Theory of $W$ and $Z$ boson production at the Tevatron and LHC

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“Theoretically clean” features of vector boson production

- Production from quark initial states
  \[ ud \rightarrow W, \ u\bar{u} \rightarrow Z, \ d\bar{d} \rightarrow Z \]

  \[ \Rightarrow \text{parton distribution functions (PDFs) known from deep inelastic scattering} \]

- Decay into easily identifiable leptonic states
  \[ W \rightarrow e\nu, \ W \rightarrow \mu\nu, \ Z \rightarrow e^+e^-, \ Z \rightarrow \mu^+\mu^- \]

- Relatively simple (but sizable) QCD radiative corrections

- Relatively small electroweak radiative corrections
Tevatron and LHC are sensitive to

- initial-state gluons (up to $-15\%$ at the LHC)
- recoil from QCD and EW radiation
- Breit-Wigner line shape
- spin correlations between hadrons and leptons
- ...

The framework used in to describe these effects in the Tevatron Run-1 is $\mathcal{O}(\alpha_s)$ QCD with elements of $\mathcal{O}(\alpha)$ EW corrections
Base theory framework in the Tevatron Run-1

QCD corrections

- Full $\mathcal{O}(\alpha_s)$
- Breit-Wigner propagator with a running width $\Gamma_W(Q)$

Electroweak corrections

- $\mathcal{O}(\alpha)$ QED corrections ($\gamma$) to the final state (Berends, Kleiss; Wagner)

↑ Calculated in independent computer programs ↑

Adequate for comparison to the Run-1 data
Theory requirements for Tevatron Run-2

Experimental targets:

\[ \delta \sigma_{\text{tot}} / \sigma_{\text{tot}} \sim 2 - 3\% \]
\[ \delta M_W \sim 30 \text{ MeV} \]

Many factors contribute at a percent level:

- \( \mathcal{O}(\alpha_s^2) \) (NNLO-QCD) corrections
- \( \mathcal{O}(\alpha) \) (NLO-EW) corrections
- uncertainties in parton distributions (PDFs)
- power corrections to resummed cross sections

The near-future challenge: consistent and efficient implementation of these effects
Total $W$ and $Z$ cross sections

- Monitors of the beam and parton luminosity at future colliders
  (Dittmar, Pauss, Zurcher; Khoze, Martin, Orava, Ryskin; Giele, Keller)
Total cross sections: NNLO QCD corrections

\[ \sigma_{tot}(p\bar{p} \rightarrow V) = \sum_{\text{partons}} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/\bar{p}}(x_2) \hat{\sigma}_{tot}(ab \rightarrow V) \]

- NNLO hard cross section \( \hat{\sigma}_{tot}(ab \rightarrow V) \)
  (Hamberg, van Neerven, Matsuura, 1991; Harlander and Kilgore, 2002)
- Partial NNLO results for parton distributions \( f_{a/p}(x) \)

Talks by W.-K. Tung and Eric Laenen

- Scale dependence of order 1%
- NNLO \( K \)-factor is about 1.04 at the Tevatron and 0.98 at the LHC (MRST’03)
Precision prediction for $\sigma_{tot}$ depends on understanding of

- uncertainties in PDF’s
  - W.-K. Tung’s talk

- electroweak effects
  - tree-level approximation
    insufficient!
  - EW corrections, updated
  - EW parameters

- acceptance
  (Frixione, Mangano, 2004)
Cancellation of PDF uncertainties in $\sigma_{tot}(Z)/\sigma_{tot}(W)$: new results from CTEQ

(Huston, P. N., Pumplin, Stump, Tung, Yuan, 2004)

In spite of different quark flavors, a measurement of $\sigma(Z)$ will constrain $\sigma(W)$ (and possibly other quark-dominated cross sections)!
Rapidity distributions and W charge asymmetry
Rapidity distributions at $O(\alpha_s^2)$
(Anastasiou, Dixon, Melnikov, Petriello, 2004)

New method for calculation of two-loop cut diagrams

- Transformation of phase space constraints into propagators via unitarity relations

- Recursive reduction to known integrals (Tkachov; Chetyrkin, Tkachov; Laporta; Gehrmann, Remiddi)
  - using Lorentz invariance
  - integration by parts

- Solution of master integrals with the help of
  - differential equations (Kotikov; Gehrmann, Remiddi; Bern, Dixon, Kosower)
  - Mellin-Barnes method (Smirnov; Tausk)
NNLO rapidity distributions at the Tevatron

- Tiny scale dependence (< 1%)
- For $|y| < 2$, NNLO leads to a uniform enhancement

$$\sigma_{NNLO} \approx K \cdot \sigma_{NLO}$$

$K(Z) \sim 3 - 5\%$, $K(W) \sim 2.5 - 4\%$

- Larger corrections in forward regions
Charge lepton asymmetry

\[ A_{ch}(y_e) = \frac{d\sigma^{W^+}_{y_e} - d\sigma^{W^-}_{y_e}}{d\sigma^{W^+}_{y_e} + d\sigma^{W^-}_{y_e}} \]

- related to the boson Born-level asymmetry \((y_W=\text{rapidity of } W)\)

\[ A_{ch}(y_W) \quad y_W \rightarrow y_{\max} \quad \frac{r(x_b) - r(x_a)}{r(x_b) + r(x_a)}, \quad r(x) = \frac{d(x, M_W)}{u(x, M_W)} \]

- constrains the PDF ratio \(d(x, M_W)/u(x, M_W)\) at \(x \rightarrow 1\)

- In experimental analyses, a selection cut \(p_{Te} > p_{Te}^{\text{min}}\) is imposed
Charge asymmetry: CDF Run-2 vs. CTEQ6.1 and ResBos

(Stump et al; Balazs, Yuan; Brock, Landry, P. N., Yuan)

\[ A(y_e) \]

\[ 0 \leq \mu \leq 2M \]

\[ \sqrt{s} = 1.96 \text{ TeV} \]

\[ \mu = M_e \]

\[ 40 \text{ extreme pdfsets} \]

\[ C. \text{ Balazs, C.-P. Yuan, Phys. Rev. D56, 5558 (1997)} \]

\[ pT_e \text{ cut introduces dependence of } A_{ch}(y_e) \text{ on QCD corrections} \]
Measurement of $W$ boson mass $M_W$ and width $\Gamma_W$

- Test of the standard model (SM)
Standard model relates $M_W$, top mass $m_t$, and Higgs boson mass $M_H$:

\[ M_W = 80.3827 - 0.0579 \ln\left(\frac{M_H}{100 \text{ GeV}}\right) - 0.008 \ln^2\left(\frac{M_H}{100 \text{ GeV}}\right) + 0.543 \left(\left(\frac{m_t}{175 \text{ GeV}}\right)^2 - 1\right) - 0.517 \left(\frac{\Delta \alpha_{\text{had}}(M_Z)}{0.0280} - 1\right) - 0.085 \left(\frac{\alpha_s(M_Z)}{0.118} - 1\right) \]

- Measurement of $M_W$ and $m_t$ constrains $\log M_H$ in SM
  \[ \delta M_W \sim 30 \text{ MeV} \quad \delta m_t \sim 2 \text{ GeV} \quad \Rightarrow \quad \frac{\delta M_H}{M_H} \sim 35\% \]

- Measurement of $M_W$, $m_t$ and $M_H$ tests consistency of SM
Observables sensitive to $M_W$

1. Leptonic transverse mass

$M^{\ell\nu}_T \equiv 2 |\vec{p}_{Te}| |\vec{p}_{T\nu}| - 2 (\vec{p}_{T\ell} \cdot \vec{p}_{T\nu})$

Sensitivity region

- $M_W$: $M^{\ell\nu}_T \sim 60 - 100$ GeV
- $\Gamma_W$: $M^{\ell\nu}_T > 100$ GeV

2. Transverse momentum of the charged lepton ($p_{T\ell}$)

- $M_W$: $p_{T\ell} \sim 35 - 45$ GeV

3. $p_T$ of the neutrino ($E_T$)

4. $M^{e\nu}_T(W)/M^{\ell\ell}_T(Z)$ (Rajagopolan, Rijssenbeek; Giele, Keller; Shpakov)

5. $\sigma_{tot}(W)/\sigma_{tot}(Z)$ (R. Brock et al., 2001)
Determination of $M_W$ from distributions of transverse momenta

Kinematical (Jacobian) peaks... 
...located exactly at $M_W$ ($M_W/2$) 
at Born level 
...smeared by EW and QCD radiation 
...sensitive to 

- EW radiative corrections
- PDF parametrizations
- the shape of $q_T$ distributions

↑ Sources of the largest theory uncertainties on $M_W$ (tens of MeV) in Run-1
NLO electroweak corrections to $W$ boson production

Born-diagram:

\[ \begin{array}{c}
q_1(p_1) & W^+ (q) & \nu_\ell (p_1) \\
\bar{q}_\ell (p_\ell) & W^+ (p_\ell) & l^+ (p_\ell) \\
\end{array} \]

pure weak contribution:

\[ \begin{array}{c}
Z & W^+ & Z \\
\vdots & \vdots & \vdots \\
Z & \gamma & Z \\
\end{array} \]

virtual $\gamma$ contribution:

\[ \begin{array}{c}
\gamma & W^+ & \gamma \\
\gamma & \gamma & \gamma \\
\gamma & \gamma & \gamma \\
\end{array} \]

real $\gamma$ contribution:

\[ \begin{array}{c}
W^+ & \gamma & W^+ \\
\gamma & \gamma & \gamma \\
\gamma & \gamma & \gamma \\
\end{array} \]
Hierarchy of electroweak radiative corrections

- Effective Born approximation (EBA)
  - used in QCD programs in Run-1
- NLO corrections in the pole approximation
  - large effect at $Q \sim M_V$
  - Can be classified into initial-state, final-state, and interference terms
- Final-state QED radiation dominates (Baur, Keller, Wakeroth, 1998)
- Full NLO (including non-resonant terms)
  - required at $Q \gg M_V$
- Radiation of two (Baur, Stelzer, 2000) and many photons (Placzek, Jadach, 2003; Carlone Calame et al., 2003)

\[
\delta M_{W}^{EW} = -65 \pm 20 \, \text{MeV} \quad \text{and} \quad -168 \pm 10 \, \text{MeV}
\]
Factorization at small $q_T$
(resummation)

☞ E. Laenen’s talk

Relevant momentum scales:
$Q^2 \gg 1 \text{ GeV}^2, q_T \ll Q, x \sim 1$

Trouble:
The series $\frac{1}{q_T^2} \alpha_S^n \ln^m \frac{q_T^2}{Q^2}, \ m = 0, ..., 2n - 1$ lose convergence

Solution: summation of logarithms through all orders of $\alpha_S$
$q_T$ resummation: mainstream approaches

- Formalism in impact parameter $(b)$ space \textit{(Collins, Soper, Sterman)}
  - theory symmetries preserved automatically
  - conservation of momentum
  - fast and accurate evaluation of Fourier-Bessel transform possible \textit{(ResBos, Balazs, P. N., Yuan)}

- Formalism in $q_T$ space \textit{(Altarelli, Ellis, Greco, Martinelli; Ellis, Ross, Veseli)}
  - straightforward identification of logs for matching with the fixed-order result
Recent developments

- Hybrid methods:
  - analytical evaluation of Fourier-Bessel transform \((\text{Kulesza, Stirling})\)
  - threshold-\(q_T\) resummation \((\text{Kulesza, Sterman, Vogelsang})\)
  - \(q_T\) resummation for \(c, b\) quarks in a variable-flavor number (ACOT) scheme \((\text{Berge, P.N., Olness})\)
    * flavor dependence of \(W\) and \(Z\) cross sections
  - \(q_T\) resummation with small-\(x\) effects \((\text{Berge, P.N., Olness, Yuan})\)
    * broadening of \(d\sigma/dq_T\) at the LHC

- Structure of the resummed form factor
  \((\text{Collins, Soper; CSS; Catani, de Florian, Grazzini})\)

- Application to polarized \(W\) and \(Z\) production at RHIC
  \((\text{Weber; P.N., Yuan})\)
Sensitivity of $q_T$ cross sections to nonperturbative contributions

Resummed $d\sigma/dq_T^2$ is expected to include a universal nonperturbative function $S_{NP}(b, Q)$ (analogous to universal PDF’s)

$S_{NP}(b, Q)$ is non-negligible in any non-pert. model at $q_T < 10$ GeV

Comparison of models for nonpert. terms ($b_*$ and extrapolation)

Variation of non-pert. terms moves the peak of $d\sigma/dq_T$ by 200-500 MeV

* Valid models for $S^{NP}(b, Q)$ must provide accuracy comparable to $\delta M_W \sim 30$ MeV
Models for nonperturbative contributions

- $b_*$ anzatz (CSS, 1985)
  - simultaneous agreement with all fixed-target Drell-Yan and $Z^0$ boson data (Landry, Brock, P. N., Yuan, 2002)
  - strong evidence for universality of $S_{NP}(b, Q)$

- freezing $\alpha_s(\mu)$ at $\mu \sim \Lambda_{QCD}$
- renormalon analysis (Korchemsky, Sterman, 1995)
- extrapolation of leading power terms (Qiu, Zhang, 2000)
- principal value resummation (Sterman; Kulezsa, Sterman, Vogelsang, 2002)
- dispersive equations (Guifanti, Smye, 2000)
- $k_T$-dependent factorization (X. Ji, J. Ma, F. Yuan, 2004)

In all models, incalculable power correction terms are required for agreement with data ($\sim \exp{-gb^2}$, with $g \sim 0.8$ (2.7) in extrapolation ($b_*$) model)
Combined effects of electroweak corrections, resummation, and PDF uncertainties

(Active ongoing work)
QCD resummation + final-state QED radiation
(Q. Cao, C.-P. Yuan, 2004)

- New version of the resummation program ResBos (ResBos-A)

- Includes the dominant EW contribution from final-state QED radiation

- Without detector smearing, QCD and QED corrections to $M_T$ are of similar magnitude and opposite sign

Detector smearing reduces QED correction without strong effect on QCD corrections
CTEQ error analysis for $W$ and $Z$ observables
(Huston, P. N., Pumplin, Stump, Tung, Yuan, in progress)

1. Effect of PDF uncertainties on $W$ and $Z$ distributions

2. Nonperturbative corrections to $q_T$ distributions
   - explore form and universality of $S_{NP}(b, Q)$
   - determine “tolerance range” for $S_{NP}(b, Q)$ and uncertainties in $d\sigma/dq_T$, etc.

3. Simultaneous global analysis of PDF’s and $q_T$ distributions
   - correlated errors for the PDF’s and $d\sigma/dq_T$
Vector boson production as the background for new physics

- Searches for contact interactions, $W'$ and $Z'$, extra dimensions...

- $WW$ and $ZZ$ production
  - background for heavy Higgs production ($M_H > 160$ GeV)

- $\gamma\gamma$ production:
  - background for light Higgs production ($M_H \sim 120 – 140$ GeV)
  - soft gluon radiation in $gg$ channel
  - photon fragmentation contributions
Conclusions

1. NNLO for $\sigma_{tot}$ & rapidity distributions
   - global effect: rescaling of $\sigma_{NLO}$ by $K \sim 2.5 - 5\%$
   - full NNLO is needed to describe details (forward rapidity, high $p_T$, angular distributions (?))

2. Requirements for the Tevatron Run-2 and LHC ($\delta \sim 1\%$)
   - Simultaneous implementation of leading NNLO-QCD and NLO-EW effects
   - Reduction of uncertainties in nonperturbative inputs (PDF’s, $p_T$ power corrections)
   - Correlated theory uncertainties (PDF’s vs. $p_T$)
   - Attention to minor details: old electroweak parameters or low computer accuracy can now make big difference!
Backup slides
$q_T$ resummation for vector boson production

Resummation: W boson production at the Tevatron

- No QCD radiation
- QCD radiation

Needed to precisely measure $W$-boson mass

Resummation describes all $q_T$ range in one unified framework
QCD factorization in hard and soft regions (CSS)

Finite-order (FO) factorization
\[ \Lambda_{QCD}^2 \ll q_T^2 \sim Q^2 \]

Small-\(q_T\) factorization
\[ \Lambda_{QCD}^2 \ll q_T^2 \ll Q^2 \]

Solution for all \(q_T\) (matching):
Factorization at $q_T \ll Q$

Realized in the space of the impact parameter $b$ (conjugate to $q_T$)

$$\frac{d\sigma_{AB\rightarrow VX}}{dQ^2 dy dq_T^2} \bigg|_{q_T^2 \ll Q^2} = \sum_{a,b=\text{g, u, d},\ldots} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}} \tilde{W}_{ab}(b, Q, x_A, x_B)$$

In the perturbative region ($b \ll 1 \text{ GeV}^{-1}$):

$$\tilde{W}_{ab}(b, Q, x_A, x_B) = \sum_j |\mathcal{H}_j|^2 e^{-S(b,Q)} \overline{P}_a(x_A, b) \overline{P}_b(x_B, b)$$

$\mathcal{H}_j$ is the hard vertex, $S$ is the soft (Sudakov factor), $\overline{P}_a(x, b)$ is the unintegrated PDF,

$$\overline{P}_a(x, b) = \int d^{n-2}k_T e^{-i\vec{k}_T \cdot \vec{b}} P_a(x, \vec{k}_T)$$

$\overline{P}_a(x, b)$ factorizes as

$$\overline{P}_a(x, b) \equiv \sum_{i=g,u,d\ldots} [C_{ai} \otimes f_i](x_A, \mu_F, b)$$

$S(b, Q)$, $\overline{P}_a(x, b)$, and $C_{ai}(x_A, \mu_F b)$ are calculable in perturbative QCD