$. (r and θ are the radius and polar angle in spherical coordinates)

4.3 Consider an empty spherical shell of radius a with charge spread about its surface. Outside the shell, the potential is given by:

$$V(r, \theta, \phi) = A\frac{a^2 \cos\theta}{r^2}, \quad r > a$$

(a) Find $V(r < a)$

(b) Find the charge density on the surface of the shell, $\sigma(\theta)$.

HELPFUL INFO FOR THIS PROBLEM AND NEXT: Solutions to Laplace's equation in spherical coordinates are:

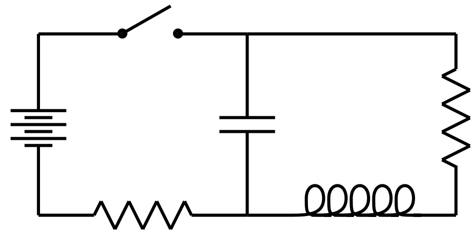
$$\begin{aligned} \phi(\vec{r}) &= P_\ell(\cos\theta)\left\{\frac{A}{r^{\ell+1}} + Br^\ell\right\} \\ P_0(x) &= 1,\ P_1(x) = x,\ P_2(x) = (3x^2 - 1)/2, \cdots \end{aligned}$$

4.4 A dielectric sphere of radius a and relative permittivity $\kappa = \epsilon/\epsilon_0$ is inserted into a region of constant electric field, i.e. $U(\vec{r}) = -Ez$. Express the potential at all points in space (both $r < a$ and $r > a$).

4.5 Consider an infinite conducting plane at $z = 0$ which is grounded ($U = 0$). An infinite string of positive charge density per unit length, λ, is placed parallel to the x axis at $z = a$ and $y = 0$. Find the electric field, $\vec{E}(x, y, z)$ for all $z > 0$.

4.6 A point charge q is brought a distance r from a conducting plane. Find the magnitude of the force acting on the charge.

4.7 A poloarized electromagnetic wave has electric field $\vec{E} = \hat{x}E_0\cos(kz - \omega_t)$. Using Maxwell's equations, determined the magnetic field $\vec{B}$. Show your work.



4.8 Consider the circuit above. After being closed for a long time, the switch is opened at t=0. Each resistor has resistance R, the battery has voltage V, the capacitance is C and the inductance is L. Find the current through the inductor as a function of t for $t > 0$.

4.8 Consider two current loops of radius a, each carrying current I. The first loop is centered at $x = y = 0$ and lies in the $z = 0$ plane. The second is also centered at $x = y = 0$ and lies in the $z = r$ plane. Assume $r >> a$. Find the force between the loops as a function of r.

4.9 Consider a long straight wire along the line $x = y = 0$ carrying a current

$$I_w(t) = I_0 \cos(\omega t).$$

Nearby one finds a square loop in the $y = 0$ plane. The square is defined by: $a < x < b, 0 < z < c$. If the loop has resistance r, find the induced current in the loop as a function of time.

4.10 Consider a square loop of length a in the $z = 0$ plane centered about $x = y = 0$. If the wire carries a current I, what is the magnitude of the magnetic field at the origin?

5 Thermodynamics

From introductory texts, you should know basic principles of engines and the manifestations of the laws of thermodynamics. Students should also be able to write and apply partition functions for simple systems. Combined with the thermodynamics covered in typical introductory texts, the first four chapters of Kittel's *Thermal Physics* should suffice.

5.1 A spin 1/2 system in a magnetic field has two states with energies $\pm\mu B$, and equilibrates according to a temperature T.

(a) What is the average energy at temperature $T = 0$?

(b) What is the average energy at temperature $T \to \infty$?

(c) Give an expression for the average energy as a function of T.

5.2 Two identical spin-zero bosons are placed in a system with 3 single-particle energy levels, $-\epsilon, 0$ and $+\epsilon$.

(a) What is the average energy of the system for $T = 0$?

(b) What is the average energy of the system for $T = \infty$?

(c) List all the system energy levels along with their degeneracy.

(d) Find a closed expression for the average energy as a function of temperature.

5.3 A heat pump moves heat from inside a home with temperature T_{in} to the outside where the temperature is $T_{\text{out}} < T_{\text{in}}$. Beginning with the laws of thermodynamics, e.g., $TdS = dE + PdV$, DERIVE the maximum efficiency attainable for such a heat pump in terms of T_{in} and T_{out}.

5.4 An ideal gas has a specific heat of $dE/dT = (5/2)nR$, where n is the number of moles of gas. Such a gas expands isentropically from $T = T_0$ to $T = T_f$. Find the ratio V_f/V_0.