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Hadronic Transport Properties in Intermediate Energy Heavy Ion Collisions

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Abstract

We introduce a set of coupled transport equations for nucleons, pions, and Deltas for the description of hadronic matter in heavy ion collisions up to beam energies per nucleon of 2 GeV. By investigating pion production, pion transverse momentum spectra, baryon collective flow, and two-proton intensity interferometry, we are able to obtain insight on the sensitivity of these observables to the parameters of the nuclear equation of state and to nuclear transport properties.

1. TRANSPORT THEORY

A theoretical description of heavy ion reactions in terms of transport equations for nucleon phase space distribution functions has proven to be immensely successful during the last five years [1–4]. These transport equations predict the change of the nucleon phase space distribution function due to the the one-body effect of nucleons moving in the collectively generated mean field potential of all other nucleons and due to the two-body effect of hard collisions between pairs of individual nucleons. At beam energies around 10–1000 MeV/nucleon, this theory has reproduced many experimental observables such as spectra of emitted nucleons, collective nuclear deflections, nuclear stopping, and particle production. However, for beam energies above the pion production threshold, a consistent theory incorporating the pionic degrees of freedom as well as relativistic effects is called for, and during the last few years a number of such theories has been proposed. For a list of references see [5].

We have recently put forward one such extension of nuclear transport theory [5] from the nucleonic level to the hadronic level by including pions and Δ-resonances in our description. We do this by starting out with Lagrangian of the Walecka type, including σ, ω, and π mesons. We integrate the Euler-Lagrange equations of motion over the degrees of freedom of the virtual mesons and obtain a field description for the baryon dynamics. Our theory also contains real pions, and we are able to incorporate the formation and decay of Δ resonances. Our final set of coupled transport equations is:

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$$
\begin{aligned}
\frac{\partial f_b(xp)}{\partial t} + \frac{\Pi^i}{E_b^*(p)}\nabla_i^x f_b(xp) &- \frac{\Pi^\mu}{E_b^*(p)}\nabla_i^x U_\mu(x)\nabla_p^i f_b(xp) + \frac{M_b^*}{E_b^*(p)}\nabla_i^x U_s\nabla_p^i f_b(xp) \\
&= I_{bb}^b(xp) + I_{b\pi}^b(xp) \,, \qquad (1)
\end{aligned}
$$

where f_b is the baryon (nucleon or Δ) phase space distribution function, and where we have used the notation $x = (t, \boldsymbol{r})$. For (any charge state of) the pion we have

$$
\frac{\partial f_\pi(xk)}{\partial t} + \frac{\boldsymbol{k}}{E_\pi} \cdot \nabla_r f_\pi(xk) = I_{b\pi}^\pi(xk). \qquad (2)
$$

In equations (1) and (2) we have used the notation $p = (E, \boldsymbol{p})$ for the four-momentum of baryons and $k = (E_\pi, \boldsymbol{k})$ for that of pions.

The collision terms on the right-hand sides of these equations are calculated by truncating the many body correlations at the two-body level and using the G-matrix method to solve the equation of motion for the two-body correlation function [5,6] The baryon-baryon collision term can be written as

$$
\begin{aligned}
I_{bb}^b(xp) &= \pi \sum_{\alpha_1\alpha_2\alpha_3, m_s^b} \iiint \frac{M_b M_{\alpha_1} M_{\alpha_2} M_{\alpha_3}}{E_b E_{\alpha_1} E_{\alpha_2} E_{\alpha_3}} W_{bb}^b(p_1\alpha_1, p_2\alpha_2, p_3\alpha_3, p\alpha_b) \cdot \qquad (3) \\
& [f_{\alpha_2}(xp_2) f_{\alpha_3}(xp_3) \overline{f}_{\alpha_1}(xp_1) \overline{f}_b(xp) - \overline{f}_{\alpha_2}(xp_2) \overline{f}_{\alpha_3}(xp_3) f_{\alpha_1}(xp_1) f_b(xp)] \cdot \\
& \delta^{(4)}(p + p_1 - p_2 - p_3) \frac{1}{(2\pi)^9} dp_1 dp_2 dp_3 \,,
\end{aligned}
$$

where the label $\alpha = (b, m_s, m_t)$, $b = N$ or Δ, and m_s/m_t is spin/isospin of the baryon.

The pion-baryon collision terms can be written as

$$
\begin{aligned}
I_{b\pi}^b(xp) &= \frac{\pi}{8} \sum_{\pi\alpha' m_s^b} \iint \frac{M_b M_{\alpha'}}{E_b(p) E_{\alpha'}(p')} W_{b\pi}^b(\alpha' p', \pi k, \alpha p) \cdot \\
& \{[\overline{f}_\pi(xk) f_{\alpha'}(xp') \overline{f}_b(xp) - f_\pi(xk) \overline{f}_{\alpha'}(xp') f_b(xp)] \delta^{(4)}(p' - k - p) \qquad (4) \\
& + [f_\pi(xk) f_{\alpha'}(xp') \overline{f}_b(xp) - \overline{f}_\pi(xk) \overline{f}_{\alpha'}(xp') f_b(xp)] \delta^{(4)}(p' + k - p)\} \cdot \\
& \frac{1}{(2\pi)^6} \, dp' dk \,,
\end{aligned}
$$

and

$$
\begin{aligned}
I_{b\pi}^\pi(xk) &= \frac{\pi}{16} \sum_{\alpha\alpha'} \iint \frac{M_\alpha M_{\alpha'}}{E_\alpha(p) E_{\alpha'}(p')} W_{b\pi}^\pi(\alpha p, \alpha' p', \pi k) \cdot \\
& [\overline{f}_\pi(xk) f_{\alpha'}(xp') \overline{f}_\alpha(xp) - f_\pi(xk) f_\alpha(xp) \overline{f}_{\alpha'}(xp')] \cdot \\
& \delta^{(4)}(p' - p - k) \frac{1}{(2\pi)^6} dp dp'. \qquad (5)
\end{aligned}
$$

It is worth mentioning that the appearance of the Fermi-Dirac factors $\overline{f}_\alpha$ for baryons and Bose-Einstein factors $\overline{f}_\pi$ for pions,

$$
\begin{aligned}
\overline{f}_\alpha(xp) &= 1 - f_\alpha(xp) \,, \qquad (6) \\
\overline{f}_\pi(xk) &= 1 + f_\pi(xk) \,, \qquad (7)
\end{aligned}
$$

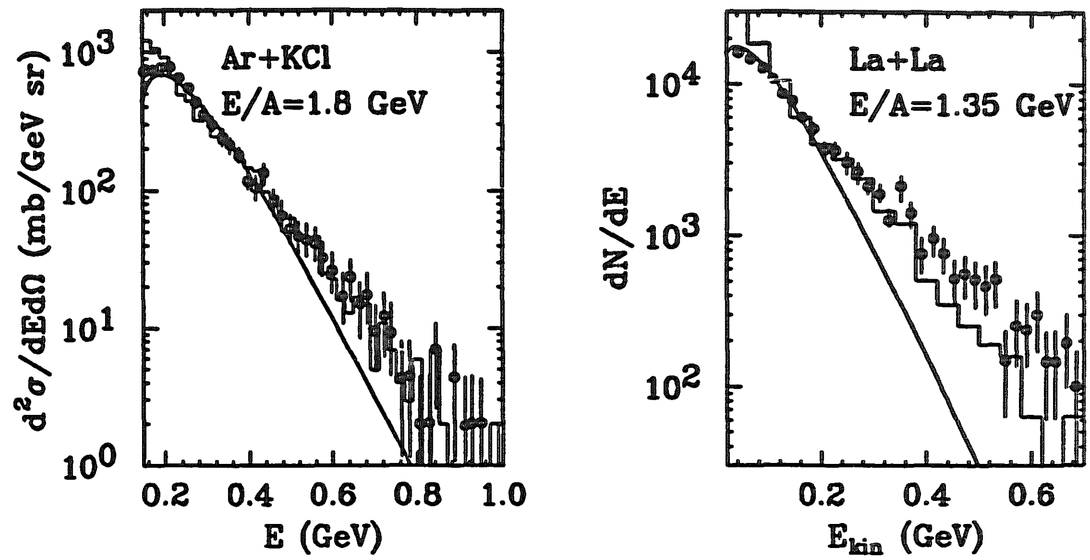

Figure 1: Comparison between theoretical calculations for π^- energy spectra produced in central heavy ion collisions (histograms) and data (circles, taken from Ref. [9]).

follows from our derivation, which can be found in Ref. [5].

The above equations form a complete set of coupled transport equations for hadronic matter and can be solved (within certain approximations) on a present-day computer by utilization of the test-particle method. For a detailed account of the numerical techniques involved we refer to [7]. In the following sections, we will examine a few exploratory calculations performed with a computer program based on the above formalism.

2. TWO-TEMPERATURE PION SPECTRA

The phenomenon that pion spectra produced in ultra-relativistic heavy ion reactions do not form a single exponential, but show a concave shape with an enhancement at low pion energies, has recently attracted attention [8]. However, the occurrence of this so-called two-temperature puzzle was first observed at relativistic heavy ion collisions of beam energies per nucleon around 1 GeV at the BEVALAC [9].

We have investigated this effect within our theoretical framework. We find that we are able to quantitatively reproduce the magnitude and concave shape of the experimental data in our calculations. In Figure 1, we show a comparison between our calculations (histograms) and the data of Ref. [9] (circles) for the reactions La + La at 1.35 GeV/nucleon and Ar + KCl at 1.8 GeV/nucleon. Also displayed are the best one-temperature fits to the data (solid lines), corresponding to T = 49 MeV and 63 MeV, respectively. Clearly, a deviation from a one-temperature spectrum can be observed for experiment as well as for theory.

We find that this concave feature of the pion spectrum is due to dynamical effects during the course of the heavy ion collision. At the beam energies of interest here, most of

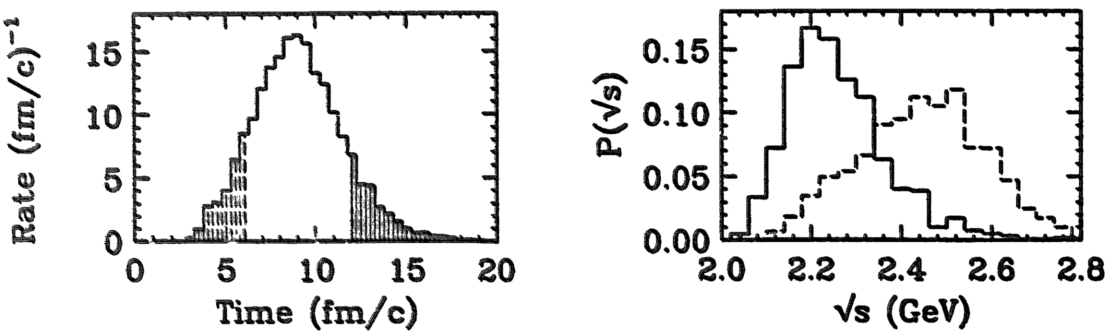

Figure 2: Left: Rate of nucleon-nucleon collisions leading to the formation of a Δ in the La+La reaction of Figure 1. Right: Probability distribution for the total center of mass energy in nucleon-nucleon collisions for $t \leq 6$ fm/c (dashed line) and $t \geq 12$ fm/c (solid line), corresponding to the dashed and solid hatched histograms on the left-hand side.

the pion production cross section results from the excitation and decay of Δ-resonances. At the early stage of the reaction, the produced Δ's have on average higher energies, because they are mainly created in collisions between nucleons at target and at projectile rapidity; whereas Δ's produced during the later stages of the reaction are also produced in collisions between nucleons at target or projectile rapidity and nucleons at mid-rapidity, which were transported there due to earlier nucleon-nucleon collisions (compare Figure 2). Of equal importance is the dynamical effect on the effective absorption cross section for the Δ's. The higher energy Δ's produced in the early part of the reaction are generated during the high baryon density phase of the reaction and are therefore more strongly reabsorbed than the lower energy Δ's produced later and at lower baryon densities.

We found that varying the nuclear compressibility had very little effect on the shape of the pion spectrum. Therefore, we conclude that the concavity is *not* predominantly caused by collective flow of the Δ's in matter. We also found that the inclusion of N*-resonances and pions produced from Dalitz decays, N+N$\rightarrow$N+N+π, does not change the resulting pion spectrum noticeably.

3. PION COLLECTIVE FLOW

Looking for collective flow signatures in produced pions, the DIOGENE collaboration found that the in-plane transverse momentum, $\langle p_x \rangle$, as a function of rapidity y is always positive in asymmetric (Ne or Ar) + (Nb or Pb) systems, while Intra-Nuclear-Cascade calculations show results compatible with $\langle p_x \rangle = 0$ over the whole rapidity range [10]. Calculations with the QMD model show that inclusion of mean field effects can describe the effect in a qualitative manner, but show less asymmetry than observed in the experiment.

To address this question further, we calculated the in-plane pion transverse momentum spectrum in our theoretical framework [12]. The main results of this study are summarized in Figure 3. Here we compare the calculated π^+ in plane transverse momen-

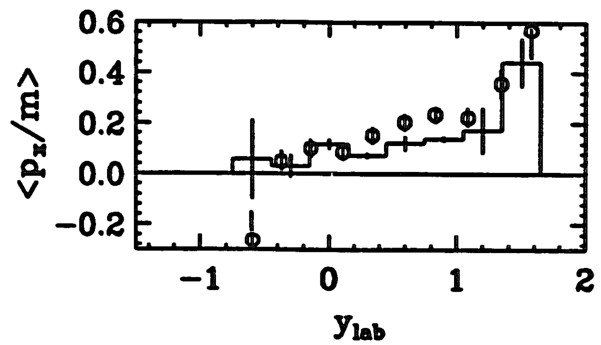

Figure 3: Comparison between the experimental transverse momentum distribution (circles) [10] and the model calculation (histogram) for the Ne+Pb → π^++X reaction at $E/A = 800$ MeV.

tum distribution for an impact parameter of $b = 3$ fm (histogram) with the experimental data (circles, taken from Ref. [10]). The in-plane transverse momentum of pion j is

$$p_{x,j} = \left\{ \boldsymbol{Q} \cdot \boldsymbol{p}_{\perp,j} \right\} / |\boldsymbol{Q}| \tag{8}$$

where the reaction plane vector $\boldsymbol{Q}$ is determined from the transverse momenta of the detected protons

$$\boldsymbol{Q} = \sum_i (y_i - \bar{y}) \boldsymbol{p}_{\perp,i}. \tag{9}$$

As can be seen from Figure 3, our calculations are able not only to reproduce the tendency in the data that the average in-plane transverse momentum of the produced π^+ is positive for all rapidities, but also the *magnitude* of the effect. We find that it is primarily caused by the asymmetric distribution of baryons in the reaction plane in these asymmetric mass systems resulting in asymmetric Δ and pion reabsorption (target shadowing). We can prove this by switching off all Delta and pion rescattering and reabsorption channels in our calculations, which results in a $\langle p_x \rangle$-distribution compatible with 0 for all rapidities. We also infer from this study that collective flow of Δ's in matter, although in principle present in our calculations, has very little influence on the observed pion spectra.

4. NUCLEON COLLECTIVE FLOW

The effect that nucleons show a collective sidewards flow in heavy ion collisions was discovered at the BEVALAC [13]. The theoretical interest in this problem has been generated by the possibility of determining the stiffness of the nuclear equation of state from the experimental data. The general consensus seems now to be that the data are best fit by a soft ($\kappa \approx 210$ MeV) equation of state with momentum dependent mean field potentials [14].

It was pointed out by Molitoris et al. [15] that at lower beam energies collective sidewards flow should diminish and eventually reach a beam energy at which it vanishes.

The reason for the disappearance of nuclear collective flow is the balance between the repulsive nuclear interaction due to compression (volume effect, $\propto A$, decreasing with beam energy) and the attractive interaction of the two nuclear surfaces. We have studied the disappearance of flow and its relevance to nuclear matter physics [16] in the framework of our theoretical model for the system Ar+V.

We find that the beam energy for which the disappearance of flow occurs, E_{bal}, shows only little sensitivity to the stiffness of the nuclear equation of state. However, E_{bal} exhibits a strong sensitivity to the value of the in-medium nucleon-nucleon cross section. This is the cross section entering Eq. 4 after using the Born approximation, where the corrections due to the Pauli principle on the final state nucleons, $(1-f_{\alpha_2})(1-f_{\alpha_3})$ are already taken into account. From our calculations, we are able to narrow down the in-medium nucleon-nucleon cross section for intermediate energy heavy ion collisions to

$$\sigma_{\rm medium} = (1.0 \pm 0.2)\, \sigma_{\rm free} \tag{10}$$

5. TWO-PROTON INTERFEROMETRY

Small angle correlations between emitted particles in heavy ion reactions have recently attracted attention because of the possibility of extracting sizes and lifetimes of the emitting sources from them [17]. We have conducted a first study of two proton intensity interferometry in intermediate energy heavy ion collisions based on our transport theory [18].

One can obtain the two-particle correlation function C from the time integrated one-body phase space distribution function g via [19]

$$C(\boldsymbol{P},\boldsymbol{q}) = \frac{\int d^4x_1 d^4x_2 g(\boldsymbol{P}/2,x_1)g(\boldsymbol{P}/2,x_2)|\phi|^2}{(\int d^4x g(\boldsymbol{P}/2,x))^2} = \int d^3r F_{\boldsymbol{P}}(\boldsymbol{r})|\phi(\boldsymbol{q},\boldsymbol{r})|^2 \,. \tag{11}$$

Here $\boldsymbol{P}$ and $\boldsymbol{q}$ are the sum and difference momenta, respectively, of the two emitted protons, and $\phi(\boldsymbol{q},\boldsymbol{r})$ is the two-proton relative wave function. It is calculated by solving a one-dimensional Schrödinger equation including the effects of the strong interaction, the ^{2}He-'resonance', the Coulomb interaction, and the antisymmetrization due to the Pauli principle. The function $F_{\boldsymbol{P}}(\boldsymbol{r})$ contains all information on the space-time extension of the source and can be calculated with our theory.

We studied two-proton correlation functions for a variety of intermediate energy heavy ion reactions. One example, the reaction ^{27}Al(^{14}N,pp) at $E/A = 75$ MeV is shown in Figure 4. On the left-hand side, we show that the shape of the two-proton correlation function is strongly dependent on the value of the in-medium nucleon-nucleon cross section, and only weakly on the nuclear compressibility, for the reaction investigated here. A comparison of apparent source radii as a function of the sum momentum $\boldsymbol{P}$ as extracted from theory and experiment shows very good agreement, if one uses an in-medium cross section according to Eq. (10).

Our studies thus show that two-proton intensity interferometry can be used as a quantitative tool for the characterization of heavy ion collisions, and that transport theories

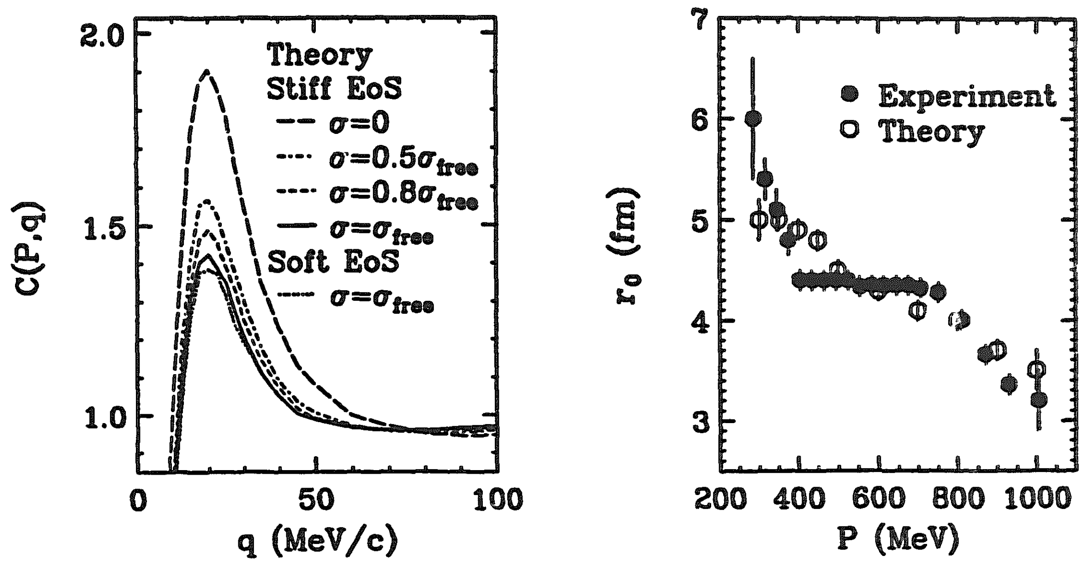

Figure 4: Left: Sensitivity of the two-proton correlation function $C(P,q)$ for $P = 500$ MeV/c to the parameters of the nuclear equation of state. Right: Comparison of the apparent source sizes extracted from experiment and theory. In both cases, the reaction is ^{27}Al(^{14}N,pp) at $E/A = 75$ MeV/nucleon.

are able to quantitatively reproduce the experimental data at intermediate energies, enabling us to extract valuable information on nuclear transport properties of hot nuclear matter.

6. SUMMARY

We have introduced a new hadronic matter transport theory for heavy ion collisions up to beam energies of $E/A \approx 2$ GeV. We studied several transport properties of hadronic matter within this framework. By comparing to experimental data, we were able to clarify the mechanism of the two-temperature puzzle and of pion collective transverse momentum in relativistic heavy ion collisions. Our studies of the disappearance of nuclear collective flow and of two-proton intensity interferometry enabled us to learn more about the space-time structure of heavy ion collisions and to place limits on the in-medium nucleon-nucleon cross section.

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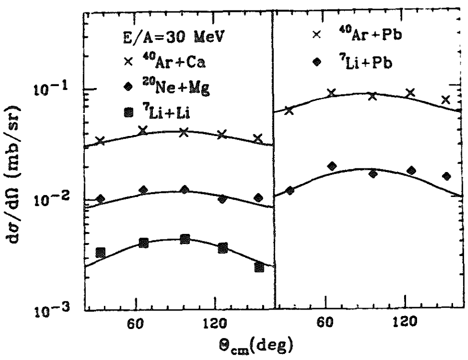

Figure 5 Angular distributions as observed in the half velocity frame.[27]

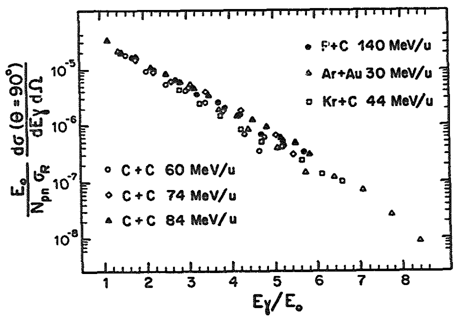

Figure 6 Plot of the invariant quantity $\frac{d\sigma(\theta=90^\circ)}{dx}\frac{1}{\sigma_R<N_{np}>}$ as a function of $x = \frac{E}{E_0}$ for different systems.