Gravitational Lensing

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Abstract

Deflection of light was predicted by the General Theory of Relativity and was confirmed observationally in 1919. This led to the idea of a gravitational lens in which a massive object like a galaxy or clusters of galaxies come in between the source and the observer and cause multiple imaging or magnification, or both, of the source. In the first section we will give some historical background about this phenomena. In the second section we will derive the lens equation and describe some properties of the image. In the third section we will describe different groups of gravitational lensing observations.

1 History of Gravitational Lensing

The first written account of the deflection of light by gravity appeared in the article "On The Deflection Of Light Ray From Its Straight Motion Due To The Attraction Of A World Body Which It Passes Closely," by Johann Soldner in 1804. In his article Soldner predicted that a light ray passing close to the solar limb would be deflected by an angle $\tilde{\alpha} = 0.84$ arcsec.

More than a century later,in 1919, Albert Einstein directly addressed the influence of gravity on light. At this time Einstein's General Theory of Relativity was not fully developed. This is the reason why Einstein obtained - unaware of the earlier result - the same value for the deflection angle as Soldner had calculated with Newtonian physics. In this paper Einstein found $\tilde{\alpha} = 2GM_\odot/c^2 R_\odot = 0.83$ arcsec for the deflection angle of a light ray grazing the sun (here $M_\odot$ and $R_\odot$ are the mass and radius of the sun, c and G are the speed of light and the gravitational constant respectively).

Einstein emphasized his wish that astronomers investigate this equation. In 1913 Einstein contacted the director of Mount Wilson observatory, George Hale, and asked if it will be possible to measure the positions of stars near the sun during the day in order to establish the angular deflection effect of the sun.

The first observational attempt to test Einstein's prediction for the deflection angle was in 1914. This attempt was never accomplished which was fortunate for Einstein whose predicted value for the angular deflection was actually wrong.

With the completion of the General Theory of Relativity in 1916, Einstein was the first to derive the correct formula for the deflection angle $\tilde{\alpha}$ of a light passing at a distance r from an object with mass M as

$$\tilde{\alpha} = \frac{4GM}{c^2} \frac{1}{r^2} \tag{1}$$

The additional factor of two (compared to the "Newtonian" value) is due to the spatial curvature which is missed if photons are just treated as particles.

For the sun, Einstein obtained

$$\tilde{\alpha} = \frac{4GM_\odot}{c^2} \frac{1}{R_\odot^2} = 1.74 \text{ arcsec} \tag{2}$$

This value was first confirmed to within 20% by Arthur Eddington and his group during a solar total eclipse in 1919. Recently this value was confirmed to within 0.02%.

In 1924 Chwolson mentioned the idea of a "factious double star." He also mentioned the symmetric case of star exactly behind star, resulting in a circular image. In 1936 Einstein also reported about the appearance of a "luminous" circle of perfect alignment between the source and the lens, such a configuration is called "Einstein Ring". In 1937 Fritz Zwicky pointed out that galaxies are more likely to be gravitationally lensed than a star and that one can use the gravitational lens as natural telescope.

2 Basics of Gravitational Lensing

In this section we'll study the "Thin Lens Approximation" in which we assume that the lensing action is dominated by a single matter inhomogeneity between the source and the observer and that this action takes place at a single distance. This approximation is valid only if the relative velocities of the source, image and observer are small compared to the velocity of light $v \ll c$ and if the Newtonian potential is small $|\Phi| \ll c^2$. These assumptions are justified in all astronomical cases of interest. The size of a galaxy or cluster of galaxies is of the order of a few Mpc. This "lens thickness" is small compared to the typical distance between the lens and the source or the lens and the observer which is of order of a few Gpc. Here we assume that the underlying spacetime is well described by a perturbed Friedmann-Robertson-Walker metric :

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 - a^2(t) \left(1 - \frac{2\Phi}{c^2}\right) d\sigma^2 \tag{3}$$

2.1 Lens Equation

The basic setup for a gravitational lens scenario is displayed in Figure (1). The three ingredients in such a lensing situation are the source S, the lens L and the observer O. In this scenario light rays emitted by the source are deflected by the lens which will produce two images, S_1 and S_2.

Figure (2) shows the corresponding angles and angular diameter distances D_L, D_S, D_{LS}.

Assuming a spherical-symmetric lens, the underlying spacetime around the lens is well described by the Schwarzschild metric :

$$ds^2 = \left(1 - \frac{2m}{r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 \tag{4}$$
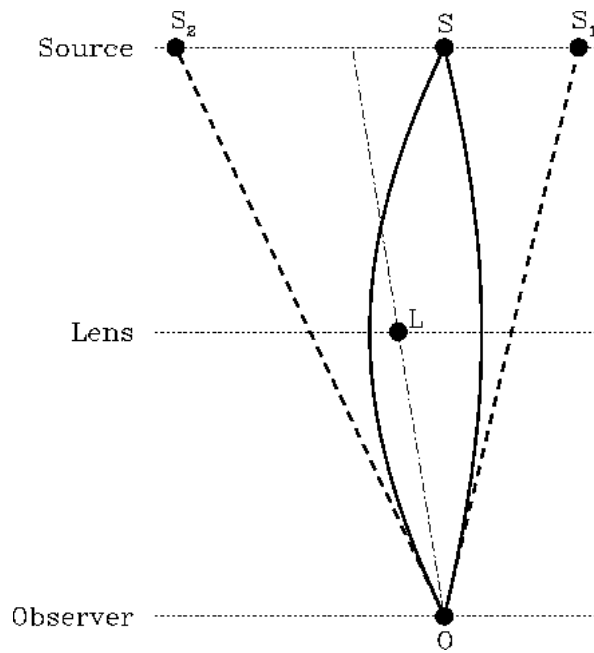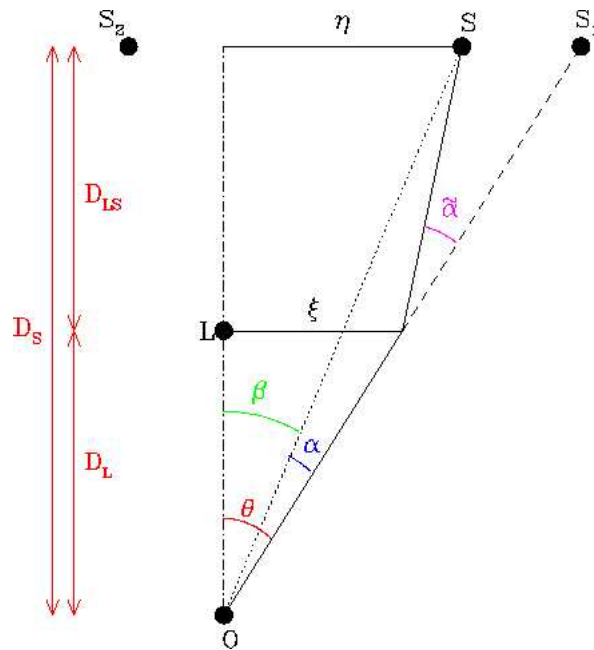
Figure (1): setup of a gravitational lens.

Figure (2): Corresponding angles and angular diameters

In this case the deflection angle is given as

$$\tilde{\alpha} = \frac{4GM(\xi)}{c^2} \frac{1}{\xi^2} \tag{5}$$

where M(ξ) is the mass inside a radius ξ.

From figure 2 we see that the following relations hold :

$$\theta D_S = \beta D_S + \tilde{\alpha} D_{LS} \tag{6}$$

$$\alpha(\theta) = \left(\frac{D_{LS}}{D_S}\right) \tilde{\alpha}(\theta) \tag{7}$$

$$\beta = \theta - \alpha(\theta) \tag{8}$$

(this is true for all astrophysical relevant situations in which $\theta, \beta, \tilde{\alpha} \ll 1$) where equation (8) is the lens equation.

2.2 Einstein Radius

Plugging equation (5) into equation (7) and noticing that $\xi = \theta D_L$, we get for the lens equation :

$$\beta(\theta) = \theta - \frac{D_{LS}}{D_L D_S} \frac{4GM}{c^2 \theta} \tag{9}$$

For the special case in which the source lies exactly behind the lens $\beta = 0$ and we have :

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}} \tag{10}$$

where θ is the angular radius of the ring-like image, which we call **Einstein Ring**. For a massive galaxy with a mass $M = 10^{12} M_\odot$ at a redshift of $Z_L = 0.5$ and a source at redshift $Z_S = 2.0$ the Einstein radius is

$$\theta_E \approx 1.8 \sqrt{\frac{M}{10^{12} M_\odot}} \quad \text{arcsec} \tag{11}$$

2.3 Image positions and magnifications

In the case of a single point lens, the lens equation can be reformulated to :

$$\beta = \theta - \frac{\theta_E^2}{\theta} \tag{12}$$

Solving for θ we get

$$\theta_\pm = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right) \tag{13}$$

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Equation (13) says that for an isolated point source one always gets two images of a background source with the corresponding angular positions θ_+ and θ_-.

The magnification μ of an image is defined as the ratio between the solid angles of the image and the source

$$\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta} \tag{14}$$

By using the lens equation, we find :

$$\mu_\pm = \left(1 - \left[\frac{\theta_E}{\theta_\pm}\right]^4\right)^{-1} = \frac{u^2 + 2}{2u\sqrt{u^2 + 4}} \pm \frac{1}{2} \tag{15}$$

where $u = \beta/\theta_E$

The sum of the absolute values of the two image magnifications is the measurable total magnification μ

$$\mu = |\mu_+| + |\mu_-| = \frac{u^2 + 2}{2u\sqrt{u^2 + 4}} \tag{16}$$

Note that this value is always larger than one. (This does not violate energy conservation, since this is the magnification relative to an "empty" universe and not relative to a "smoothed out" universe.) The difference between the two image magnifications is unity:

$$\mu_+ - \mu_- = 1 \tag{17}$$

2.4   Non-Singular isothermal sphere

Here we assume singular isothermal sphere with a three-dimensional density distribution

$$\rho = \frac{\sigma_v^2}{2\pi G}\frac{1}{r^2} \tag{18}$$

where σ_v is the one-dimensional velocity dispersion. By projecting the matter in a plane, one obtains the circularly-symmetric surface mass distribution

$$\Sigma(\xi) = \frac{\sigma_v}{2G}\frac{1}{\xi} \tag{19}$$

The mass inside a sphere of radius ξ is given by

$$M(\xi) = \int_0^\xi \Sigma(\xi')2\pi\xi' \, d\xi' \tag{20}$$

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Plugging this into equation (5) one obtains :

$$\tilde{\alpha}(\xi) = 4\pi \frac{\sigma_v^2}{c^2} \tag{21}$$

From the above equation we see that the deflection angle for an isotheral sphere is constant(i.e. independent of ξ)

This can be simplified to :

$$\tilde{\alpha} = 1.15 \left(\frac{\sigma_v}{200\text{kms}^{-1}}\right)^2 \text{arcsec} \tag{22}$$

2.5 Lens mapping

For a non-symmetric mass distribution, all angles become vector-valued and the lens equation reads

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) \tag{23}$$

In the vicinity of an arbitrary point, equation (23) can be described by its Jacobian matrix A:

$$A = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j}\right) = \left(\delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}\right) \tag{24}$$

Where we used the fact that the deflection angle can be expressed as the gradient of a two-dimensional scalar potential ψ :

$$\vec{\nabla}_\theta \psi = \vec{\alpha} \tag{25}$$

and

$$\psi(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(\vec{r}) dz \tag{26}$$

and $\Phi(\vec{r})$ in the Newtonian potential of the lens.

The magnification is the inverse of the determinant of the Jacobian

$$\mu = \frac{1}{det A} \tag{27}$$

Let's define

$$\psi_{ij} = \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \tag{28}$$

The Laplacian of the effective potential ψ is twice the convergence :

$$\psi_{11} + \psi_{22} = 2\kappa = \text{tr}\psi_{ij} \tag{29}$$

With the definitions of the components of the external shear γ

$$\gamma_1(\vec{\theta}) = \frac{1}{2}(\psi_{11} - \psi_{22}) = \gamma(\vec{\theta})\cos[2\varphi(\vec{\theta})] \tag{30}$$

and

$$\gamma_2(\vec{\theta}) = \psi_{12} = \psi_{21} = \gamma(\vec{\theta})\sin[2\varphi(\vec{\theta})] \tag{31}$$

Where the angle φ reflects the direction of the shear-inducing tidal force relative to the coordinate system.

The Jacobian matrix can be written as

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma\begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{pmatrix} \tag{32}$$

The magnification can now be expressed as a function of the local convergence κ and local shear γ :

$$\mu = (\det A)^{-1} = \frac{1}{(1-\kappa)^2 - \gamma^2} \tag{33}$$

We notice that locations for which det $A = 0$ have infinite magnification. These locations are called **critical curves**. For a spherically symmetric mass distribution the critical curves are circles. For a point lens, the caustic degenerates into a point. For elliptical lenses or spherically symmetric lenses plus external shear, the caustics can consist of cusps and folds.

2.6   Time delay

Using equation (25), the lens equation becomes :

$$\vec{\nabla}_\theta \psi = (\vec{\theta} - \vec{\beta}) \tag{34}$$

or

$$\vec{\nabla}_\theta \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi\right) = 0 \tag{35}$$

The term in the brackets appears in the physical time delay function for gravitationally lensed images:

$$\tau(\vec{\theta}, \vec{\beta}) = \tau_{geom} + \tau_{grav} = \frac{1+Z_L}{c}\frac{D_L D_S}{D_{LS}}\left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\theta)\right) \tag{36}$$

The time delay τ is a function of the image geometry $(\vec{\theta}, \vec{\beta})$, the distances D_L, D_S, D_{LS} and the gravitational potential ψ. Here the geometrical time delay reflects the extra path length compared to the direct line between the observer and the source. The gravitational time delay is the retardation due to the gravitational potential of the lensing mass.

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3   Lensing Phenomena

In this section we describe some of the gravitational lens observations.

3.1   Einstein rings

In section (2.2) we showed that for the special case in which the source lies exactly behind the lens, a ring-like image will be produced, we also showed that the angular radius of such image is given by equation (10). There are two conditions to be satisfied in order to be able to observe such image; first, the mass distribution of the lens needs to be axially symmetric, as seen form the observer; and second, the source must lie exactly on top of the resulting degenerate point-like caustic.

The first observed "Einstein ring" was in 1988. The extended radio source MG1131+0456 was observed with high resolution radio observations, it turned out to be a ring with a diameter of about 1.75 arcsec. The source was at redshift $Z_S = 1.13$, whereas the lens was a galaxy at $Z_L = 0.85$. Figure (3) shows a remarkable image of Einstein ring 1938+666 by the infrared HST, we can see the perfect circular ring with two bright parts plus the bright central galaxy.

The gravitational lens JVAS B1938+666

Left: HST/NICMOS greyscale with MERLIN radio contours
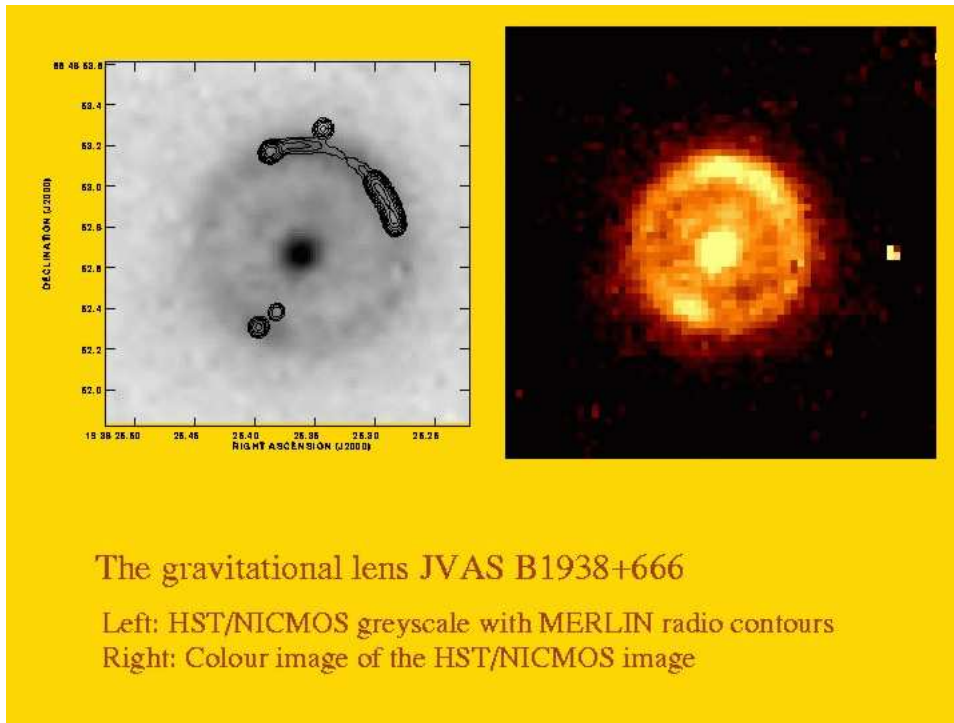Right: Colour image of the HST/NICMOS image

Figure (3): Einstein ring 1938+666, the diameter of the ring is about 0.95 arcseconds .

By now about half a dozen cases have been found that qualify as Einstein rings. Their diameters vary between 0.33 to 2 arcseconds. All of them are found in the radio regime, some have optical or infrared counterparts as well. Broken Einstein rings have also been observed. In these "broken" rings at least one interruption is observed along the ring circle. The sources of most Einstein rings have both an extended and a compact component. The latter is always seen as a double image, separated by roughly the diameter of the Einstein ring. In some cases monitoring of the radio flux showed that the compact source is variable. This gives the opportunity to measure the time delay and the Hubble constant H_o in these systems.

3.2   Giant luminous arcs

This phenomena was first discovered in 1986 by Lynds and Petrosian: magnified, distorted and strongly elongated images of background galaxies which happened to lie behind foreground clusters of galaxies.

Clusters of galaxies at $Z \approx 0.2$ with masses of order $10^{14} M_\odot$ are very effective lenses. Their Einstein radii are of the order of 20 arcseconds. No complete Einstein ring has been found around clusters due to the facts that most clusters are not really spherical mass distributions and since the alignment between the lens and source are not perfect.

Giant arcs can be exploited in two ways, as is typical for many lensing phenomena. Firstly they provide us with strongly magnified galaxies at (very) high redshifts. These galaxies are faint to be detected or analyzed in their unlensed state. But with the aid of gravitational lenses we can study these galaxies in their early evolutionary ages, possibly as infant or proto-galaxies, relatively shortly after the Big Bang. The other practical application of the arcs is to take them as tools to study the potential and mass distribution of the lensing galaxy cluster. In the simplest model of a spherically symmetric mass distribution for the cluster, giant arcs form very close to the critical curve, which marks the Einstein ring. So with the redshifts of the cluster and the arc, it is easy to determine a rough estimate of the lensing mass by just determining the radius of curvature and interpreting it as the Einstein radius of the lens system.

Gravitational lensing is one of three methods used for the determination of masses of galaxy clusters. The first method is the mass determinations by X-ray analysis; and the second one is by using the virial theorem and the velocity distribution of the galaxies. Although there are still some discrepancies between the three methods, it appears that in relaxed galaxy clusters the agreement between these different mass determinations is very good.

An interesting result from the analysis of giant arcs in galaxy clusters is

that Clusters of galaxies are dominated by dark matter. The typical "mass-to-light ratios" for clusters obtained from strong lensing analysis are $M/L \geq 100 M_\odot / L_\odot$.

Distribution of dark matter follows roughly the distribution of the light in the galaxies, in particular in the central part of the cluster. The fact that we see such arcs shows that the central surface mass density in clusters must be high. The radii of curvature of many giant arcs is comparable to their distance to the cluster centers; this shows that core radii of clusters - the radii at which the mass profile of the cluster flattens towards the center - must be of order of this distance or smaller.

Figures (4) and (5) show two of the most spectacular cluster lenses producing arcs: Clusters Abell 2218 and CL0024+1654. Close inspection of the HST image of Abell 2218 reveals that the giant arcs are resolved (Figure (4)), structure can be seen in the individual components and used for detailed mass models of the lensing cluster. In addition to the giant arcs, more than 100 smaller "arclets" can be identified in Abell 2218. They are farther away from the lens center and hence are not magnified and curved as much as the few giant arcs. These arclets are all slightly distorted images of background galaxies. With the cluster mass model it is possible to predict the redshift distribution of these galaxies. This has been successfully done in this system with the identification of an arc as a star-forming region, opening up a whole new branch for the application of cluster lenses.
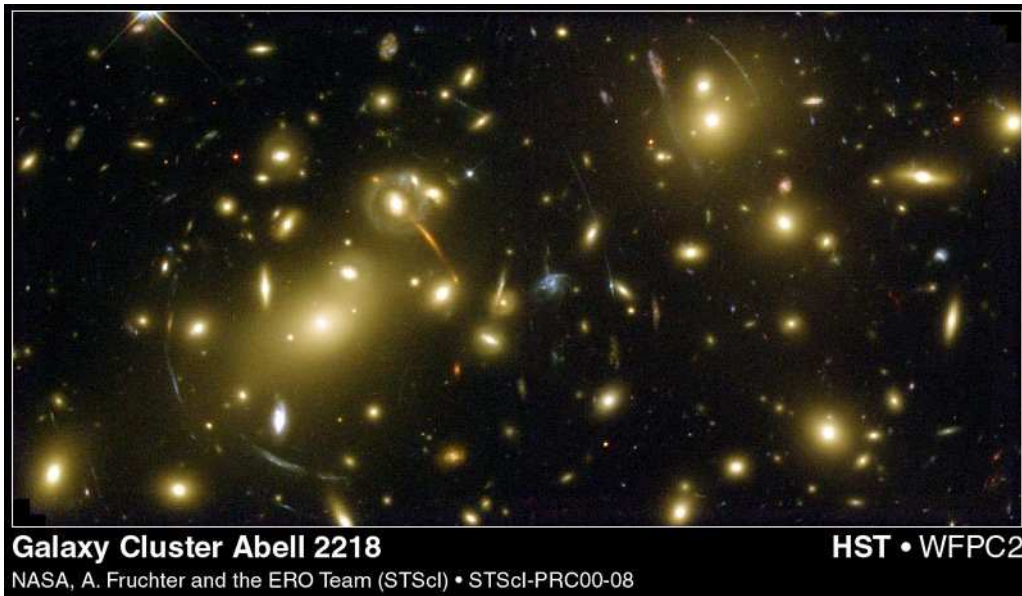

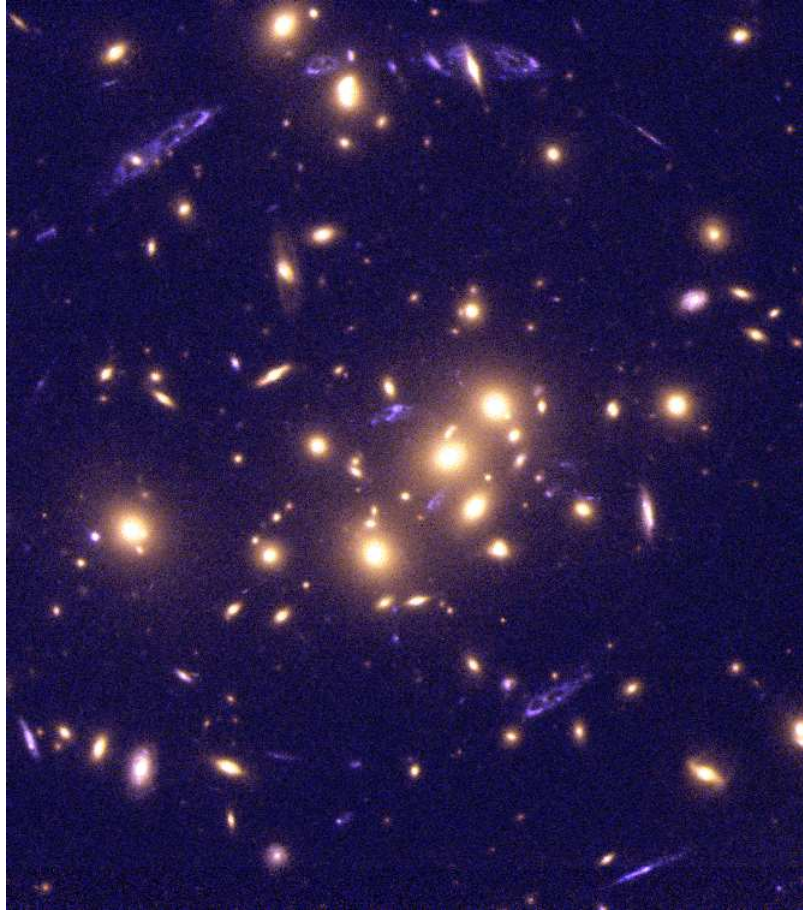Figure (4): Galaxy Cluster Abell 2218 with Giant Luminous Arcs

Figure (5): Galaxy Cluster $CL0024 + 1654$ with multiple images of a blue background galaxy

Another impressive exposure is the galaxy cluster CL0024+1654 (redshift Z = 0.39). Figure (5) shows very nicely the reddish images of cluster galaxies, the brightest of them concentrated around the center, and the bluish arcs.

3.3 Multiply-imaged quasars

In 1979, gravitational lensing became an observational science when the double quasar Q0957+561 was discovered. This was the first example of a lensed object. It was not entirely clear at the beginning, though, whether the two quasar images really were an illusion provided by curved spacetime or rather physical twins. But intensive observations soon confirmed the almost identical spectra. The intervening "lensing" galaxy was found, and the "supporting" cluster was identified as well. Later very similar lightcurves of the two images confirmed this system beyond any doubt as a bona fide gravitational lens.
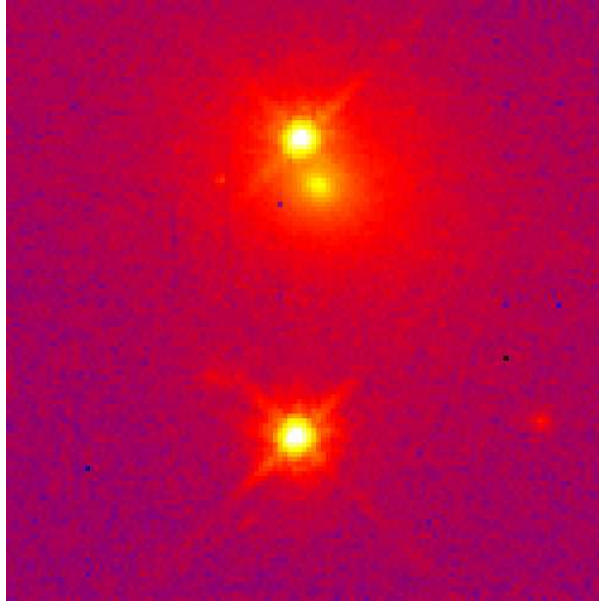
Figure (6): Hubble Space Telescope image of the double quasar Q0957+561

By now about two dozen multiply-imaged quasar systems have been found, plus another ten good candidates. This is not really an exceedingly large number, considering a 20 year effort to find lensed quasars. The reasons for this "modest" success rate are:

1. Quasars are rare and not easy to find (by now roughly 10^4 are known).

2. The fraction of quasars that are lensed is small (less than one percent).

3. It is not trivial at all to identify the lensed (i.e. multiply-imaged) quasars among the known ones.

The quasar Q0957+561 was originally found in a radio survey; subsequently an optical counterpart was identified as well. After the confirmation of its lens nature, this quasar attracted quite some attention. Q0957+561 has been looked at in all available wavebands, from X-rays to radio frequencies. More than 100 scientific papers have appeared on Q0957+561, many more than on any other gravitational lens system.

In the optical light, Q0957+561 appears as two point images (figure (6)). The spectra of the two quasars reveal both redshifts to be $Z = 1.41$. Between the two images, not quite on the connecting line, the lensing galaxy (with redshift $Z = 0.36$) appears as a fuzzy patch. This galaxy is part of a cluster of galaxies at about the same redshift. This is the reason for the relatively large separation for a galaxy-type lens. In this lens system, the mass in the galaxy cluster helps to increase the deflection angles to this large separation.

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