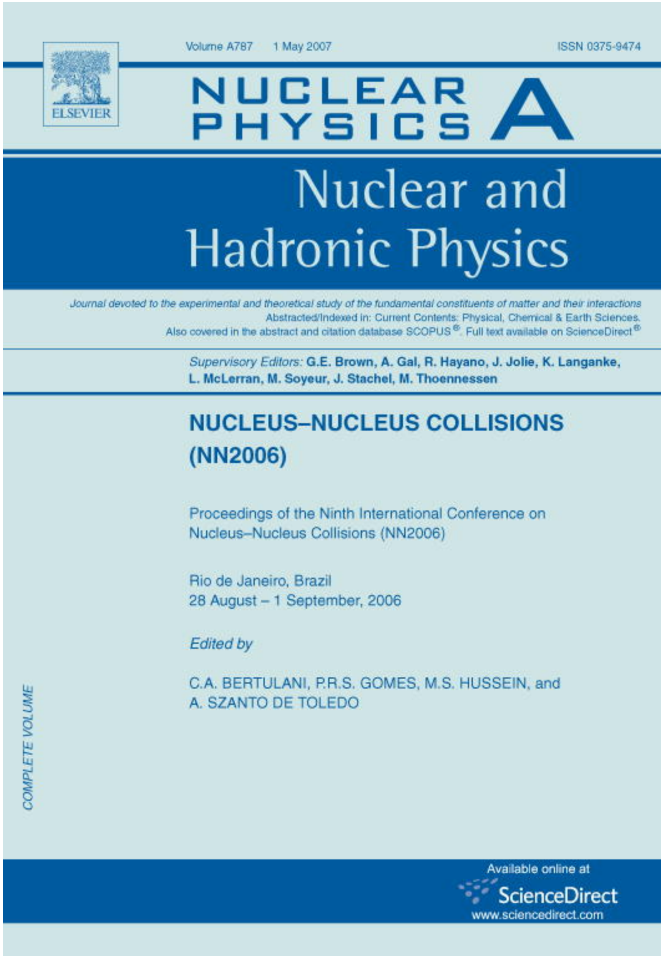
Volume A787 1 May 2007
ISSN 0375-9474
ELSEVIER
NUCLEAR PHYSICS A
Nuclear and Hadronic Physics
Journal devoted to the experimental and theoretical study of the fundamental constituents of matter and their interactions
Abstracted/Indexed in: Current Contents: Physical, Chemical & Earth Sciences.
Also covered in the abstract and citation database SCOPUS®. Full text available on ScienceDirect®
Supervisory Editors: G.E. Brown, A. Gal, R. Hayano, J. Jolie, K. Langanke, L. McLerran, M. Soyeur, J. Stachel, M. Thoennessen
NUCLEUS–NUCLEUS COLLISIONS (NN2006)
Proceedings of the Ninth International Conference on Nucleus–Nucleus Collisions (NN2006)
Rio de Janeiro, Brazil
28 August – 1 September, 2006
Edited by
C.A. BERTULANI, P.R.S. GOMES, M.S. HUSSEIN, and A. SZANTO DE TOLEDO
COMPLETE VOLUME
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ScienceDirect
Nuclear Physics A 787 (2007) 595c–602c

Fragmentation and the Nuclear Equation of State

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Progress on the determination of the order of the fragmentation phase transition, the location of its critical point in the nuclear matter phase diagram, the values of the critical exponents that determine the universality class of the transition, and finite size scaling effects is discussed. Evidence for the presence of Zipf-Mandelbrot-scaling in the relative size of the largest clusters is examined, and the connection to the value of the critical exponent τ is established.

1. Nuclear Phase Diagram

Since the introduction of the nuclear Fireball model [1–4] it has been well established that thermal concepts are applicable to heavy ion collisions. In the process of these collisions finite chunks of nuclear matter are heated and compressed, and equilibrium concepts can be applied. This allows us to ask questions on the phase structure of nuclear matter. The exploration of the nuclear matter phase diagram has been for the last two decades and still is the premier goal of nuclear reaction physics.

There is now general agreement that we should expect at least two phase transitions in nuclear matter, one between a gas of hadrons and a quantum-liquid of nuclear matter, and another between a gas of hadrons and a deconfined phase of quarks and gluons, combined with spontaneous breaking of chiral symmetry [5]. While the quark-gluon plasma [6] phase transition was long thought to be of first order, now hints of a much richer phase structure begin to emerge [7,8].

The "liquid-gas" phase transition can be probed by nuclear fragmentation experiments, and its exploration is much more mature. We now understand that this phase transition is of first order, terminating in a critical point. This transition and its critical point can be probed with symmetric heavy ion collisions with beam energies per nucleon close to the Fermi energy, and with pion or proton beam of 10 GeV or higher. The latter approach is preferable, because it avoids problems with dynamic instabilities due to large radial flow, the transient formation of non-compact structures, and an associated imaginary sound velocity in nuclear matter [9].

There are many interesting physics question that had to and in some case still have to be answered, and by which nuclear science can inform a much broader science community. A sample of these is:

*Research supported by the U.S. National Science Foundation under grants PHY-0245009 and PHY-0555893; email: bauer@pa.msu.edu

doi:10.1016/j.nuclphysa.2006.12.089

∗ How do finite-size effects influence the outcome?
Phase transitions are strictly only defined in the infinite-size limit. In computational approaches to phase transition questions one has to take great care to understand finite-size scaling. Nuclear collisions present a great opportunity to experimentally study the influence of extreme finite-size effects on the character of the phase transition.
∗ Is there equilibrium, and how do we know?
Heavy ion collisions in the energy regime of interest proceed on time scales of 10^{-22} seconds, and it is not clear *a priori* that equilibration times are sufficiently short to speak of a meaningful establishment of thermal equilibrium. Thermal models are successful in reproducing the data [10–12], but what remains to be shown is that thermal and not ensemble averages are the cause for this agreement.
∗ How do we measure the variables of the thermodynamic state?
Since we cannot observe the heavy ion collision directly, we are relegated to collect indirect evidence on the values of the state variables (temperature, density, pressure, entropy, ...), and we can only extract time-averaged quantities experimentally.
∗ What does the migration of the system through the phase diagram entail?
Nuclei cannot be fixed at one point in the phase diagram, as opposed to macroscopic systems that one studies conventionally in other fields. If we hope to extract signatures of critical behavior, for example, we need to be sure that the part of the trajectory through the nuclear phase diagram that is spent in the vicinity of the critical point has essential influence on the final time-integrated experimental observables.
∗ How do sequential decays influence the observed distributions of reaction products?
Here we have to rely on extensive modeling of the known and in many cases unknown decay channels. Reaction network calculations are essential in interpreting the experimental data [13,14].

2. Universality Class of the Fragmentation Phase Transition

If we want to explore the nuclear fragmentation phase transition in model calculations, we need to provide a model that has a well-defined infinite-size limit, in which we can explore critical behavior, and that can also be used for a finite number of constituents, as is the case in nuclear systems. Our model needs to account for the short range character of the strong nuclear force, with its nearest neighbors interactions. It also needs to allow us to model the nuclear surface, an inevitable consequence of the finiteness of nuclei. One such model is the percolation model, which has been applied to nuclear fragmentation reactions with great success [15–20].

Percolation models can be applied to nuclear fragmentation, many other exploding or fragmenting systems, such as the fragmentation of buckyballs [21,22], combustion processes, movement of liquids through porous media, traffic flow, and biological systems. The percolation model has a well-defined infinite size limit with its universality class defined by sets of critical exponents that are functions of the spatial dimensionality of the problem. For example, the cluster size distribution near the critical point can be expressed as

$$N(p, A) = a\ A^{-\tau} f(A^{\sigma}(p - p_c)/p_c) \qquad (1)$$

where p_c is the critical value of the percolation transition, f is a monotonic function with

$f(0) = 1$, and σ and τ are critical exponents. (a is a normalization constant). Percolation models can be realized on lattices with various fixed coordination numbers or even without underlying lattices in the form of random percolation. Since all percolation calculations are performed on finite lattices, it is easy to study finite size effects.

Percolation models come in several basic classes: site percolation, bond percolation, and site-bond percolation. We favor bond-percolation models, in which the nucleons are represented by the (fixed number of) lattice sites, and their nearest-neighbor interaction is represented by bonds. These bonds are broken due to the deposition of energy into the nuclear system, and the remaining connected clusters are identified as fragments.

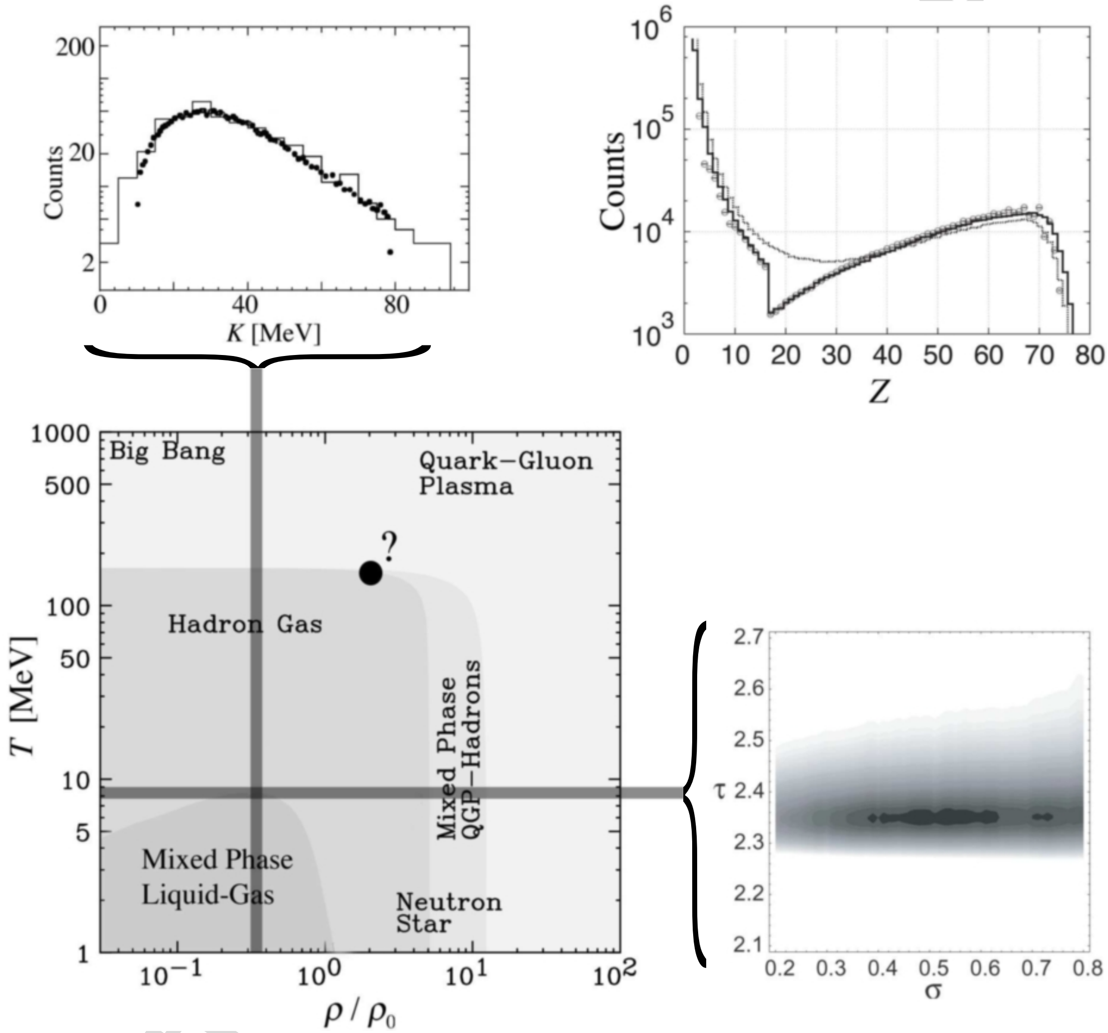

Figure 1. Upper right: Percolation calculation compared to ISiS charge yield data [25]; Lower left: Nuclear matter phase diagram, with constraints of the location of the fragmentation critical point for finite nuclei imposed by scaling analysis of the ISiS data (lower right) and fragment kinetic energy spectra (upper left).

One of the greatest difficulties in connecting percolation calculations with experimental results is to model the pre-equilibrium energy deposition in the nuclear residue. This can be done via a Glauber-ansatz [20], an initial step transport or cascade simulation [23,24], or by an experimental determination of the nuclear temperature [25] and then using the relationship [26–28] between the nuclear temperature T and the probability that a given bond is broken, p

$$p = 1 - \frac{2}{\sqrt{\pi}}\Gamma\left[\frac{3}{2}, 0, \frac{B}{T}\right] \tag{2}$$

Percolation models have been successfully used to reproduce experimental fragment mass or charge distributions (upper right part of Figure 1; circles: data [25], thin histogram: unfiltered percolation calculations, thick histogram: filtered calculations), fragment energy spectra (upper left part of Figure 1; circles: data [29], histogram: calculation), dependence of the moments of the fragment distribution on multiplicity, fragment-fragment correlations, and the size of the largest fragment (a measure for the order parameter) as a function of multiplicity (an approximate measure for the control parameter).

In particular, it has been shown that the experimental results of the EOS [30–32] and ISiS [33–35] collaborations are consistent with a subset of their completely characterized events having reached the critical point of the nuclear matter phase diagram. Furthermore, the extracted critical exponents are consistent with the universality class of the percolation type. In [23,24] it was found that the values of the critical exponents β and γ that can be extracted from the EoS data set [30–32] are consistent with those known from 3d percolation, $\gamma = 1.80$, $\beta = 0.41$, and in [25] the values of the critical exponents τ and σ and the critical temperature T_c were determined as $\tau = 2.35 \pm 0.05$, $\sigma = 0.5 \pm 0.05$, and $T_c = 8.3 \pm 0.2$ MeV (see lower right part of Figure 1).

The value of the critical density for nuclear fragmentation cannot be determined from percolation considerations alone, but one has to extract it from the freeze-out density of a two-step model, with percolation fragmentation as the first step and with a multi-particle Coulomb expansion simulation as a second step [36]. The information on the location of the critical point of nuclear fragmentation obtained from all of the above considerations is summarized in the lower left part of Figure 1. We find

$$T_c = 8.3 \pm 0.2 \text{ MeV}, \quad \rho_c/\rho_0 = 0.35 \pm 0.1 \tag{3}$$

Also shown in Figure 1 is the approximate location of another critical point at which the first-order hadron gas - QGP transition ends. This point in the nuclear matter phase diagram has been studied to a much lesser extent, and at present there are only theoretical and computational hints for this point, which are obtained from lattice-QCD calculations [37]. But we predict that future experimental and theoretical analysis of the physical properties of this critical point has to proceed very much along the same lines as outlined above for the critical point of nuclear fragmentation.

3. Zipf's Law

In 1949 the Harvard linguist Zipf published an astounding result [38]: If one counts the frequency f of words in a text (in the English language) and ranks the words by decreasing frequency, f_1 being the frequency of the most-used word, f_2 that of the second-most used word, and so on, then one finds empirically that

$$f_n \propto 1/n \tag{4}$$

Zipf arrived at his result before the age of computers, by simply hiring people to count words manually. We have redone the Zipf analysis with a much larger data set, the British National Corpus (BNC) [39], which is a "100 million word collection of samples of written and spoken language from a wide range of sources". The 4124 texts in this collection are selected to represent wide cross-section of current British English, both spoken and written. The most frequent words are (in order): the, of, and, a, in, to, it, is, was, to, I, for, you, he, be, with, on, that, by, at, ... If Zipf's Law (Equation 4) indeed holds, then we can form the ratio $f_1/f_n = n$, i.e. the ratio of the frequencies should be strictly linear. In figure 2 we show that this is indeed the case, albeit with the caveat that we had to multiply the frequency of the occurrence of the word "the" with a factor of 1.4.

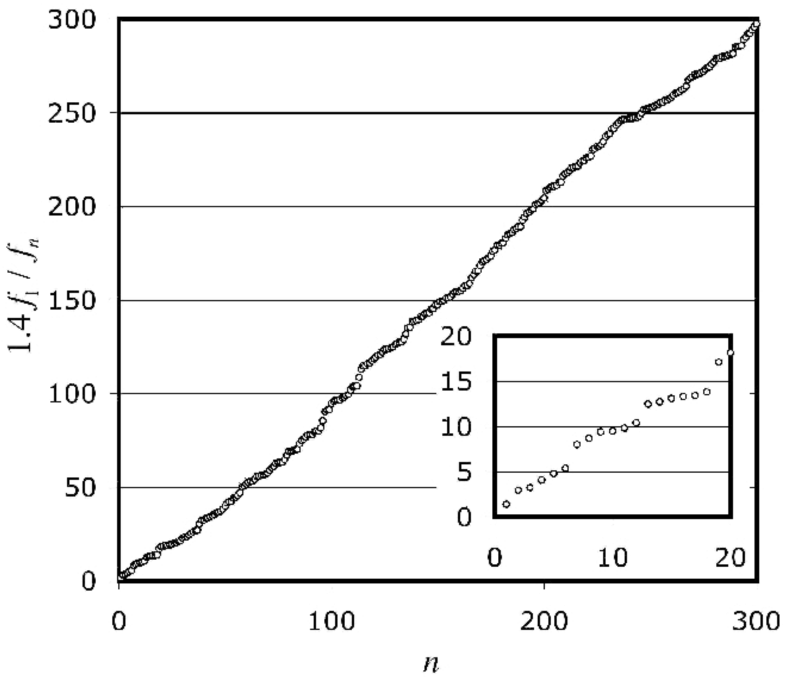

Figure 2. Frequency of the most-used word in the English language ("the") divided by the frequency of the n^{th}-most-used word for the 300 most frequently used words in the English language. Inset: 20 most frequently used words.

One can ask why Zipf's Law works [40]. Amazingly, one finds many manifestations of this law in vastly different fields, such as the population of the largest cities in the US, or the total number of citations for a given author [41]. And in fact one can argue that Pareto [42] discovered the same law first when he examined how people's annual incomes rank. A connection to nuclear fragmentation was established when Watanabe [43] claimed that Zipf's Law also holds for the sizes of the rank-ordered largest clusters in percolation at the critical point, and when Ma [44,45] proposed to search for the critical point by examining the Zipf-Law exponent as a function of excitation energy.

We can test if the above claims hold strictly true for nuclear fragmentation, provided that we use a phase transition model with a scaling function of the kind given in Equation 1. This can be done for a system of finite size and also be extrapolated to infinite size, without the need for Monte-Carlo type computer simulations. The theoretical methods needed to solve this problem have been developed in [46], and we find that the extracted cluster size distributions do not exactly follow Zipf's Law, but instead follow the more general distribution (Zipf-Mandelbrot)

$$\langle A_{r\text{th}} \rangle = \frac{c}{(r+k)^{\lambda}} \tag{5}$$

where $\langle A_{r\text{th}} \rangle$ is the average size of the r^{th}-largest cluster in a fragmentation event. This function is not a simple power law in the rank r, but has an offset k. The power-law exponent λ does not, in general have the value of 1 that is expected from Zipf's Law. Instead, one finds that this exponent depends on the critical exponent τ as

$$\lambda \approx \frac{1}{\tau - 1} \tag{6}$$

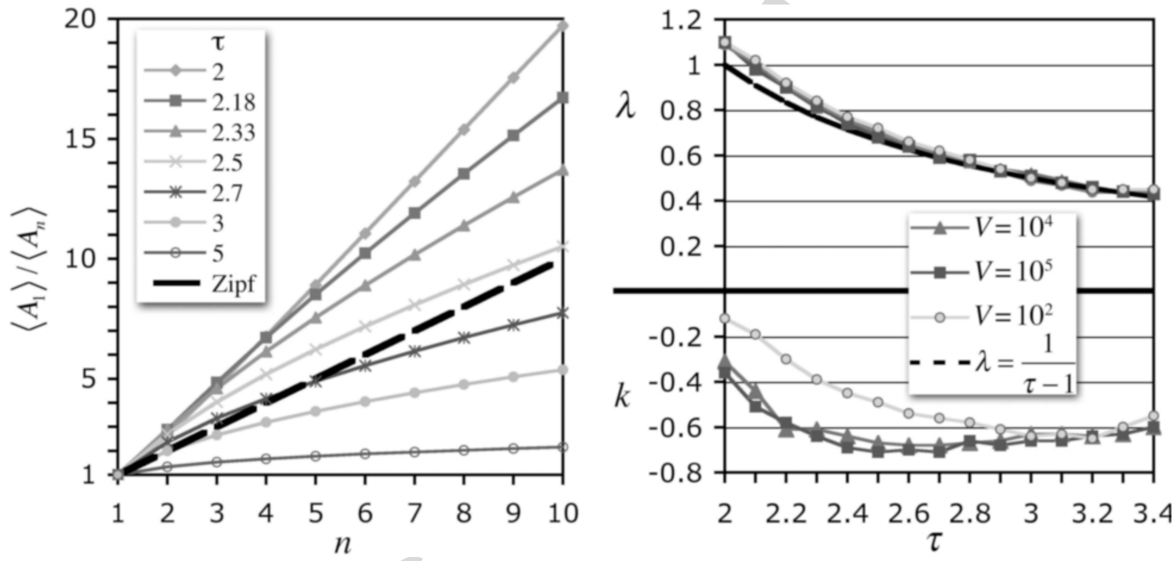

Figure 3. Left: Ratio of the size of the largest cluster to the size of the n^{th} largest cluster as a function of the rank n for systems of size $V = 100,000$ at the critical point, with various values of the critical exponent τ. The straight dashed line is the expected result if Zipf's law were exactly valid. Right: Extracted fits for the parameters λ (upper part) and k (lower part) in Equation 5 as a function of the value of the critical exponent τ, for three different system sizes.

In Figure 3 (left side) we show the ratio of the average size of the largest cluster to the average size of the r^{th}-largest cluster as a function of the rank r for different values of the critical exponent τ (plot symbols), together with a fit to the distribution of equation 5. We compare to the expectation from a straightforward application of Zipf's Law (dashed line) and find that cluster size distributions at the critical point does not exactly follow Zipf's Law for any value of the critical exponent τ. On the right side of figure 3 we show the values of the parameters λ (upper part) and k (lower part) in Equation 5 as a function of the value of the critical exponent τ. The three curves in each of the two plots are the values extract for three system sizes. For all system sizes the values of the parameter λ are nearly identical and fall monotonically with τ. Shown in the upper right part of this figure is also the asymptotic behavior (Equation 6, dashed line), which provides an excellent description of the numerical results. In the lower right part of Figure 3 we show that the extracted values of the parameter k are generally in the range between $-1/3$ and $-2/3$ for very large systems (shown are $V = 10^4, 10^5$). For the $V = 10^2$ system we see strong finite-size corrections.

From this exercise it is clear that it is not possible to apply Zipf's Law in a straightforward fashion to nuclear fragmentation. (Similar results were reported by Campi and Krivine [47]). However, the more general form of the Zipf-Mandelbrot distribution is very much applicable, and we have shown here that Zipf-Mandelbrot scaling provides a viable method to characterize the cluster size distributions at the critical point, for arbitrary values of the critical exponent τ (provided that $\tau \geq 2$). At present, it is not clear what additional information beyond the already known values of the critical exponents this scaling law contains, but interesting research in this direction is in progress.

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