## Nuclear Multifragmentation Critical Exponents

In a recent Letter [1] the EOS Collaboration presented data of fragmentation of 1*A* GeV gold nuclei incident on carbon, claiming to determine the critical exponents $\gamma$, $\beta$, and $\tau$ for finite nuclei. However, the analysis presented in [1] is not sufficient to support the claim that these exponents have been unambiguously determined.

Nuclei cannot be prepared and held near the temperature and density associated with the critical point. Instead, reactions must be used to excite the nuclei, which may then expand to conditions sufficiently close to the critical point. There are problems in measuring the "temperature" of the decaying system, and in identifying which of the particles came from the equilibrated system near the critical conditions and which came from the "preequilibrium" stage of the reaction. These two problems are interrelated in the procedure used in Ref. [1], where the authors assume that the observed multiplicity $m$ can be used as an indicator of temperature. Here we use the percolation model of nuclear fragmentation [2] to show how these problems can affect the determination of critical exponents.

In percolation models one uses a bond breaking parameter $p$, $0 \le p \le 1$, including $p_c$, the "critical value." In this model, for $p$ near $p_c$, the size ($Z_{\max}$) of the largest fragment is given by $Z_{\max} \propto (p - p_c)^\beta$, and the second moment of the fragment size distribution by $M_2 \propto |p - p_c|^{-\gamma}$, with $\beta = 0.41$ and $\gamma = 1.8$.

One would only expect $Z_{\max} \propto (m - m_c)^\beta$ and $M_2 \propto |m - m_c|^{-\gamma}$, if the relationship between $m$ and $p$ were strictly linear. In reality, however, there is a distribution in the values of $m$ for each value of $p$. Thus, any translation of an observable, $O$, in terms of $p$ into terms of $m$ involves a nontrivial convolution $O(m) = \int dp\, m(p) \otimes O(p)$. Furthermore, percolation calculations show that while $\langle m \rangle$ rises monotonically with $p$ it does not rise linearly near $p_c$.

To examine the analysis of [1] we constructed a sample of $10^4$ events, from a lattice of 79 sites. We find, as in [1], roughly identical values of $\gamma$ and $\gamma'$ for the liquid and gas branches, if we examine $\ln M_2$ vs $\ln|m - m_c|$ and use $m_c = 26$, the cut employed in [1]. However, $m = 26 < \langle m(p_c) \rangle$, and the numerically extracted value of $\gamma = \gamma'$ is approximately 1.2 to 1.4, significantly lower than 1.8, the critical exponent for the percolation model. These two observations indicate some of the problems in trying to find the critical exponents via the analysis employed in [1].

From our sample of percolation events we plot in Fig. 1 $\ln Z_{\max}$ vs $\ln|m - m_c|$ (open circles). The best fit to these points results in a slope "$\beta$" = 0.55. For comparison, the solid line has a slope of $\beta = 0.41$, the nominal value for percolation. This demonstrates that the slope extracted from the points is not the critical exponent $\beta$.

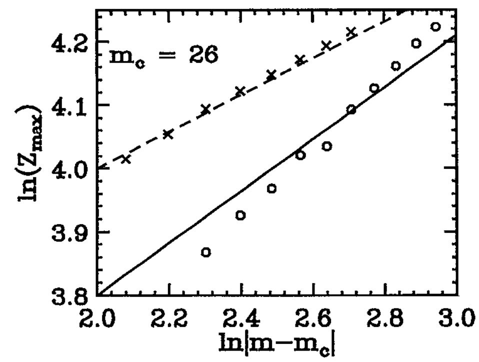


FIG. 1. Percolation model simulation of the fragmentation of gold. Circles: $Z = 79$ system fragmenting; crosses: $Z = 69$ system fragmenting plus 10 preequilibrium protons. For comparison, the solid line has a slope of $\beta = 0.41$, and the dashed line $\beta = 0.29$.

One also expects that a portion of the observed multiplicity is comprised of preequilibrium particles—up to $\approx 15$ in cascade simulations. To illustrate their influence on the extraction of values for the critical exponent $\beta$, we show the results of a calculation in which we assume that 10 preequilibrium particles exist along with an equilibrated system of 69 charges, which is then fragmented, again by using the percolation model. The total observed multiplicity includes the preequilibrium particles, and the plot of $\ln Z_{\max}$ vs $\ln|m - m_c|$ results in the crosses in Fig. 1. For comparison, the dashed line has a slope of $\beta = 0.29$. Similarly, when following the steps of analysis in [1] for determination of $\gamma$, we find significant contamination of the "liquid" branch due to preequilibrium emission. Since the analysis of [1] does not work for a simple model with known values of critical exponents, one might not expect it to yield the correct critical exponents for the data either.

In summary, while we applaud the effort and beautiful data of [1], we do not agree with the conclusion that the critical indices of nuclear fragmentation have been determined yet.

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