NEW APPROACH TO FRAGMENTATION REACTIONS:
THE NUCLEAR LATTICE MODEL ☆

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A model of fragmentation reactions is proposed which describes the projectile and target as nuclear lattices of nucleons. By allowing the bonds of the lattice to be broken with a certain probability, the essential features of proton-induced reactions can be described.

In recent years a large amount of data has become available on a class of processes that can be described as "fragmentation reactions":

$$A_p + A_t \rightarrow A_f + X. \tag{1}$$

Here A_p, A_t, and A_f are the projectile, target, and fragment masses, respectively.

These fragmentation reactions have been studied with many theoretical approaches, which are based on quite different physical assumptions. The Purdue group [1] claims that the collision produces a hot nucleus which then condenses into droplets. Other authors [2] have developed models based on statistical and chemical equilibrium. Bohrmann et al. [3] propose that the projectile cleaves the target into basically two fragments, whereas Friedman and Lynch [4] have described some data on the basis of sequential evaporation.

Additionally, in an attempt to describe the microscopic dynamics of the process, TDHF descriptions of fragmentation have been given [5].

Much of the discussion of fragmentation processes has focussed on the yield of medium-mass fragments ($A_f < A_t/3$), for which the data can often be described by a power law,

$$\frac{dS}{dA_f} = \text{const.} \cdot A_f^{-\lambda}, \tag{2}$$

with $2 \leqslant \lambda \leqslant 4$.

The formal similarity between the above equation and that describing the yield of different sized droplets of mass A in gas condensing at the critical temperature has led to the hypothesis of critical phenomena being involved in fragmentation reactions [1,6].

It is by no means clear which reaction mechanism is appropriate for fragmentation processes. Indeed, by simply demanding that charge is conserved and then applying the principle of minimal information, Aichelin and Hüfner [7] are able to reproduce the gross features of the mass-yield curve. This shows that the available data are rather insensitive to the theoretical input. Campi et al. [8] also use a minimal-information method, but include in addition an evaporation stage in an attempt to describe the mass-yield curve for all values of A_f.

It seems that only with the advent of complete measurements (as obtained, for example, with the plastic ball [9]) will the insensitivity of experimental results to the theoretical approach be reduced.

In view of the large number of rather disparate models, we have developed a method with which we can test many different assumptions. The goal here is to decide which parts of the data so far assembled are only due to statistical effects, and which parts contain new physics. Our first results are presented here.

We consider a nucleus to be a lattice, in which every lattice site represents a single nucleon. We focus our interest not on the nucleon—nucleon collisions,

but on the bonds between the nucleons.

We then introduce a single parameter p, the probability of a bond being broken. Eventually p should be related to some physical input [10]. It is also reasonable to assume that p is an increasing function of the excitation energy E^* of the spectators.

Using Monte Carlo techniques, we decide which bonds of the lattice are broken and then, as a final step, count the number of clusters of a given size. We have developed a very efficient algorithm for this purpose, which enables us to obtain good statistics in the minimum of computing time.

Identifying the cluster size with the mass number of the fragment, one can obtain mass and also multiplicity distributions. Clearly this approach is related to percolation theory [11], but it differs in that we use a finite (and, indeed, rather small) lattice. Thus the effects of finite particle number and also surface properties are accounted for. In the infinite-lattice limit, percolation theory shows that the lattice type plays no role: only the dimensionality of the system is important. While this is no longer true for finite lattices, we limit ourselves for computational simplicity to the simple cubic lattice. Apart from those at the surface, each lattice site (i.e. "nucleon") therefore has six nearest neighbours.

In this paper we restrict ourselves, for reasons indicated below, to proton-induced reactions, but we stress that the lattice can also be used to simulate heavy-ion collisions. The effect of varying the impact parameter can also be considered, so that we are not restricted to central collisions. We can prescribe arbitrary shapes for the target and projectile, as well as different spatial dependences of the breaking probability p to mock-up various physical situations.

The results below were obtained with an approximately spherical representation of the nuclei on the lattice, which means that those A lattice sites within a certain distance of the chosen origin are initially occupied. In addition, we use a geometry similar to that used in the nuclear fireball model [12], in that we assume that the proton generates a cylindrical fireball in the target. The breaking probability is then taken to be constant over the spectator nucleons. Our results show that the position of the fireball in the target is unimportant, as is the number of nucleons contained within it, provided that we only consider reactions with a very small projectile such as a proton.

In fig. 1 we show some of our results for the reaction p + Xe with different breaking probabilities from 0.35 to 0.80.

Some features of this figure are worth commenting on:

(a) For small p, which corresponds to small kinetic energy of the projectile, there are some low-mass fragments, but the major part of the yield lies in particles with mass similar to that of the target nucleus.

(b) At high p large fragments are no longer stable and so only small objects are observed;

(c) In the intermediate case the distribution is broad with all kinds of mass numbers being observed;

(d) The mean multiplicity is a monotonically-increasing function of p.

The striking feature of these results is that they show characteristics that are also obtained in more complicated theories, and yet we have used rather few assumptions. In particular the present model is able to generate U-shape fragment-mass distributions for intermediate breaking probabilities (see fig. 1).

For breaking probabilities in the range from 0.5 to 0.7 fits can be obtained which agree reasonably well with the experimental mass spectra [10]. The ratio of the mass yields in the p + Xe and p + Kr reactions [1] is also very well reproduced [10]. This is a consequence of the scaling properties of percolation theory [8,11].

As mentioned earlier the mass-yield curves of proton-induced reactions can be fitted by a power law such as eq. (2). Such a power law will be represented by a straight line in a doubly-logarithmic plot of ln(mass-number) versus ln(mass-yield), and the slope of the straight line will be $-\lambda$, the apparent exponent. In fig. 2 we use such a plot as an example of the results of our calculations, and, as one can see, our results do agree nicely with such a power-law fit. This behaviour is by no means trivial and gives some support to the underlying statistical description.

In the phase-transition model of fragmentation [6], it is expected that this apparent exponent is a minimum if the fragmentation occurs at the critical temperature, T_c. Thus

$$\min(\lambda) = \lambda(T_c) = \tau, \tag{3}$$

where τ is a critical exponent (= 2.33 in a "classical" three-dimensional system). A compilation of experimental data [13] suggests that λ has a minimum at a
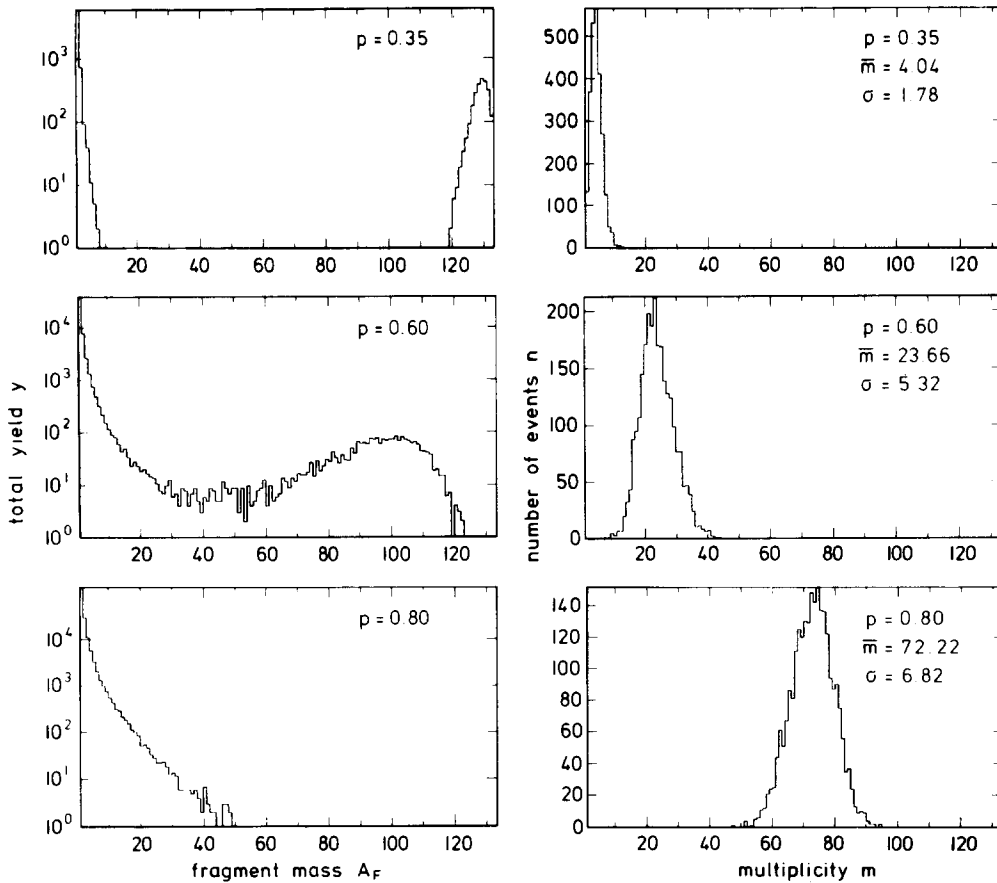
Fig. 1. Dependence of mass yield (left) and multiplicities (right) on the breaking probability, p, for the reaction p + Xe. For each value of p 2500 Monte Carlo runs were performed. Also indicated are the values of the mean ($\bar{m}$) and the full width at half maximum (σ) of the multiplicity distributions.
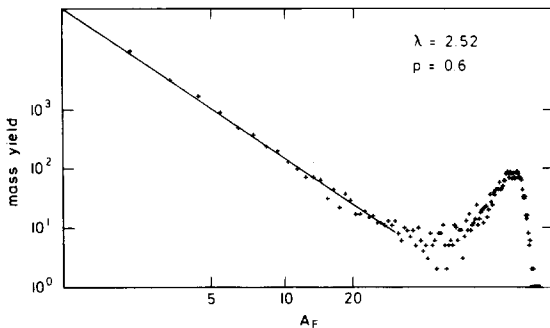

Fig. 2. Doubly-logarithmic plot of the mass yield for breaking probability 0.6 in the p + Xe system. Again 2500 runs were performed. For low-to-medium mass fragments the yield varies as $A^{-2.52}$.

temperature of around 12 MeV, which has therefore been identified with T_c [13].

There has also been an attempt to explain the general behaviour of $\lambda(T)$ using purely Coulomb-tunneling effects [14].

We can use our model to calculate λ as a function of p, and in fig. 3a we show our results. It is clear that there is a minimum at $p = 0.7$, $\lambda = 2.17$, similar to that described above for the liquid–gas phase-transition theory. However, this result is only a consequence of the percolation-like ingredients of our model, and we make no connection of p with a "temperature".

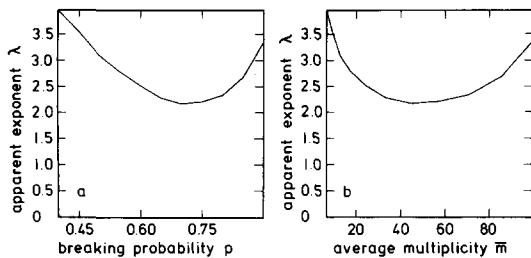In fig. 3b we use the result from fig. 1 that the average multiplicity, $\bar{m}$, of fragments also monotoni-

Fig. 3. The apparent exponent, λ, as a function of (a) breaking probability, p, and (b) mean multiplicity, $\bar{m}$, for the p + Xe system.

cally increases with p. In this way we are able to eliminate the breaking probability, p, and hence predict the behaviour of $m(\lambda)$ in a parameter-free manner.

As far as we are aware, there are no existing data with which this prediction can be compared, since accurate multiplicity measurements have only recently become feasible. It would, however, be extremely interesting for a variety of systems to be examined to see whether our statistical model is sufficient to describe the data, or whether the input of additional physics is required.

In conclusion, we have found that a purely statistical model of fragmentation reactions is able to describe the behaviour of inclusive mass yields in such reactions. In particular the characteristic features of fragment-mass distributions, such as the power-law dependence of the yield for small to medium mass fragments and the U-shape are obtained, with only minimum input of physical properties. However, quantities other than the mass yield, such as the particle multiplicities, may be more sensitive to the actual dynamical process. We are, therefore, currently performing a wide array of calculations, e.g. giving p a spatial dependence, relating p to the dynamics of the

fragmentation process, calculating coincidence multiplicities, and considering heavy-ion reactions with an allowance for different impact parameters. In this way we have a tool which may help in the separation of the essential physics from purely statistical elements contained in the data.

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