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LIGHT PARTICLE CORRELATIONS IN HEAVY ION COLLISIONS*

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Abstract: The existing BUU theory of heavy ion collisions is modified to conserve total momentum and retain the two-particle correlations induced by the collision term. In this framework the two-proton correlation function is investigated for heavy ion collisions. For light heavy-ion systems (^{16}O, ^{12}C) and intermediate beam energies (25-85 MeV/nucleon) the experimental features of the correlation function are reproduced.

During the last few years many experiments investigating correlated light particle emission in heavy ion reactions have been performed for intermediate [1-6]) and high beam energies [7,8]). For high energies the dominant contribution to the proton-proton correlation function is found to be the direct knock-out [7]). In this energy domain correlations between protons emitted with nearly equal momentum can be used to determine size and lifetime of the collision zone [9,10]). For intermediate beam energies (20-200 MeV/nucleon) the importance of the conservation of total momentum has been pointed out [6,11,12]).

Theoretical models based on the assumption of local thermal equilibrium like the "fireball" model [13,14]) as well as models based on semiclassical transport equations ("BUU", "VUU") [15-18]) have been quite successful in reproducing protons singles spectra in heavy ion collisions at the energies in question. However, neither in the fireball model nor in BUU/VUU total momentum is explictly conserved. Thus both approaches cannot address the question of the importance of this conservation law for light particle correlations.

In this paper we modify the equations used in the BUU-approach to take momentum conservation into account. Our starting point is the BUU equation:

$$\frac{\partial f_1}{\partial t}-\nabla_p f_1 \cdot \nabla_r U+\nabla_r f_1 \cdot \boldsymbol{p}/m=\frac{4}{(2\pi)^3}\int \mathrm{d}^3p_2\,\mathrm{d}^3p_3\,\mathrm{d}\Omega\, v_{12}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,\delta^3(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_3-\boldsymbol{p}_4)$$
$$\times(f_3f_4(1-f_1)(1-f_2)-f_1f_2(1-f_3)(1-f_4)) \qquad (1)$$

for the time evolution of the phase space density $f_i=f(\boldsymbol{r}_i,\boldsymbol{p}_i,t)$. In (1) v_{12} is $|\boldsymbol{p}_1-\boldsymbol{p}_2|/m$ and the integration $\mathrm{d}\Omega$ is performed over the relative angles between $\boldsymbol{p}_1-\boldsymbol{p}_2$ and

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$p_3 - p_4$. Commonly this equation is solved in the test particle simulation which results in the set of coupled differential equations:

$$\frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t} = \boldsymbol{p}_i / m_i ,$$

$$\frac{\mathrm{d}\boldsymbol{p}_i}{\mathrm{d}t} = -\boldsymbol{\nabla}_r U(\rho(\boldsymbol{r}_i)) + C(\boldsymbol{p}_i) ,$$

$$i = 1, \ldots, n \cdot (A_\mathrm{T} + A_\mathrm{P}) \tag{2}$$

for the test particles. A_T and A_P are the target and projectile mass. n is the number of test particles per nucleon. This number should for practical purposes at least be 100 to guarantee a smooth spatial behaviour of the density ρ. The term $C(\boldsymbol{p}_i)$ represents the contribution of nucleon-nucleon collisions to the change of the momenta of the test particles. The collisions are treated using a cascade approach [19]) in which the NN cross section is parametrized as a function of energy and angle in the NN center-of-mass system. The effects of the Pauli principle for the final states of the nucleons after the collision are also taken into account (compare reference [20])). The mean field potential is taken from a density dependent Skyrme parametrization [20,21])

$$U(\rho) = -218\ \mathrm{MeV} \cdot \frac{\rho}{\rho_0} + 164\ \mathrm{MeV} \cdot \left(\frac{\rho}{\rho_0}\right)^{4/3} . \tag{3}$$

The corresponding equation of state in the one-body limit yields a nuclear matter binding energy of -15.75 MeV, saturates at $\rho = \rho_0$, and yields a nuclear compressibility of $\kappa = 235$ MeV. There is no Coulomb interaction included. Since we will only apply our calculations to small systems, the Coulomb interaction is not too important. The potential used does not properly describe the nuclear surface. However, it turns out in the calculations that impact parameters around $\frac{1}{2} b_{\max}$ deliver the maximum contribution to the observables extracted. The peripheral reactions which are not properly described with the potential of eq. (3) do not contribute very much to, for example, the total proton cross section.

Whereas total momentum is exactly conserved in the NN collisions this is not the case for the interaction of the individual test particles with the mean field $U(\rho)$ resulting in small fluctuations of the total momentum $\boldsymbol{P}$ of the system. Furthermore, different subsets of $(A_\mathrm{T} + A_\mathrm{P})$ test particles can exchange momentum which results in a wide spread of total momenta $\boldsymbol{\Pi}_l$ for these subsets l around the value $\boldsymbol{P}/n$.

Here we propose a modification of the set of equations (2) which also results in a slight physical reinterpretation. Instead of propagating $n(A_\mathrm{T} + A_\mathrm{P})$ test particles, we simultaneously propagate n events of $A_\mathrm{T} + A_\mathrm{P}$ nucleons each. The phase space density $f(\boldsymbol{r}, \boldsymbol{p}, t)$ is averaged over all n events to assure the same accuracy in the evaluation of the mean field potential and the Pauli-blocking factors $(1 - f(\boldsymbol{r}, \boldsymbol{p}, t))$ as in the conventional BUU approach. In this reinterpretation the cascade part remains practially unchanged. However, in deriving the BUU equation, two-particle correlations are calculated perturbatively to obtain the collision integral. These correlations will be included in the modified BUU theory by restricting collisions to collisions between particles in the same event simulation. It is now possible to enforce conservation of momentum in every event separately at every time instant via:

$$\frac{\mathrm{d}\boldsymbol{r}_j}{\mathrm{d}t} = \boldsymbol{p}_j/m_j\,,$$

$$\frac{\mathrm{d}\boldsymbol{p}_j}{\mathrm{d}t} = -\nabla U(\rho(\boldsymbol{r}_j)) + \boldsymbol{q} + C(\boldsymbol{p}_j)\,,$$

$$\boldsymbol{q} = \frac{\sum_{k=1}^{A_\mathrm{T}+A_\mathrm{P}} \nabla U(\rho(\boldsymbol{r}_k))}{A_\mathrm{T}+A_\mathrm{P}}$$

$$j = 1, \ldots, (A_\mathrm{T}+A_\mathrm{P})\,. \tag{4}$$

It is easily verified that indeed

$$\boldsymbol{\Pi}_l = \sum_{j=1}^{A_\mathrm{T}+A_\mathrm{P}} \boldsymbol{p}_j = \text{constant}\,. \tag{5}$$

The correction introduced via $\boldsymbol{q}$ is merely a shift of the total momentum of the system and compensates for the error made in the evaluation of the equations (2). BUU calculations with sets of ensembles were already performed before [15]), but without enforcing conservation of total momentum.

The ansatz of eq. (4) violates energy conservation. A similar correction could be introduced to assure the explicit conservation of total energy of the system. However, in numerical studies we found that over a time interval of 200 fm/c total energy was conserved down to 1 MeV/nucleon. This value was not much different from what we found using an original BUU calculation. Thus this effect is negligible in the beam energy region of interest in this paper.

We have compared our calculations to experimental data for the systems 25 MeV/nucleon $^{16}O+^{12}C$ [ref. [3])], 40 MeV/nucleon $^{12}C+^{12}C$ [ref. [2])], and 85 MeV/nucleon $^{12}C+^{12}C$ [ref. [4])]. It would be very much desirable to also compare to heavier systems. However, to extract proton-proton correlation functions with reasonably small statistical errors, we need to generate a large number of events. This number is of the order of 10 000 to 100 000. In addition, the emission of protons is not limited to the early phase of the heavy ion reaction like (for example) the emission of photons [ref. [20])]. In all experiments under consideration contributions of projectile evaporations were not negligible. Therefore every event has to be traced for a comparatively long time. Both of these requirements result in a rather large consumption of CPU time and have limited us to the study of these smaller systems.

Before we approach two-proton correlations we wish to present our simulation of the singles spectra. In fig. 1, we compare our results (histograms) to the experimental data for the reaction 25 MeV/nucleon $^{16}O+^{12}C \rightarrow p+X$ as a function of

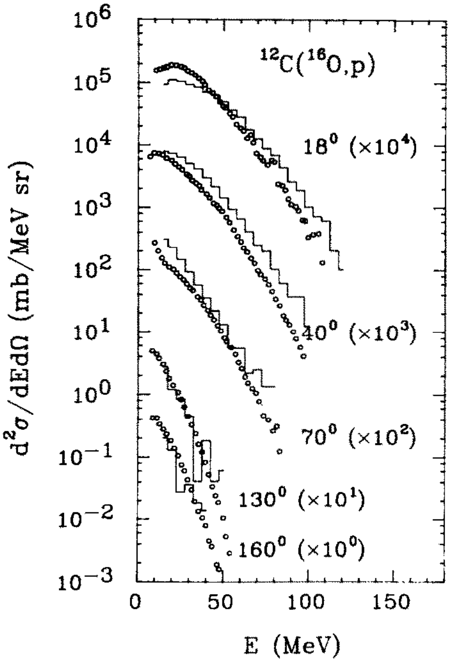

Fig. 1. Double differential proton cross sections for the reaction 25 MeV/nucleon $^{16}O+^{12}C$. Our calculations (histograms) are compared to the data (circles) from ref. [3]).

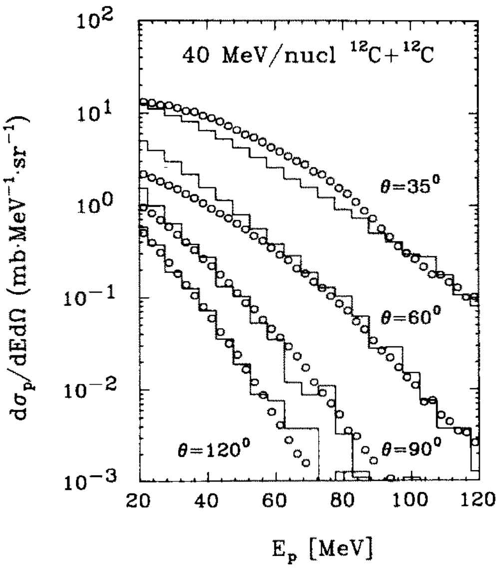

Fig. 2. Double differential proton cross sections for the reaction 40 MeV/nucleon $^{12}C+^{12}C$. The calculations are represented by histograms and the data (circles) are taken from ref. [2]).

proton energy and emission angle θ. In fig. 2, a similar comparison is made for the reaction 40 MeV/nucleon $^{12}C+^{12}C \rightarrow p+X$. For each figure 40 000 events were generated.

It can be seen from figs. 1 and 2 that the overall normalization of the cross section is well reproduced. The angular and energy dependences of the experimental cross sections agree with our calculations in all cases.

We define the two-particle correlation function

$$C(\theta_1, \theta_2, \phi, \Delta E_1, \Delta E_2)$$
$$= \frac{\sigma_R \cdot \sigma_{12}}{\sigma_1 \cdot \sigma_2}$$
$$= \sigma_R \cdot \int_{\Delta E_1} dE_1 \int_{\Delta E_2} dE_2 \frac{d^4\sigma_{12}(E_1, E_2, \theta_1, \theta_2, \phi)}{dE_1\, d\Omega_1\, dE_2\, d\Omega_2} \Big/ \int_{\Delta E_1} dE_1 \frac{d^2\sigma_1(E_1, \theta_1)}{dE_1\, d\Omega_1}$$
$$\times \int_{\Delta E_2} dE_2 \frac{d^2\sigma_2(E_2, \theta_2)}{dE_2\, d\Omega_2}. \tag{6}$$

In this definition $\sigma_R = \pi \cdot (R_T + R_P)^2$ is the total reaction cross section. ΔE_1 and ΔE_2 are the energy intervals over which the cross sections are integrated. ϕ is the relative azimuthal angle between the two coincident particles.

It is useful to write eq. (6) in terms of emission probabilities ε. With

$$\sigma_{12} = \sigma_R \int_{\Delta E_1} dE_1 \int_{\Delta E_2} dE_2 \frac{d^4\varepsilon_{12}(E_1, E_2, \theta_1, \theta_2, \phi)}{dE_1\, d\Omega_1\, dE_2\, d\Omega_2}, \tag{7}$$

$$\sigma_i = \sigma_R \int_{\Delta E_i} dE_i \frac{d^2\varepsilon_i(E_i, \theta_i)}{dE_i\, d\Omega_i}; \qquad i = 1, 2, \tag{8}$$

eq. (6) reads:

$$C(\theta_1, \theta_2, \phi, \Delta E_1, \Delta E_2)$$
$$= \int_{\Delta E_1} dE_1 \int_{\Delta E_2} dE_2 \frac{d^4\varepsilon_{12}(E_1, E_2, \theta_1, \theta_2, \phi)}{dE_1\, d\Omega_1\, dE_2\, d\Omega_2} \Big/ \int_{\Delta E_1} dE_1 \frac{d^2\varepsilon_1(E_1, \theta_1)}{dE_1\, d\Omega_1}$$
$$\times \int_{\Delta E_2} dE_2 \frac{d^2\varepsilon_2(E_2, \theta_2)}{dE_2\, d\Omega_2}. \tag{9}$$

In case of statistically independent emission of particle 1 and 2 the emission probability ε_{12} factorizes

$$\varepsilon_{12}(E_1, E_2, \theta_1, \theta_2, \phi) = \varepsilon_1(E_1, \theta_1)\varepsilon_2(E_2, \theta_2) \tag{10}$$

and it is then obvious from eq. (9) that for this case

$$C(\theta_1, \theta_2, \phi, \Delta E_1, \Delta E_2) \equiv 1. \tag{11}$$

Values of $C<1$ mean a suppression and $C>1$ an enhancement of the emission of particle-2 into the phase space region $\{\theta_2, E_2\}$ due to the coincident emission of particle-1 into the phase space region $\{\theta_1, E_1\}$.

For intermediate energy systems there are plenty of data available. For the reaction 25 MeV/nucleon $^{16}O+^{12}C \rightarrow 2p+X$ we have compared our results to the experimental data [3]. In this experiment, a trigger detector was placed at an angle $\theta_1=40°$ with respect to the beam axis. A second detector was placed at the angles $\theta_2=15°$, 40°, and 70° with respect to the beam axis. Its azimuthal angle ϕ relative to detector 1 was varied between 0° and 180° to obtain azimuthal correlations. The energy intervals used for both detectors were $\Delta E_1=\Delta E_2=[36\text{ MeV}, 120\text{ MeV}]:=E_{36}$. In fig. 3 the experimental results are represented by circles and the histograms stand for the corresponding calculations.

The most obvious feature from this figure is the fact that the two-proton correlation function is always smaller than 1. The emission of a second proton is always suppressed due to the emission of the first one. Adopting the language of statistical mechanics one can say that this result is a microcanonical effect due to the finiteness of the number of particles and available energy. Naively one could argue that at a beam energy of 25 MeV/nucleon there are 400 MeV total kinetic energy available and the emission of a single proton with an energy of, for example, 40 MeV could

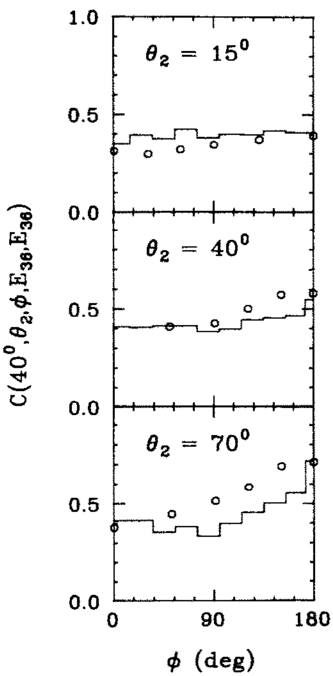

Fig. 3. Two proton correlation function C as a function of relative azimuthal angle ϕ for three different angles θ_2 and $\theta_1=40°$ for the reaction 25 MeV/nucleon $^{16}O+^{12}C$. The data (circles) are taken from ref. [3].

only be considered as a small perturbation. However, one has to keep in mind that not all of the 400 MeV kinetic energy of the projectile is thermalized, especially not in peripheral collisions. The experimentally observed anticorrelation is well reproduced by the calculations for all three angles.

The data also show an increasing value of the correlation function of the two emitted protons with increasing relative azimuthal angle ϕ between them. This effect is due to the conservation of overall momentum. Due to the emission of the trigger particle the remaining system experiences a 'recoil'. This experimental result is also reproduced in the calculations. For $\theta_2=15°$ and $\theta_2=40°$ the agreement is very satisfying. For $\theta_2=70°$ we see differences of up to 0.15 in the values of the experimentally and theoretically extracted correlation functions. This could be partially due to the poor statistics for this angle in the theoretical study.

In two other experiments [2,4]) the in-plane/out-of-plane ratio $R(\theta_1, \theta_2, \Delta E_1, \Delta E_2)$ was investigated. This ratio is defined as

$$R(\theta_1, \theta_2, \Delta E_1, \Delta E_2) := \frac{C(\theta_1, \theta_2, 180°, \Delta E_1, \Delta E_2)}{C(\theta_1, \theta_2, 90°, \Delta E_1, \Delta E_2)} . \tag{12}$$

In both experiments the system $^{12}C+^{12}C$ was investigated. The trigger detector was placed at $\theta_1=45°$, and θ_2 was varied. In fig. 4 the beam energy was 40 MeV/nucleon. The low-energy cutoff for both detectors was 10 MeV. The data were taken from reference [22]) and are represented by the symbols. Our calculation (histogram) can roughly reproduce the experimental findings. The ratio R is always between 1 and 1.5. In fig. 5 the same quantity is plotted for the beam energy 85 MeV/nucleon. In this experiment [4]) a low energy cutoff $\Delta E_1=35$ MeV was used for the trigger detector and $\Delta E_2=55$ MeV for the other detector. Again the calculations roughly reproduce the magnitude of the experimental ratio for all angles θ_2.

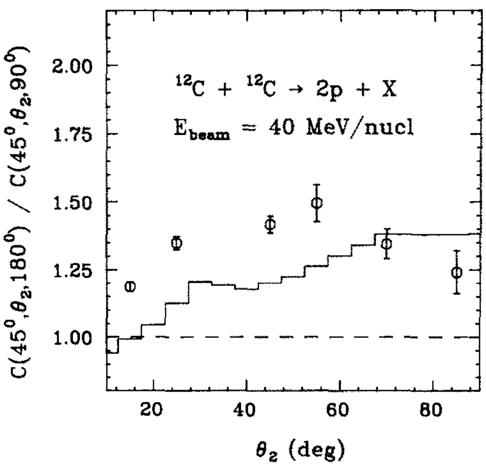

Fig. 4. In-plane/out-of-plane ratio as a function of θ_2 for $\theta_1=45°$ for the reaction 40 MeV/nucleon $^{12}C+^{12}C$. Our calculations (histograms) are compared to the data (circles) from ref. [22]).

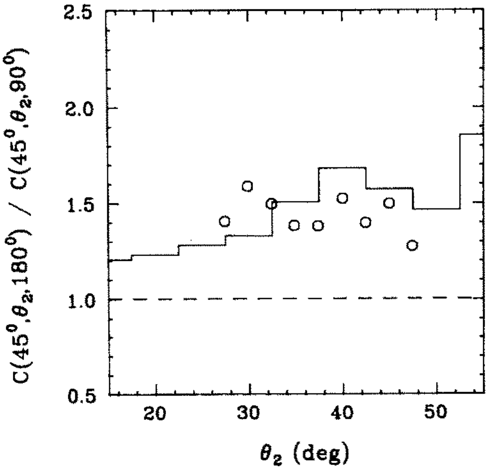

Fig. 5. In-plane/out-of-plane ratio as a function of θ_2 for $\theta_1 = 45°$ for the reaction 85 MeV/nucleon $^{12}C + ^{12}C$. The data from ref. [4]) are represented by circles and our calculations by histograms.

In both cases the fact that the in-plane/out-of-plane ratio is bigger than 1 has to be attributed to the effects of the overall momentum conservation. A BUU-calculation without the explicit conservation of momentum gave values of R which did not deviate significantly from 1.

In both systems there is no particular enhancement of the ratio R at $\theta_2 = 45°$ visible which would correspond to a quasielastic pp scattering. In reference [4]) it is estimated that such a pure quasielastic pp scattering process would give values of R up to 9. In both experiments no such enhancement was observed. This also agrees with our calculations. This suppression of the quasielastic pp scattering contribution might surprise at first, but the reason is probably to be found in the wide distributions of NN center-of-mass momenta for individual collisions due to the Fermi motion of the nucleons. In all of the reactions considered, the Fermi spheres of target and projectile had at least partial overlap. For higher beam energies the direct knock-out component should be relatively enhanced and become visible both in the calculations and the experimental data.

To sum up we can state the following: The observed in-plane/out-of-plane ratios can be understood in terms of overall momentum conservation for the emitting system. The two-proton correlation function mainly reflects effects of this conservation law and the finiteness of the available energy. In the reactions considered, there are no obvious signatures for the presence of a direct knock-out component as expected from a pure quasielastic pp scattering. The calculations agree with all of these experimental findings. Thus we can also conclude that in the mass and beam energy range investigated here the effects of additional two-particle correlations in the interaction (which are beyond the scope of a mean field theory like the one applied here) are unimportant for the observables considered so far.

It would be interesting, however, to perform the following experiment in which the pure effects of momentum conservation could be filtered out and other types of correlations could become important. With identical detector setups and beam energies one could measure $\alpha+\alpha \rightarrow p+p+X$, $^{12}C+^{12}C \rightarrow t+t+X$, and $^{16}O+^{16}O \rightarrow \alpha+\alpha+X$. In all these reactions the fraction of mass carried away by the two coincident particles is the same. Choosing the right detector energy intervals, one could also fix the mean momentum carried away by the emitted particles to be the same in all three cases. By comparing the results of these three setups one could eliminate the effects of the conservation laws and possibly detect other types of correlations.

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