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A General Approach for Optimizing Dynamic Response for Buck Converter

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APPLICATION NOTE

Abstract

A general approach for optimizing dynamic response for buck converters is presented. The basic theory to stabilize a buck converter with different types of compensation networks is introduced in detail. Using an averaging model and a computer program, three types of compensation networks for the buck converter are examined and analyzed. The K−factor approach to determine the compensation network components is explained. Finally, a practical experiment with a popular buck controller IC for computing applications is introduced. By using the presented approach, a fast transient response system using type−III compensation network is evaluated and results in a high performance system with sufficient stability margin.

INTRODUCTION

The subject of stability, which pertains to the closed−loop frequency response of switching regulators, has received much attention and many papers have been published on and around the subject. All of them have their own implementation method and considerations. To most practicing engineers, it seems a cloud of mystery shrouds feedback control loop stability. This paper seeks to remove that shroud, blending theory, simulation tools and practical experiment illustrating a general approach to stabilize the buck converter with least effort.

ANALYSIS OF THE OPEN LOOP BUCK CONVERTER

Figure 1 shows the feedback system for a buck converter. First, transfer functions $G_p(s)$ for the Pulse Width Modulation (PWM) stage and the power stage are identified. These two blocks are commonly grouped as the modulator. $G_c(s)$ is the compensation network transfer function. It will be discussed in the next section. The modeling of the low−frequency behavior of power switches in square−wave power converters is explained in [1]. The circuit of a buck modulator is shown in Figure 2 and the model of its power switches is shown in Figure 3. That represents the low frequency equivalent circuit for the buck modulator. The transfer function for the output, V_{out} with respect to duty ratio, D is:

$$\frac{dV_{out}(s)}{dD(s)} = V_{in}\frac{\dfrac{1}{(1/R_o)+\dfrac{1}{R_{esr}+(1/sC)}}}{sL+R+\dfrac{1}{(1/R_o)+\dfrac{1}{R_{esr}+(1/sC)}}} \quad \text{(eq. 1)}$$

When $R_o >> R_{esr}$ and $R_o >> R$, equation (1) can be simplified to:

$$\frac{dV_{out}(s)}{dD(s)} = V_{IN}\frac{R_{esr}Cs+1}{LCs^2+\left(\frac{L}{R_o}+C(R+R_{esr})\right)s+1} \quad \text{(eq. 2)}$$

where the double poles are located at:

$$f_p = \frac{1}{2\pi\sqrt{LC}} \text{ and} \quad \text{(eq. 3)}$$

the zero is located at:

$$f_z = \frac{1}{2\pi R_{esr}C} \quad \text{(eq. 4)}$$

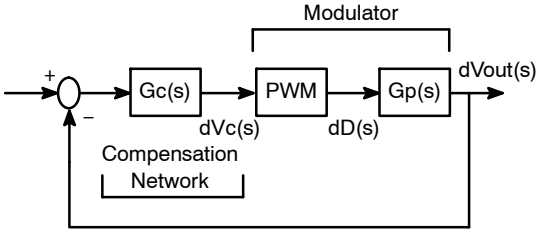


Figure 1. Feedback Control Loop for Buck Converter

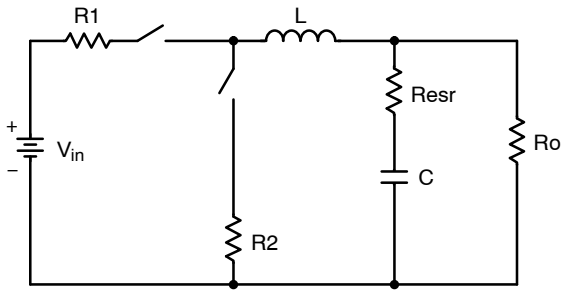


Figure 2. Buck Modulator Switches Model

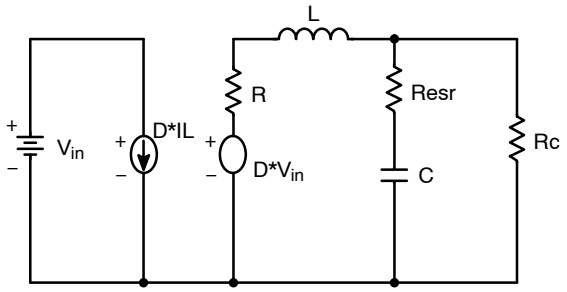


Figure 3. Low−Frequency Equivalent Circuit of Buck Modulator, R = DR1 + (1−D) R2

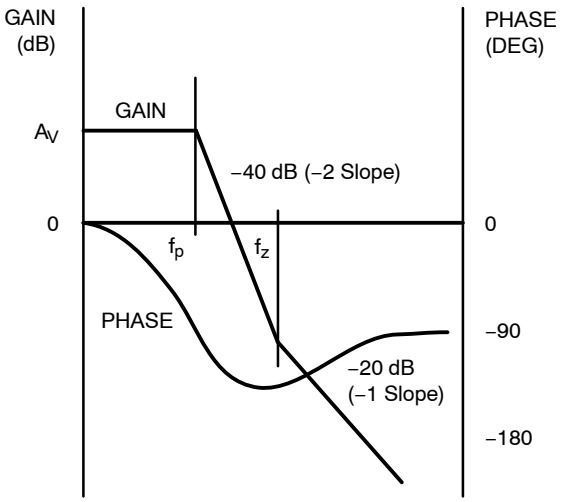


Figure 4. Frequency Response of Buck Modulator

Figure 4 shows the frequency response of a typical buck modulator. Note that the effect of the complex conjugate poles of L−C, f_p will make the gain curve rolls off at –40 dB/decade (−2 slope) and phase curve towards –180°. The roll−off continues until the frequency reaches region around f_z, where the ESR (Equivalent Series Resistance) of the output capacitor introduces a zero. At this point the gain curve slope changes to –20 dB/decade (−1 slope) and the phase curve turns back towards –90°.

For PWM shown in Figure 1, the PWM stage transfer function is:

$$\frac{dD(s)}{dV_C(s)} = \frac{1}{V_M} \qquad \text{(eq. 5)}$$

where V_M is the amplitude of the ramp in the PWM stage. Therefore, the DC gain for this stage is simply the input voltage V_{in} divided by V_M.

From (2) and (5), the open loop transfer function for the output V_{out} with respect to the compensation network control voltage V_c is:

$$\frac{dV_{out}(s)}{dV_C(s)} = \frac{V_{IN}}{V_M} \frac{R_{esr}Cs + 1}{LCs^2 + \left(\frac{L}{R_o} + C(R + R_{esr})\right)s + 1} \qquad \text{(eq. 6)}$$

COMPENSATION NETWORK TYPE

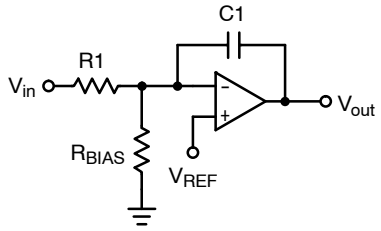
Type−I

The simplest form of compensation network with single−pole roll off is shown in Figure 5. This is called a Type−I compensation network. The transfer function of the compensation network in Figure 5 is:

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_1C_1s} \qquad \text{(eq. 7)}$$

with a crossover frequency, $f_c = \frac{1}{2\pi R_1 C_1}$.

A Type−I compensation network provides a single pole at the origin and the gain rolls off at –20 dB/decade (–1 slope) forever, crossing unity gain at the frequency where the reactance of C_1 is equal in magnitude to the resistance of R_1. The Type−I compensation network has –270° (−180° phase shift with the inverting compensation network included) of phase shift throughout the –1 slope region. Type−I compensation network is used for systems where the phase shift of the modulator is minimal.



where $R_{BIAS} = \frac{V_{REF}R_1}{V_{in} - V_{REF}}$

Figure 5. Type−I Compensation Network Schematic Diagram

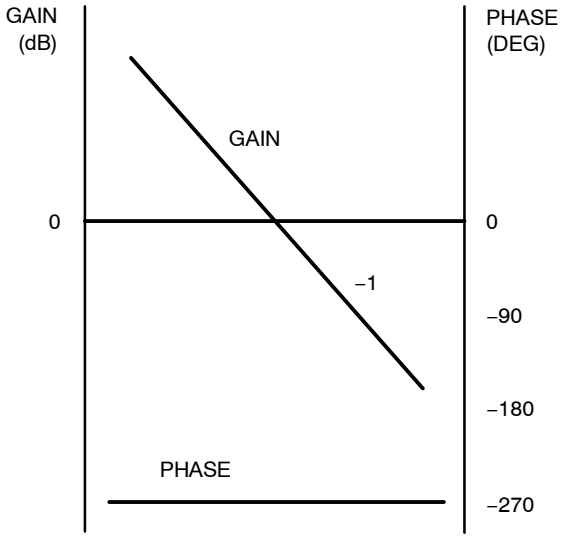
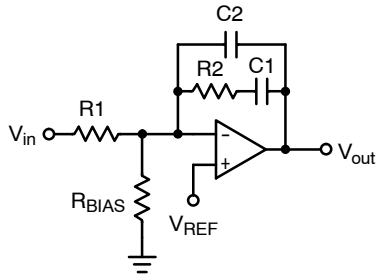


Figure 6. Frequency Response of Type−I Compensation Network

Type−II

The compensation network of Figure 7 offers improved buck converter transient response when the converter is subject to output load changes, as opposed to the slow response of the Type−I compensation network. Figure 8 shows the frequency response of the Type−II compensation network. A zero−pole pair has been introduced to give a region of frequency where the gain is flat and no phase shift is introduced. The region with constant gain occurs between the break frequencies f_1 and f_2. This region must be used for loop gain crossover. The gain and break frequencies are presented below.



where $R_{BIAS} = \frac{V_{REF}R_1}{V_{in}-V_{REF}}$

Figure 7. Type−II Compensation Network Schematic Diagram

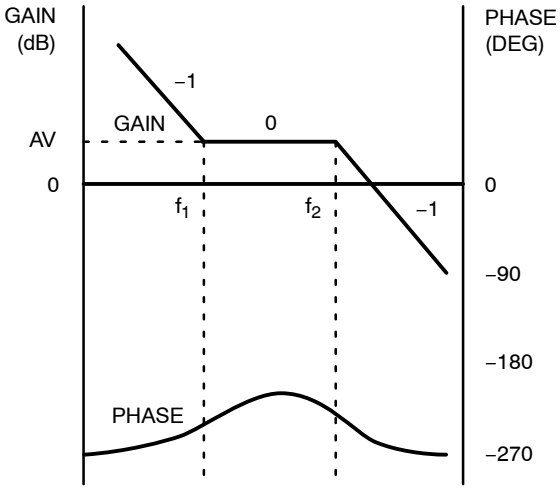


Figure 8. Frequency Response of Type−II Compensation Network

$$A_V = \frac{R_2}{R_1} \quad \text{(eq. 8)}$$

$$f_1 = \frac{1}{2\pi R_2 C_1} \quad \text{(eq. 9)}$$

$$f_2 = \frac{C_1 + C_2}{2\pi R_2 C_1 C_2} \approx \frac{1}{2\pi R_2 C_2} \quad \text{(eq. 10)}$$

where $C_2 << C_1$

Type−III

The compensation network depicted in Figure 9 can give superior transient response. In this circuit, the network provides a pole at the origin with two zero−pole pairs. As shown in Figure 10 shows how the low frequency gain decreases at −20 dB/decade (−1 slope) due to the pole at the origin. The gain becomes constant between the two zero frequencies, f_1 and f_2. After f_2, the effects of second zero cause the gain to increase at +20 dB/decade (+1 slope) until approaching f_3. It is flat again after f_3. After f_4, the magnitude response decreases at a rate of –20 dB/decade (−1 slope). The closed loop compensation crossover should occur in between f_2 and f_3 for best results. The gains and pole−zero frequencies can be calculated from the following equations:

$$A_{V1} = \frac{R_2}{R_1} \quad \text{(eq. 11)}$$

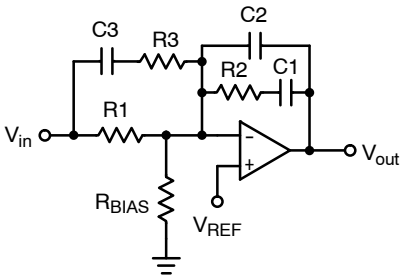
$$A_{V2} = \frac{R_2(R_1 + R_3)}{R_1 R_3} \quad \text{(eq. 12)}$$

$$f_1 = \frac{1}{2\pi R_2 C_1} \quad \text{(eq. 13)}$$

$$f_2 = \frac{1}{2\pi(R_1 + R_3)C_3} \quad \text{(eq. 14)}$$

$$f_3 = \frac{C_1 + C_2}{2\pi R_2 C_1 C_2} \quad \text{(eq. 15)}$$

$$f_4 = \frac{1}{2\pi R_3 C_3} \quad \text{(eq. 16)}$$



where $R_{BIAS} = \frac{V_{REF}R_1}{V_{in}-V_{REF}}$

Figure 9. Type−III Compensation Network Schematic Diagram

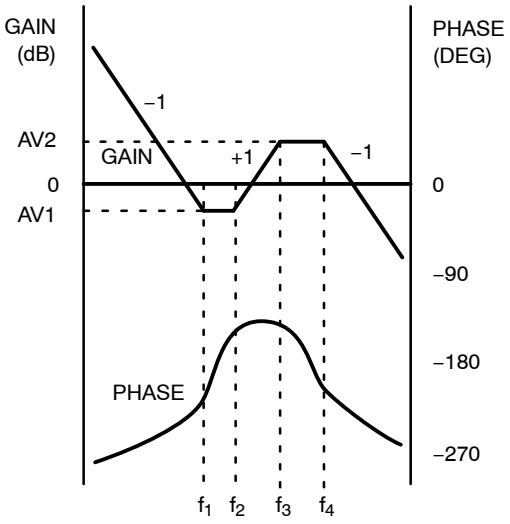


Figure 10. Frequency Response of Type−III Compensation Network

K−Factor

The K−factor [2] is a simple mathematical tool for defining the shape and characteristics of a transfer function, regardless of the type of compensation network used. The K−factor is a measure of the reduction of gain at low frequencies and increase of gain at high frequencies, arrived at by controlling the location of poles and zeros of the feedback compensation networks Bode plot in relation to the loop crossover frequency f_c. Figure 11a shows that, for Type−I compensation network K is always 1. This is due to a total lack of phase boost or corresponding increase or decrease in gain. For Type−II and Type−III compensation networks, as shown in Figures 11b and 11c, the zero frequency is placed a factor of K below the loop crossover frequency and the pole frequency a factor of K above. Since f_c is the geometric mean of the zero and pole locations, peak phase boost will occur at the crossover frequency. It is widely known that phase boost due to a zero−pole pair is the inverse tangent of the ratio of the measurement frequency to the zero or pole frequency. The total phase shift then is the sum of all individual zero and pole phase contributions. For type−II compensation network, the phase boost θ_{boost} at frequency f_c is given by the equation:

$$\theta_{boost} = \tan^{-1}(K) - \tan^{-1}\left(\frac{1}{K}\right) \quad \text{(eq. 17)}$$

From this equation it can be shown that:

$$K = \tan\left[\left(\frac{\theta_{boost}}{2}\right) + 45^\circ\right] \quad \text{(eq. 18)}$$

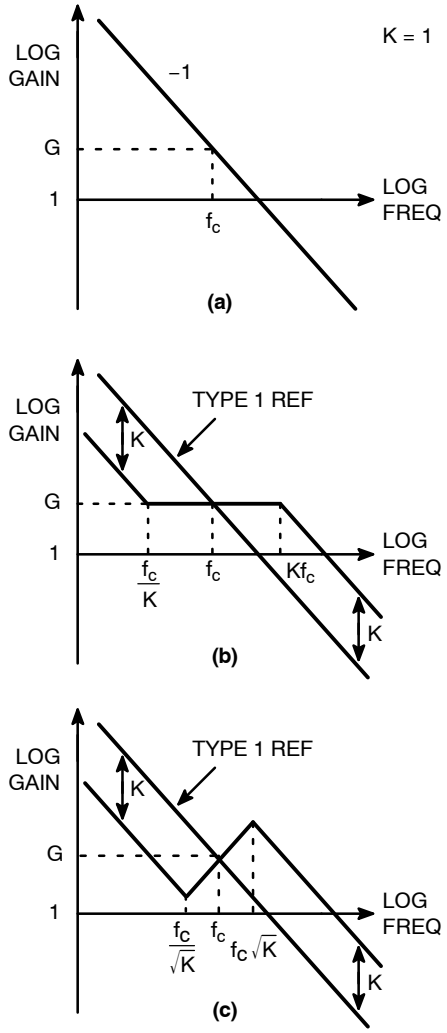


Figure 11. The Bode plot characteristics of (a) the Type−I compensation network, (b) Type−II compensation network, and (c) Type−III compensation network, in relation to the K factor.

For Type−III compensation network, the phase boost θ_{boost} at frequency f_c is given by the equation:

$$\theta_{boost} = \tan^{-1}\left(\sqrt{K}\right) - \tan^{-1}\left(\frac{1}{\sqrt{K}}\right) \quad \text{(eq. 19)}$$

and subsequently,

$$K = \tan 2\left[\left(\frac{\theta_{boost}}{4}\right) + 45^\circ\right] \quad \text{(eq. 20)}$$

Equations shown in Table 1 provide a convenient way to calculate the component values for each compensation network type discussed in the previous section. For the corresponding schematic, please refer to Figure 9. The gain G is the required compensation network gain at the crossover frequency and must equal the modulator loss.

Table 1. Components for Type−I, Type−II and Type−III Compensation Networks

	Type−I	Type−II	Type−III
R_1	User−Selected		
R_2	Not Used	$\frac{K^2}{K^2-1}GR_1$	$\frac{\sqrt{K}}{K-1}GR_1$
R_3		Not Used	$\frac{R_1}{K-1}$
C_1	$\frac{1}{2\pi f_c GR_1}$	$\frac{K^2-1}{K}\frac{1}{2\pi f_c GR_1}$	$\frac{K-1}{2\pi f_c GR_1}$
C_2	Not Used	$\frac{1}{K}\frac{1}{2\pi f_c GR_1}$	$\frac{1}{2\pi f_c GR_1}$
C_3		Not Used	$\frac{K-1}{\sqrt{K}}\frac{1}{2\pi f_c R_1}$

NOTE: $R_{BIAS} = \frac{V_{REF}R_1}{V_{in} - V_{REF}}$

Synthesis of Compensation Networks

The basic steps to synthesize a compensation network to stabilize a feedback loop are recommended as follows:

Step 1: Choose a crossover frequency and determine the phase shift and gain. The crossover frequency is the point where you want the overall loop gain to be unity. Remember that the higher the crossover frequency, the better the transient response of the power converter. As a rule of thumb, the crossover frequency should be high enough to provide good dynamic regulation and low enough to avoid sub−harmonic instability and noise amplification. However, practical limitations restrict the range of the crossover frequency. The theoretical limit is half of the switching frequency, but practical considerations have proven that a crossover frequency figure of less than one−fifth of the switching frequency is a good choice. Determine the phase shift, P_m and modulator gain, G_m at the crossover frequency, f_c.

Step 2: Determine the required compensation network gain. The gain, G is the required compensation network gain at crossover frequency and must be equal to the modulator loss. If the gain is expressed in dB, then the compensation network gain is simply the negative of the modulator gain, that is:

$$G = 1/G_m \quad \text{(eq. 21)}$$

Step 3: Choose the desired phase margin (using the K−factor approach). This margin is the amount of phase desired at unity gain. The phase margin should be large enough to provide well−damped transient response and accommodate unforeseen excess phase shift due to all possible variations. Phase margin may have a range of 30° to 90°, with 60° being a good compromise.

Step 4: Calculate the required phase boost and determine the K value (using the K−factor approach). The amount of phase boost required from the zero−pole pair in the compensation network is given by the formula:

$$\theta_{boost} = M - P_m - 90^\circ \quad \text{(eq. 22)}$$

where:

M = desired phase margin (degrees)

P_m = modulator phase shift (degrees)

Step 5: Choose the compensation network type and determine the K value (using the K−factor approach). Choose compensation network Type−I when no phase boost is required, compensation network Type−II when the required boost is less than 90° (a more practical requirement is less than 70°), and compensation network Type−III when the required phase boost is greater than 70° and less than 180°. K value can be calculated from Equation (18) or (20) for Type−II and Type−III respectively. For Type−I, K is always equal to 1.

Step 6: Calculate component values. Based on Equations (7)−(16), calculate values for the compensation network. Otherwise, derive the values using the K−factor approach and Table 1 as described in previous sections.

SELECTION OF COMPENSATION NETWORK TYPE

The Type−I compensation network uses a minimum number of components to achieve necessary phase margin. The phase margin can be adjusted by choosing the unity gain crossover frequency. This type of compensation network is used for converter topologies that exhibit a minimal phase shift prior to the anticipated unity gain crossover frequency. Topologies include forward−mode regulators, such as buck, push−pull, half−bridge and full−bridge using either voltage or current mode control techniques. These converters exhibit a relatively low phase shift below the pole contributed by the output filter, so no phase boost is required from the compensation network stage. Type−I compensation network has a relatively poor transient load response time as the unity gain crossover frequency normally occurs at a low frequency. Its load regulation is outstanding due to its very high DC gain. This type of compensation network is not commonly used in systems that require rapid transient load respond.

The Type−II compensation network is used for converters that exhibit a single filter pole at low frequency and a maximum phase shift of 90°. These converters are the boost, buck−boost and the fly−back topologies operating in the discontinuous mode (DCM) of operation. Forward−mode converters with current−mode control are also included. The pole caused by the output filter capacitor and the load resistance occurs at an extremely low frequency. In order to improve the transient response characteristic, the loop bandwidth needs to be extended. By adding an additional zero before the first pole, the loop bandwidth can be greatly extended with phase boost and hence the overall transient response time can be greatly improved.

The Type−III compensation network is intended for converters that exhibit a –40 dB/decade roll−off above the poles of the output filter and a −180° phase lag. These include the forward−mode converters such as buck, push−pull, half−bridge and full−bridge topologies using voltage mode control techniques. Like the Type−II compensation network method, Type−III compensation network introduces zeros into the error amplifier to reduce the steep gain slope above the double pole caused by the filter and its associated −180° phase shift. This extends the loop bandwidth. Type−III compensation network can achieve very fast transient response and may provide more than 70° phase boost. They are commonly used for systems requiring very fast transient respond.

CLOSED FEEDBACK LOOP SYSTEM

Closed Loop System with Different Types of Compensation Network

A real life buck converter shown in Figure 2 with V_{in} = 5.0 V, L = 1.8 μH, R_{esr} = 5.0 mΩ and C = 3.5 mF, R_o = 0.25 Ω is considered. First of all, the converter with different compensation networks is evaluated. With the averaged model proposed by Sam Ben−Yaakov [3], we can simulate the open loop transfer function and the whole closed feedback loop system response. The schematic of the buck converter compensated with a Type−I compensation network is shown in Figure 5. With steps suggested in previous sections and equations listed in Table 1 or Equation (7), component values are calculated. With R_{BIAS} = 9.23 kΩ, R_1 = 10 kΩ and C_1 = 100 nF, the break frequency is 159 Hz as shown in Figure 12, the ramp size of the PWM stage is 1.0 V and the reference for the error amplifier, V_{REF} = 1.2 V. From simulation results shown in Figure 12, the closed loop system has 79° of phase margin, but the unity gain bandwidth is only 1.1415 kHz. The high phase margin results in very stable system, but the low bandwidth will result in very slow transient response. Another compensation network type should be considered.

The buck converter is then compensated with a Type−II compensation network as shown in Figure 7 with R_2 = 20 kΩ, R_1 = 2.2 kΩ, C_2 = 165.8 pF and C_1 = 3.96 nF by placing a zero around the L−C resonant frequency of the buck modulator and a pole around 1/5 switching frequency. Again by simulation, the frequency response is shown in Figure 13. The open loop response has 40° of phase margin and the unity gain bandwidth of 19.78 kHz. The transient response is much better then last trial when type−II compensation network is used, however, the phase margin is not good enough.

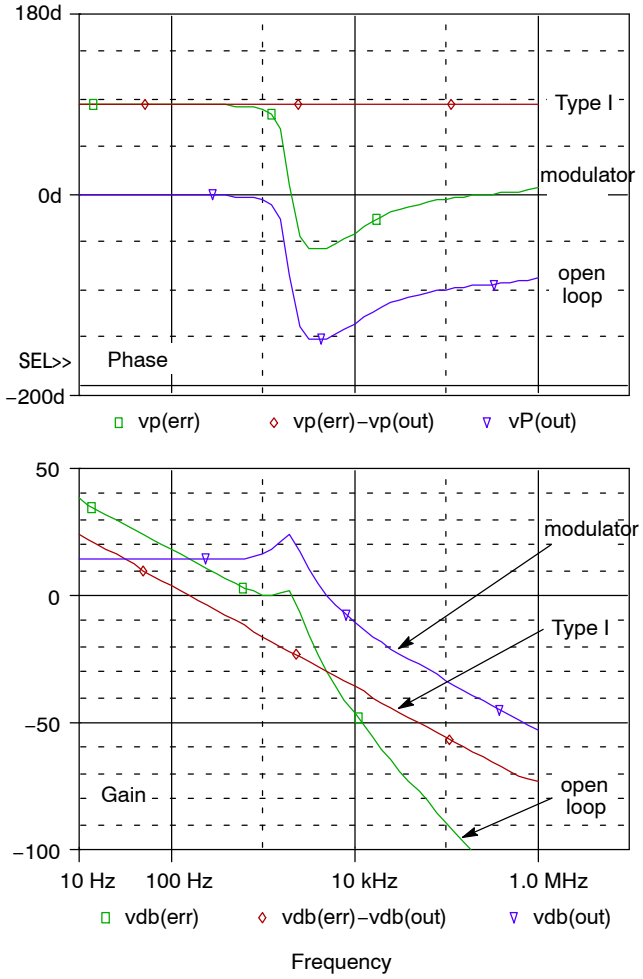


Figure 12. Open Loop System of Buck Converter with Type−I Compensation Network

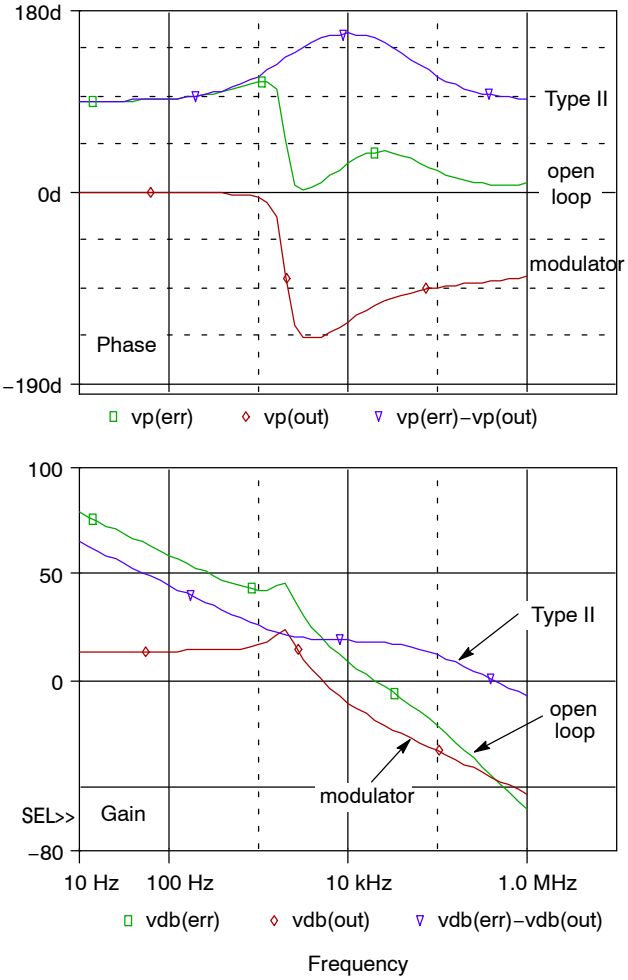


Figure 13. Open Loop System of Buck Converter with Type−II Compensation Network

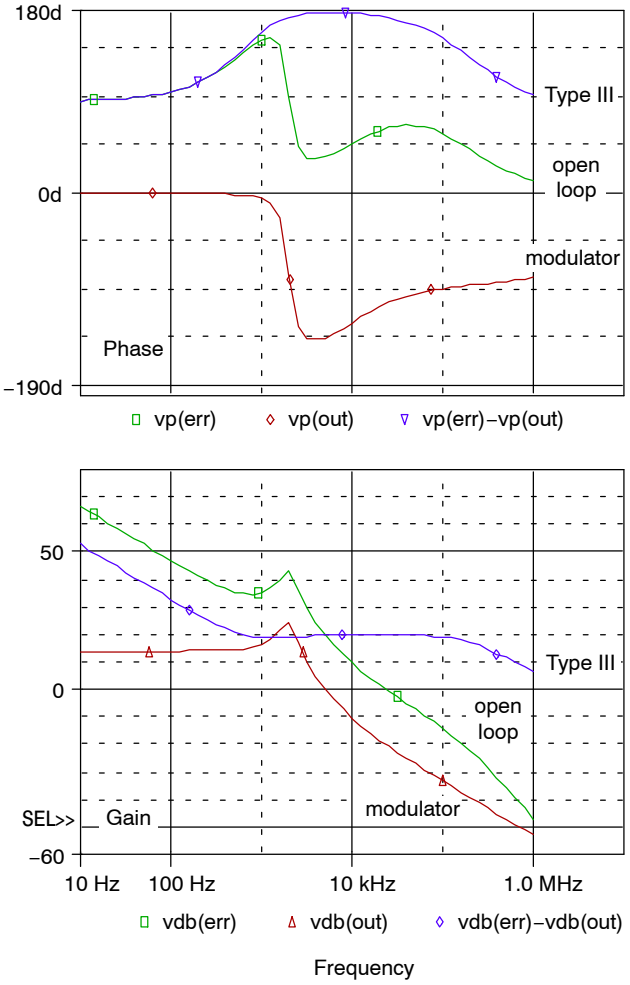


Figure 14. Open Loop System of the Buck Converter with Type−III Compensation Network

For the Type−III compensation network shown in Figure 9, applying the equations in Table 1 and choosing a crossover frequency less than 1/5 of the switching frequency, the gain loss of the modulator is obtained from the open loop Bode plot shown in Figure 14. Then the compensation network gains and break frequencies are calculated. The double zero is placed around the resonant frequency of the modulator. The first pole is placed around the ESR zero of the modulator and the second pole is placed around 1/2 of the switching frequency. Component values are calculated as R_2 = 20 kΩ, C_1 = 6.8 nF, C_2 = 10 nF, R_3 = 8.0 Ω, C_3 = 100 nF, R_1 = 2.2 k and R_{BIAS} = 2.0 kΩ. The frequency response of the Type−III compensated buck converter is shown in Figure 14. The open loop system provides 63.66° of phase margin and unity gain bandwidth of 23.48 kHz. The moderate phase margin and significantly higher bandwidth provide an excellent trade−off between stability and fast transient response.

Although the compensation network type is selected, based on the phase boost requirement, for most cases, the converter actually can be designed with all three types of compensation networks. The major difference is the transient response of the closed loop system. The Type−III compensation network can give the fastest transient response among three types of compensation network. Choosing the value of R_1 depends on the modulator output current. When the output current is large, the value of R_1 can be arbitrary. If the output current is small, R_1 should not be too small in order to avoid loading effects at V_{out}.

Testing Closed Loop System with Numerical Tool

ON Semiconductor has developed software [4] for simulating buck converter behavior with all three types of compensation network reviewed in this paper. The example in the section, Selection of Compensation Network Type, is next evaluated using this software. Figure 15 shows the open loop Bode plot of the converter modulator itself. Figures 16, 17 and 18 illustrate the open loop converter response with Type−I, Type−II and Type−III compensation networks, respectively. Results of these simulations are agreed well with the results from PSpice simulation with the averaging model in above−mentioned section.

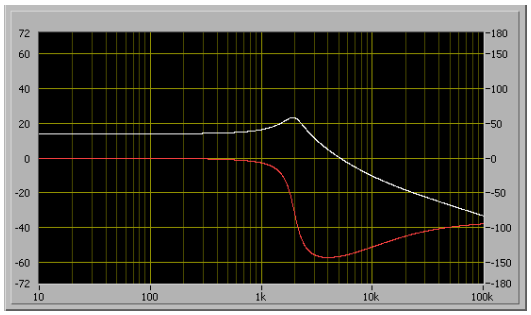
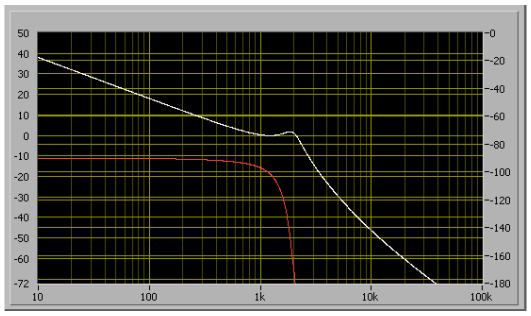


Figure 15. Modulator Bode Plot of the Buck Converter



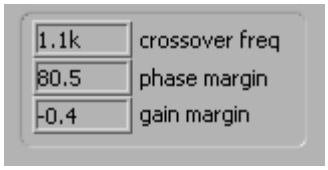
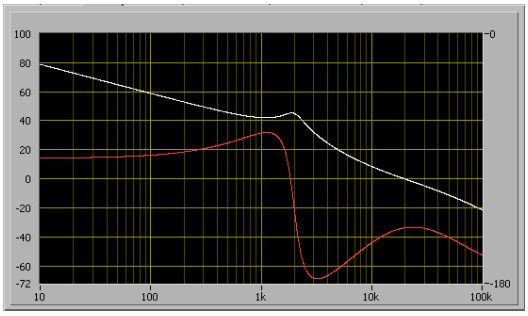


Figure 16. Open Loop Bode Plot of the Buck Converter with a Type−I Compensation Network



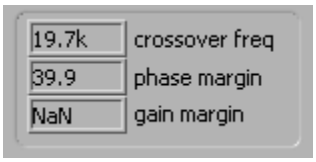
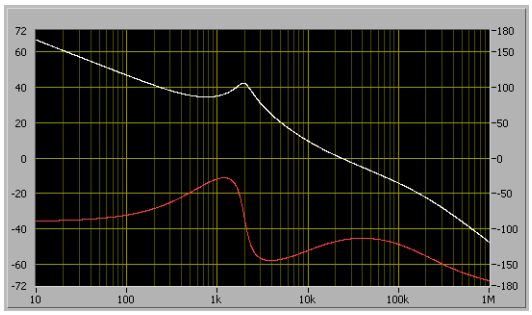


Figure 17. Open Loop Bode Plot of the Buck Converter with a Type−II Compensation Network



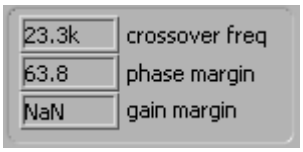
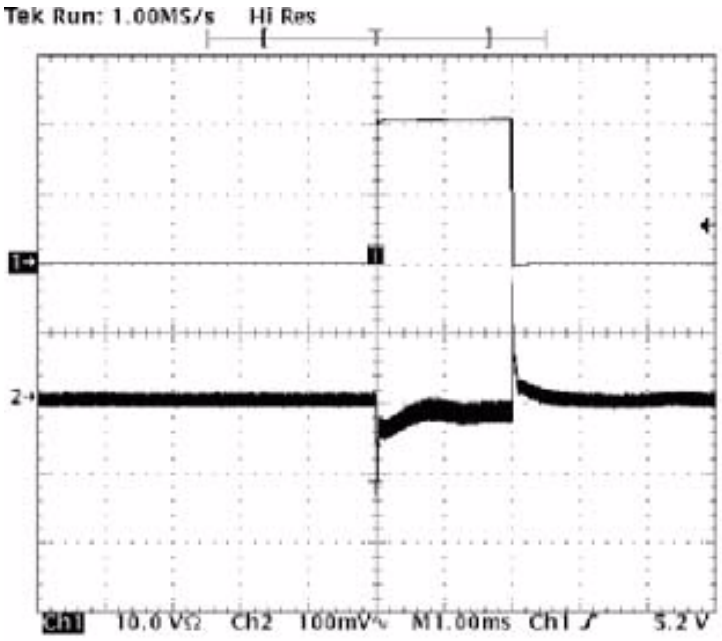


Figure 18. Open Loop Bode Plot of the Buck Converter with a Type−III Compensation Network

Feedback Loop System with Practical Controller

ON Semiconductor provides a series of synchronous buck controller. The NCP5210 for computing applications require very fast transient response. It is well known that type−III compensation network can give very fast transient response and have good phase margin at the same time. For systems requiring fast response, the device designer obviously uses the Type−III compensation network rather than the other two types of compensation network. The NCP5210 design includes a high bandwidth amplifier. This high bandwidth error amplifier can provide fast transient response, but, for the fastest transient response, it should be compensated with a Type−III compensation network. By using the component values of the Type−III compensation network derived in previous sections, an actual circuit was set up. The experimental transient response of this converter with a Type−III compensation network was captured in Figure 19.



Channel 1: Current sourced into buck converter, 10 A/div
Channel 2: Buck converter output voltage, 100 mV/div

Figure 19. Transient Response of the Buck Converter with a Type−III Compensation Network

CONCLUSION

A closed loop system can be implemented with different types of compensation network. The Type−I compensation network can give good phase margin, but bandwidth is usually too low for fast transient systems. The Type−II compensation network can improve the transient response but phase boost is limited to less than 90°. The Type−III compensation network provides fast transient response and sufficient phase margin to ensure system stability, but at the cost of circuit complexity. Selection of compensation type requires detailed understanding of the target system. In this paper, the theory of compensation and types of compensation networks are explained in detail. The K−factor approach for feedback loop design is introduced, and, through examples and simulations, the benefits of the tool are highlighted.

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Transfer Functions Revisited

We are going to have a brief refresher here about transfer functions because several of the later chapters will use transfer functions for analyzing system stability.

Let us remember our generalized feedback−loop transfer function, with a gain element of K, a forward path Gp(s), and a feedback of Gb(s). We write the transfer function for this system as:

$$H_{cl}(s) = \frac{KGp(s)}{1 + H_{ol}(s)}$$

Where H_{cl} is the closed−loop transfer function, and H_{ol} is the open−loop transfer function. Again, we define the open−loop transfer function as the product of the forward path and the feedback elements, as such:

$$H_{ol}(s) = KGp(s)Gb(s)$$

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