ELSEVIER

Nuclear Physics A583 (1995) 93–100

NUCLEAR PHYSICS A

Regular and Chaotic Dynamics in Giant Nuclear Oscillations

Wolfgang Bauer,[a] David McGrew,[a] Vladimir Zelevinsky[a*] and Peter Schuck[b]

[a]National Superconducting Cyclotron Laboratory
and Department of Physics and Astronomy
Michigan State University
East Lansing, Michigan 48824-1321, USA

[b]Institut de Physique Nucléaire
Université de Grenoble
53 avenue des Martyrs, 38026 Grenoble Cedex, France

We study the problem of giant nuclear oscillations by performing self-consistent calculations in semiclassical approximation utilizing a multipole-multipole interaction of the Bohr-Mottelson type for quadrupole and octupole deformations. In all cases considered, we find regular motion of the collective coordinate, the multipole moment of deformation. This is in contradiction to the predictions of the wall formula and suggests that this type of one-body dissipation might not be realized in real nuclear systems. In addition, we find chaotic single particle motion in coexistence with the regular collective dynamics.

1. INTRODUCTION

What causes dissipation in the collective motion of finite Fermi systems? This question is relevant in the physics of atomic nuclei [1] or small metallic clusters and is up to now not completely satisfactorily solved. The contribution of one- and two-body processes to dissipation in nuclei is still a question of debate. Two-body dissipation is caused by collisions of pairs of nucleons and strongly suppressed at low excitation energies due to the action of the Pauli exclusion principle. For one body dissipation there are in principle two leading candidates. One is the escape of nucleons from the collective potential well into the continuum. And the other is collisions of individual nucleons with the nuclear surface ('wall'), generated collectively by the mean field of all nucleons. This leads to pseudo-random motion of the nucleons (heating) and causes the loss of collective energy [2,3]. The fact that deformed static nuclear potentials may exhibit chaotic motion was recognized early by Arvieu and co-workers [4].

In a recent study, Blocki *et al.* [3] consider a purely classical gas of particles contained in a deformed billiard undergoing periodic shape oscillations of frequencies much smaller than a typical single particle frequency. They study the single particle kinetic energy as a function of time and find that for ellipsoidal shape deformations ($\ell = 2$) the net total kinetic energy increase over an entire shape oscillation period is 0. However, for

*On leave from: Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia.

SSDI 0375-9474(94)00637-7

$\ell \geq 3$ the kinetic energy in the single particle motion is not completely 'given back', but rather steadily increases in time, thus indicating dissipation of collective energy in this case. This is explained by the fact that in an $\ell = 2$ potential the motion of the particles remains non-chaotic and therefore synchronized, whereas in the $\ell \geq 3$ the scattering of the segments of the wall with positive curvature leads to chaotic motion similar to the one observed in a Sinai [5,6] billiard and thus a destruction of synchronization. This scenario is very similar to the so-called Fermi acceleration, proposed to explain the occurrence of very high energy cosmic radiation [7,8].

In this paper we present an attempt to include *selfconsistency* into the problem of motion in multipole-deformed nuclear potentials. More details can be found in ref. [9]

2. A SELFCONSISTENT MODEL

We have chosen a selfconsistent, but schematic, model of separable forces. We chose an interaction of the Bohr-Mottelson type [10] with static r^2 and r^6 potentials and multipole-multipole interactions as studied, for example, by Stringari *et al.* [11–13]. In the semi-classical small amplitude limit, this model has recently been investigated [13], and the low-lying quadrupole and octupole frequencies come out to be in reasonable agreement with experimental data. Our single-particle Hamiltonian is

$$\mathcal{H} = \mathcal{H}_0 + V^{(\ell)}(\boldsymbol{r}, t) = \frac{p^2}{2m} + V_0 + V^{(\ell)}(\boldsymbol{r}, t) \tag{1}$$

$V^{(\ell)}(\boldsymbol{r}, t)$ is the potential associated with the (separable) multipole-multipole force [11,13]

$$V^{(\ell)}(\boldsymbol{r}, t) = \mu_\ell \, q_\ell(\boldsymbol{r}) \, Q_\ell(t) \ , \tag{2}$$

and V_0 is the static external potential. We take here $V_0 = \frac{1}{2} m \omega_0^2 r^2$, resulting in the Bohr-Mottelson Hamiltonian [10], and $V_0 = \frac{1}{2} m \omega_0^6 r^6$, to also investigate anharmonic static potentials.

For the r^2 static potential the coupling constants μ_ℓ can be calculated using a selfconsistent normalization condition [10,13]. $q_\ell(\boldsymbol{r})$ is given by $q_2(\boldsymbol{r}) = r_y r_z$ (quadrupole) or $q_3(\boldsymbol{r}) = r_x r_y r_z$ (octupole), and the multipole moments $Q_\ell(t)$ are selfconsistently calculated via

$$Q_\ell(t) = \int \frac{d^3r \, d^3p}{(2\pi)^3} q_\ell(\boldsymbol{r}) f(\boldsymbol{r}, \boldsymbol{p}, t) \ , \tag{3}$$

where $f(\boldsymbol{r}, \boldsymbol{p}, t)$ is the one-body phase space distribution function of nucleons, the Wigner transform of the one-body density.

We treat this problem in semi-classical approximation by a Wigner transformation of the von Neumann equation of motion for the density matrix, $i\,\partial_t \rho = [\mathcal{H}, \rho]$, to obtain a Vlasov equation, $\partial_t f = \{\mathcal{H}, f\}$. We then solve the Vlasov equation in the test particle method [14,15] using a fourth-order Runge-Kutta algorithm with typical time step sizes of 1 fm/c. Our numerical simulation is fully selfconsistent and conserves total energy to better than 0.1%.

3. INITIAL CONDITIONS

In order to generate selfconsistent initial deformations in coordinate and momentum space, we start with a spherically symmetric configuration generated in local Thomas-Fermi approximation without the deformation potential $V^{(\ell)}$. We then apply a time-dependent external potential of the form

$$V_0^{(\kappa)}(\ell, \boldsymbol{r}, t) = \kappa_0 \sin(\omega_D t)\, s(t)\, q_\ell(\boldsymbol{r}) \tag{4}$$

where ω_D is the driving frequency, and where $s(t)$ is a differentiable spline interpolation function on the time interval $[0,\tau]$ with vanishing first derivatives at both ends, which is monotonically increasing from 0 to 1. This procedure results in a giant oscillation of the nucleus at $t = \tau$, provided that τ is chosen $\tau \gg \omega_D^{-1}$. The deformation is dependent on the value of the coupling κ_0 chosen.

Figure 1 shows, as an example, the time evolution of the quadrupole moment in coordinate (Q_2) and momentum space (P_2) resulting from this procedure.

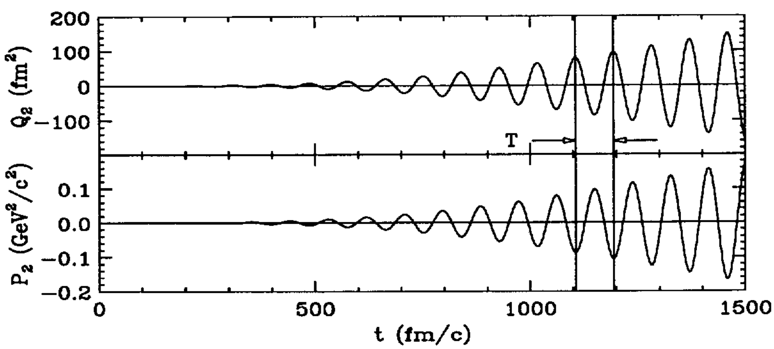

Figure 1. Generation of a selfconsistent giant quadrupole oscillation

4. TIME EVOLUTION OF THE COLLECTIVE COORDINATES

We now use the initial conditions generated in the way described in the last section (with $\tau = 1500$ fm/c) to study the time evolution under the action of our Hamiltonian as defined in Eq. (1). The four panels of Figure 2 contain the results of our calculations. The two upper panels show giant quadrupole oscillations, and the two lower panels show giant octupole oscillations. The two left panels are generated with an r^2, and the two right panels with an r^6 static potential. In all four cases, one can clearly observe a regular undamped oscillation of the multipole moment in coordinate space as a function of time. For the octupole oscillation, our result is in contradiction to the wall-formula prediction of a strongly damped oscillation.

Fig. 2 indicates that no chaoticity is present in the collective multipole coordinates. We have also used slightly different initial conditions (by using a different number of test particles in the simulation) and obtained only slightly different results. This means that there is no sensitive dependence on the initial conditions present here as would be the case for chaotic motion. As an additional test, we performed a Fourier transform of the time signals ($Q_2(t)$ and $Q_3(t)$) and found one peak at the dominant frequency and no ω^{-1} noise. This result is surprising, because the collective multipole coordinates are coupled to and generated (see eq. 3) from the integration over the single-particle coordinates, which in most cases exhibit chaotic motion – as shown below.

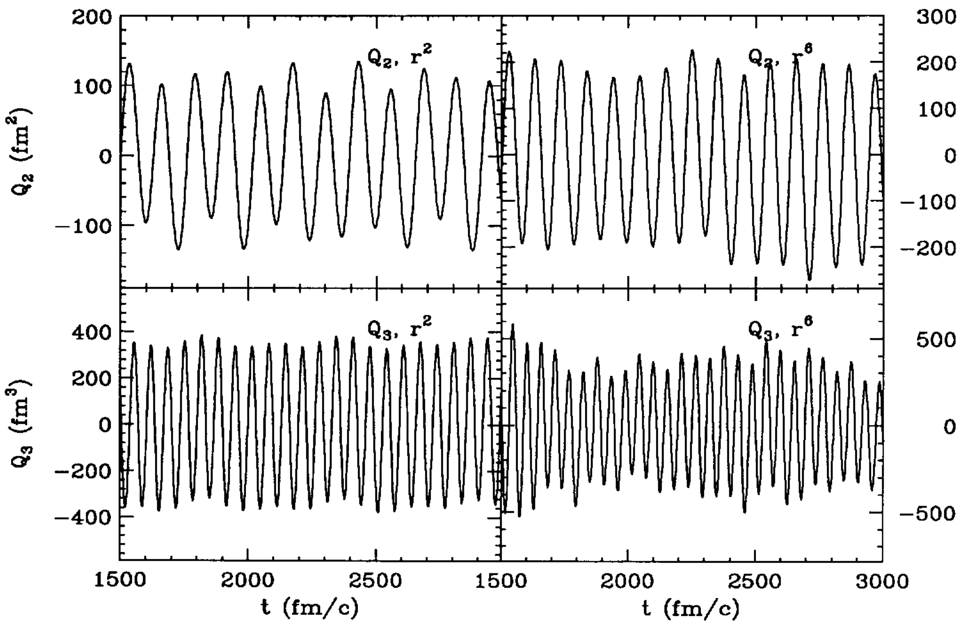

Figure 2. Time oscillations of collective multipole moments under the action of the Hamiltonian of Eq. (1).

Examining the upper left panel, one further sees that the period of oscillation has been stretched from the 0-coupling value of 88.4 fm/c ($= T_{2,0} = 2\pi/(2\omega_0)$) to 128 fm/c. This is consistent with the analytic calculations for infinitesimal deformations [16,10,13], which yield

$$\begin{aligned} Q_2(t) &= Q_2(t_0)\,\exp(i\,\omega_{2+}\,t) \\ \omega_{2+} &= \sqrt{4\omega_0^2 + \mu_2 \frac{2\,A\,\langle r^2\rangle}{3m}} = \sqrt{2}\,\omega_0 \end{aligned} \qquad (5)$$

for the giant quadrupole frequency, and consequently $T_2 \approx 125$ fm/c for $\omega_0 = 0.0355$ c/fm used in this example.

In the lower left panel of Figure 2, the observed oscillation period is $T_3 \approx 66$ fm/c, in good agreement with the analytical result of [13] $\omega_{3-} = \sqrt{7}\,\omega_0$ for the giant octupole oscillation frequency, which results in $T_3 \approx 66.8$ fm/c for our value of ω_0.

5. SINGLE PARTICLE MOTION

To analyze the single particle motion in our 6-dimensional single particle phase space we determine the set of 6 Lyapunov exponents by observing the long-term evolution of an infinitesimal 6 sphere around the particle. Since we are dealing with a Hamiltonian system the volume of this sphere - averaged over all particles - will not change. However, it may deform into a 6-ellipsoid. The i^{th} Lyapunov exponent is then defined as

$$\lambda_i = \left\langle \lim_{t\to\infty} \left\{ t^{-1} \log_2 \frac{\ell_i(t)}{\ell_i(0)} \right\} \right\rangle_A , \tag{6}$$

where $\ell_i(t)$ is the length of the i^{th} principal axis of the ellipsoid and the symbol $\langle ... \rangle_A$ indicates averaging over all single particles. In our calculation we use the standard numerical techniques of continuous rescaling and Gram-Schmidt reorthonormalization to extract the numerical values of the exponents [17,18]. As a control of the numerical accuracy we checked the value of the sum of the exponents (theoretically expected to be exactly 0), and found $|\sum_{i=1}^{6} \lambda_i| < 10^{-7}$ bits c/fm in all cases considered.

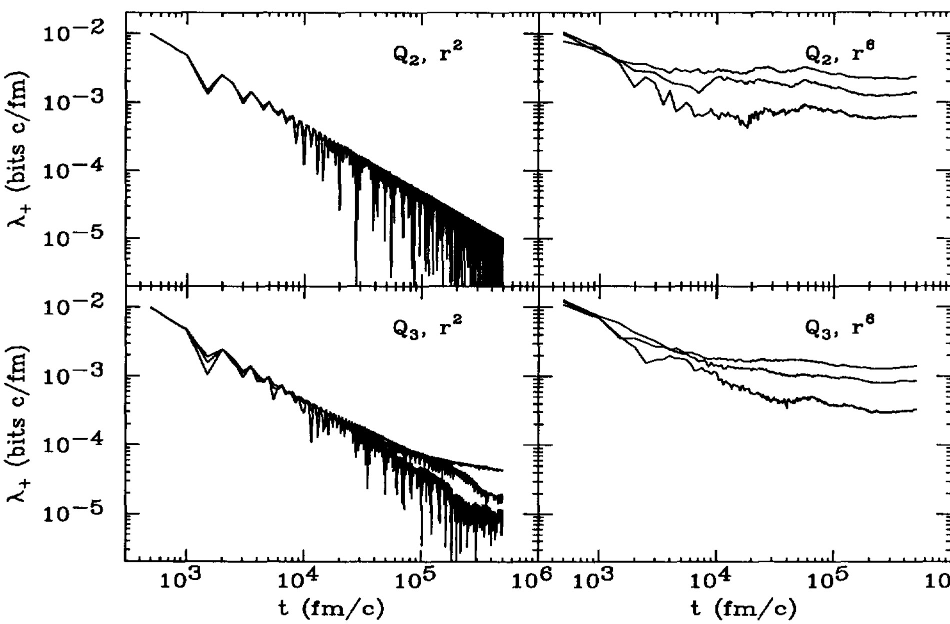

Figure 3. Lyapunov exponents for the single particle trajectories.

In Fig. 3 we display the 3 positive Lyapunov exponents as a function of time for the four cases considered here. The panels are again ordered as in Fig. 2. On the left we show the results for the harmonic static potential (r^2), and on the right for the anharmonic (r^6). For the top the quadrupole-quadrupole interaction (Q_2) was used, and for the bottom

Table 1
Values of the largest positive Lyapunov exponents obtained in the full selfconsistent calculations and in the calculations with static external multipole potentials with maximum deformations.

	selfconsistent		static	
	r^2	r^6	r^2	r^6
Q_2	0	$(2 \pm 1) \times 10^{-3}$	0	0
Q_3	$(4 \pm 1) \times 10^{-5}$	$(1.5 \pm 0.5) \times 10^{-3}$	$(8 \pm 2) \times 10^{-5}$	$(1 \pm 0.3) \times 10^{-3}$

panels we used the octupole-octupole interaction (Q_3). All cases except for the (Q_2, r^2) have at least one positive Lyapunov exponent and therefore show chaotic single particle dynamics. The values of the maximum positive Lyapunov exponents, λ_1, are listed in Table 1. (The large fluctuations in the individual Lyapunov exponents extracted for the calculations with static r^2 potential are as expected in these nearly harmonic systems.)

In order to analyze the causes for the observed single particle chaoticity we also performed calculations with non-selfconsistent fixed multipole potentials, where the deformations were frozen at the maximum values obtained from the self-consistent calculations. The values of the largest Lyapunov exponents obtained from this procedure are also listed in Table 1.

For the static octupole deformation one can show analytically the presence of weak destruction of integrability; and for the static quadrupole one obtains integrability. This is reflected in the obtained values of the largest Lyapunov exponents in these cases. The in our opinion most remarkable change from the maximum static deformation to the self-consistent calculation occurs for the (Q_2, r^6) case. Here the calculations with static quadrupole deformation yield no chaos, but the self-consistent calculations show chaotic single particle behavior. We attribute the origin of this chaoticity to the exchange of energy between the motion of the individual test particles and the collective motion of the multipole coordinate. This exchange of energy is possible, because the individual test particles oscillate with frequencies, which do not have a rational ratio with the frequency of the collective coordinate. This results in the particles reaching meta-stable or unstable points in phase space during the course of its time evolution. At these points small changes in the initial conditions will have a large effect on the subsequent dynamics. An example for this would be the decision if the particle will temporarily oscillate in or out of phase with the collective coordinate. Consequently, these points provide large positive contributions to the Kolmogorov entropy, and chaotic single particle dynamics results.

In turn one also expects each single test particle to have a randomly fluctuating effect on the energy contained in the motion of the collective coordinate. However, since there are quite many test particles, these chaotic random fluctuations are averaged out leaving only a smooth sinusoidal oscillation of the collective coordinate. This is qualitatively new in our investigation: the generation of regular dynamics for the collective variable, the multipole moment of the collective oscillation, from the ensemble of single particles with chaotic trajectories. This is an example of how ordered macroscopic motion can result from underlying chaotic microscopic dynamics. (To obtain this result, it was crucial to employ a self-consistent treatment of the dynamics entailing conservation of total energy.)

6. CONCLUSIONS

We have performed a fully selfconsistent calculation of giant nuclear oscillations to investigate the mechanism of one-body dissipation. In our semiclassical calculation we found no damping of the collective quadrupole or octupole motion. Both collective coordinates undergo regular sinusoidal oscillations. For the octupole motion, this is in clear contrast to the prediction of the wall-formula. We attribute this discrepancy to total energy conservation, which is contained in our model due to the use of selfconsistency.

Even though the collective coordinates are obtained from the single particle coordinates and show regular motion, individual single particle trajectories are found to show chaoticity.

One may speculate that this interplay between chaoticity in individual single particle degrees of freedom and regularity in certain collective coordinates may also play a role in the time evolution of other physical systems. Examples that come to mind as likely candidates are plasmas in a tokamak, the human brain wave activity, the weather. Chaos on a microscopic level need not necessarily lead to a catastrophic breakdown of the system on the macroscopic scale.

This work was supported in parts by the US National Science Foundation under grants number PHY 90-17077 and PHY 94-03666. W.B. acknowledges support form a US NSF Presidential Faculty Fellow award. We are grateful for useful discussions with J. Blocki, E. Heller, O. Meplan, and A. Rapisarda.

REFERENCES

1. D.L. Hill and J.A. Wheeler, Phys. Rev. **89** (1953) 1102.
2. J. Blocki et al., Ann. Phys. (N.Y.) **113** (1978) 330; J. Randrup and W. Swiatecki, Ann. Phys. (N.Y.) **125** (1980) 193.
3. J. Blocki et al., Nucl. Phys. **A545** (1992) 511c; Nucl. Phys. **A554** (1993) 387.
4. R. Arvieu et al., Phys. Rev. A **35** (1987) 2389.
5. Ya.G. Sinai, Russ. Math. Surveys **25(2)** (1970) 137.
6. W. Bauer and G.F. Bertsch, Phys. Rev. Lett. **65** (1990) 2213.
7. E. Fermi, Phys. Rev. **75** (1949) 1169.
8. A.J. Lichtenberg and M.A. Lieberman, *Regular and Stochastic Motion (Applied Mathematical Sciences, Vol. 38)* (Springer, New York, 1983).
9. W. Bauer et al., Phys. Rev. Lett. **72** (1994) 3771.
10. A. Bohr and B.A. Mottelson, *Nuclear Structure*, Volume II, p. 350 ff (W.A. Benjamin, Reading, Mass., 1975).
11. S. Stringari, Nucl. Phys. **A325** (1979) 199; S. Stringari, Phys. Lett. **103B** (1981) 5.
12. H. Reinhardt and H. Schulz, Nucl. Phys. **A391** (1982) 36.
13. H. Kohl, P. Schuck, and S. Stringari, Nucl. Phys. **A459** (1986) 265.
14. G.P. Maddison and D.M. Brink, Nucl. Phys. **A378**, (1982) 566.
15. C.Y. Wong, Phys. Rev. C **25** (1982) 1460.
16. T. Suzuki, Nucl. Phys. A **217** (1973) 182.
17. G. Bennetin et al., Meccanica **15** (1980) 9.
18. A. Wolf et al., Physica **16D** (1985) 285.