Thermal production of the ρ meson in the $\pi^+\pi^-$ channel

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Recent measurements of the $\pi^+\pi^-$ invariant-mass distribution at RHIC show a shifted peak for the ρ meson in 100*A* GeV in peripheral Au+Au and even in $p+p$ collisions. A recent theoretical study based on a picture of in-medium production rates of pions showed that a large shift could result from a combination of the Boltzmann factor and the collisional broadening of the ρ. Here we argue that the two-pion density of states is the appropriate quantity if one assumes a sudden breakup of the system. Methods for calculating the density of states which include Bose effects are derived. The resulting invariant-mass distributions are significantly enhanced at lower masses and the ρ peak is shifted downward by ~35 MeV.

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I. INTRODUCTION

One of the most compelling motivations for studying heavy-ion collisions is the prospect for observing the restoration of chiral symmetry. The spontaneous breaking of chiral symmetry is accompanied by the creation of a quark-antiquark condensate whose coupling to nucleons is responsible for the great bulk of the nucleon mass, and is therefore responsible for most of the observed mass of the universe. The transient nature of the heavy-ion reaction precludes a detailed investigation of all the quasiparticle modes in the highly excited collision volume. However, the ρ meson is unique for it typically decays inside the spatial region where the vacuum structure might undergo novel changes. A neutral ρ decays with 99% probability into a $\pi^+\pi^-$ pair and decays with a small probability into an e^+e^- or $\mu^+\mu^-$ pair. The electromagnetic channels are especially useful because dilepton pairs will largely leave the collision volume unscathed by interactions with the thousands of other constituents. Since the ρ has the same quantum numbers as the photon, the invariant-mass spectrum of dileptons is dominated by the ρ for masses between 600 and 800 MeV. Experiments at the CERN SPS for e^+e^- [1] and $\mu^+\mu^-$ [2,3] suggest that the ρ has either dissolved [4] (as would be expected in a quark-gluon plasma), or has moved down a few hundred MeV [5] (due to chiral symmetry restoration), or has been broadened via collisions by many hundreds of MeV [6].

Recently, the possibility of studying in-medium properties of the ρ meson through the $\pi^+\pi^-$ channel has been discussed [7,8]. Unlike dileptons, pions are not penetrating probes and are likely to reinteract before they escape. Since temperatures fall to near 100 MeV at breakup, where the ρ/π ratio falls to a few percent, the chance that a π^+ is accompanied by a π^- that originated from the same ρ, rather than a charged pion from a different source, is only a few percent. Thus, a background subtracted invariant-mass distribution should have a ρ peak that comprises only a few percent of the integrated distribution.

The STAR collaboration at RHIC has measured such a peak in *pp* collisions, and for the first time, in peripheral relativistic heavy-ion collisions [17]. A surprisingly significant downward shift of the mass was observed even in *pp* collisions, especially at low p_t, and an even larger shift was observed in peripheral Au+Au collisions. Results are not yet available for central collisions where it is more difficult to observe the peak since the ρ/π ratio falls. Eventually, the ρ peak should also be measured for central collisions given sufficient statistics.

In Refs. [7,8], the mass distribution was predicted by considering the in-medium rate of ρ decays into $\pi^+\pi^-$ pairs, dN/dMd^3xdt. This is the same approach as has been applied for dilepton studies. In Ref. [8], these rates were corrected for collision broadening and for Bose effects. Collision broadening was shown to be particularly important in moving strength to lower-lying masses. However, emission of pions is of a fundamentally different character than that of dileptons. First, the final-state distribution is not necessarily proportional to the decay rate since the decay rate is often balanced by a formation rate of similar magnitude. Second, collisional broadening cannot be applied in the same manner since measurements are made in the asymptotic state. Finally, the presence of the ρ alters the two-pion scattering partial waves at nonzero separations which should affect the mass distribution. As we will demonstrate, the production-rate calculations of Refs. [7,8] provide different results than a freeze-out prescription which is governed by the available phase space such as in Ref. [9].

If the last strong interactions felt by the two pions used in the distribution can be considered sufficiently hard to statistically sample the outgoing phase space, the two-pion density of states should govern the invariant-mass distribution. By definition, the sudden breakup scenario requires the density

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being lowered to the point where collisions have ceased and collision broadening can be ignored. However, this might not be the same criterion for demanding that the real parts of the self-energies (the mean field) have returned to vacuum values. Whereas the real part of the self-energy has one-loop contributions that scale as the square of the coupling constant, the imaginary part due to collision broadening is a two-loop effect that scales as the fourth power of the coupling constant. For the modeling of neutrinos through the sun it is well justified to consider only the real part of the self-energy since the coupling constants are effectively small. However, for strongly interacting particles, it is rather inconsistent to ignore collision broadening while simultaneously discussing mass shifts in the final state.

Although these ρ's probably decayed during the breakup stage, which is well below the critical density, the decaying ρ mesons might still sample a region where mean-field effects, i.e., in-medium mass shifts, are not negligible. Since pions are Goldstone bosons, they probably leave the region with their energy and momenta unchanged during their exiting trajectory, and one expects that a modification of the two-pion invariant-mass distribution would reflect the in-medium modifications of the ρ rather than those of the pion.

Whereas, this mass shift may be of the order of 100 MeV at high temperature, it is unlikely to be much more than 25 MeV at breakup when densities have fallen well below the nuclear density. Theory has not provided a definitive estimate of the mass shift, and even the sign of the shift is somewhat controversial. In a pion gas the ρ meson is shifted upwards by $\approx$15 MeV by the mixing with $\pi\pi$ states [10]. More sophisticated models include excited baryonic states and may also consider the ρ and a_1 to be part of chiral multiplet [11]. Such models also result in mass shifts of the order of 10 MeV. From a different perspective, the mass of the ρ is affected by the dissolution of the $q\bar{q}$ condensate which couples to the ρ and provides much of its mass. According to lattice gauge calculations [12], the condensate should largely dissolve at temperatures near the deconfinement transition. It should also be significantly reduced by the presence of other heavy hadrons, e.g., nucleons. QCD sum rules have been applied to estimate the reduction of the ρ mass due to the presence of nucleons, and the effects can be parametrized by the form

$$\frac{m_\rho^*}{m_\rho} = 1 - \alpha\frac{\rho}{\rho_0}, \tag{1}$$

with α being quoted as 0.16 ± 0.06 in Ref. [13]. Other treatments based on QCD sum rules [14,15] result in similar shifts, but the value of α remains uncertain at the factor of two level. At RHIC, the breakup density of (anti)nucleons is $\approx$10% of normal nuclear density and the expected lowering of the mass from coupling to the reduced $q\bar{q}$ condensate should be of the order of 20 MeV. Since multiple competing effects are of the order of 20 MeV and each is fairly uncertain, the net shift remains an open question for theoretical debate which can hopefully be resolved by experiment. It is not the aim of the current paper to model the in-medium mass shift of the ρ, but rather to investigate how the invariant-mass distribution would look in a thermal description based entirely on the outgoing two-pion density of states and related Bose effects. Changes to the observed mass distribution beyond what is executed from these considerations might then be associated with modifications [16] of the in-medium properties.

In contrast to the preceding discussion for strong interactions, it is not necessarily inconsistent to consider final-state Bose effects from third bodies while neglecting other three-body interactions. Whereas the effects of third-body interactions for providing mass shifts and collisions are proportional to the density, multiparticle Bose effects are determined by the phase-space density. During an isentropic expansion, or during free streaming, the average phase-space density, as sampled by the particles themselves, stays constant even though the density is changing rapidly. Thus, we argue that it is reasonable, and in fact important, to include Bose effects even though the description is based on two-body phase space.

In the following section, methods for calculating the two-particle density of states are presented along with a comparison with the functional forms one would expect from rate calculations. After convoluting with the Boltzmann weighting, we find that the ρ peak is shifted downward by $\sim$30 MeV relative to the nominal ρ mass. The shift is due to three factors, the Boltzmann weighting [18], the fact that the density of states peaks below the ρ mass, and the inclusion of other partial waves. Bose-Einstein effects also enhance the distribution at lower masses [8,19,20], especially for heavy-ion collisions where the pionic phase-space filling factors are approaching unity [21,22]. In Sec. III methods are presented for including Bose effects into the two-pion density of states. The resulting mass distribution is strengthened at lower invariant masses, but the peak does not shift appreciably.

II. INVARIANT-MASS DISTRIBUTIONS FROM THE TWO-PION DENSITY OF STATES

Since the first measurements of the ρ meson [23,24], the masses and widths have fluctuated by several MeV depending on the analysis. Currently, the Particle Data Group assigns a nominal mass of 771.1 MeV and a width of 149.2 MeV [25], with uncertainties for each number being near 1 MeV. The ρ mass has been determined from a number of means, $e^+e^-\to\pi^+\pi^-$ reactions, pp collisions, and $\pi p\to\pi\pi p$ reactions [26]. Electroproduction of the ρ is complicated by the interference with the $\omega\to2\pi$ channel [27] which constructively interferes with the ρ^0 channel since the electromagnetic coupling violates isospin conservation. Since pp collisions are typically highly inelastic, extracting the ρ mass is complicated by the same factors that complicate the study in a heavy-ion environment. In the $\pi p\to\pi\pi p$ reaction, the proton is treated as a source of pions which are assumed to scatter elastically with the incoming pions. In fact, $\pi^+\pi^-$ phase shift analyses have been successfully performed.

Considering only scattering through ρ^0, the π^+ π^- cross section should have a Breit-Wigner form,

$$\sigma(M) = 3\frac{4\pi}{q^2}\frac{(\Pi_I)^2}{(M^2-M_0^2)^2+(\Pi_I)^2}, \tag{2}$$

where q is the momentum of either pion in the center-of-mass frame ($M=2\sqrt{m_\pi^2+q^2}$) and Π_I is the imaginary part of the one-loop self-energy of the relativistic propagator:

$$\Pi_I = \Pi_{I,0}\frac{M_0}{M}\left(\frac{q}{q_0}\right)^3 F(q,q_0), \tag{3}$$

$$\Pi_{I,0} = \Gamma_0 M_0. \tag{4}$$

Here, Γ_0 is the nominal width and q_0 is the momentum required to provide the nominal mass M_0. The last term $F(q/q_0)$ is a form factor whose exact form is in doubt [19]. The product $q^2\sigma$ peaks precisely at the nominal mass irrespective of the form factor.

The spectral function of the ρ is related to the imaginary part of the propagator,

$$S_\rho(M) = \frac{2M}{\pi}\mathrm{Im}\frac{1}{(M^2-M_0^2)+i\Pi_I} \tag{5}$$

$$=\left(\frac{2M}{\pi\Pi_I}\right)\mathrm{BW}(M), \tag{6}$$

$$\mathrm{BW}(M) = \frac{(\Pi_I)^2}{(M^2-M_0^2)^2+(\Pi_I)^2}. \tag{7}$$

Here, S_ρ is usually associated with the number of states available to the ρ with a given mass. The real part of Π is being ignored for the current discussion. Since the Breit-Wigner function BW(M) always peaks at $M=M_0$, and since Π_I/M is rapidly growing with M near the ρ mass, the spectral function always peaks below the ρ mass. Setting the form factor in Eq. (3) to unity, the peak of the ρ spectral function shifts downward by 5 MeV. Applying some of the different expressions for Π_I discussed in Ref. [19] may result in the peak being shifted further downward, perhaps as much as an additional 5 MeV.

The change in the total density of states can be expressed in terms of phase shifts, [9,28,29]:

$$\Delta\rho(M) = \frac{1}{\pi}\sum_\ell (2\ell+1)\frac{d\delta_\ell}{dM}. \tag{8}$$

Given the relation between the phase shift and the self-energy, one can express $\Delta\rho$ in terms of the self-energy:

$$\tan\delta = \frac{\Pi_I}{M_0^2-M^2}, \tag{9}$$

$$\Delta\rho_{\pi\pi}(M) = \frac{3}{\pi}\frac{2M\Pi_I}{(M_0^2-M^2)^2-\Pi_I^2}\left(1+\frac{M_0^2-M^2}{2\Pi_I M}\frac{d\Pi_I}{dM}\right). \tag{10}$$

The first term is the spectral function of the ρ, which is often associated with the probability of having a ρ meson of mass M. Together, the two terms describe the entire correction to the density of states, including the effects of modifying the outgoing partial waves.

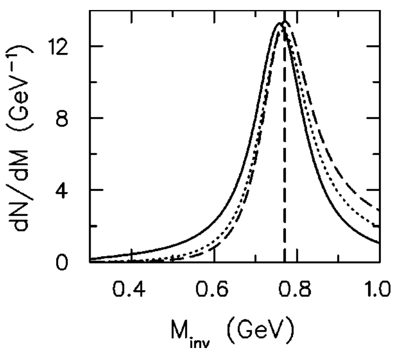

FIG. 1. The spectral function of the ρ (dotted line) is broader than the Breit-Wigner form (dashed line). The total density of states, including effects of modifying the outgoing partial waves, is noticeably shifted to the left relative to the other forms. The difference is especially noticeable at small invariant masses, where the three forms rise proportional to q^6, q^3, and q respectively.

Figure 1 illustrates the importance of using the correct expression for the density of states. The spectral function of the ρ is peaked below the Breit-Wigner function, and the total density of states is peaked even lower. The difference is especially strong at low invariant masses, as the Breit-Wigner function rises as q^6, the ρ spectral function rises as q^3, and the pionic density of states rises as q. This relative scaling with q would hold for any p-wave interaction.

Thus far, the distribution of masses has not incorporated the Boltzmann factor, which should push the peak even lower with the thermal weight, $e^{-M/T}$ [18]. More precisely, one needs to integrate over the modes in momentum space due to relativistic effects,

$$\Delta\frac{dN_{\pi\pi}}{dMd^3x} = \int\frac{d^3P_\rho}{(2\pi)^3}e^{-\sqrt{P_\rho^2+M^2}/T}\Delta\rho_{\pi\pi}(M). \tag{11}$$

This should represent the background-subtracted two-pion invariant-mass distribution. As can be seen in Fig. 2, the Boltzmann weight pushes the distribution increasingly downward for lower temperatures. The upper panel shows the mass distribution assuming a temperature of 170 MeV, which is a reasonable temperature for thermal models of pp collisions, while the lower panel shows the result for a temperature of 110 MeV, which may be reasonable for the breakup temperature in central heavy-ion collisions. Calculations using both the ρ spectral function and the two-pion density of states are displayed to illustrate the importance of choosing the appropriate form for the density of states. The Boltzmann factor greatly magnifies the enhancements at low M, to the point that a second peak appears for lower temperature.

The $\pi^+\pi^-$ density of states is also affected by phase shifts in other channels. For $\pi^+\pi^-$, the s-wave channel is split into two isospin components, 2/3 weight for $I=0$ and 1/3 weight for $I=2$. The $I=0$ channel is particularly important as it corresponds to the mythical σ meson. Although phase shift

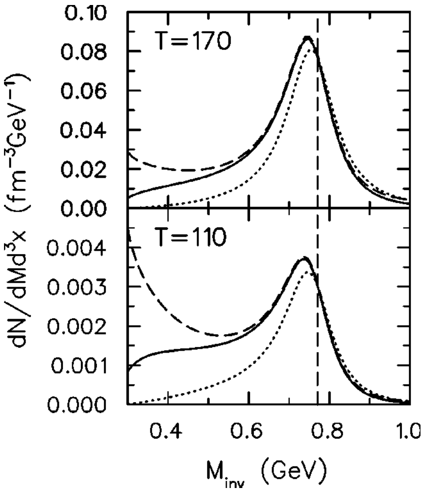

FIG. 2. The thermal mass distribution of $\pi^+\pi^-$ pairs is shown for three calculations, both at T=110 MeV and T=170 MeV. Using the full density of states as calculated from taking derivatives of phase shifts results in a broader distribution for the ℓ=1 channel (full line) than using the ρ spectral function (dotted line). Including all s, p, and d channels (dashed line) provides significant strength at low invariant masses due to the s wave channels and moderate strength at higher masses from the d wave channels.

analyses do not reveal a sharp peak as in a resonance [30–34], the phase shifts are considerable, rising steadily from zero at threshold to $\approx 90°$ at $M=2M_K\sim 1$ GeV, where the kaon channel opens. At the two-kaon threshold, the behavior of the phase shifts becomes complicated and an inelastic treatment becomes warranted. Since one uses derivatives of the phase shifts to find the density of states, interpolating data for phase shifts can be dangerous due to noise in the experimentally determined phase shifts. Thus, we apply a simple form that describes the general behavior

$$\delta_{I=0,S=0} = aq + b(M - 2m_\pi). \tag{12}$$

The first coefficient a is the scattering length, which is small due to constraints from chiral symmetry. The number varies throughout the literature by several tens of percent. We use the value $a=0.204/m_\pi$ [35]. The second term does not contribute to the scattering length, as $(M-2m_\pi)\sim q^2$ at low q. Choosing $b=9.1\times 10^{-4}$ GeV^{-1} crudely reproduces experimental phase shifts, which are reviewed in Ref. [30]. Since these phase shifts rise half as far as those in the δ channel, have one-third the spin degeneracy, and have a 2/3 weight in the $\pi^+\pi^-$ channel, they are noticeably less important than the ρ channel in affecting the overall density of states, unless one is near the two-pion threshold where p-wave interactions vanish.

Other phase shifts also contribute: $(I=2,\ell=0)$, $(I=0,\ell=2)$, and $(I=2,\ell=2)$. Since none of these phase shifts exceed more than a few degrees, they make nearly negligible contributions to the density of states. For the $(I=2,\ell=0)$ channel, we apply an effective range expansion [36]

$$\cot\delta = \frac{1}{qa} + \frac{1}{2}Rq, \tag{13}$$

where $a=-0.13$ MeV^{-1} and $R=1.0$ MeV^{-1}. The d wave is also composed of $I=0$ and $I=2$ pieces. For the $(I=0,\ell=2)$ piece, the data [37] are rough, and we make a simple expansion

$$\delta_{I=0,\ell=2} = cq^5, \tag{14}$$

where $c=6.2$ GeV^{-1}. The parameter a is uncertain to the 50% level. For the $(I=2,\ell=2)$ partial wave, we use an expansion [36]

$$\delta_{I=2,\ell=2} = -8.4q^5 + 12.5q^6 \text{ GeV}^{-1}. \tag{15}$$

None of these three channels are well understood, but none have a substantial impact at or below the ρ region of invariant mass.

Figure 2 also shows the invariant-mass distribution of a thermal ensemble with T=110 MeV using all the s, p, and d channels. The s-wave contributions are non-negligible near the ρ mass, and dominate near the two-pion threshold. The d-wave contributions matter only for masses near or greater than 1.0 GeV.

III. BOSE-EINSTEIN CORRECTIONS

Bose-Einstein corrections should preferentially enhance low-mass pairs since low-mass pairs are more likely to include a low-momentum pion. This has been investigated within the context of the ρ peak as well as the influence on Z boson decay modes [19]. In this section, we present a means to include Bose enhancement effects which are consistent with the statistical picture described in the preceding section.

In order to demonstrate Bose enhancement effects, we revert to the fundamental definition of the two-particle density of states:

$$\rho(M) = \frac{1}{2\pi}\text{Im Tr}\frac{1}{M-H+i\epsilon} \tag{16}$$

$$= \frac{1}{2\pi}\text{Im Tr}\sum_{n=0}\frac{1}{M-H_0+i\epsilon}\left(V\frac{1}{M-H_0+i\epsilon}\right)^n. \tag{17}$$

We will work in the two-pion rest frame, so the trace would cover all two-pion states that have total momentum zero. When including Bose effects, one would sum all such two-pion states, plus average over the distribution of other identical particles whose probability of being populated is

$$f(\mathbf{q}) = \frac{f_0(\mathbf{q})}{1-f_0(\mathbf{q})} = f_0(\mathbf{q})[1+f(\mathbf{q})]. \tag{18}$$

Thus, $(1+f)$ can be considered as a Bose enhancement factor while f_0 is the phase-space filling factor if Bose statistics were neglected.

Using the cyclic property of the trace, Eq. (16) can be written in terms of a derivative with respect to M,

$$\rho(M) = \rho_0(M) + \frac{1}{2\pi}\mathrm{Im}\frac{d}{dM}\mathrm{Tr}\sum_{n=1}\frac{1}{n}\left(\frac{V}{M-H_0+i\epsilon}\right)^n \tag{19}$$

$$=\rho_0(M) + \frac{1}{\pi}\mathrm{Im}\frac{d}{dM}\mathrm{Tr}\sum_{n=1}\frac{1}{n}\left(\frac{\mathcal{P}}{M-H_0}V + i\pi\delta(M-H_0)V\right)^n. \tag{20}$$

One can expand the n terms and note that the sum includes all possible orderings of n factors where each factor is either the principal value piece, which is real, or the imaginary part, which is proportional to the density of states. One could restrict this sum to cover only those terms where the first factor is the imaginary part and multiply by a factor of n/N_ρ, where N_ρ is the number of times that $i\pi\delta(M-H_0)$ appears in the term. The sum over n can then be transformed into a sum of all possible numbers of appearances of the real part:

$$\rho(M) = \rho_0(M) + \frac{1}{2\pi}\mathrm{Im}\frac{d}{dM}\mathrm{Tr}\sum_{N_\rho=1}\frac{1}{N_\rho}[i\pi\delta(M-H_0)\mathcal{R}]^{N_\rho}, \tag{21}$$

$$\mathcal{R} \equiv V + V\frac{\mathcal{P}}{E-H_0}\mathcal{R}. \tag{22}$$

Here, $\mathcal{R}$ is often referred to as the R matrix. This can be written in terms of a logarithm,

$$\Delta\rho(M) = \frac{1}{\pi}\mathrm{Im}\frac{d}{dM}\mathrm{Tr}\ \ln\left(\frac{1+i\pi\delta(M-H_0)\mathcal{R}}{1-i\pi\delta(M-E)\mathcal{R}}\right). \tag{23}$$

Thus, the density of states is determined completely by a single matrix

$$\tau \equiv \pi\rho_0(M)\mathcal{R}, \tag{24}$$

which is evaluated only for those states whose energy equals M. In a partial-wave basis, τ is related to the phase shift $\tau = \tan\ \delta$. In a plane-wave basis, the matrix τ links one direction of the relative momentum with another, i.e., the matrix should be written with indices τ_{Ω_1,Ω_2}.

The presence of other particles alters τ. Each matrix element V used to construct τ is modified by the presence of other particles by the Bose enhancement factor

$$V(\mathbf{q}_1,-\mathbf{q}_1;\mathbf{q}_2,-\mathbf{q}_2) \to V(\mathbf{q}_1,-\mathbf{q}_1;\mathbf{q}_2,-\mathbf{q}_2)\sqrt{[1+f(\mathbf{q}_1)][1+f(-\mathbf{q}_1)][1+f(\mathbf{q}_2)][(1+f(-\mathbf{q}_2)]}. \tag{25}$$

If the intermediate states contained in the definition of $\mathcal{R}$ are not affected by the phase-space density, one can scale τ in the same manner as V. Then, given the fact that each state appears in both the bra and ket, one can modify τ in a simple manner to account for Bose effects,

$$\tau(\mathbf{q}_1,-\mathbf{q}_1;\mathbf{q}_2,-\mathbf{q}_2) = \tau_0(\mathbf{q}_1,-\mathbf{q}_1;\mathbf{q}_2,-\mathbf{q}_2)[1+f(\mathbf{q}_1)] \times[1+f(-\mathbf{q}_1)]. \tag{26}$$

The density of states is comprised of integrals of a cyclic nature, I_n,

$$\Delta\rho(M) = \frac{1}{\pi}\mathrm{Im}\frac{d}{dM}\sum_{n=1,3,5,\ldots} I_n/n, \tag{27}$$

$$I_n(M) = \int \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi}\cdots\frac{d\Omega_n}{4\pi} \times\tau_0(\Omega_1,\Omega_2)\tau_0(\Omega_2,\Omega_3)\ldots\tau_0(\Omega_n,\Omega_1)[1+f(\mathbf{q}_1)] \times[1+f(-\mathbf{q}_1)]\cdots[1+f(\mathbf{q}_n)][1+f(-\mathbf{q}_n)]. \tag{28}$$

Unless the momentum of the pair $P=0$, the phase-space densities will be sensitive to the direction of the relative momentum Ω.

For a purely s-wave interaction, τ_0 has no angular dependence and I_n easily incorporates Bose effects,

$$I_n = \left(\tau_0\int\frac{d\Omega}{4\pi}[1+f(\mathbf{q})][1+f(-\mathbf{q})]\right)^n. \tag{29}$$

The correction to the density of states is then

$$\Delta\rho(M) = \frac{1}{\pi}\frac{d\tau/dM}{1+\tau^2}, \tag{30}$$

$$\tau = \tan\ \delta\int\frac{d\Omega}{4\pi}[1+f(\mathbf{q})][1+f(-\mathbf{q})], \tag{31}$$

where δ is the phase shift as measured in the absence of Bose modifications.

For a p wave interaction, τ_0 has the angular dependence,

$$\tau_0(\Omega_1,\Omega_2) = \bar{\tau}_0\hat{q}_1\cdot\hat{q}_2. \tag{32}$$

By choosing a coordinate system where the z axis is parallel to the total pair momentum, there is reflection symmetry about the x, y, and z planes. By making use of the identity

$$\int\frac{d\Omega_b}{4\pi}(\vec{A}\cdot\hat{b})(\hat{b}\cdot\vec{C})F(\Omega_b) = \vec{A}'\cdot\vec{C}, \tag{33}$$

$$A_i' = A_i F_i, \tag{34}$$

$$F_x = \int \frac{d\Omega}{4\pi} F(\Omega)\cos^2\phi \sin^2\theta, \tag{35}$$

$$F_y = \int \frac{d\Omega}{4\pi} F(\Omega)\sin^2\phi \sin^2\theta, \tag{36}$$

$$F_z = \int \frac{d\Omega}{4\pi} F(\Omega)\cos^2\theta, \tag{37}$$

one can iteratively perform the integral in Eq. (28):

$$I_n(M) = (\bar{\tau}_0 F_x)^n + (\bar{\tau}_0 F_z)^n + (\bar{\tau}_0 F_z)^n. \tag{38}$$

Using $F(\Omega)=[1+f(\mathbf{q})][1+f(-\mathbf{q})]$, one can calculate $\Delta\rho$:

$$\Delta\rho(M) = \frac{1}{\pi}\left(\frac{d\tau_x/dM}{1+\tau_x^2} + \frac{d\tau_y/dM}{1+\tau_y^2} + \frac{d\tau_z/dM}{1+\tau_z^2}\right), \tag{39}$$

$$\tau_x = \tan\,\delta \int \frac{d\Omega}{4\pi}[1+f(\mathbf{q})][1+f(-\mathbf{q})]3\sin^2\phi\sin^2\theta, \tag{40}$$

$$\tau_y = \tan\,\delta \int \frac{d\Omega}{4\pi}[1+f(\mathbf{q})][1+f(-\mathbf{q})]3\cos^2\phi\sin^2\theta, \tag{41}$$

$$\tau_z = \tan\,\delta \int \frac{d\Omega}{4\pi}[1+f(\mathbf{q})][1+f(-\mathbf{q})]3\cos^2\theta. \tag{42}$$

The calculation of $\Delta\rho(M)$ must be repeated for each value of the total momentum since $f(\mathbf{q})$, which is defined in the two-pion rest frame, changes when the total momentum is changed.

The p-wave and s-wave corrections to the density of states do not interfere with one another since they have opposite parities and $[1+f(\mathbf{q})][1+f(-\mathbf{q})]$ has even parity. However, calculation of the $\ell=2$ contributions would be complicated by the fact that the elliptical distortion of the Bose enhancement factors would mix the $\ell=0$ and $\ell=2$ contributions. For the calculations here, the $\ell=2$ contributions were calculated by assuming that the Bose enhancement factors were independent of Ω, then using enhancement factors which had been averaged over all directions of Ω.

The mean Bose enhancement $\langle(1+f_1)(1+f_2)\rangle$ is shown as a function of the invariant mass and momentum of the decaying ρ^0 in Fig. 3 assuming a breakup temperature of 110 MeV and an effective chemical potential of 90 MeV. The enhancement has been averaged over the directions of the relative momentum. The enhancement is largest for low-momentum, low-mass pairs since these pions most strongly sample the region of high phase-space density. For higher invariant masses, the Bose enhancement is actually stronger for higher pair momenta, as it allows one of the outgoing pions to have low p_t and sample the high phase-space density region. From viewing Fig. 3, it is clear that the Bose modifications to the invariant-mass distribution would be more acute if experiments were to focus on pion pairs with low total momentum.

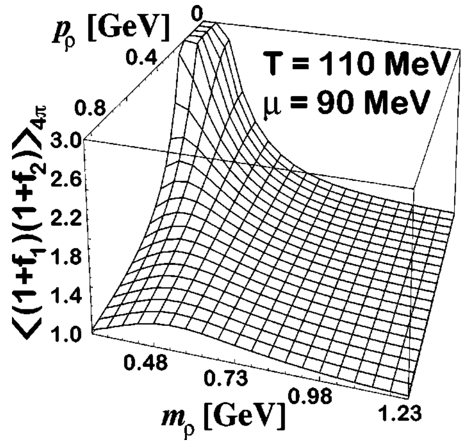

FIG. 3. The mean values of $\langle(1+f_1)(1+f_2)\rangle$ are shown as a function of the invariant mass and total momentum of the outgoing pion pair. The values have been averaged over all directions of the relative momentum. The enhancement factors exceed 2.0 for a low values of the pair momentum and invariant mass.

Bose corrected densities of states are shown in Fig. 4 for T=110 MeV and μ=90 MeV. A nonzero chemical potential was used to account for the relative overpopulation of pionic phase space which may result from rapid cooling [38] and might be magnified by the effects of chiral symmetry restoration [39]. Analyses of $\pi\pi$ correlations from RHIC indeed point to high phase-space densities [21,22], especially for central collisions of heavy ions. As expected, lower-mass states were more enhanced by Bose effects. Since the density of states was proportional to the derivative of tan $\delta\langle(1+f)(1+f')\rangle$, and since the averaged phase-space filling factors generally fall as M increases, the density of states was less enhanced for intermediate masses as compared to the no-Bose case. The peak of the distribution shifted downward by only 1 MeV after the inclusion of Bose effects.

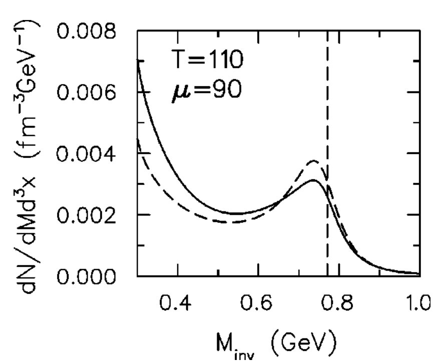

FIG. 4. The mass distribution is shown with (solid line) and without (dashed line) Bose effects for T=110 MeV,μ=90 MeV. Bose effects enhance the probability of producing low invariant-mass pairs since they are more likely to have low momenta and stronger Bose enhancement factors.

Although the position of the peak was not much affected by Bose effects shown in Fig. 4, Bose effects led to a near doubling of the distribution at low masses. These effects are most important at low p_t where the phase-space densities are higher. In pp collisions, a movement of the ρ peak was observed for low p_t pairs [40,17] which is suggestive of Bose effects. However, at the ρ peak each pion has a relative momentum of $\sim$300 MeV/c and will largely sample phase-space regions with moderate to low phase-space densities. Although the invariant-mass distribution is mainly altered at invariant-masses below the ρ peak, Bose effects should contribute to washing out the peak by increasing the slope of the background structure in Fig. 4.

IV. SUMMARY

Our principal finding is that the ρ peak in the $\pi^+\pi^-$ invariant-mass distribution should be $\approx$35 MeV lower than the nominal ρ mass, if one accepts the scenario of a sudden breakup that thermally samples the two-pion density of states. The shift was the result of convoluting the density of states which is shifted by $\sim$10 MeV below the ρ mass with the Boltzmann factor. Given the extra cooling inherent to heavy-ion collisions, the breakup temperature is probably near 110 MeV, well below the characteristic temperatures used to describe pp collisions. The low temperature provides an additional downward shift of the peak in heavy-ion collisions. In addition to the shift of the peak, the distribution showed significant additional strength at invariant masses near the two-pion threshold. This additional strength hinged on using the correct expressions for the density of states, especially in the $\ell=0$ channels. Although the position of the peak was not much affected by Bose effects, Bose effects led to a near doubling of the distribution at low masses.

The thermal model presented here rests critically on a pair of assumptions. First, we have assumed that the breakup is sudden, i.e., the last strong interaction experienced by the particles samples the outgoing two-particle phase space. Indeed, interferometric measurements do suggest a sudden breakup [22,41,42]. If emission were gradual, e.g., surface evaporation, this picture would be invalid. An appropriate treatment of the surface would include the dynamics of surface penetration and absorption, and might include collisional broadening. For instance, spectral lines in stars are affected by collisional broadening. The "truth" of the breakup at RHIC probably has elements of both volumelike breakup and surfacelike evaporation. Thus, the effect of collisions, which played a pivotal role in moving the distribution downward in Ref. [8], requires more study.

The second assumption inherent to these calculations is related to the negligence of finite-size effects. The enhancement factors applied to small-angle correlation studies are usually based on the outgoing wave function, $|\phi(\mathbf{q},\mathbf{r})|^2$ [43,44]. For large sources, there is a straightforward correspondence between the integrated wave functions and the phase shifts [45,46]:

$$\frac{1}{\pi}\sum_{\ell}(2\ell+1)\frac{d\delta_\ell}{dE}=\frac{qM}{8\pi^2}\int d^3r[|\phi(\mathbf{q},\mathbf{r})|^2-|\phi_0(\mathbf{q},\mathbf{r})|^2]. \tag{43}$$

Since the modification of the wave function is mostly confined to a region where $qR<\pi$, one expects that the treatments shown here should work well for $q>100$ MeV/c, or for masses greater than 400 MeV. For masses near threshold, a different approach, based on the actual scattered wave functions, would be warranted. Such an approach could also incorporate the effects of the Coulomb interaction between pions.

Finally, it should be emphasized that other correlations, besides those resulting from the change in the two-pion density of states, will play a role in any experimental measurement. Experimental analyses are typically based on a like-sign subtraction. This should eliminate global correlations such as elliptic flow which correlate same-sign and opposite-sign pairs equally. However, any correlation based on charge conservation should survive the subtraction [47,48]. For every π^+, there is a $\sim$75% chance that local charge conservation will result in an extra π^- being emitted with a similar rapidity. This should provide a bump in the like-sign-subtracted invariant-mass distribution that peaks for masses near 400 MeV. The ratio of the ρ peak in the like-sign subtracted distribution to the bump from charge correlation is approximately determined by the chance that a given π^+ had its last interaction with other hadrons through the decay of a ρ^0. This ratio should be smaller for central collisions since the breakup temperature is lower which reduces the ρ/π ratio.

The in-medium mass of the ρ might be altered by $\sim$20 MeV at breakup. Given that this peak is also spread out and distorted as shown in the calculations presented here, it is certainly challenging to isolate the contribution from the ρ and to quote a peak height to a better accuracy than 20 MeV. Upcoming runs at RHIC may increase the statistics by more than an order of magnitude. Thus, we believe that there remains a good chance that the ρ can be studied in detail, even in the central collision Au+Au environment.

Finally, we compare the experimentally observed mass shifts to results of our model. In Ref. [17] it was reported that the ρ shifts downward in pp collisions by $\sim$20 MeV at higher p_t while shifting downward by $\sim$45 MeV at low p_t. The shift appeared to be 5–10 MeV larger for high-muliplicity pp collisions and perhaps another 5 MeV lower for peripheral Au+Au reactions. A similar behavior for pp collisions had been reported for $\sqrt{s}=27$ GeV pp collisions [40]. The shifts that we extracted were as large 35 MeV, but these calculations assumed a lower temperature, 110 MeV, and a higher effective chemical potential, 90 MeV, than would be appropriate for pp phenomenology. For a temperature of 170 MeV, and zero chemical potential, the shift was in the range of 20 MeV, a somewhat smaller shift than what was observed by STAR. It appears that the experimental mass shift is 10–20 MeV stronger than what we would expect from our approach. But, before this discrepancy can be attributed to novel in-media phenomena, i.e., a mass shift of the ρ, it should be stressed that systematic uncertainties described in Ref. [17] are of the order of 10 MeV. This problem would be served well by both a higher statistics experimental analysis and a more detailed theoretical modeling. An improved calculation would consider finite-size effects, the influence of other resonances, and the effects of experimental acceptances and efficiencies.

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