Momentum space simulation of proton spectra in heavy ion reactions

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A new and transparent model for the theoretical simulation of proton singles spectra from heavy ion collisions is introduced. Working with few assumptions about momentum space geometry in heavy ion collisions, this model is able to explain the double differential singles cross sections. Predictions for proton-proton coincidence data are also obtained in the framework of this model and compared with data.

To study medium energy heavy ion reactions, a modification of the Vlasov equation with the Uehling-Uhlenbeck collision integral [Vlasov-Uehling-Uhlenbeck (VUU) and Boltzmann-Uehling-Uhlenbeck (BUU)] recently has been brought forward.[1-3] In this approach the goal is to combine the concepts of mean field theories, such as the time dependent Hartree-Fock (TDHF) theory[4,5] which has been successful in describing single particle observables in low energy heavy ion collisions, and intranuclear cascade models[6,7] which have been applied successfully to high energy collisions where the effects of individual nucleon-nucleon collisions become dominant. At medium energies the effects of the mean field potential and of nucleon-nucleon collisions are both important. In this energy range (20–100 MeV/nucleon), the VUU approach has been reasonably successful in describing the shape of, for example, double differential proton spectra from ^{12}C-^{12}C collisions.[3]

However, the major drawback to the VUU/BUU approach is the large amount of computer time required to carry out numerical calculations based on VUU theory. This time arises from the fact that the trajectories of approximately 100 pseudoparticles per nucleon must be followed through the six-dimensional phase space.

We propose a model which is based on simple considerations of momentum space geometry. This model requires a small amount of computer time and thus enables us to generate many thousands of events. The results of the model are compared to the experimental data of ^{12}C-^{12}C collisions at $E_{\rm lab}=40$ MeV/nucleon beam energy which recently have been measured.[8]

Our model is presented in a nonrelativistic formulation which is sufficient for the energies under consideration, but is is straightforward to include the effects of relativistic kinematics. We begin by assigning every nucleon a random momentum p_i inside a sphere with radius $p_F(r)$ using local Thomas-Fermi approximation in the rest frame of its nucleus.

$$\begin{aligned} p_i &= (p_{x,i}, p_{y,i}, p_{z,i}) \\ &= (\xi_1,\xi_2,\xi_3)p_F(r) + (0,0,\alpha(2m_{\rm N}E/A)^{1/2}) . \end{aligned} \tag{1}$$

Here the beam axis is assumed to be parallel to the z axis. The quantities ξ_1, ξ_2, and ξ_3 are random numbers between -1 and 1 with the constraint

$$\xi_1^2+\xi_2^2+\xi_3^2 \le 1 . \tag{2}$$

The constant α is 0 for the target and 1 for the projectile. The local Fermi momentum is given by

$$p_F(r) = \hbar[1.5\pi^2\rho(r)]^{1/3} , \tag{3}$$

where

$$\rho(r) = \rho_0/\{1+\exp[(r-R_\alpha)/a]\} . \tag{4}$$

In Fig. 1(a) the occupation of the longitudinal and transversal momentum space resulting from this method are shown.

One can include the effects of nucleon-nucleon collisions in a simple way by considering only elastic collisions which largely dominate the cross section at the beam energies under investigation. The total number of individual nucleon-nucleon collisions, N, is treated as a free input parameter. As will be seen later, the spectra obtained are sensitive to N and thus allow a determination of the actual number of nucleon-nucleon collisions which have occurred during the nucleus-nucleus collision.

For each one of the N nucleon-nucleon collisions we randomly pick two nucleons i and j. Energy and momentum conservation for the collision require

$$(p_i^b)^2+(p_j^b)^2=(p_i^a)^2+(p_j^a)^2 , \tag{5}$$

$$p_i^b+p_j^b=p_i^a+p_j^a , \tag{6}$$

where the superscripts b and a stand for "before the collision" and "after the collision." In this geometry a collision is simply represented by a rotation of the relative momentum p_i-p_j around the nucleon-nucleon center of mass momentum $\frac{1}{2}(p_i+p_j)$ by a random pair of angles θ and ϕ. This treatment results in isotropic nucleon-nucleon cross sections in the nucleon-nucleon rest frame which is a reasonable approximation for these energies.

It is possible to reject nucleon-nucleon collisions, if either p_i^a or p_j^a ends up in an already occupied region of momentum space. This is the method with which Pauli blocking is treated in the VUU approach. However, for our purposes this approach is not used because every collision depopulates some area of already occupied momentum space and populates another area randomly.

Thus, on the average, the Pauli principle is not violated any more than in the random initialization procedure based on local Thomas-Fermi theory used above.

In Fig. 1(b) the population of transverse and longitudinal momentum space after $N=3$ random nucleon-nucleon collisions per nucleus-nucleus reaction is shown for the ^{12}C-^{12}C system at 40 MeV/nucleon. The most obvious effect is that higher transverse momenta are available after the collisions. Thus the isotropy in the single particle spectra will increase with N.

If a nucleon is to be emitted from the overlap region of the two nuclei, it has to escape from a mean field potential. This means that its kinetic energy has to be bigger than an energy V in the center of velocity frame. In our momentum space geometrical picture this corresponds to a sphere around p_{cv} with radius $p_{pot}=(2m_N V)^{1/2}$ from which nucleons cannot be emitted. All other nucleons will be emitted with an energy of

$$\begin{aligned} E_{kin} &= E_{kin}(0) - V \\ &= (p_x^2+p_y^2+p_z^2)/2m_N - V \\ &= Q^2/2m_N , \end{aligned} \tag{7}$$

where Q is the absolute value of the momentum vector of the particle after escaping from the potential.

The absolute number of particles which are able to escape from the potential is sensitive to the magnitude of V for collisions with a beam energy around the Fermi energy. So we should be able to investigate some properties of the nuclear mean field in finite systems by looking at the absolute normalization of the proton cross section $\sigma(E,\Omega)$.

To obtain the normalized cross sections we use a prescription similar to the one used in the fireball model.[9] The output of our numerical simulation is the number of emitted protons into an energy interval ΔE and an angle interval $\Delta\Omega$, $N(\Delta E,\Delta\Omega,b)$, as a function of the impact parameter b. We obtain the double differential cross section from this via

$$\frac{d^2\sigma}{dE\,d\Omega} = \int \frac{2\pi b N(\Delta E,\Delta\Omega,b)db}{N_{tot}\Delta E\,\Delta\Omega}(1-\lambda) . \tag{8}$$

Here the factor $(1-\lambda)$ is used to correct the absolute normalization of the spectra for the effect that a fraction λ of the escaping protons will be trapped in larger fragments and thus not be counted as single protons. The fraction λ is taken to be 0.5 based on experimental composite particle yields from Ref. 8. In principle our simulation could be combined with a coalescence part which should then also allow us to predict the yield of larger mass fragments. For the purpose of this investigation, however, we tried to keep the simulation simple and transparent and avoided the inclusion of composite fragments.

The depth of the mean field potential, V, is the second free parameter in our model. This parameter determines the overall normalization of the cross section. In the following we shall estimate the magnitude of this potential energy V on theoretical grounds. We start out with a standard Skyrme parametrization for the mean field potential[10]

$$U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^\sigma , \tag{9}$$

in which the exponent σ is determined by the desired nuclear compressibility and the coefficients A and B are then fixed by the requirement that the binding energy per nucleon in nuclear matter has a minimum of -16 MeV at density $\rho=\rho_0$. In Fig. 2 the values of A, B, and the compressibility κ are displayed as a function of the exponent σ.

Two particular choices for σ have recently received special attention. A value of $\sigma=\frac{7}{6}$ yields the so-called "soft" equation of state (EOS)

$$U(\rho) = -359\text{ MeV}\times(\rho/\rho_0) + 305\text{ MeV}\times(\rho/\rho_0)^{7/6} , \tag{10}$$

with a nuclear matter compressibility of 200 MeV. For a value of $\sigma=2$ we obtain the "stiff" equation of state

$$U(\rho) = -124\text{ MeV}\times(\rho/\rho_0) + 70\text{ MeV}\times(\rho/\rho_0)^2 , \tag{11}$$

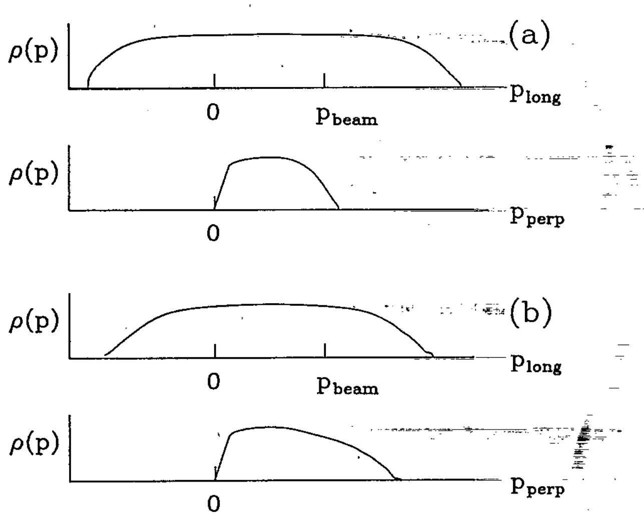

FIG. 1. Occupation of the longitudinal and transverse momentum space for the system $^{12}C+^{12}C$ at a beam energy of 40 MeV/nucleon before (a) and after $N=3$ nucleon-nucleon collisions. $\rho(p)$ is displayed on a logarithmic scale.

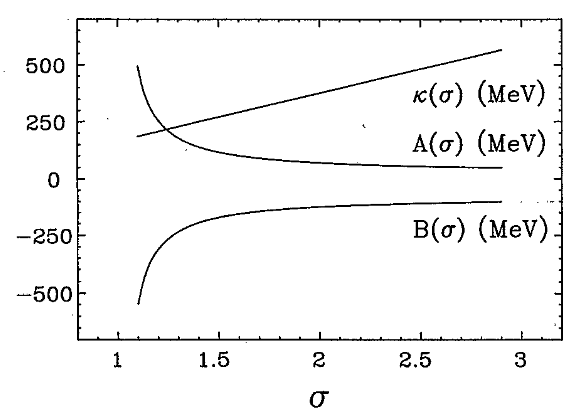

FIG. 2. The coefficients A and B in the Skyrme parametrization for the equation of state and the nuclear compressibility κ as a function of the exponent σ.

with a nuclear compressibility of 375 MeV. In the following we will use the latter to obtain numerical estimates, but the numbers do not change very much by using the soft EOS or any value of σ between $\frac{7}{6}$ and 2.

From $U(\rho)$ we obtain the average potential energy V by integration

$$V_\infty(\rho_{\min},\rho_{\max})=\frac{1}{\rho_{\max}-\rho_{\min}}\int_{\rho_{\min}}^{\rho_{\max}} U(\rho)d\rho \ . \quad (12)$$

Taking $\rho_{\min}=0$ and $\rho_{\max}=\rho_0$ we obtain the nuclear matter value of the mean potential energy

$$V_\infty(0,\rho_0)=-39 \text{ MeV}=-\frac{3}{5}\frac{\hbar^2}{2m_N}k_{F,\infty}^2+\frac{E_{\text{bind}}(\infty)}{A} \ . \quad (13)$$

For finite nuclei we have to use the actual binding energy, e.g., $E_{\text{bind}}(^{12}\text{C})/A=-8$ MeV, instead of the nuclear matter value of $E_{\text{bind}}(\infty)/A=-16$ MeV to determine V. From the fact that local Thomas-Fermi theory was used to initialize the occupation of momentum space, we also get a correction for the average kinetic energy value of nuclear matter,

$$\bar{E}_{\text{kin}}=\tau^{-1}\int_\tau \frac{3}{5}\frac{\hbar^2}{2m_N}k_F^2(r)d^3r \ . \quad (14)$$

For the case of ^{12}C this results in $\bar{E}_{\text{kin}}(^{12}\text{C})=18$ MeV instead of the value of $\bar{E}_{\text{kin}}=23$ MeV for infinite nuclear matter. So we obtain for the average potential energy of a ^{12}C nucleon

$$V(0,\rho_0)=-26 \text{ MeV} \ . \quad (15)$$

This simple estimate can be compared to a self-consistent Hartree-Fock calculation for ^{12}C which yields the values $\bar{E}_{\text{kin}}=15.87$ MeV and $V(0,\rho_0)=-24.06$ MeV.

During the collision, it is not a good approximation to use $\rho_{\min}=0$ and $\rho_{\max}=\rho_0$ for the overlap region. $\rho_{\min}=0$ would correspond to a complete disintegration of the overlap region which is unlikely for a beam energy of 40 MeV/nucleon. Also the value for $\rho_{\max}$ has to be corrected, since we expect the maximum density in the overlap region to exceed ρ_0. Since these minimum and maximum values of the density are not exactly known, we choose another approach and treat the proper value of the average potential energy for the overlap region as an input parameter into out simulation. Using the arguments given above, we expect the value of V roughly to be between 20 and 35 MeV. By comparing to the experimental data we find that a value of $V=20$ MeV yields the correct absolute normalization for the proton cross section.

In our momentum space geometrical simulation, the only other input parameter besides the average potential energy is the average total number of individual nucleon-nucleon collisions per nucleus-nucleus reaction. A large number of collisions will lead to a thermalized system, whereas a smaller number of collisions will cause steeper energy spectra. In Fig. 3 we show the comparison of our model predictions with an impact parameter

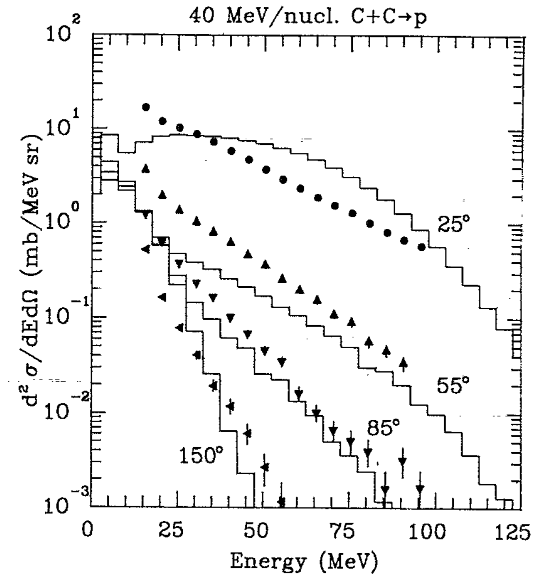

FIG. 3. Comparison of the results of our calculations (histograms) to the experimental data from Ref. 8 for the double differential proton cross sections from ^{12}C+^{12}C collisions at 40 MeV/nucleon as a function of proton kinetic energy for the proton emission angles of 25, 55, 85, and 150 deg.

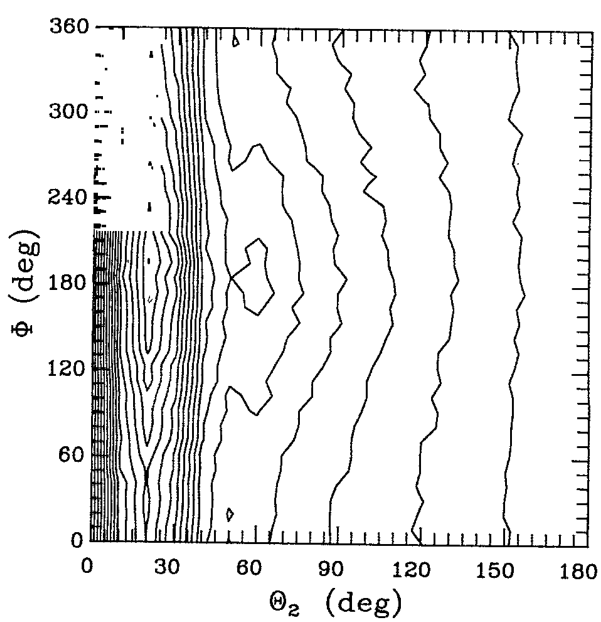

FIG. 4. $\bar{\sigma}=\int \sigma(E,\theta_2,\phi)dE$ as a function of the two angles θ_2 and ϕ. θ_2 is the emission angle of the second particle with respect to the beam axis, and ϕ is the relative azimuthal angle between the trigger particle and the second particle. The contour plot is the result of our calculation.

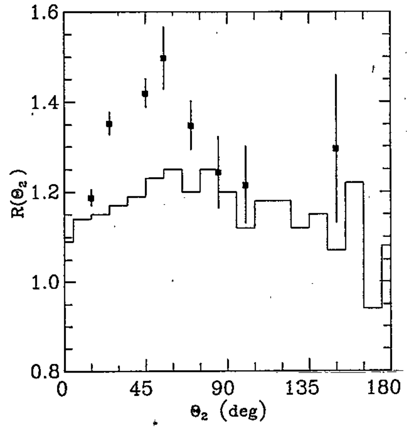

FIG. 5. The in-plane to out-of-plane ratio $R(\theta_2)$ as a function of the emission angle θ_2 with respect to the beam axis. The histogram represents our calculations. The data are taken from Ref. 8 and are represented by squares.

averaged number of $N=3$ unblocked nucleon-nucleon collisions per event with the experimental data from Ref. 8 for double differential proton spectra from 40 MeV/nucleon $^{12}C+^{12}C$ collisions as a function of proton energy for the proton emission angles 25, 55, 85, and 150 deg. The theoretical simulation shows a degree of agreement with the data which is surprising considering the simplifying assumptions going into the model. The magnitudes of the experimental cross sections are well reproduced. The relative yields for different angles as well as the angular dependence of the falloff constant in the observed energy spectra are in agreement with the experimental values and thus well understood. However, due to the fact that we do not include evaporation from exited target and projectile residues our calculations underpredict the data at low proton energies.

Recently, measurements of particle-particle correlations in heavy ion collisions at intermediate energies have been performed.[8,11-13] In light systems (C+C and N+C) it has been shown that the two-particle correlations are dominated by momentum conservation. The correlation functions show a strong preference for the two particles to be emitted in the same plane and on opposite sides of the beam.[8,13] Since total momentum is conserved in our model simulation, it is possible to study exactly this effect. We again use the example of a ^{12}C-^{12}C collision at E_{beam}/nucleon=40 MeV and compare our results to the experimental data of Ref. 8. In the experiment a trigger detector was used at a laboratory angle $\theta=45°$ relative to the beam axis. The momentum vector of a particle detected in this detector and the beam momentum define a plane. In the experiment the in-plane to out-of-plane ratio for a second particle emitted with an angle θ_2 relative to the beam axis and an angle ϕ relative to the plane was measured. This ratio is defined as

$$R(\theta_2)=\frac{\int \sigma(E,\theta_2,\phi=180°)dE}{\int \sigma(E,\theta_2,\phi=90°)dE} .$$

In Fig. 4 we show $\bar{\sigma}=\int \sigma(E,\theta_2,\phi)dE$ as a function of the two angles θ_2 and ϕ. Two features are visible. First, due to the fact that the calculations, as well as the experiment, are done in the laboratory system, we observe a strong enhancement of the counts in forward direction. Another feature is that it is almost completely symmetric with respect to $\phi=180°$, as expected. For fixed θ_2 the graph has maxima around ϕ 180°. This effect will cause the in-plane to out-of-plane ratio $R(\theta_2)$ to be larger than 1. In Fig. 5 the function $R(\theta_2)$, as calculated from the results of our model simulation, is compared to the experimental results from Ref. 8. The calculations show a smaller value of R than the experiments, but it seems that our model can nicely explain the tendency of the data by only using momentum conservation.

In summary, we have presented a simple model which enables us to understand the experimental features of proton spectra from heavy ion collisions at beam energies per nucleon around the Fermi energy. Its main advantages are the transparency of its concepts and the very small CPU-time requirements which enable us to generate statistics which are comparable to the experimental ones. Since momentum and energy are exactly conserved in every nucleon-nucleon collision, we are also able to investigate proton-proton coincidences in the light of momentum conservation. The results of our simulation for the double differential single particle cross section as well as the values for the in-plane to out-of-plane ratios are in reasonable agreement with the data.

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[1]G. Bertsch, H. Kruse, and S. Das Gupta, Phys. Rev. C **29**, 673 (1984).
[2]H. Kruse, B. Jacak, and H. Stoecker, Phys. Rev. Lett. **54**, 289 (1985).
[3]J. Aichelin and G. Bertsch, Phys. Rev. C **31**, 1730 (1985).
[4]K. T. R. Davies, R. S. Devi, S. E. Koonin, and M. R. Strayer, in *Heavy Ion Science,* edited by D. A. Bromley (Plenum, New York, 1984).
[5]J. Negele, Rev. Mod. Phys. **54**, 913 (1982).
[6]J. Cugnon, Phys. Rev. C **22**, 1885 (1980).
[7]Y. Yariv and Z. Fraenkel, Phys. Rev. C **29**, 2227 (1979).
[8]D. Fox, D. A. Cebra, J. Karn, C. Parks, G. D. Westfall, and W. K. Wilson, Phys. Rev. C **36**, 640 (1987).
[9]J. Gosset, H. H. Gutbrod, W. G. Meyer, A. M. Poskanzer, A. Sandoval, R. Stock, and G. D. Westfall, Phys. Rev. C **16**, 629 (1977); G. D. Westfall, J. Gosset, P. J. Johansen, A. M. Poskanzer, W. G. Meyer, H. H. Gutbrod, A. Sandoval, and R. Stock, Phys. Rev. Lett. **37**, 1202 (1976).
[10]G. Bertsch, in *Frontiers in Nuclear Dynamics,* edited by R. A.

Broglia and C. H. Dasso (Plenum, New York, 1985).
[11]M. B. Tsang, W. G. Lynch, C. B. Chitwood, D. Fields, D. R. Klesch, C. K. Gelbke, G. R. Young, T. C. Awes, R. L. Ferguson, F. E. Obenshain, F. Pasil, and R. L. Robinson, Phys. Lett. **148B**, 265 (1984).
[12]P. Kristiansson, L. Carlen, H.-Å. Gustafsson, B. Jakobsson, A. Oskarsson, H. Ryde, J. P. Bondorf, O.-B. Nielsen, G. Lovhoiden, T.-F. Thorsteinsen, D. Heuer, and H. Nifenecker, Phys. Lett. **81B**, 147 (1979).
[13]C. B. Chitwood, D. J. Fields, C. K. Gelbke, D. R. Klesch, W. G. Lynch, M. B. Tsang, T. C. Awes, R. L. Ferguson, F. E. Obenshain, F. Plasil, R. L. Robinson, and G. R. Young, Phys. Rev. C **34**, 858 (1986).