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Pre-equilibrium particle emission and critical exponent analysis

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In two different phase transition models of nuclear fragmentation we show that the emission of pre-equilibrium particles and mixing of events from different classes cannot be ignored in the analysis of nuclear fragmentation data in terms of critical exponents, and we show how the apparent values of the extracted exponents are affected.

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The premier goal of medium and high energy heavy-ion reactions is the exploration of the nuclear phase diagram. On theoretical grounds we expect infinite nuclear matter to undergo at least two distinct phase transitions. One is the deconfinement or quark-gluon-plasma phase transition. The other is a "liquid-gas" type phase transition. It is believed to be of first order, terminating at the critical point in a second order transition. In nuclear multifragmentation reactions one attempts to map out the liquid-gas coexistence region and locate the critical point.

The first data set to be interpreted in terms of critical exponents resulted from proton-induced spectator-fragmentation of krypton and xenon targets [1,2]. However, it was later shown in the framework of the percolation model [3] that value of the "critical" exponent τ observed in [1,2] was predominantly a result of mixing of different event classes and integration over impact parameter. In addition, similar power-law behavior was seen in classical reaction dynamics simulations, where it was shown to be inconsistent with matter going through the critical point [4–6].

A significant step forward was then taken by performing event-by-event analysis of the moments of the mass or charge distributions of the fragments [7,8]. The result of this analysis of emulsion data suggested that nuclei break up similar to percolation clusters [7].

It was hoped that the problem of impact parameter selection would be less severe and therefore the critical exponents easier to extract for participant fragmentation in symmetric heavy ion collisions. By focusing on very central collisions as a function of beam energy, a minimum of the fit parameter λ, with $\sigma(Z_f) \propto Z_f^{-\lambda}$ was observed [9]. This confirmed similar observations obtained from reverse-kinematics reactions [10]. It had been predicted that this minimum would correspond to the actual value of the critical exponent τ [3]. However, theoretical calculations showed that there may be significant topology effects such as the formation of bubbles and toroids [11–13] at work, changing the values of the measured exponents [14].

At present, significant effort is also directed at the study of the moments of the fragment charge distribution in reverse-kinematics reactions, where the rise and fall of multifragmentation was observed [10,15,16]. Recently the EOS TPC Collaboration used the reaction 1 A GeV Au + C in an attempt to reach the critical point of nuclear matter and determine the critical exponents in the spectator fragmentation of the residue of the gold nucleus [17,18].

It is generally agreed that the fragmentation process in proton-induced or reverse-kinematics reactions proceeds in two steps—a first pre-equilibrium step in which the participants interact and deposit excitation energy into the spectators, and a second equilibrium step in which the excited (and hopefully equilibrated) spectator residue decays. To this end we have constructed a hybrid model, in which the pre-equilibrium energy deposition and resulting residue size are calculated in the framework of an intranuclear cascade (INC) model [19], and in which we calculate the decay of the residue within a percolation model [3,8] or statistical multifragmentation model (SMM) [20] framework. For any given impact parameter the INC provides the charge and mass as well as the excitation energy per nucleon of the spectator residue by calculating a sequence of individual nucleon-nucleon collisions and single-particle removals from the nuclear potential well. It is worth noting that in central collisions the average residue charge is less than 49, implying the emission of 30 pre-equilibrium charges.

The excitation energy can be converted into a percolation bond-breaking probability via [9]

$$p_b = 1 - \frac{2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, 0, \frac{B}{T}\right), \tag{1}$$

where Γ is the generalized incomplete gamma function, B is the binding energy per nucleon in the residue (taken as 6 MeV here), and T is the temperature as calculated from the INC model, $T = \sqrt{E^*/a}$. The SMM is microcanonical statistical multifragmentation model where the decay probabilities into channel j is proportional to the statistical weight $W_j \propto \exp[S_j(T,A,Z)]$, where S_j is the entropy in channel j. In the SMM we used a standard set of parameters applied before for analysis of multifragmentation in peripheral heavy-

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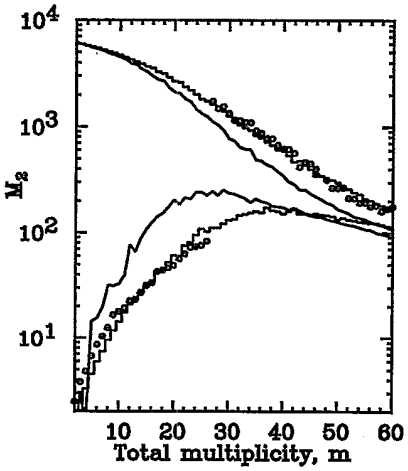

FIG. 1. Second moment of the charge distribution as a function of the charged particle multiplicity for the reaction 1 A GeV Au + C. Histogram: INC/percolation; line: INC/SMM; circles: data [18].

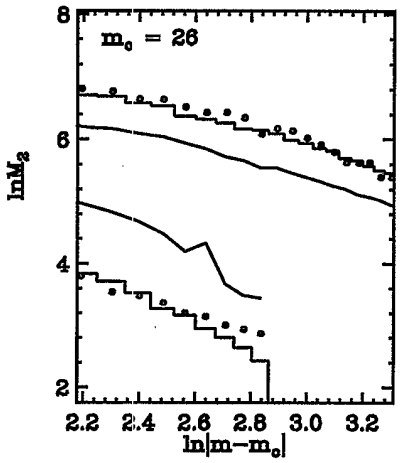

FIG. 2. Second moment of the charge distribution as a function of the charged particle multiplicity for the reaction 1 A GeV Au + C. Histogram: INC/percolation; line: INC/SMM; circles: data [17].

ion collisions [16]. In particular the breakup density is assumed to be 1/3 of the normal nuclear density.

We then arrive at the final inclusive mass distribution (and its moments) as follows: For any given event we select an impact parameter with geometrical probability weighting, calculate $E^* \rightarrow T \rightarrow p_b$ and the pre-equilibrium multiplicity, m_{pre}, within the INC stage, and then use this p_b and residue charge, $Z_{\text{res}}=79-m_{\text{pre}}$, to obtain the multiplicity, m_{eq} (and charge/mass distribution) of the equilibrium particles from the percolation stage or SMM stage. The total multiplicity is then $m=m_{\text{pre}}+m_{\text{eq}}$. A standard integration over impact parameter yields the desired distributions. Within the assumptions stated above, both hybrid models are then free of adjustable parameters.

In Fig. 1 we compare the results of our models to the data of [18]. Displayed is the second moment of the charge distribution, computed event by event, and averaged over all events with identical total charged particle multiplicity,

$$M_2(m)=\sum_i \delta_{m,m_i}\left\{\sum_{Z=1}^{Z_{\max}} Z^2 N_i(Z)\right\} \Big/ \sum_i \delta_{m,m_i}. \quad (2)$$

Here $N_i(Z)$ is the number of fragments of charge Z emitted in event i and m_i is the total charged particle multiplicity of this event. Σ_i is a summation over all events and δ_{m,m_i} is the usual Kronecker δ. $Z_{\max}$ is the upper cutoff used in the summation over all fragment charges. Here we should point out the usefulness of displaying multiplicity-sorted data of $M_2(m)$ by including and excluding the largest, 2nd largest, 3rd largest, etc., fragment. This information is available in a model-independent way, and it shows the most probable partition of the system in each multiplicity bin. In the EOS data [18] displayed here (circles) only part of this important information is given: The largest fragment was included for multiplicities above 26 and excluded below. For the models we display in the lower curves $M_2(m)$ excluding the largest fragment and in the higher curves including the largest fragment for all multiplicities. Fission events were eliminated from both the data and the calculations.

We observe an astonishing degree of agreement between the results of the INC/percolation hybrid (histogram) and the data for all multiplicities. We should point out here that the fragmentation of a $Z=79$ residue with the same excitation energy distribution results in average moments very different from the data, indicating the importance of the emission of pre-equilibrium charged particles for this observable. In addition we performed a calculation fragmenting a $Z=79$ residue at a fixed excitation energy corresponding to the critical point of the percolation model. This resulted in events distributed over the multiplicity interval from 12 to 28, having approximately constant values of $M_2 \approx 100$ when excluding and $M_2 \approx 3000$ when including the largest fragment. This also indicates the importance of mixing between events of different excitation energy classes in the observed values of the moments.

The INC/SMM hybrid shows also qualitatively the same features as the data, but is generally too high on the lower branch and too low on the upper branch by about a factor of 2. Presumably one could generate better agreement for the INC/SMM by fitting an excitation energy and residue size distribution to the data along the method used in [16]. This, however, is not the goal of the present work.

Rather we would like to point out that the data and both models have similar slopes of M_2 versus m over the entire multiplicity interval. These slopes were used by the EOS Collaboration to extract the "critical" exponent γ. This is even more obvious in Fig. 2, where we display $\ln M_2(m)$ as a function of $|m-m_c|$ with $m_c=26$, according to the analysis of [17]. We see that data and both models have roughly parallel "liquid" and "gas" branches over the interval displayed here. However, in contradiction to the analysis of [21], we do not recover the known critical exponent γ (=1.8) of the percolation model from this figure. Instead both models yield about "γ"=1.4, in accordance with the experimental observation.

We have also performed an analysis of the size of largest fragment as a function of the total multiplicity. From an analysis like this, the authors of Ref. [17] claim to have extracted the critical exponent β. Recently, one of us [22]

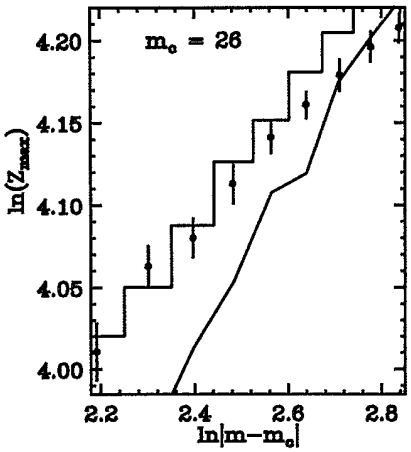

FIG. 3. Charge of the largest fragment as a function of the charged-particle multiplicity for the reaction 1 A GeV Au + C. Histogram: INC/percolation; line: INC/SMM; circles: data [17].

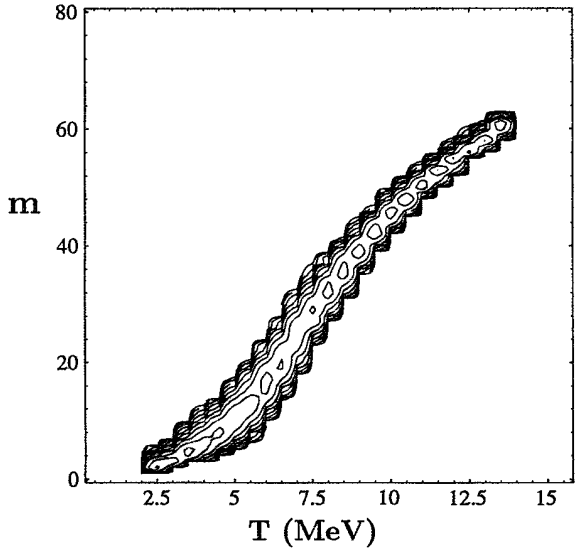

FIG. 4. Histogram of the number of events as a function of the temperature, T, and the total charged particle multiplicity, m, as predicted by the INC/percolation model. Each contour line is offset by a factor of 2 from the previous one.

pointed out that the presence of pre-equilibrium particles may change the value of the extracted apparent exponent. Within the framework of the present hybrid models we can now perform a quantitative comparison to the data, as shown in Fig. 3. Performing a linear regression fit to the output of the INC/percolation model in the interval shown yields a slope of "β"=0.35, reasonably close to the value of 0.29 $\pm$0.02 extracted from the data, and not close to the nominal value of β=0.41 for the percolation model. A fit to the INC/SMM output yields "β"=0.50.

It is dangerous to interpret these exponents extracted in the above way as the true critical exponents characterizing the universality class of the actual nuclear phase transition. Three effects play important roles in this: the extreme finite size of the fragmenting system, the use of $\ln|m-m_c|$ instead of $\ln|T-T_c|$ (or $\ln|p-p_c|$) for the abscissa, and, most importantly, the mixing of different event classes of size and excitation in the same multiplicity bin. This mixing is caused by the use of inclusive, impact parameter integrated data with different admixtures of pre-equilibrium particles.

The mixture of events with different "temperature" into the same multiplicity bins represents a convolution

$$O(m)=\int dT\, N(m,T)\otimes O'(T), \tag{3}$$

where $O(m)$ and $O'(T)$ is the physical observable as a function of multiplicity and temperature, and $N(m,T)$ is the (integral normalized) number distribution function of events with a given temperature and multiplicity. In Fig. 4, we show $N(m,T)$ as a contour plot for the INC/percolation model. Each contour level represents a factor of 2 higher value than the one surrounding it. One can clearly see that $N(m,T)$ is rather broad, indicating a large degree of mixing of events with different temperature into the same multiplicity bin.

While the scaling laws of critical behavior are valid for $O'(T)$ in the vicinity of the critical point, no such statement can *a priori* be made for the same observable as a function of the multiplicity. To accomplish this, the inverse of the convolution function, $N(m,T)^{-1}$ has to be known. For obvious reasons, the function $N(m,T)$ cannot be extracted from data in a model-independent way, and therefore the inverse is also not accessible. For all these reasons, a precise model-independent determination of critical exponents, as suggested in Refs. [21,17,18], is not correct.

In conclusion, taking the data of the EOS Collaboration we have shown that an interpretation in terms of critical phenomena should be carried on very carefully. Mixing of different event classes into the same multiplicity bins and the contributions of pre-equilibrium particle emission make a model-independent extraction of critical exponents questionable.

What is possible, however, is a detailed comparison of the data to models which include pre-equilibrium as well as equilibrium components, and which emulate as closely as possible the experimental trigger conditions. We have used two of the models here to perform this kind of comparison. We have found, e.g., that the INC/percolation model achieves surprising agreement with the data. An analysis of the kind employed in [17] for the model output yields apparent exponents of "β"=0.35 and "γ"=1.4, whereas the known critical exponents for $3d$ percolation in the infinite size limit are β=0.41 and γ=1.8. The next step, in our opinion, could be analysis of exclusive events aimed at finding actual equilibrated residues. After that we can really investigate manifestations of the phase transition in the residue nuclei. There is hope that this direction of research will be fruitful. The rather surprising agreement of the INC/percolation model (which explicitly contains a phase transition) with the data has given rise to this hope.

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