

## Nuclear flow excitation function

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We consider the dependence of collective flow on the nuclear surface thickness in a Boltzmann-Uehling-Uhlenbeck transport model of heavy ion collisions. Well-defined surfaces are introduced by giving test particles a Gaussian density profile of constant width. Zeros of the flow excitation function are as much influenced by the surface thickness as the nuclear equation of state, and the dependence of this effect is understood in terms of a simple potential scattering model. Realistic calculations must also take into account medium effects for the nucleon-nucleon cross section, and impact parameter averaging. We find that balance energy scales with the mass number as  $A^{-\nu}$ , where  $\nu$  has a numerical value between 0.35 and 0.5, depending on the assumptions about the in-medium nucleon-nucleon cross section.

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### I. INTRODUCTION

Broadly speaking, nuclear collective flow in a heavy ion collision is the deflection of nuclear matter perpendicular to the beam axis during the course of the reaction. Experimentally, one observes that flow disappears at a well-defined beam energy [1–5], the so called balance energy  $E_{\text{Bal}}$ , whose value depends on the system and impact parameter range being considered. These zeros in the flow excitation function were predicted by the Boltzmann-Uehling-Uhlenbeck (BUU) transport model [6, 7], and an analysis of scale invariant quantities [8], and may be understood as an overall cancellation of the attractive part of the mean field interaction with repulsive contributions from the mean field and collisional kinetic pressure.

Thus, as has been shown explicitly for BUU simulations in Ref. [3],  $E_{\text{Bal}}$  is expected to depend on both the nuclear equation of state and the magnitude of the in-medium nucleon-nucleon cross section. By making a systematic study of the balance energy as a function of the nuclear mass, one therefore hopes to gain insight into these properties. However, other parameters might well influence the balance energy. In particular, we wish to investigate in this note the effect of finite nuclear surface thicknesses and impact parameter variations on  $E_{\text{Bal}}$ .

We begin by defining the flow variable to be used here, and point out the importance of obtaining well-defined nuclear surfaces that are independent of the grid size used to compute density gradients. We then show how the strong surface dependence of flow in a Vlasov simulation may be understood in terms of a simple potential scattering model. This dependence persists for full BUU calculations that include a nonzero collision integral. Last, we

consider the effect on the balance energy when the impact parameter, the nucleon-nucleon cross section, and the mass number are varied.

### II. SURFACE DEPENDENCE OF FLOW

To analyze flow quantitatively in experiments one of the main problems that has to be addressed is the determination of the reaction plane (see, for example, Refs. [9–11]). In model calculations, on the other hand, knowledge of the reaction plane immediately allows one to define a flow variable such as the average in-plane transverse momentum

$$\langle wP_x \rangle = \frac{1}{N} \sum_{i=1}^N w_i P_i^x, \quad (1)$$

where the weight  $w_i = 1$  or  $-1$  if the test particle  $i$  is emitted into the forward or backward center-of-mass hemispheres, respectively, and  $P_i^x$  is the transverse momentum of the test particle in the reaction plane. In this note, we shall refer to “flow” in the sense of the above equation.

We begin by examining the way the local particle density  $\rho$  is calculated for the mean field dynamics in a Vlasov simulation. While the mean field is momentum dependent [12–14], we shall consider here, for illustrative purposes, a Skyrme-like parametrization that is a function of the density alone:

$$U(\rho) = A \frac{\rho}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^\sigma, \quad (2)$$

and take values of the parameters  $A = -124$  MeV,  $B = 70$  MeV, and  $\sigma = 2$ . This choice reproduces known

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nuclear matter properties, with a rather stiff compression modulus at saturation of  $K_0 = 380$  MeV.

We choose to represent the nucleon phase space distribution function  $f(\mathbf{r}, \mathbf{p}, t)$  by an ensemble of test particles. If  $f(\mathbf{r}, \mathbf{p}, t)$  is to satisfy the Vlasov equation, the equations of motion of a test particle with coordinates  $(\mathbf{r}_i, \mathbf{p}_i)$  are given by Hamilton's equations of motion with potential (2):

$$\dot{\mathbf{p}}_i = -\nabla_r U(\rho(\mathbf{r}_i)) \text{ and } \dot{\mathbf{r}}_i = \frac{\mathbf{p}_i}{\sqrt{m_N^2 + p_i^2}}, \quad (3)$$

where  $m_N$  is the free nucleon mass. The local particle density  $\rho$  is often calculated on a grid, and the gradient obtained as a finite difference. In this procedure, each test particle counts a certain fraction towards the density of the cell it occupies, while neighboring cells receive a smaller contribution [15]. This stabilizes the numerics, but also introduces a nuclear surface whose thickness is roughly given by the grid size  $\Delta x$ . This grid size must be larger than the maximum distance traversed by a test particle in one time step, but small enough to be able to compute the gradients in Eq. (3) to sufficient accuracy.

Instead of "smearing" a test particle in steps over only the nearest neighboring cells [16], one may choose to supply each test particle with, say, a Gaussian density profile of constant width. This also introduces a finite surface thickness, but one that is well-defined and independent of the grid size. Thus one may study the effect of varying the nuclear surface thickness without the external numerical constraints imposed on the choice of  $\Delta x$ .

As an illustrative example, we consider flow in pure Vlasov dynamics as a function of the size of the grid for  $^{139}\text{La}$  on  $^{139}\text{La}$  collisions at a beam energy of 200 MeV/nucleon and impact parameter  $b = 2.7$  fm. Figure 1 shows that  $\langle wP_x \rangle$  depends very strongly on the grid size, i.e., the effective nuclear surface (dotted line). The calculations with finite  $\Delta x > 0.5$  fm gave an overall attraction, whereas the extrapolation to  $\Delta x = 0$  predicts zero flow. This strong dependence persists at beam energies of 800 MeV/nucleon. At this energy, the extrapolated value to zero grid size (zero surface thickness) is twice as large as the value at  $\Delta x = 1$  fm, a frequently used grid size.

In Fig. 1, we also show the flow obtained with Gaussian test particle density profiles. The solid line represents the variation with  $\Delta x$  for a surface thickness of 2 fm, while the dashed line is for a surface of 1 fm. Clearly, as long as the grid size is smaller than the Gaussian smearing width (i.e., the nuclear surface), the flow is independent of  $\Delta x$ . Also, the dotted line crosses the solid and dashed lines at  $\Delta x \sim 2$  and 1 fm, respectively. This indicates that at least most of its rise with decreasing grid size is directly attributable to the changing surface thickness, and is not a "numerical artifact." We conclude that  $\langle wP_x \rangle$  depends rather strongly on the surface thickness, a thinner surface producing more than a thicker one. Of course, quantitatively the results obtained here are not reliable, since we have, for instance, ignored the momentum dependence and hard collisions, but they do show the important influence of the nuclear surface.

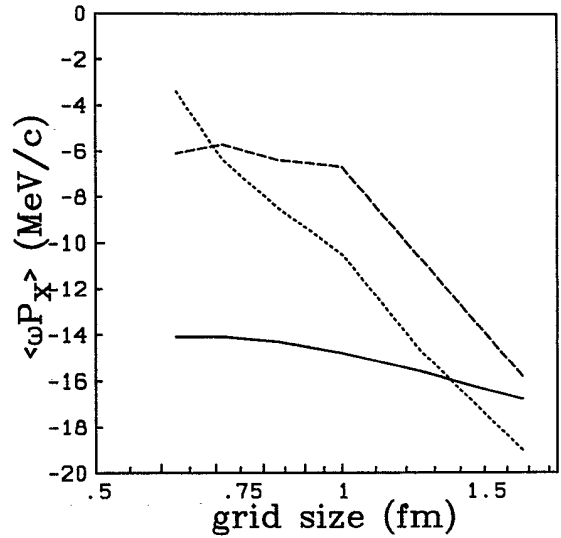


FIG. 1. Flow [as defined in Eq. (1)] versus grid size for pure Vlasov dynamics with smearing over neighboring cells only (dotted line). The dashed and solid lines show results if each test particle is given a Gaussian density profile of constant width. The dashed line corresponds to a nuclear surface thickness of 1 fm, the solid line to 2 fm. All curves shown are for  $^{139}\text{La} + \text{La}$ ,  $E = 200$  MeV/nucleon, and  $b = 2.7$  fm.

### III. AN INSTRUCTIVE MODEL

The surface dependence of flow can be understood in terms of a simple potential scattering model [14]. Assuming that the nuclei pass through each other without changing their shape in phase space, the centers of mass of the nuclei move according to the Hamilton function

$$H = \frac{P_1^2}{2M} + \frac{P_2^2}{2M} + \tilde{V}(R), \quad (4)$$

where the potential  $\tilde{V}$  is given by

$$\tilde{V}(R) = \int V(\rho_R(r)) dr^3, \quad (5)$$

$M$  is the nuclear mass,  $P_1$  and  $P_2$  the center-of-mass momenta, and  $R$  the separation of the nuclei. In (5),  $V(\rho_R)$  is the potential energy density corresponding to Eq. (2)

$$V(\rho) = \frac{A \rho^2}{2 \rho_0} + \frac{B}{\sigma + 1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma}. \quad (6)$$

To describe nuclei with a surface, we choose the density profile to be

$$\frac{\rho_R(\mathbf{r})}{\rho_0} = [1 + \exp(|\mathbf{r} - \mathbf{R}/2| - R_0)/a)]^{-1} \\ + [1 + \exp(|\mathbf{r} + \mathbf{R}/2| - R_0)/a)]^{-1}, \quad (7)$$

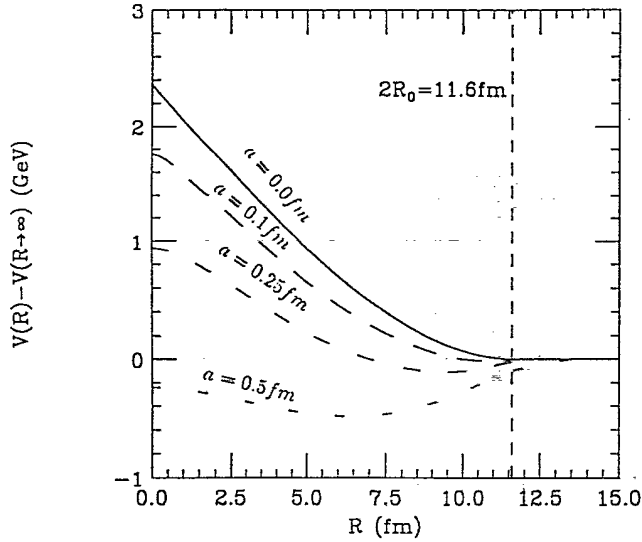


FIG. 2. Potential Eq. (5) as a function of the separation of the two nuclei. The radius  $R_0 = 5.8$  fm corresponds to  $^{139}\text{La}$ .

where  $R_0$  is the nuclear radius, and  $4a$  the surface thickness.

The potential (6) is shown in Fig. 2 for various values of the parameter  $a$ . As expected, for an increasing surface thickness, the potential  $\tilde{V}$  decreases, and we expect a larger surface to produce less flow. This can be shown explicitly in the time evolution of the  $\langle wP_x \rangle$  (see Fig. 3). We find reasonable agreement of the potential scattering model with the test particle Vlasov calculation at both values of the surface thickness. Of course, differences are seen in the details, and are expected because of the crude assumptions made in Eq. (4). For example, a Vlasov calculation shows that  $E_{\text{Bal}}$  occurs between 200 MeV and 300 MeV, depending on the surface, while the scattering model shows no zeros in the flow excitation function.

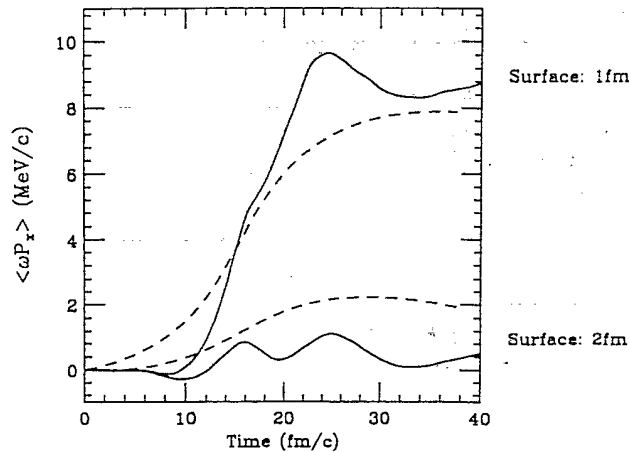


FIG. 3. The time evolution of flow for pure Vlasov dynamics (solid line), and the potential scattering model, Eq. (4) (dashes). The surface thicknesses are 1 fm (upper two curves) and 2 fm (lower two curves), and the system is as for Fig. 1.

#### IV. REALISTIC CALCULATIONS

In the previous sections, we have studied the surface dependence of the flow variable under the very clean conditions of pure Vlasov dynamics. We now focus on the disappearance of flow in a more realistic calculation, i.e., a full BUU simulation that includes collisions. First we want to look at surface effects in  $^{139}\text{La}$  on  $^{139}\text{La}$  and  $^{12}\text{C} + ^{12}\text{C}$ , and at flow as a function of the impact parameter for  $\text{Cl} + \text{Cl}$ . In the next section, we will present a study of the balance energy for nuclei ranging from a combined mass of  $A = 24$  to  $A = 302$ , including an estimate of medium effects on the cross section.

For the reaction  $^{139}\text{La}$  on  $^{139}\text{La}$  [1] we find that the balance energy is shifted by  $\approx 10$  MeV when the surface thickness increases from 1 fm to 2 fm. This is comparable to the shift expected when one changes from a stiff to a soft equation of state. For example, in  $^{40}\text{Ar}$  on  $\text{V}$  reactions,  $E_{\text{Bal}}$  changes by only 8 MeV if the incompressibility is increased from  $K_0 = 200$  MeV to 380 MeV by adjusting the parameters in Eq. (2) [3].

This becomes even more apparent in smaller systems such as  $^{12}\text{C} + ^{12}\text{C}$ , for which values of  $E_{\text{Bal}}$  were recently measured at the National Superconducting Cyclotron Laboratory [17]. Figure 4 shows the flow obtained in simulations as a function of beam energy, for surface thicknesses of 1 fm (left panel) and 2 fm (right panel). The stiff EOS gives  $E_{\text{Bal}} \approx 140$  MeV and 190 MeV, respectively, while the soft EOS yields  $E_{\text{Bal}} \approx 170$  MeV and 220 MeV, respectively [18].

The surface thickness dependence was already discussed in the preceding section. The flow excitation function in pure collisionless Vlasov dynamics shows a zero at about 250 MeV, depending on the surface thickness. The collisions merely add a repulsive component to the forces acting on the nucleons hence adding a negative term to the flow excitation function and shifting the zero to very much lower values. However, the importance of the surface in the precise value of the balance energy is main-

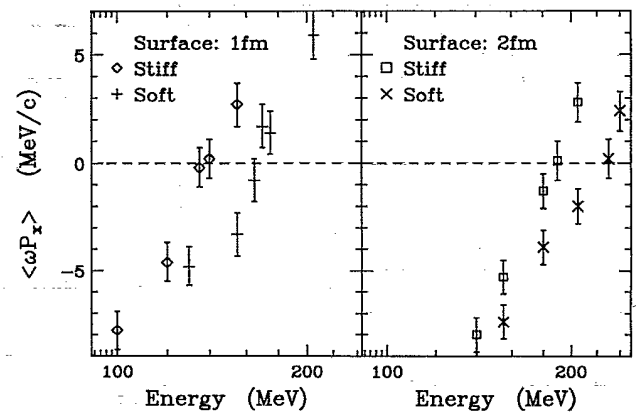


FIG. 4. Flow versus beam energy in the vicinity of the zeros of the excitation function. The system is  $^{12}\text{C} + ^{12}\text{C}$  at an impact parameter  $b = 1.4$  fm. The left panel is for a surface thickness of 1 fm, the right for 2 fm. Both panels show results for stiff (diamonds) and soft equations of state (plus signs).

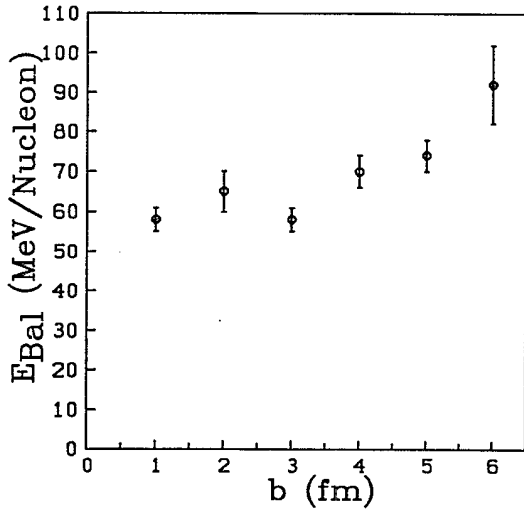


FIG. 5. The balance energy for Cl + Cl as a function of impact parameter.

tained. Since the collision-induced repulsion is weaker in smaller systems, the relative importance of the surface thickness is enhanced in smaller systems.

In addition to effects mentioned previously,  $E_{\text{Bal}}$  also depends on the impact parameter  $b$  [19], as can be seen in Fig. 5. For peripheral collisions, the contribution of the nucleon-nucleon collisions becomes relatively less important for the flow production than the mean field, because the repulsion generated by the nucleon-nucleon collisions is proportional to the overlap volume. Of course, for a quantitative comparison with experiment an impact-parameter-weighted averaged flow has to be considered. However, calculating  $E_{\text{Bal}}$  for several  $A$  and  $b$  is computationally prohibitive. In addition, the precise distribution of impact parameters contributing to the data collected with different experimental triggers is not known. And so one usually performs the calculations at the average impact parameter determined from the experimental trigger conditions. We also follow this approach.

## V. MASS DEPENDENCE OF THE BALANCE ENERGY

In this section we conduct a systematic study of  $E_{\text{Bal}}$  as a function of mass  $A$ . Table I gives an overview of the

TABLE I. Performed systematic calculations.

System	Combined mass	Range of energies (MeV)
C+C	24	50 - 240
O+O	32	50 - 200
Ne + Ne	40	50 - 200
Al + Al	54	50 - 200
Cl + Cl	74	50 - 200
Ca + Ca	96	50 - 200
Zn + Zn	132	50 - 150
Y + Y	178	45 - 150
Ag + Ag	214	45 - 150
Eu + Eu	302	40 - 150

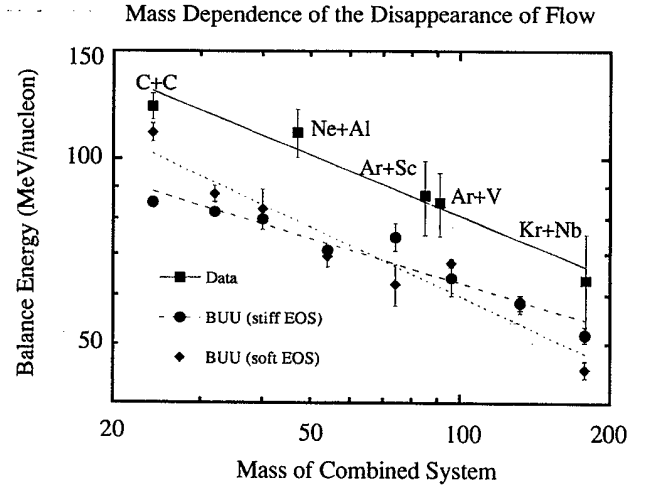


FIG. 6. The calculated values of  $E_{\text{Bal}}$  as a function of the mass of the system. Only symmetric systems are considered. Diamonds and circles correspond to a soft and stiff EOS, respectively. For comparison, experimental data from Ref. [5] (squares) are shown. The solid and dashed curves are power-law fits to the calculations and data.

calculations performed. For every system, flow was determined for five to ten beam energies within the ranges given in the table, for a surface thickness of 1 fm and 2 fm, and also for a range of in-medium nucleon-nucleon cross sections. We note that the results in Figs. 6 and 7 use only one impact parameter, obtained from the average value in the experiment, and proportionally scaled  $b$  for calculations at mass numbers  $A$  for which no data exists.

Figure 6 shows the balance energy as a function of the combined mass of the system. Experimental data (squares and diamond) suggests that the dependence of the balance energy on the combined mass,  $A$ , of the colliding nuclei follows a power law,

$$E_{\text{Bal}} = xA^{-y}, \quad (8)$$

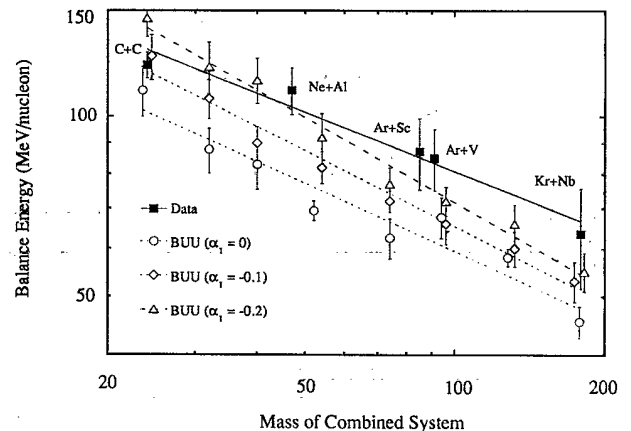


FIG. 7. Same as Fig. 6, but varying the in-medium cross section according to Eq. (9) with  $\alpha_1$  defined in Eq. (10). For all calculations, a soft equation of state was used. The lines represent power-law fits to the calculations and data.

where the exponent  $y$  has a numerical value of  $0.33 \pm 0.04$ . This power law dependence is reproduced by the theoretical calculations (triangles and circles correspond to a soft and stiff EOS respectively) [20]. In the present calculation, we have parametrized the nucleon-nucleon cross sections in terms of a least squares fit to the experimental data of Ref. [21]. The resulting values are somewhat different from the isospin averaged expressions used at higher beam energies. In particular,  $\sigma_{pp} \neq \sigma_{pn}$ , and the cross sections are larger than the ones described in Ref. [15]. This results in a shift of the balance energy to lower values.

In Fig. 7, we investigate the dependence of the balance energy on the value of the in-medium nucleon-nucleon cross section, where we look for medium corrections beyond the effect of the Pauli principle on the outgoing scattering states. In previous studies the free nucleon-nucleon cross section was multiplied with an overall constant scaling factor [3,7,22]. However, this approach fails when one has collisions in low-density nuclear matter, where the in-medium cross section should approach its free-space value. A more realistic approach uses a Taylor expansion of the in-medium cross section in the density variable:

$$\begin{aligned} \sigma_{NN}(\sqrt{s}, \rho) &= \sigma_{NN}(\sqrt{s}, 0) + \rho \left. \frac{\partial \sigma_{NN}(\sqrt{s}, \rho)}{\partial \rho} \right|_{\rho=0} + \dots \\ &= \left( 1 + \alpha_1 \frac{\rho}{\rho_0} + \dots \right) \sigma_{NN}(\sqrt{s}, 0) \end{aligned} \quad (9)$$

where we have introduced the dimensionless parameter  $\alpha_1$ , given by

$$\alpha_1 = \rho_0 \left. \frac{\partial}{\partial \rho} \{ \ln \sigma_{NN}(\sqrt{s}, \rho) \} \right|_{\rho=0}. \quad (10)$$

In principle,  $\alpha_1$  is dependent on  $\sqrt{s}$ , but we have here — as a first approximation — taken  $\alpha_1$  as an energy-independent constant. Figure 7 shows the mass dependence of the calculated balance energy as a function of different values of  $\alpha_1$ , where we have used a soft nuclear equation of state. It is clear that we obtain the best overall agreement with the experimental data for a value of  $\alpha_1 = -0.2$ , corresponding to a 20% reduction of the nucleon-nucleon cross section at  $\rho = \rho_0$ . However, the power-law exponent  $y$  from Eq. (8) depends on the choice of  $\alpha_1$ , with  $\alpha_1 = 0$  (no medium modification) yielding the best agreement ( $y = 0.38 \pm 0.05$ ) with the experimental value of  $y = 0.33$ . For  $\alpha_1 = -0.1$  one extracts  $y = 0.42 \pm 0.03$ , and for  $\alpha_1 = -0.2$  we obtain  $y = 0.47 \pm 0.03$ . That  $y$  increases with  $|\alpha_1|$  is due to the fact that at higher beam energies higher densities are reached, and therefore the reduction of the in-medium cross section is stronger for the lighter systems, which have higher balance energies. This effect may, however, be at least partially compensated once one incorporates

a more realistic energy dependence of  $\alpha_1$ .

Of course, the use of momentum-dependent mean fields will also influence the balance energy. However, we find that momentum-dependent mean fields give results that are only weakly dependent on  $A$ . For example for carbon on carbon we get a balance energy of about 60 to 80 MeV and for La+La a value of 50 MeV. The La+La calculation agrees well with the one published by Krofcheck *et al.* [1]. This shortcoming of the currently used parametrization of the momentum dependence of the nuclear mean field needs a separate detailed analysis and will be the subject of future work.

## VI. CONCLUSIONS

In conclusion, we have shown that in Vlasov and BUU simulations reliable results for nuclear collective flow must take into account the finite thickness of the nuclear surface, a conclusion also reached by Koch *et al.* [16]. This is best done by giving the test particles a Gaussian density profile, with a width that is larger than the grid size used to obtain the density gradients. The value of the nuclear surface thickness has a strong effect on the balance energy. Therefore, more quantitative BUU predictions have to not only take into account the equation of state and medium effects on the nucleon-nucleon cross section, but also proper initial conditions in phase space and impact parameter averaging. In addition, we find that a realistic variation of the in-medium nucleon-nucleon cross-section with density has a clear effect on the mass dependence of the balance energy. We find the best overall agreement with the experimental data for  $\alpha_1 = -0.2$ , where  $\alpha_1$  is defined in Eq. (10).

From our studies it seems apparent that one should be able to extract the information on the nuclear compressibility and information on the in-medium nucleon-nucleon cross sections from a careful comparison of the experimental mass dependence of the balance energy, i.e., the zero in the nuclear collective flow excitation function, to theoretical predictions. To accomplish this we have to, however, obtain better parametrizations of the momentum dependence of the nuclear mean field than the ones in current use. This method of extraction of information on the nuclear equation of state may then rival other complimentary methods such as the investigation of the monopole data [23] or astrophysical evidences [24, 25]

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