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Phase transitions in a two-component site-bond percolation model

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A method for analyzing a N-component percolation model in terms of one parameter p_+ is presented. In Monte Carlo simulations on 16^3, 32^3, 64^3, and 128^3 simple cubic lattices the percolation threshold p_+^c is determined for $N=2$. Continuous transitions of p_+^c are reported in two limits for the bond existence probabilities $p_=$ and $p_{\neq}$. In the same limits, empirical formulas for the percolation threshold p_+^c as a function of one-component concentration f_b are proposed and links to existing percolation models are established. In the limit $p_==0$ a different site percolation model is proposed and its threshold, $f_b^c \simeq 0.145$, is reported.

The percolation model goes back to Flory[1] who introduced it in the context of polymer gelation. Since then it has been used in a wide range of approaches and techniques: Exact percolation probabilities in two dimensions were given by Sykes and Essam.[2] Recently, universal formulas for site and bond percolation thresholds have been postulated, which essentially read as a power law in the coordination number and the space dimension.[3] These formulas have also been extended to anisotropic and aperiodic lattices.[4] As to applications of percolation theory,[5] there have been, for example, approaches to nuclear physics, fragmentation processes in particular, using a nuclear lattice model. It was used to well reproduce mass-yield curves of proton-induced multifragmentation reactions,[6] and also an application to the question of fragment multiplicity distributions[7] has proven successful. Furthermore, quite recently, a stock-market model relying on percolation theory was presented.[8] In all these cases, standard percolation models were used, in which either bond or site percolation is dealt with.[9,10] Site-bond percolation, which was introduced by Coniglio, Stanley, and Klein,[11] goes a step further, combining the two formulations, dealing with randomly occupied sites (vertices) and randomly existing bonds (open edges) connecting these sites. However, in this version of the model only one active component exists, the other sites are considered unoccupied. A further generalization is to consider several components, which was done for site percolation as well as bond percolation by Zallen[12] and called polychromatic percolation. Zallen focused on the coexistence of percolating species in highly connected lattices, giving a criterion for the occurrence of a panchromatic regime where all species percolate. Generalizing the nuclear lattice model, site-bond percolation using two components was previously applied by one of us[13] to the question of the nuclear liquid-gas phase transition. Site-bond percolation with several species was considered by another author[14] and an approximate percolation criterion was given.

Here we investigate a two-component site-bond percolation model on a simple cubic lattice, which is a generalization of site-bond and AB percolation.

We will show that this model contains important effects, giving rise to interesting behavior but also reproducing known results of other percolation models. This allows for an interpretation of transitions shown in the present model as effective transitions between different percolation models. It will also be shown that in a specific parameter regime of this AB site-bond percolation model a different two-component site percolation process, which can be seen as inverse site percolation, is obtained.

Let us begin by describing the approach we have taken, in the general case of N different component flavors. No assumption concerning topological dimensions or lattice structure is made. We have N component concentrations f_i with

$$\sum_{i=1}^{N} f_i = 1, \qquad (1)$$

giving rise to $(N-1)$ freely adjustable concentrations. Furthermore we have different bond probabilities p_{ij} to connect all possible combinations of sites. The bonds are assumed to be directionless, meaning that their probabilities only depend on the species of the sites they are connecting: $p_{ij}=p_{ji}$. This results in an overall number of

$$A_{par} = (N-1) + \binom{N+2-1}{2} = N^2/2 + 3N/2 - 1 \qquad (2)$$

free parameters a_i. We now want to know in which region of this A_{par}-dimensional parameter space an infinite network C_∞ of connected bonds occurs, that is, where the probability for a given site to belong to the infinite network, $P_\infty(\{a_i\})$, is nonzero. The particular type of a bond shall be irrelevant in order for it to belong to the infinite network. For a system with $N \geq 3$ components, however, this approach is quite impractical. It would be preferable to be able to reduce the dependence of the order parameter P_∞ to one variable at fixed particle concentrations. In one-component bond percolation this one variable is the bond probability p, which gives the bond density in the system under observation. In a multicomponent system, several bond probabilities p_{ij} are in-

volved. To find the bond density which corresponds to a given set of bond existence probabilities $\{p_{ij}\}$, one also needs to know the probability α_{ij} to have a nearest-neighbor edge connect a vertex of type i and a vertex of type j. With the component concentrations f_i these probabilities α_{ij} read

$$\alpha_{ij} = 2f_i f_j \quad \text{for } i \neq j \tag{3}$$

and

$$\alpha_{ii} = f_i f_i. \tag{4}$$

Obviously $\alpha_{ij} = \alpha_{ji}$ holds in both cases. To simplify the notation, we rewrite Eqs. (3) and (4) as

$$\alpha_{ij} = (2 - \delta_{ij}) f_i f_j, \tag{5}$$

where δ_{ij} is the Kronecker Delta. As the α_{ij}'s are related to the component concentrations, the constraint, Eq. (1), induces a similar constraint for the α_{ij}'s:

$$\sum_{i \geqslant j = 1}^{N} \alpha_{ij} = 1. \tag{6}$$

This states that the probability to find an edge irrespective of its endvertices is unity. Thus with probability α_{ij} we have a nearest-neighbor edge that connects sites of flavor i and j, and with probability p_{ij} this edge is occupied by an ij bond. Therefore the bond density in the system, which we denote by p_+, is given as the sum over all possible bond types:

$$p_+ = \sum_{i \geqslant j = 1}^{N} \alpha_{ij} p_{ij}. \tag{7}$$

This is a generalization for N components of the p_+ proposed in Ref. 13 for the case of two components, that is, AB site-bond percolation.

    For our computations, we consider this model in the special case of $N = 2$ components on a simple cubic lattice, which is a generalization of a site bond, AB percolation model on a simple cubic lattice. We utilize system sizes of 16^3, 32^3, 64^3, and 128^3. For simplicity we call one species blue, the other red, and change the notation appropriately to indexing with $_b$ and $_r$. We now have four free parameters to vary: The fraction of one of the components, say of the blue sites, f_b and three bond activation probabilities: p_{bb} for bonds connecting two blue sites, p_{rr} for bonds connecting two red sites, and $p_{\neq}$ for b-r bonds. However, we shall set $p_= \equiv p_{bb} = p_{rr}$, introducing a symmetry in the system. This is motivated by considerations of, for example, isospin symmetry, where the $e^+ e^+$ and $e^- e^-$ interactions are identical. Equations (7) and (3) now read

$$p_+ = \alpha_{\neq} \, p_{\neq} + \alpha_= \, p_=, \tag{8}$$

with

$$\alpha_{\neq} = 2f_b(1 - f_b), \tag{9}$$

and

$$\alpha_= = f_b^2 + (1 - f_b)^2 = (1 - \alpha_{\neq}), \tag{10}$$

respectively, where we have replaced the double indices by a more intuitive notation for only two components. Again the
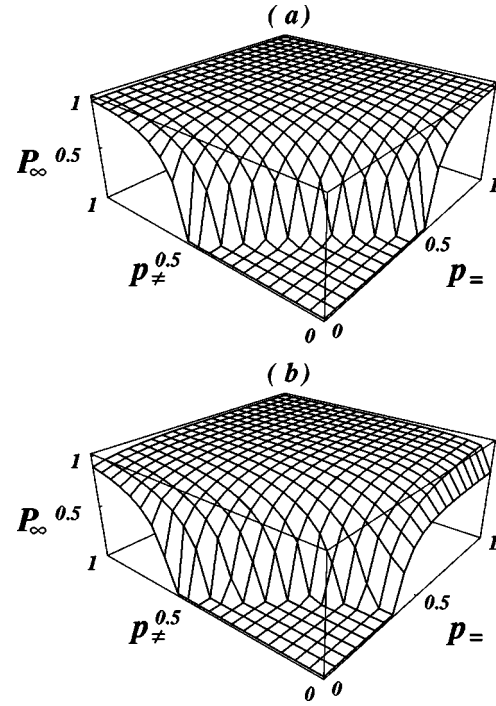

FIG. 1. Probability to belong to the infinite cluster P_∞ in a simple cubic two component site-bond percolation lattice of size 128^3 as a function of the two parameters $p_=$ and $p_{\neq}$ calculated for a fraction of the blue species of (a) $f_b = 0.5$ and (b) $f_b = 0.3$.

question is in which region of the three-dimensional $p_=$—$p_{\neq}$—f_b space an infinite network of bonds appears in the lattice. We defer the question of the concentration dependence to later and for the moment set f_b to some fixed value. Before we proceed, we would like to turn the attention to another question: Why not consider two dimensions first, where some quantities, as for example, the bond percolation threshold on the square lattice are known exactly? Consideration of the three-dimensional case was motivated by the argument that on the simple cubic lattice a panchromatic regime, a region in the parameter space, where all components can percolate simultaneously, exists. This comes about as the site percolation threshold on the simple cubic lattice is given[9] as $f_{site}^c = 0.3116$. This allows for both species concentrations f_b and f_r to be above f_{site}^c and thus allows both species to percolate with bond probabilities of unity or less. In two dimensions we have[9] $f_{2d-site}^c = 0.592\,746$ for the site percolation threshold on the square lattice. Therefore in two dimensions at most one of the two species can be in the percolation regime. We therefore focus our attention entirely on the model in three dimensions, which presents a more general case.

    In the simulation the lattice is populated at random, without correlations, according to f_b and bonds are formed for varied values of $p_=, p_{\neq} \in [0,1]$ using a Monte Carlo algorithm. The resulting cluster structure is analyzed using a cluster-find-algorithm described in Ref. 6 and $P_\infty(p_=, p_{\neq})$ is recorded. As always, a cluster is defined as a set of vertices connected by open edges. In Fig. 1 we show P_∞ as a function of the two control parameters, $p_=$ and $p_{\neq}$, for two different concentrations of the blue species f_b. In Fig. 1(a) we have a concentration of $f_b = 0.5$, in Fig. 1(b) we show results for
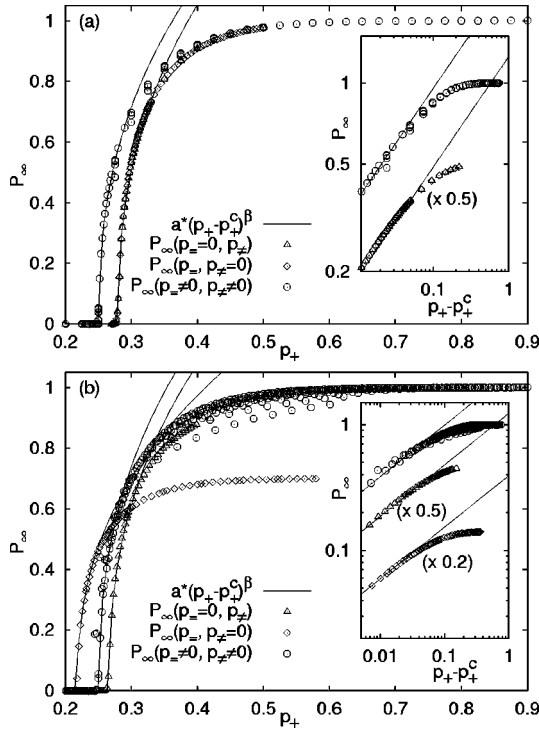
FIG. 2. Probability to belong to the infinite cluster P_∞ as a function of the linear combination p_+ of the two control parameters $p_=$ and $p_{\neq}$ in a 128^3 simple cubic lattice for values of $p_=, p_{\neq} \in [0,1]$, fitted with $P_\infty \propto (p_+ - p_+^c)^\beta$ (solid line). Here it is (a) $f_b = 0.5$ with $p_+^c \simeq 0.251$ for the upper branch and $p_+^c \simeq 0.280$ for the lower branch. In (b) it is $f_b = 0.3$ with $p_+^c \simeq 0.251$ for the "$p_= \neq 0$, $p_{\neq} \neq 0$" branch, $p_+^c \simeq 0.266$ for the "$p_= = 0$" branch and $p_+^c \simeq 0.218$ for the "$p_{\neq} = 0$" branch. In all cases we have $\beta = 0.41$. The insets show the same data in a double logarithmic representation (please note that the $p_= = 0$ and the $p_{\neq} = 0$ branches have been displaced in the insert by factors of 0.5 and 0.2 to provide better visual separation of the plot symbols for the different cases).

$f_b = 0.3$. We can see that for both values of f_b the percolation probability P_∞ changes from 0 (front corner) to 1 (back corner), with a critical line of a continuous phase transition in the $(p_=, p_{\neq})$-plane. Figure 1(a) obviously represents, with $f_b = 0.5$, a special case, since $f_b = 0.5$ implies $f_r = 0.5$ also, leading to a system that is symmetric with respect to the two types of particles. This symmetry manifests itself in Fig. 1(a) in the fact that the critical line of the phase transition is of the form $p_{\neq} = \text{const} - p_=$ (with $\text{const} \approx 0.5$) that is, encloses the same angle with the $p_=$ and the $p_{\neq}$ axis. This symmetry is broken for $f_b = 0.3$ in Fig. 1(b), yet qualitatively the same behavior is retained.

We now follow our previous consideration concerning the density of bonds in the system and analyze the percolation probability data of Figs. 1(a) and (b) in terms of the parameter p_+, the result of which is displayed in Figs. 2(a) and (b) respectively. [In Figs. 2(a) and (b) we show all data points contained in Figs. 1(a) and (b), but added additional data points in the transition region by employing a finer grid.] In the symmetric case of $f_b = 0.5$ in Fig. 2(a) three branches are seen, two of which are identical. They are all in the shape of a power law, characteristic for a continuous phase transition. In the nonsymmetric case of Fig. 2(b) with $f_b = 0.3$ the three

branches appear completely distinct. In both cases most of the data points fall on one curve, only a relatively small number of data points constitutes the two remaining curves. This fact that most data points collapse on one curve and the remaining points on only two other curves suggests that p_+ is a meaningful linear combination of the two control parameters. It is clear, however, that the question as to which points do not collapse on the "main curve" has to be investigated. This will be done in the following.

Further analysis of the data for $f_b = 0.5$ shows that the "lower" branches constitute of points with $(p_= = 0, p_{\neq} \neq 0)$ and $(p_= \neq 0, p_{\neq} = 0)$ whereas all other points, that is, points with $p_=$ and $p_{\neq}$ nonzero, fall on the "upper" branch. For $f_b = 0.3$ this observed degeneracy of the lower branches does not exist, and $(p_= = 0, p_{\neq} \neq 0)$ and $(p_= \neq 0, p_{\neq} = 0)$ each have their own branch.

Also shown in Fig. 2 are the expectations from one-component bond percolation theory, $P_\infty \propto (p_+ - p_+^c)^\beta$, (smooth lines). For the finite $(p_=, p_{\neq})$ regime in both Figs. 2(a) and (b) the critical value p_+^c has the same numerical value as the bond existence probability in one component bond percolation (aside from a small difference due to finite-size effects), $p_+^c = 0.251 \pm 0.002$. This means that as long as both bond types are active, the system under observation here and the one component bond percolation model show an identical phase transition behavior, which is consistent with the findings presented in Ref. 13. In the zero limits of $p_=$ or $p_{\neq}$, however, p_+^c is shifted to $p_+^c = 0.280 \pm 0.002$ in the case of $f_b = 0.5$. For the system with $f_b = 0.3$ we find a shift of the critical bond concentration to $p_+^c = 0.218 \pm 0.002$ in the case of $(p_= \neq 0, p_{\neq} = 0)$ and $p_+^c = 0.266 \pm 0.002$ in the case of $(p_= = 0, p_{\neq} \neq 0)$. It has to be noted that the thresholds p_+^c as given here were not determined from the data in Fig. 2, but rather in an independent simulation aimed at the determination of the critical values, as described below. The critical exponent $\beta = 0.41$ from one component bond percolation theory is the same for all curves displayed here, as shown in the double logarithmic plot in the insets of Fig. 2.

At this point we wish to briefly discuss the definition of the percolation probability P_∞. We first worked with P_∞ as defined usually in one-component percolation theory, taking it as the probability of a given site to belong to the infinite cluster C_∞, meaning the biggest cluster in a finite system. It turns out, however, that in the limits of the bond existence probability which we are discussing here, namely $p_= = 0$ or $p_{\neq} = 0$, this definition leads to an artifact of the following kind. We discuss one example, assuming $f_b = 0.5$ and $p_{\neq} = 0$. Both species can percolate and thus will both have a spanning cluster, C_∞^b and C_∞^r, which will, however, slightly differ in size. The percolation probability will then be found to be the probability that a given site belongs only to the biggest of the spanning clusters, say C_∞^b, resulting in $P_\infty(p_{\neq} = 0) \approx C_\infty^b / N^3$, where N^3 is the system size. Turning on the bond existence probability $p_{\neq}$ to some finite value $p_{\neq} = \epsilon$, with a very high probability creates a bond which connects the two biggest clusters, giving $P_\infty(p_{\neq} = \epsilon) \approx (C_\infty^b + C_\infty^b)/N^3$. This leads to a jump in P_∞ when letting $p_{\neq} \rightarrow \epsilon$. Using an alternative percolation criterion, defining the percolation probability P_∞ as the probability to belong to *any* spanning cluster, this jump vanishes, since then even at $p_{\neq}$
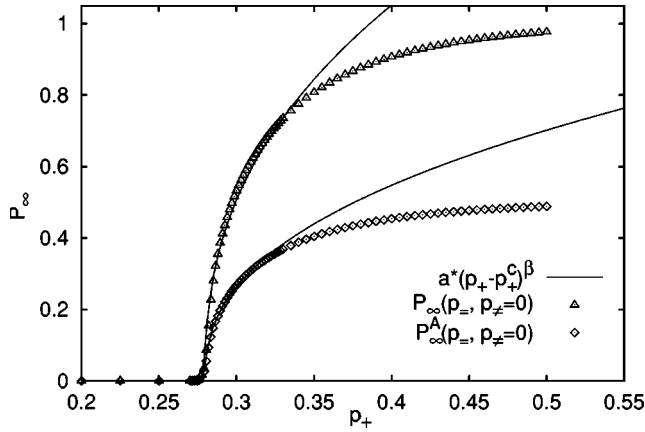
FIG. 3. Probability to belong to the infinite cluster as determined according to the usual percolation criterion in one-component percolation P_∞^A and as calculated following a criterion adjusted to AB percolation P_∞ both for $f_b=0.5$ and $p_{\neq}=0$ in a 128^3 simple cubic lattice.

$=0$ we have $P_\infty \approx (C_\infty^b + C_\infty^b)/N^3$. In Fig. 3 we show the percolation probability as calculated according to the "one-component" criterion P_∞^A, and as determined following the "new" criterion adjusted to AB percolation, P_∞. All data shown in Figs. 1 and 2 have been calculated using the adjusted criterion. The main result, however, the shift in the percolation threshold p_+^c for either $p_==0$ or $p_{\neq}=0$, remains unaffected by the definition that is employed for the percolation probability P_∞, as can be seen in Fig. 3. The independent calculations for determination of the thresholds (see below) confirm this finding.

How does this shift of p_+^c in the limits $p_=\rightarrow 0$ and $p_{\neq} \rightarrow 0$ behave as a function of the blue species concentration f_b? Furthermore, at a given concentration f_b, how does the transition of p_+^c look like as a function of $p_=$ or $p_{\neq}$, when letting $p_{\neq} \rightarrow 0$ or $p_= \rightarrow 0$, respectively? To answer this question, we undertook simulations aimed at a determination of the critical value of the bond density only. The percolation threshold p_+^c was determined by a nested interval method.[9] We started with given values of the two control parameters $p_=$ and $p_{\neq}$, holding one of the two fixed and then approaching the critical value of the other from both sides of the threshold. We found that at fixed f_b the percolation threshold p_+^c, as a function of $p_=$ or $p_{\neq}$, continually changes to a new value for either $p_{\neq} \rightarrow 0$ or $p_= \rightarrow 0$. In Fig. 4(a) we show this continuous transition for the case of $f_b=0.5$ and $p_{\neq} \rightarrow 0$. We also show a fit to the curve, which is of the form

$$p_+^c(p_=, p_{\neq} \rightarrow 0) = \frac{1}{(u + v\, p_{\neq})} + t, \tag{11}$$

where the fit parameters were found to be $u=34.6 \pm 0.3$, $v=1823 \pm 72$, and $t=0.246 \pm 0.001$. It has to be noted that this formula only stands on empirical grounds, fits by exponential functions may also be useful. As stated earlier, the same line of thought is applicable to the $p_= \rightarrow 0$ limit and indeed do our simulations present exactly the same results in the system with $f_b=0.5$. For $f_b \neq 0.5$ this direct symmetry is broken (cf. Figs. 1 and 2), but qualitatively the results are still the same.
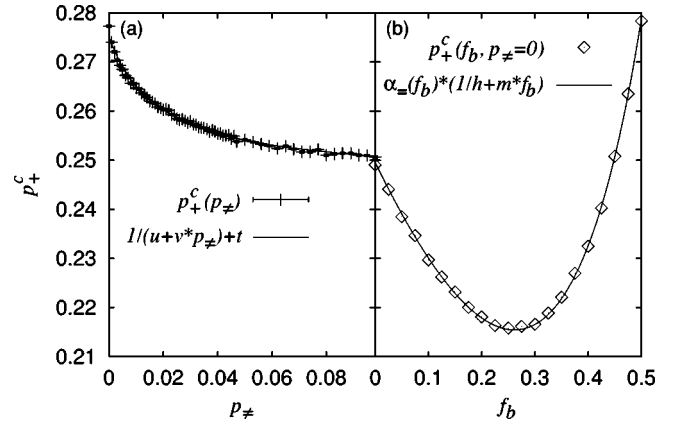

FIG. 4. Critical value p_+^c of the linear combination of the two control parameters $p_=$ and $p_{\neq}$ in a simple cubic lattice, (a) with $L=128$ and $f_b=0.5$, plotted as a function of $p_{\neq}$, and (b) plotted as a function of the fraction of blue sites f_b in the limit $p_{\neq}=0$ and obtained by a fit to the scaling law $|p_+^c(L) - p_+^c| \propto L^{-1/\nu}$. The errors, estimated as described in Ref. 17, are smaller than the symbol sizes in (b).

We now turn our attention to the question of how p_+^c behaves as a function of f_b in the limits $p_{\neq}=0$ and $p_==0$. We will first discuss results for the first case. For $p_+^c(f_b, p_{\neq}=0)$ we find the functional form depicted in Fig. 4(b). The data points shown in this phase diagram are results of fits of p_+^c to the scaling relation

$$|p_+^c(L) - p_+^c| \propto L^{-1/\nu} \tag{12}$$

as given in Ref. 9, where we kept ν fixed at 0.88 and lattice sizes $L=16$, $L=32$, $L=64$, and $L=128$ were taken into account. The simulation data are fitted with

$$p_+^c(f_b, p_{\neq}=0) = \frac{2\left(f_b - \frac{1}{2}\right)^2 + \frac{1}{2}}{h + m f_b}. \tag{13}$$

This is Eq. (8) with the purely empirical assumption of a hyperbola for $p_=^c(f_b)$. In Fig. 7(a) we show a plot of the differences between the values as predicted by Eq. (13) and the simulation results. The hyperbolic form of Eq. (13) is in agreement with the results of Heermann and Stauffer for a one-component site-bond model.[15,16] In their work, these authors investigated a generalized form of the Coniglio-Stanley-Klein[11] model, with Ising-correlated bonds on a simple cubic lattice. In this model, the temperature enters as additional parameter. Stauffer and Heermann found a phase diagram exhibiting hyperbolas for $p_=^c$ with a temperature-dependent critical concentration $f_c(T)$ entering the formula. Letting $T \rightarrow \infty$ reproduces the random percolation critical concentration f_c^{site}, which applies in our case. In this limit their formula reads

$$p_=^c = \frac{1}{h + m f_b}. \tag{14}$$

Transforming to p_+^c according to Eq. (8) by multiplication with $\alpha_=$ yields Eq. (13). In Eq. (14) it is
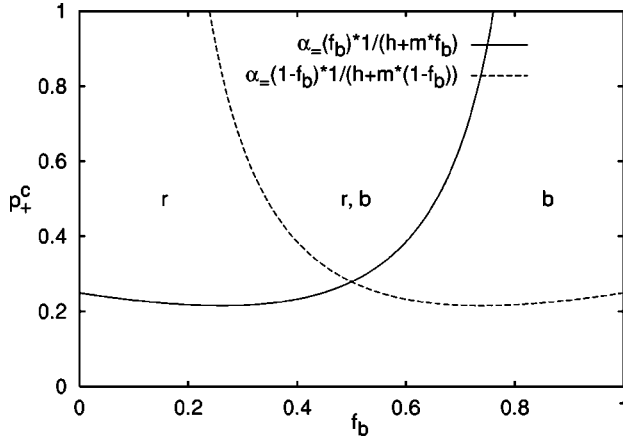
FIG. 5. Critical value p_+^c of the linear combination of the two control parameters $p_=$ and $p_{\neq}$ in a simple cubic lattice, as given by Eq. (13), plotted as a function of the fraction of blue sites f_b in the limit $p_{\neq}=0$. The different percolative phases are indicated by the minuscules, ''b'' referring to the blue and ''r'' to the red species. Below both curves, none of the two species can percolate.

$$h = \frac{1}{p_{bond}^c} = 4.019, \qquad (15)$$

and

$$m = \frac{1 - p_{bond}^c}{p_{bond}^c(f_{site}^c - 1)} = -4.386. \qquad (16)$$

Here, $p_{bond}^c = 0.2488$ and $f_{site}^c = 0.3116$ are the percolation thresholds for one component bond and site percolation on a three-dimensional simple cubic lattice, respectively. Fitting the parameters of Eq. (13) to our simulation data results in $h = 4.007 \pm 0.002$ and $m = -4.428 \pm 0.005$. However, in contrast to the one component site-bond percolation model of Heermann and Stauffer, due to the symmetry introduced in the system here, notably that $p_= \equiv p_{bb} = p_{rr}$, we only have $f_b \in [0.0, 0.5]$ as independent regime, with the interval $f_b \in [0.5, 1.0]$ being mirror symmetric to the one shown here with respect to the $f_b = 0.5$ line. This comes about since when varying f_b, the roles of blue and red sites switch at $f_b = 0.5$. Thus the whole phase diagram is characterized by two superposed scaled hyperbolas as given in Eq. (13), one taking f_b, the other $f_r = (f_b - 1)$ as argument. This fact is a manifestation of the two components behaving like two superposed, noninterfering one-component site-bond percolation systems and lets us conclude that the continuous transition in p_+^c for $p_{\neq} \rightarrow 0$ discussed above may be interpreted as an effective transition from a one-component bond percolation model to a one-component site-bond percolation model, since for $p_{\neq} \neq 0$ the system was characterized by a behavior similar to a one-component bond percolation model. We show the whole phase diagram in Fig. 5.

For the case of the second limit discussed, $p_= = 0$, the results are shown in Fig. 6. In Fig. 6, upper curve, we go back to the bond existence probability $p_{\neq}$. Its critical value, as a function of the concentration, $p_{\neq}^c(f_b, p_= = 0)$, is well reproduced with an exponential fit:
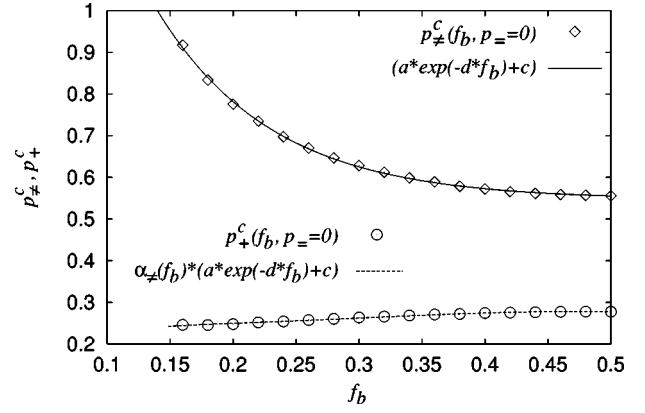

FIG. 6. Critical value of $p_{\neq}$ and of p_+, plotted as a function of the fraction of blue sites f_b in the limit $p_= = 0$. The data have been obtained by a fit to $|p_+^c(L) - p_+^c| \propto L^{-1/\nu}$. Again the errors, estimated as described in Ref. 17, are smaller than the symbol sizes.

$$\varphi = p_{\neq}^c(f_b, p_= = 0) = a \exp(-d f_b) + c. \qquad (17)$$

Fitting this empirical formula to the simulation data gives $a = 2.1 \pm 0.07$, $d = 10.9 \pm 0.2$, and $c = 0.547 \pm 0.002$. An interesting interpretation is possible for $p_{\neq} = 1$ and the component concentration f_b as free parameter. We define $f_b^c \equiv \varphi^{-1}(p_{\neq} = 1)$ and numerically get $f_b^c \simeq 0.14$ from Eq. (17), whereas in an independent simulation aimed at finding the
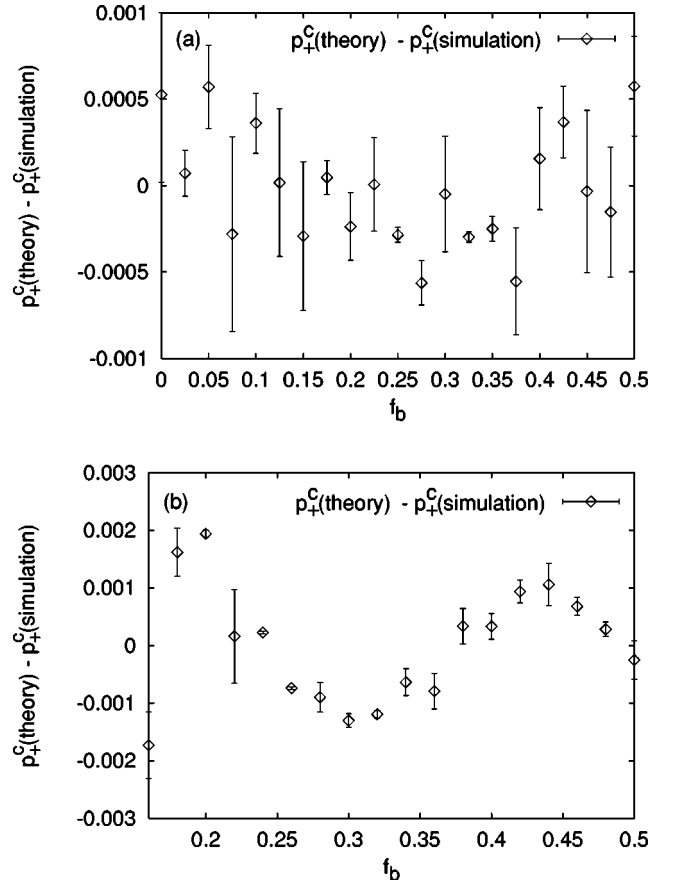

FIG. 7. Differences between the critical values of p_+^c, (a) as predicted by Eq. (13) and as obtained in the simulations for $p_{\neq} = 0$ and (b) as predicted by Eq. (17) and as obtained in the simulations for $p_= = 0$.

critical value of the component concentration, we find f_b^c $=0.145\pm0.001$, which again is a result from a fit of values for the four different lattice sizes to the scaling relation Eq. (12). The parameter f_b^c can be regarded as a different threshold in a model which we would like to refer to as the inverse site percolation model, corresponding to a simple two-component site percolation model in which nearest neighbors only belong to the same cluster if they are of opposite flavor, unlike the normal site percolation model which yields f_{site}^c $=0.3116$. In Fig. 6, lower curve, f_b^c determines the critical point of the phase transition line $p_+^c(f_b,p_=0)$. In Fig. 7(b) we also show a plot of the differences between the values of p_+^c as predicted by Eq. (17) and the simulation results. Another argument in favor of a qualitatively different behavior arises by considering the density of accessible edges at the critical concentrations in the two models. For the one considered here it is given by $\alpha_{\neq}(f_b^c)$, see Eq. (9), for the usual site percolation model we might define one in an analogous manner: $\alpha_{site}=(f_{site}^c)^2$. By setting $\alpha_{\neq}(f_b^c)=\alpha_{site}$ one would expect f_b^c to be 0.051 which stands in contradiction to our findings.

Concluding, we introduced a different way to treat N-component percolation. This approach was applied to a two-component site-bond percolation model. Different aspects of the percolative phase behavior of this specific model were investigated using computer simulations. We could reaffirm the findings of Ref. 13 which state that for both active bond types, the phase behavior of the usual one-component bond percolation model and the AB site-bond percolation model coincide. However, a different behavior was found in the case where only one type of bond was kept active in the lattice, which means $p_{\neq}\to0$ or $p_=\to0$. In this limit, the percolation thresholds show a continuous change to different, concentration dependent values (see Fig. 4), with the critical exponents being retained. For $p_==0$ we could establish an empirical formula for the percolation threshold p_+^c as a function of component concentration f_b whereas for $p_{\neq}=0$ earlier findings of a one-component site-bond percolation model were found to apply in the two-component model too. This observation led to the conclusion that the transition observed in p_+^c for $p_{\neq}\to0$ of the two-component site-bond percolation model can be interpreted as a transition from a one-component bond percolation model to a one component site-bond percolation model. The results presented in the case of $p_=\to0$ led to the introduction of an inverse site percolation model for which the site percolation threshold f_b^c could be determined.

In the future one might try to apply the same methods to multicomponent systems. This approach might also find a broad range of possible applications. For example, one may think of special networks or gelation phenomena with several components involved, which only interact with each other, as well as wetting phenomena. Furthermore, an application to stock-market simulations seems possible and is being undertaken by the authors.

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