Unified calculation of photon and pion spectra in intermediate energy heavy ion reactions

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We calculate the energy spectra of high energy photons and pions produced in heavy ion collisions at intermediate energies of around 75 MeV/nucleon. We use a unified approach based on the Boltzmann-Uehling-Uhlenbeck theory and the assumption that both pions and high energy gamma rays are produced in individual nucleon-nucleon collisions during the course of the heavy ion reaction. A comparison to existing data for energy spectra of neutral pions and high energy photons shows good agreement.

To what degree do collective processes contribute to the production of particles emitted in heavy ion reactions at intermediate energies? This is one of the basic questions in intermediate energy heavy ion reaction physics.[1] Early work by the Frankfurt group suggested collective phenomena as the production mechanism for high energy photons and pions in intermediate energy (≈20–100 MeV/nucleon) heavy ion collisions.[2] They parametrized the nuclear reaction in terms of the deacceleration time τ and obtained good fits to the experimental pion spectra. Another cooperative model was put forward by Shyam and Knoll[3] who considered the contributions of many-nucleon clusters to pion production. Using a time-dependent Hartree-Fock model and the assumption of collective production of high energy photons we found, on the other hand, that one is not able to reproduce the experimental cross sections in light systems.[4]

However, different groups have shown that it is also possible to explain the total subthreshold pion cross section under the assumption that the pions are produced in independent two-body collisions between target and projectile nucleons during the course of the heavy ion collision. Using the Boltzmann master equation,[5] the Boltzmann-Uehling-Uhlenbeck approach,[6] and a quantal phase-space approach,[7] different authors were able to reproduce the total pion cross section as a function of beam energy.

Up to now, however, nobody has been able to reproduce the pion energy spectra in these models based on individual nucleon-nucleon collisions.[1] In the only reported calculation, Blann[5] found the pion spectra which he calculated with his Boltzmann master equation approach much harder than the experimental ones. Neither Aichelin nor Cassing report pion energy spectra in their papers. However, nucleon-nucleon pion production models should also be able to reproduce the experimental energy spectra to be considered valid theoretical competitors to collective production models. In this paper, we will calculate the pion energy spectra by using a perturbative approach to the production process and by properly including the role of the Δ resonance.

Using the Boltzmann-Uehling-Uhlenbeck approach, we calculated the production cross sections of high energy photons in light heavy ion collisions with beam energies between 20 and 100 MeV/nucleon under the assumption that the basic process is a superposition of independent proton-neutron collisions: $p+n \rightarrow p+n+\gamma$.[8] We were able to show that the angular distributions and the energy spectra of the high energy photons, as well as the beam energy dependence of the total cross section, were in reasonable agreement with data. Later, Aichelin and Ko,[9] and Heuer *et al.*[10] performed similar calculations and confirmed our result: ***High energy photons (E_γ>40 MeV) are produced mainly in incoherent individual proton-neutron collisions.***

In this letter, we will extend these calculations to also include the description of the production of subthreshold π^0's. To do this, we use the same perturbative approach that was used in Ref. 8.

In this perturbative approach every nucleon-nucleon collision with a total nucleon-nucleon center-of-mass energy $\sqrt{s} > 2M_N + M_\pi$ contributes to the pion production cross section weighted by its probability of producing a pion. In the conventional implementation of pion production into the Boltzmann-Uehling-Uhlenbeck theory, all pions are produced with weight 1 and then propagated along with the nucleons in the system. This results in very low statistics for the produced pions at the subthreshold energies of interest here. Therefore the perturbative calculation yields far better statistics for the pions. This is of great importance, if the calculation of pion energy spectra is desired.

From solving the Boltzmann-Uehling-Uhlenbeck (BUU) equation for the phase space distribution function $f(\mathbf{r},\mathbf{p},t)$,

$$\frac{\partial}{\partial t} f(\mathbf{r},\mathbf{p},t) + \frac{\mathbf{p}}{m}\nabla_r f(\mathbf{r},\mathbf{p},t) - \nabla_r U \nabla_p f(\mathbf{r},\mathbf{p},t)$$
$$= \frac{g}{2\pi^3 m^2} \int d^3q_{1'}\, d^3q_2\, d^3q_{2'} \delta\left(\frac{1}{2m}(p^2+q_2^2-q_{1'}^2-q_{2'}^2)\right) \cdot \delta^3(\mathbf{p}+\mathbf{q}_2-\mathbf{q}_{1'}-\mathbf{q}_{2'}) \cdot \frac{d\sigma}{d\Omega}$$
$$\cdot \{\overline{f}(\mathbf{r},\mathbf{q}_{1'},t)\,\overline{f}(\mathbf{r},\mathbf{q}_{2'},t)\,[1-\overline{f}(\mathbf{r},\mathbf{p},t)]\,[1-\overline{f}(\mathbf{r},\mathbf{q}_2,t)]$$
$$-\overline{f}(\mathbf{r},\mathbf{p},t)\,\overline{f}(\mathbf{r},\mathbf{q}_2,t)\,[1-\overline{f}(\mathbf{r},\mathbf{q}_{1'},t)]\,[1-\overline{f}(\mathbf{r},\mathbf{q}_{2'},t)]\}, \quad (1)$$

one obtains a distribution of relative momenta between pairs of colliding nucleons. The numerical details of solving this equation can be found in Ref. 8. We have used the same mean field potential as in Ref. 8, which results in a nuclear compressibility of 240 MeV. The relative momentum distribution obtained in this way can be used to calculate the π^0 production cross sections in the following way.

Neutral pions can be produced in nucleon-nucleon collisions via the channels

$$\begin{aligned} &p+p \to p+p+\pi^0,\\ &p+n \to p+n+\pi^0,\\ &n+n \to n+n+\pi^0,\\ &p+n \to d+\pi^0. \end{aligned} \quad (2)$$

The production channel via the creation of a bound proton-neutron pair (deuteron) should be suppressed inside a colliding nuclear system as compared to a free nucleon-nucleon collision, and we have therefore neglected it in the present calculation. For the pp and np production channels, we have used existing experimental cross sections.[11] The nn nucleon-nucleon cross sections are obtained from the pp cross sections assuming isospin symmetry.

Following Mandelstam's model,[12] we assume that pion production at low energies is proceeding via the $(\frac{3}{2},\frac{3}{2})$ Δ resonance. The mass distribution of the Δ resonance is taken from Kitazoe *et al.*[13] and is given by

$$p(M_\Delta) = \frac{\frac{1}{4}\Gamma^2(q)}{(M_\Delta - M_0)^2 + \frac{1}{4}\Gamma^2(q)}, \quad (3)$$

where $M_0 = 1232$ MeV, and the width $\Gamma(q)$ of the resonance is parametrized as

$$\Gamma(q) = \frac{0.47q^3}{\left[1+0.6\left(\frac{q}{M_\pi}\right)^2\right] M_\pi^2}. \quad (4)$$

q is the momentum of the pion in the Δ rest frame.

The Δ is assumed to be produced isotropically in the nucleon-nucleon center-of-mass system, and we also assume that the decay $\Delta \to \pi + \mathrm{N}$ has an isotropic angular distribution in the Δ rest frame. The pion momentum in this frame is given as

$$q = \left[\left(\frac{M_\Delta^2 - M_N^2 + M_\pi^2}{2M_\Delta}\right)^2 - M_\pi^2\right]^{1/2}. \quad (5)$$

Our calculation therefore generates pairs of colliding nucleons via the solution of Eq. (1); their center-of-mass energy, $\sqrt{s}$, is then used to calculate a production probability distribution $P(M_\Delta, \mathbf{p}_\Delta)$ for Δ resonances with mass M_Δ and momentum $\mathbf{p}_\Delta$ using the experimental cross sections for the processes (2). The decay of the Δ resonance is then calculated using a Monte Carlo integration technique. This leads to a distribution of pion momenta in the Δ's rest frame which is finally Lorentz transformed into the laboratory frame.

Following Aichelin's approach, we do not attempt to calculate the effects of the reabsorption of Δs and πs in a dynamical way, but rather use a mean pion absorption length of $\lambda_0 = 3$ fm, independent of pion energy. Then the absorption probability is given by

$$P_{\mathrm{abs}} = 1 - \exp\left(-\frac{d}{\lambda_0}\right), \quad (6)$$

where d is the distance the π or the Δ had to travel inside nuclear matter. For the beam energies considered here, we find that the average of the absorption probability in the ^{12}C + ^{12}C system is always about 50%, consistent with Aichelin's results. This average number results in a reduction of the calculated cross sections by a factor of 2. The resulting angular dependence of the created pions is then isotropic in the center-of-mass sytem, and the shadowing effects on the angular distribution are not reproduced. However, to calculate this effect properly, a dynamical description of the pion and Δ absorption should be used. Work in this direction is in progress.

We compare the results of our calculations to the experimental results of Noll *et al.*,[14] who have measured the energy spectra of neutral pions in collisions of ^{12}C with targets of ^{12}C, ^{56}Ni, and ^{238}U at beam energies per nucleon of 60, 74, and 84 MeV. Here we only calculate the cross sections for the symmetric ^{12}C + ^{12}C system. As pointed out by Cassing,[7] the scaling behavior of the number of nucleon-nucleon collisions as a function of mass in asymmetric systems is given by a $(A_1A_2)^{2/3}$ dependence. This is in good agreement with the experimental results of Ref. 14 where the exponent is given as 0.65.

In Fig. 1, we compare the angle-integrated cross section for the production of π^0 for the system ^{12}C + ^{12}C

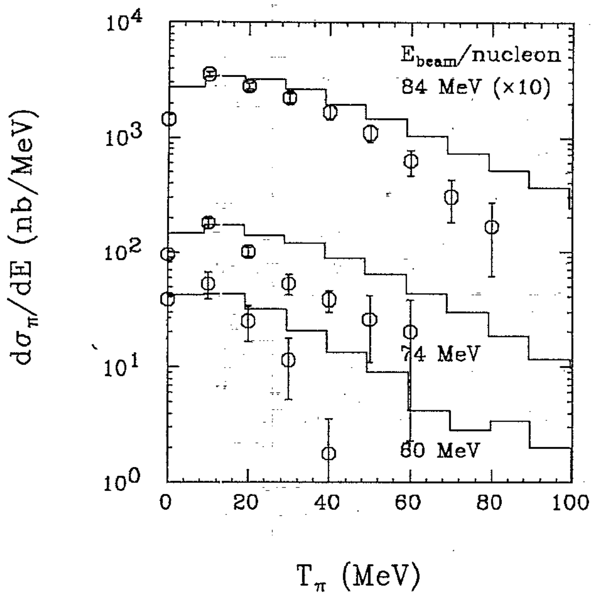

FIG. 1. Energy spectra of neutral pions from collisions of $^{12}C + ^{12}C$ at beam energies per nucleon of 60, 74, and 84 MeV. The histograms represent the data and are taken from Ref. 14. The plot symbols with statistical error bars are the result of our calculations.

to the experimental data of Ref. 14. Displayed is the cross section $d\sigma_\pi/dE'$ as a function of the kinetic energy of the pion, T_π, for the three beam energies per nucleon of 60, 74, and 84 MeV. The experimental data are represented by histograms, and the results of the calculation are shown via the octagons with their corresponding error bars. The error bars in the calculations represent the statistical errors due to the Monte Carlo integration procedure and due to the finite number of colliding test particle pairs in the numerical solution of Eq. (1). The experimental and theoretical results for the 84 MeV reaction are scaled up by a factor of 10 for better optical separation from the 74 MeV results.

It can be observed that the absolute cross sections are well reproduced in all three cases considered. This result was also obtained by Aichelin.[6] The calculated energy spectra are in rough agreement with the data. However, there is a systematic trend for the numerical results to have steeper slopes than the experimentally observed ones. The very high energy tails of the experimental data cannot be reproduced by the calculations, which are based on semiclassical momentum distributions and therefore lack the quantum mechanical high momentum components. For this reason, our calculated energy spectra cut off at $T_\pi = 40$, 60, and 80 MeV for the beam energies per nucleon of 60, 74, and 84 MeV, respectively.

It should be pointed out at this point that it is very important to calculate the pion production under inclusion of the role of the Δ resonance. If one calculates the pion spectra using the direct process $N+N \to N+N+\pi$, one finds much steeper spectra which are in strong disagreement with experiment.[15] While the total cross sections are not influenced by the specific process considered, the energy spectra are sensitive to the different kinematics of the direct and the Δ process. There are basically two reasons why the two processes yield different slopes of energy spectra. Firstly, the Δ is in a different spin-isospin state and is not subject to the Pauli principle inside nuclear matter consisting almost exclusively of protons and neutrons. Secondly, the process involving the Δ favors a kinematical situation in which a large fraction of the available nucleon-nucleon center-of-mass energy is converted into the mass of the produced Δ. This results in a different sharing of the total kinetic energy between the two final state nucleons and the produced pion in the two processes.

Even though we find that the dominant process for the production of pions and high energy photons is the individual nucleon-nucleon collision, it is unjustified to claim that no cooperative effects are in play at all. To obtain a measure for the deviation of our results from a picture of individual nucleon-nucleon collisions which are completely independent of the details of the reaction dynamics, we compare our results to a simple Fermi gas model.

The Fermi gas model was first used by Bertsch[16] to investigate threshold pion production. Here we use a simplified form of two homogeneously filled Fermi spheres in momentum space, the centers of which are separated by the beam momentum

$$\rho(\mathbf{p}) = \theta(p_F - |\mathbf{p}|)A_T + \theta(p_F - |\mathbf{p} - \mathbf{p}_b|)A_P, \qquad (7)$$

and the number of nucleon-nucleon collisions is given by[17]

$$\overline{N}(b) = \sigma_{nn} \int_{\mathcal{O}(b)} dx\, dy \int_{-\infty}^{\infty} dz_1\, dz_2\, \rho_T(x,y,z_1) \times \rho_P(x,y,z_2). \qquad (8)$$

Here $\mathcal{O}(b)$ is the coordinate space overlap of target and projectile, ρ_P and ρ_T are the coordinate space densities, and σ_{nn} is the total nucleon-nucleon scattering cross section (taken as 40 mb). The distribution of the relative momenta from this Fermi gas model is folded with the same elementary cross section for pion production that was used in the BUU calculations.

In Fig. 2, we display the results of the different models for the total neutral pion production cross section as a function of beam energy per nucleon between 50 and 100 MeV. It would be interesting to make the comparison at lower energies as well, but the numerical inaccuracies associated with the BUU approach prohibit the calculation of pion production below a beam energy per nucleon of ≈ 50 MeV. The plot symbols represent the experimental data of Noll *et al.*[14] The solid line shows the results of the BUU calculation. The two dashed lines are the result of the Fermi gas model using two different assumptions. The upper curve is calculated under the assumption that

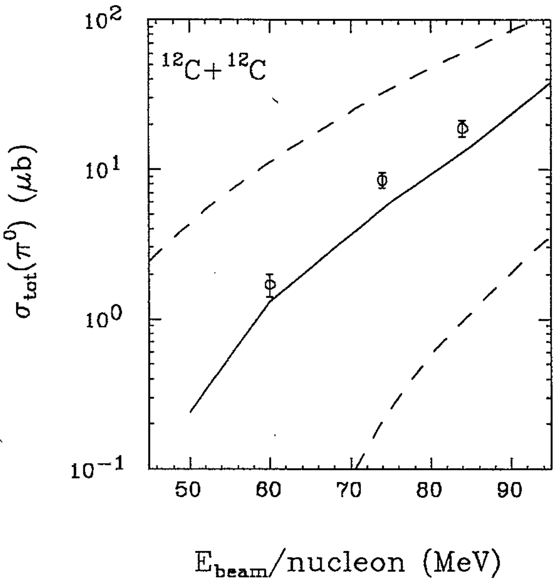

FIG. 2. Total cross section for the production of neutral pions from $^{12}C+^{12}C$ collisions as a function of the beam energy. The plot symbols with error bars are the experimental data of Ref. 14. The solid line is the result of the BUU calculation. For comparison, the two dashed lines represent the results of the Fermi gas model. The upper and lower curves stand for the result without and with final state Pauli blocking effects, respectively.

the final state phase space for the scattered nucleon in the process $N+N \rightarrow N+\Delta$ is unblocked. In the lower curve, it was assumed that the nucleon-nucleon collision is always forbidden when the final state momentum of the nucleon is inside one of the two original Fermi spheres. While the BUU calculations are in good agreement with the experimental excitation function, neither of the two models using the Fermi gas can reproduce the data.

The two main effects that enhance the result of the BUU calculation over the prediction of the Fermi gas model with final state Pauli blocking by almost an order of magnitude are as follows.

(1) A collective acceleration of the two nuclei due to the fact that the potential barrier between them is breaking down in the early penetration stages.

(2) The creation of empty phase space at $p_{c.m.}=0$ in the early stage of the collision due to the fact that the motion of the nuclear fluid is governed by the Liouville theorem. This effect was already discussed in Ref. 8 and is mainly responsible for the fact that high energy gamma rays are created in the early interpenetration stages of the nuclear collision. In later stages this phase-space hole is closed due to elastically scattered nucleons. A similar observation is made in the production of pions. However, due to reabsorption and rescattering of the pions, they cannot be used to probe the early compressional phase of the nucleus-nucleus collision.

The nucleons are deaccelerated via the mean field in the overlap region when the density exceeds ρ_0. Thus the relative momenta between colliding nucleons should be reduced, and this effect should be visible in the pion production cross section. This effect was first suggested by Stock *et al.* as a way to determine the nuclear compressibility.[18] However, in the beam energy region under investigation here this effect is suppressed. This is because the final state phase space for pion producing nucleon-nucleon collisions is almost completely closed by the time the two nuclei reach complete overlap. In spite of this, there is hope that future studies of subthreshold pion production can be used to investigate this question further, once the proper dynamical description of pion absorption is included in the calculations.

In summary, we have presented calculated energy spectra for neutral pions. They are in reasonably good agreement with experiment. Together with the results for high energy photon production, which use the same model for the nuclear dynamics and the elementary production process and which are also in good agreement with data,[8] we now have a unified picture of subthreshold particle production in heavy ion collisions at intermediate beam energies. The basic production process is the elementary nucleon-nucleon collision, but some effects of collectivity are found. Future theoretical and experimental studies may enable us to learn more about the nuclear equation of state from subthreshold particle production.

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