Intensity-Interferometric Test of Nuclear Collision Geometries Obtained from the Boltzmann-Uehling-Uhlenbeck Equation

W. G. Gong, W. Bauer, C. K. Gelbke, N. Carlin,[a] R. T. de Souza, Y. D. Kim, W. G. Lynch, T. Murakami,[b] G. Poggi,[c] D. P. Sanderson, M. B. Tsang, and H. M. Xu

National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824

S. Pratt

Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

D. E. Fields, K. Kwiatkowski, R. Płaneta,[d] V. E. Viola, Jr., and S. J. Yennello

Indiana University Cyclotron Facility, Indiana University, Bloomington, Indiana 47405

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Two-proton correlation functions measured for the $^{14}N+^{27}Al$ reaction at $E/A=75$ MeV are compared to correlation functions predicted for collision geometries obtained from numerical solutions of the Boltzmann-Uehling-Uhlenbeck (BUU) equation. The calculations are in rather good agreement with the experimental correlation function, indicating that the BUU equation gives a reasonable description of the space-time evolution of the reaction.

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Microscopic models of intermediate-energy nucleus-nucleus collisions have been successfully based on the semiclassical Boltzmann-Uehling-Uhlenbeck (BUU) equation[1] which describes the temporal evolution of the one-body density under the influence of the nuclear mean field and individual nucleon-nucleon collisions. In this paper we report the first quantitative test of the space-time geometry predicted by solutions of the BUU equation by using the technique of two-proton intensity interferometry[2] which utilizes the space-time sensitivity of the two-proton correlation function at small relative momenta.[2-11] For this purpose, we have measured two-proton correlation functions with high statistical accuracy for the relatively light projectile-target combination $^{14}N+^{27}Al$, at $E/A=75$ MeV. For such a light system, numerical calculations can be performed with good accuracy and modest amounts of CPU time.

The experiment was performed with a ^{14}N beam of $E/A=75$ MeV extracted from the K1200 cyclotron of the National Superconducting Cyclotron Laboratory at Michigan State University. An ^{27}Al target of 15 mg/cm^2 areal density was used. Protons were detected with two ΔE-E detector arrays consisting of 300–400-μm-thick silicon ΔE detectors and 10-cm-long CsI(Tl) or NaI(Tl) E detectors. An array consisting of 37 Si-CsI(Tl) telescopes[12] was centered at the polar and azimuthal angles of $\theta=25°$ and $\phi=0°$; each of its detectors had a solid angle of $\Delta\Omega=0.37$ msr and a nearest-neighbor spacing of $\Delta\theta=2.6°$. Another array consisting of 13 Si-NaI(Tl) telescopes was centered at $\theta=25°$ and $\phi=90°$; each of its detectors had a solid angle of $\Delta\Omega=0.5$ msr and a nearest-neighbor spacing of $\Delta\theta=4.4°$. Coincidence and downscaled singles data were taken simultaneously. Energy calibrations are accurate to better than 2%. Typical detector energy resolutions were of the order of 2% and 1% for protons of 40 and 100 MeV, respectively. All the data were corrected for random coincidences and had a software energy threshold of 10 MeV.

The experimental two-proton correlation function $R(q)$ is defined in terms of the coincidence yield $Y(\mathbf{p}_1, \mathbf{p}_2)$ and the single-proton yields $Y(\mathbf{p}_1)$ and $Y(\mathbf{p}_2)$:

$$\sum Y(\mathbf{p}_1,\mathbf{p}_2)=C[1+R(q)]\sum Y(\mathbf{p}_1)Y(\mathbf{p}_2). \quad (1)$$

Here, $\mathbf{p}_1$ and $\mathbf{p}_2$ are the laboratory momenta of the two protons, and $q=\frac{1}{2}|\mathbf{p}_1-\mathbf{p}_2|$ is the relative momentum of the proton pair. For each experimental gating condition, the sums on both sides of Eq. (1) are extended over all energy and detector combinations corresponding to specific relative momentum bins. The normalization constant C is determined from the requirement $R(q)=0$ at large q.

Our theoretical analysis is based upon the expression[2,10]

$$1+R(\mathbf{P},\mathbf{q})=\int d^3r\,F_{\mathbf{P}}(\mathbf{r})|\phi(\mathbf{q},\mathbf{r})|^2. \quad (2)$$

Here, $\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2$ is the total momentum of the proton pair, $\phi(\mathbf{q},\mathbf{r})$ is the relative two-proton wave function, and $F_{\mathbf{P}}(\mathbf{r})$ is defined by

$$F_{\mathbf{P}}(\mathbf{r})=\frac{\int d^3X f(\mathbf{P}/2,\mathbf{X}+\mathbf{r}/2,t_>)f(\mathbf{P}/2,\mathbf{X}-\mathbf{r}/2,t_>)}{|\int d^3X f(\mathbf{P}/2,\mathbf{X},t_>)|^2}. \quad (3)$$

Here, the Wigner function $f(\mathbf{p},\mathbf{x},t_>)$ is the phase-space distribution of particles of momentum $\mathbf{p}$ at position $\mathbf{x}$ at some time $t_>$ after the emission process. We will obtain $f(\mathbf{p},\mathbf{x},t_>)$ by solving the BUU equation.

Figure 1 shows measured two-proton correlation functions for representative ranges of the total momentum P of the proton pairs. In order to describe the dependence on P in terms of a simple variable, we have constructed experimental correlation functions for a number of narrow gates on P. Each such correlation function was characterized in terms of a simple Gaussian source of negligible lifetime,

$$f(\mathbf{P}/2,\mathbf{r},t)=\rho_0\exp[-r^2/r_0^2(P)]\delta(t-t_0)\,. \tag{4}$$

The extracted source parameters $r_0(P)$ are shown by the solid circles in Fig. 2. The error bars indicate estimated systematic uncertainties. The dashed curves in Fig. 1 show examples of fits with Eq. (4) which are used to extract the momentum dependence of $r_0(P)$ shown in Fig. 2. In these calculations, appropriate averages over total momentum and the calculated resolution of the hodoscope were taken into account.[13]

In order to compare our results with microscopic dynamical calculations, we have calculated microscopic Wigner functions by solving the BUU transport equation

$$\partial_t f(\mathbf{p},\mathbf{r},t)+\frac{\mathbf{p}}{m}\cdot\nabla_r f(\mathbf{p},\mathbf{r},t)-\nabla_r U(\mathbf{r})\cdot\nabla_p f(\mathbf{p},\mathbf{r},t)$$
$$=\frac{1}{2\pi^3m^2}\int d^3q_1'\,d^3q_2\,d^3q_2'\,\delta\left(\frac{1}{2m}(p^2+q_2^2-q_1'^2-q_2'^2)\right)\delta^3(\mathbf{p}+\mathbf{q}_2-\mathbf{q}_1'-\mathbf{q}_2')\frac{d\sigma}{d\Omega}$$
$$\times\{\hat f(\mathbf{q}_1',\mathbf{r},t)\hat f(\mathbf{q}_2',\mathbf{r},t)[1-\hat f(\mathbf{p},\mathbf{r},t)][1-\hat f(\mathbf{q}_2,\mathbf{r},t)]$$
$$-\hat f(\mathbf{p},\mathbf{r},t)\hat f(\mathbf{q}_2,\mathbf{r},t)[1-\hat f(\mathbf{q}_1',\mathbf{r},t)][1-\hat f(\mathbf{q}_2',\mathbf{r},t)]\}\,, \tag{5}$$

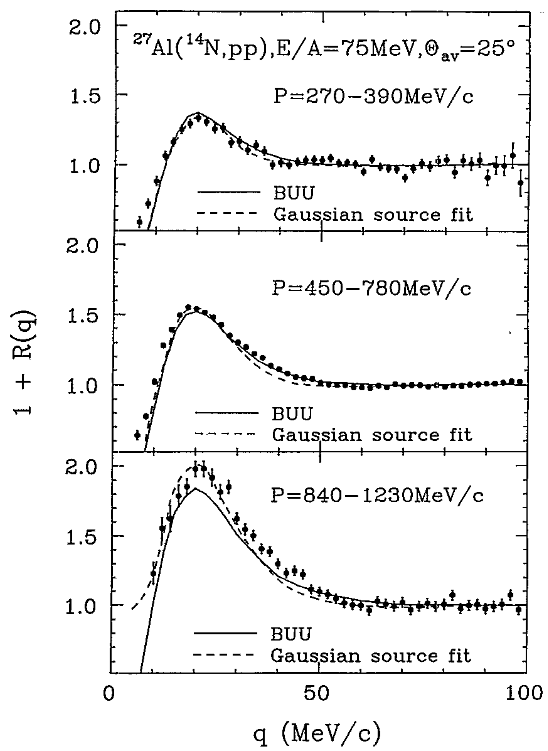

FIG. 1. Two-proton correlation functions measured for the reaction $^{14}N+^{27}Al$ at $E/A=75$ MeV. The gates placed on the total momenta P of the coincident particle pairs are indicated. The dashed curves represent calculations for Gaussian sources of negligible lifetime, assuming the momentum dependence of the radius parameter $r_0(P)$ shown by the solid circles in Fig. 2. The solid curves correspond to solutions of the BUU equation using the stiff equation of state, Eq. (6a).

where $\hat f(\mathbf{p},\mathbf{r},t)$ is the phase-space density. The in-medium nucleon-nucleon scattering cross section $d\sigma/d\Omega$ was assumed to be proportional to the experimentally known (energy-dependent) cross section for free nucleons $d\sigma_{NN}/d\Omega$. The mean field $U(\mathbf{r})$ was approximated by density-dependent, simplified local Skyrme interactions:

$$U(\rho)=-124\rho/\rho_0+70.5(\rho/\rho_0)^2\text{ MeV}\,, \tag{6a}$$
$$U(\rho)=-356\rho/\rho_0+303(\rho/\rho_0)^{7/6}\text{ MeV}\,. \tag{6b}$$

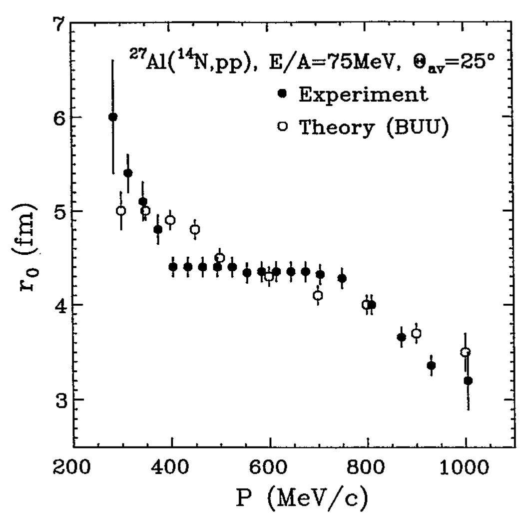

FIG. 2. Radius parameters $r_0(P)$ for Gaussian sources of negligible lifetime extracted from two-proton correlation functions gated by different total momenta P. Solid and open circles represent experimental and theoretical correlation functions, respectively.

Here, ρ_0 denotes the equilibrium density of normal nuclear matter. Equations (6a) and (6b) represent a stiff and a soft equation of state with compressibility coefficients $K=380$ and 200 MeV, respectively. The left-hand side of Eq. (5) is the Vlasov term describing the interaction of the nucleons with the mean field; the right-hand side is the collision integral which includes a semiclassical Pauli-blocking factor. We solved this equation by numerical methods which are similar to those described in Ref. 14. By explicitly storing $\hat{f}(\mathbf{p},\mathbf{r},t)$ on a six-dimensional lattice in every time step we were able to greatly speed up the computer program without relaxing the accuracy of the treatment of the Pauli exclusion principle.[15] Nucleon emission was calculated for a time interval of $\Delta t_e = 140$ fm/c following initial contact of the colliding nuclei. Nucleons were considered as emitted when the surrounding density fell below $\rho_e = \rho_0/8$ and when subsequent interaction with the mean field did not cause recapture into regions of higher density.[16] For the calculation of a correlation function, we generated phase-space points from a total of 5250 events with impact parameters distributed according to their geometrical weights; appropriate averages over impact parameter, orientation of the reaction plane, and momenta of the outgoing particles were taken into account.

Correlation functions calculated from Wigner functions predicted by the BUU equation are shown by the solid lines in Fig. 1. For these calculations, we used the stiff equation of state, Eq. (6a), and the free nucleon-nucleon scattering cross section, $d\sigma/d\Omega = d\sigma_{NN}/d\Omega$. In order to facilitate a more detailed comparison with the experimental data, the predicted correlation functions were characterized in terms of equivalent Gaussian sources, Eq. (4), following the procedure outlined above. The corresponding equivalent radius parameters $r_0(P)$ are shown by the open circles in Fig. 2. The errors represent estimates for the numerical accuracy of our calculations. The predicted correlation functions are in good agreement with the data, indicating the BUU equation gives a good overall description of the average space-time evolution of the reaction.

We have also explored the sensitivity of the calculated correlation functions to the nuclear equation of state and the magnitude of the in-medium nucleon-nucleon cross section. Figure 3 shows correlation functions predicted for intermediate-momentum proton pairs, $P=500$ MeV/c. The solid and dotted curves show correlation functions predicted for the stiff and soft equations of state, using $d\sigma/d\Omega = d\sigma_{NN}/d\Omega$. The two calculations are very similar, indicating little sensitivity to the nuclear equation of state.[17] The solid, dashed, dot-dashed, and dot-dot-dashed curves represent calculations with the stiff equation of state performed with the assumption that the in-medium nucleon-nucleon cross section is equal to 1.0, 0.8, and 0.5, and 0.0 times the free nucleon-nucleon cross section. The predicted correlation functions become more pronounced for decreasing values

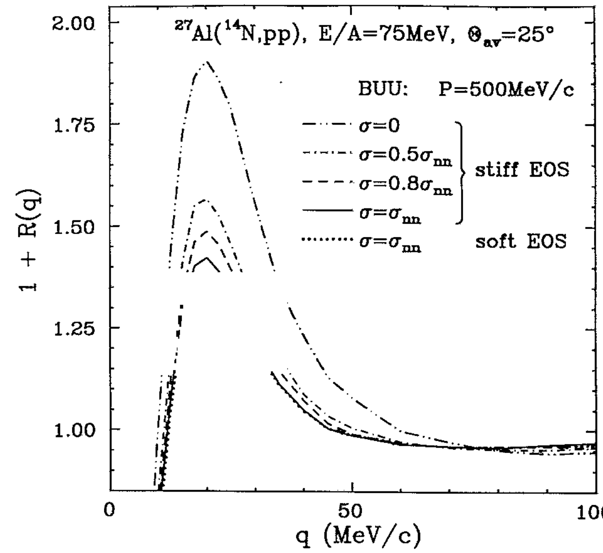

FIG. 3. Sensitivity of two-proton correlation functions at intermediate momentum to the nuclear equation of state and the in-medium nucleon-nucleon cross section.

of $d\sigma/d\Omega$, indicating that increased nucleon-nucleon scattering leads to slower emission time scales. For intermediate total momenta of the proton pairs, $P \approx 500$ MeV/c, the calculated correlation functions are rather sensitive to the magnitude of the in-medium nucleon-nucleon cross section. For the emission of very energetic particles, $P \gtrsim 800$ MeV/c, this sensitivity is reduced. The measured correlation functions are rather well reproduced by calculations using free nucleon-nucleon cross sections; see Figs. 1 and 2. The agreement is significantly worse when in-medium cross sections are used which are smaller than one-half of the free nucleon-nucleon cross section.

In summary, we have investigated two-proton correlation functions for $^{14}N + ^{27}Al$ collisions at $E/A = 75$ MeV. Microscopic calculations based upon solutions of the BUU transport equation are in good agreement with the measured correlation functions, indicating that the theory predicts reasonable space-time geometries for nonequilibrium light-particle emission. The predicted correlation functions are rather insensitive to the stiffness of the equation of state, but exhibit considerable sensitivity to the magnitude of the in-medium nucleon-nucleon cross section.[17] The measured correlation functions are rather well reproduced by calculations in which the in-medium cross section is approximated by the energy-dependent free nucleon-nucleon cross section.

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Estado de São Paulo (FAPESP), Brazil.

[(a)]Present address: Instituto de Física, Universidade de São Paulo, CP 20516, Codigo de Enderecamento Postal 01498, São Paulo, Brazil.

[(b)]Present address: Department of Physics, Kyoto University, Kyoto 606, Japan.

[(c)]Permanent address: Dipartimento di Fisica dell'Università and Istituto Nazionale di Fisica Nucleare, Largo Enrico Fermi 2, 50125 Firenze, Italy.

[(d)]Present address: Institute of Nuclear Physics, Jagellonian University, Krakow, Poland.

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[16]The calculated correlation functions depend on the specific choice of Δt_e and ρ_e. At present, these parameters must be treated as unknown model parameters which introduce uncertainties of the order of 5%–10% into the magnitude of the predicted correlation functions.

[17]Sensitivities of the predicted two-proton correlations functions to in-medium cross sections and equation of state can be expected to depend on incident energy and the size of projectile and target. In particular, the lack of sensitivity of the present calculations to the stiffness of the equation of state may be related to the small size of projectile and target investigated in this paper. Calculations for heavier systems and different incident energies would clearly be valuable.