



ELSEVIER

24 December 1998

PHYSICS LETTERS B

Physics Letters B 444 (1998) 231–236

# Fragment multiplicity distributions, a signal of true nuclear multifragmentation

Tarek Gharib <sup>1</sup>, Wolfgang Bauer, Scott Pratt

*Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University,  
East Lansing, MI 48824, USA*

Received 12 August 1998; revised 19 October 1998

Editor: W. Haxton

---

## Abstract

Multiplicity fluctuations of intermediate-mass fragments are studied with the percolation model. It is shown that super-Poissonian fluctuations occur near the percolation transition and that this behavior is associated with the fragmentative nature of the percolation model. The consequences of various choices in defining and binning fragments are also evaluated. Several suggestions for experiments in nuclear fragmentation are presented. © 1998 Published by Elsevier Science B.V. All rights reserved.

PACS: 25.70.Pq; 24.10.Pa; 24.60.-k; 24.60.Ky

---

The study of nuclear fragmentation at high excitation has proven enigmatic. Sophisticated models based on statistical equilibrium [1,2] or mean field simulations that model the growth of unstable dynamical modes [2,3] have been able to reproduce several features of fragment yields [1] as measured in intermediate heavy ion collisions. Even more ambitious microscopic models that account for the Fermi degeneracy of nuclear matter are currently under development [4–6]. However, simple percolation models [7–9] have perhaps been the most successful in reproducing fragment yields and their moments over a wide range of excitations [10], and have also

had some success in modeling the fragmentation of atomic clusters [11].

Recently, Moretto and collaborators [12] have put forth the measurement of fragment multiplicity distributions as an insightful tool for understanding the mechanisms and the driving principles of nuclear fragmentation. Experimental fragment yields have shown themselves to be well described by binomial distributions, while the interpretation of the binomial parameters has been deeply debated [13–15].

Here, we present calculations of fragment multiplicity distributions for percolation calculations. Our aim is to address the following questions:

1. Are fragment multiplicity distributions from percolation calculations of a binomial nature?
2. Is the variance of the multiplicity distribution governed by simple conservation laws or by other principles?

---

<sup>1</sup> Permanent address: Ain Shams University, Faculty of Science, Department of Physics, Abbassia 11566, Cairo, Egypt.

3. Do fluctuations near the percolative transition affect fragment distributions?
4. In analyzing nuclear experiments, should one bin distributions by multiplicity, by excitation energy, or by some other criteria?

At first glance, percolation models seem to have little in common with a nuclear multifragmentative event. No dynamics are present, and bulk properties such as pressure and specific heat do not even have analogs in a percolative description. Intermediate-sized clusters appear copiously in percolation model—only near the critical point, and at that point are characterized by fractal shapes. In contrast, the liquid-gas phase transition is first order, and clusters can appear far from the critical point, where surface energies are significant and clusters are more spherical in nature. However, percolation models do incorporate particle-number conservation and the topology associated with short-range binding, and therefore can prove insightful in modeling nuclear fragmentation. In fact, a rigorous connection between bond breaking probability, deposited energy and nuclear binding energy has been established [16], which is a generalization of the Coniglio-Klein formula of the lattice gas model [17,18]. For our studies we employ bond percolation [7–9] where a spherical section of a cubic lattice is arranged, and bonds between the sites are randomly broken with a probability  $p$ . One defines a fragment as a group of connected sites. For values of  $p$  below  $p_c = .7512$  the majority of sites belong to a single large cluster. When  $p$  exceeds  $p_c$ , the lattice is broken into many small and intermediate sized clusters. We have chosen spherical lattices of size  $N_{\text{sites}} = 123$  to address lattices of sizes relevant for nuclear fragmentation and  $N_{\text{sites}} = 4169$  to understand the behavior in larger lattices. For each event the number  $n$  of intermediate-mass fragments (IMFs) is recorded. The default definition of an IMF is that it is of size,

$$3 \leq Z \leq 20, \quad (1)$$

where  $Z$  refers to the number of sites in the cluster. By recording thousands of events multiplicity distributions were generated for given values of  $p$ . Fig. 1 displays multiplicity distributions for  $p = 0.7$  and  $p = 0.8$  for the 123-site case.

Moretto and collaborators have reported that the multiplicity distributions of IMFs in nuclear frag-

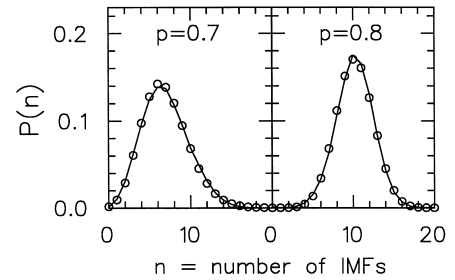


Fig. 1. The multiplicity distribution for IMFs for the case of  $p = 0.7$  (left panel) and  $p = 0.8$  (right panel). The lines represent negative binomial (left panel) and binomial (right panel) fits, where the two parameters were chosen to match the mean and variance of the distributions.

mentation are observed to be well described by binomial distributions. Binomial distributions are defined by two parameters  $p_b$  and  $N_b$ ,

$$P_b(n) = \frac{N_b!}{n!(N_b - n)!} p_b^n (1 - p_b)^{N_b - n}. \quad (2)$$

The mean and variance of binomial distributions are given by

$$\langle n \rangle = p_b N_b, \quad \sigma^2 = \langle n \rangle (1 - p_b), \quad (3)$$

with the variance always being less than the mean. Thus, by measuring the mean and variance, one can determine the binomial parameters,  $p_b$  and  $N_b$ . In the limit that the variance equals the mean the distribution becomes Poissonian, and if the variance is larger than the mean (super-Poissonian), the distribution can no longer be considered binomial. However, one might then consider the distribution to be a negative binomial,

$$P_{nb}(n) = \frac{(N_{nb} + n - 1)!}{(N_{nb} - 1)! n!} p_{nb}^n (1 - p_{nb})^{N_{nb}} \quad (4)$$

In that case the mean and variance become

$$\langle n \rangle = \frac{p_{nb} N_{nb}}{1 - p_{nb}}, \quad \sigma^2 = \frac{\langle n \rangle}{(1 - p_b)}. \quad (5)$$

The lines in Fig. 1 represent negative binomial and binomial fits for the  $p = 0.7$  and the  $p = 0.8$  cases respectively, where the parameters were chosen to match the mean and variance of the two distributions. In all the calculations discussed here, two-parameter fits were remarkably successful in describing the multiplicity distributions.

Fig. 2 displays  $\langle n \rangle / N_{\text{sites}}$  and  $\sigma^2 / \langle n \rangle$  as a function of  $p$  for the small and large lattices. The

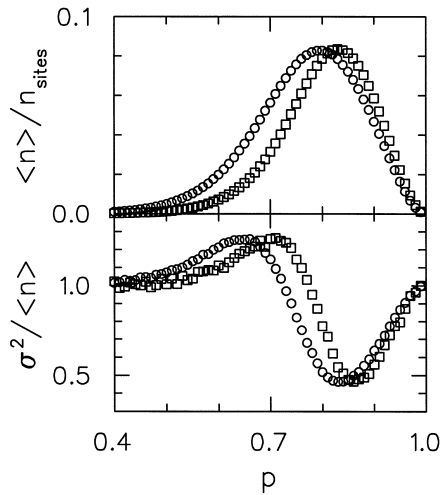


Fig. 2. The average multiplicity of IMFs divided by the number of sites, is displayed in the upper panel. The ratio of the variance to the mean is shown in the lower panel. A ratio greater than unity is super-Poissonian and signifies a positive correlation between IMFs. Both 123-site (circles) and the 4169-site cases are illustrated.

distributions are super-Poissonian for  $p < p_c$  and become sub-Poissonian just above  $p_c$ . The super-Poissonian behavior is a signal of a positive correlation between IMFs, as it signals that the presence of an IMF will be positively correlated with the production of other IMFs. We argue that this positive correlation is a signal of the fragmentative nature of the percolation model.

To understand the correlation, we rewrite the expression for the difference of the variance and mean in terms of a correlation function,

$$\begin{aligned} \sigma^2 - \langle n \rangle &= \sum_{a \neq b} \langle (n_a - \bar{n}_a)(n_b - \bar{n}_b) \rangle \\ &\quad + \sum_a \langle n_a^2 - n_a - \bar{n}_a^2 \rangle \\ &\approx \sum_{a \neq b} \langle (n_a - \bar{n}_a)(n_b - \bar{n}_b) \rangle. \end{aligned} \quad (6)$$

The sums over  $a$  and  $b$  represent the sums over all types of IMFs, where a type  $a$  refers to a specific size, shape and position. The first sum on the right-hand side of Eq. (6) represents the correlation between different IMFs. The second sum can be neglected, as the first two terms of the second sum cancel each other since  $n_a$  can only be zero or unity, and the last term, which is negative, is small. This last term becomes zero in the limit that the probabil-

ity of any specific IMF (defined by size, shape and position) is small.

Since the bond breaking is random, only fragment types that share the same sites or the same boundaries are correlated. If type  $a$  and type  $b$  share any of the same sites, the correlation is clearly negative as they can not coexist. This is related to particle number conservation. However, if  $a$  and  $b$  merely share some section of their boundaries a positive correlation can exist. This positive correlation appears only for values of  $p$  where a majority of the sites are taken up by large clusters, larger than the size of an IMF. The presence of an IMF  $a$  then creates extra surface within some larger cluster. The increased surface area eases the production of a second IMF of type  $b$  which borders the first IMF. As  $p$  is increased to the point where most of the sites are assigned to fragments the same size or smaller than IMFs, the positive correlation disappears, and the effects of particle-number conservation are dominant.

The super-Poissonian variance of the IMF multiplicity distribution signals the fragmentative aspect of the percolation model. The positive correlation arises from the additional surface created by the production of a fragment. In other models, e.g. an evaporative picture, emission does not lead to an increased surface area, and one might expect qualitatively different behavior. Thus, the study of IMF multiplicity distribution in nuclear collisions may provide insight into the general principles of the fragmentation mechanism.

Several other effects can affect the width of the IMF multiplicity distribution. One can imagine binning percolation events according to  $p$  as done above, by the number of broken bonds, or by the overall multiplicity. One might similarly consider binning experimental events by an even larger assortment of criteria: multiplicity, transverse energy, beam energy, the size of the largest fragment, or by any combination of the above. One might also consider altering the mass range that defines an IMF. All such seemingly arbitrary choices affect the width of the multiplicity distribution. For nuclear experiments, the choice of criteria is often constrained by details of the experiment such as acceptance or method of excitation, e.g. symmetric central collisions of heavy ions at intermediate energy vs. high-

energy peripheral collisions. Thus, we concentrate on the general behavior and manifestations of various binning criteria in percolation calculations. Many of the lessons learned from this effort should carry over to the analysis of nuclear experiments.

Before we proceed, we define a quantity  $R(p)$ , the difference of the variance and the mean, normalized by the ratio,  $N_{\text{sites}}/\langle n \rangle^2$ .

$$R(p) \equiv \frac{N_{\text{sites}}}{\langle n \rangle^2} (\sigma^2 - \langle n \rangle). \quad (7)$$

By dividing by  $\langle n \rangle^2$ , this normalization allows one to view the correlation even when fragment production is rare, and by multiplying by  $N_{\text{sites}}$ ,  $R$  becomes independent of the lattice size for large lattices. Positive and negative values of  $R$  refer to super and sub-Poissonian distributions respectively.

Rather than breaking each bond with probability  $p$ , one can break a fixed percentage  $p$  of the bonds. The multiplicity distributions as seen in Fig. 3 remain sub-Poissonian for the entire range of  $p$ . This extra negative correlation is expected as the presence

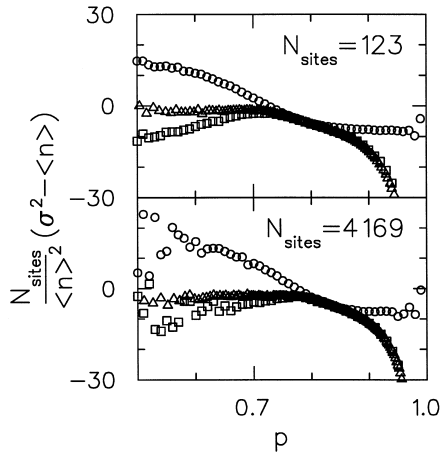


Fig. 3. Three different binnings of events are illustrated: Describing the event by the random probability  $p$  of breaking bonds (circles), Categorizing events by the actual fraction of broken bonds (squares), or binning events according to the overall multiplicity (triangles). In the last case, the average value of  $p$  for events of a given multiplicity is used to determine the horizontal axis. For the latter two cases the binnings introduce an autocorrelation with the number of IMFs that reduces the width of the multiplicity distribution.

of an IMF of type  $a$  expended some of the broken bonds. This correlation is non-local as fragments far away from  $a$  are less likely to be produced. This difference between binning by  $p$  and binning by the actual number of broken bonds can be thought of as being analogous to the difference between the canonical and microcanonical statistical distributions. In fact, if an energy is assigned to each bond, and the state of the system is completely described by whether bonds are broken or not, the two choices of binning can indeed be labeled as the canonical and microcanonical treatments.

However, it is not obvious as to which choice of binnings is more appropriate for comparison with nuclear data. Even for the case of fixed beam energy and central collisions, other degrees of freedom, such as the kinetic and internal energy of the fragments, share the energy with the spatial degrees of freedom which are more closely associated with binding. Also, several particles are typically emitted before the fragmentation stage and the portion of the original excitation energy that determines fragmentation is further randomized.

One might also bin events by the overall multiplicity of fragments of any size rather than  $p$ . This has the advantage of offering a convenient means of comparing percolative calculations to experimental results, and sidestep some of the ambiguities mentioned above. This also introduces a negative correlation as the existence of an IMF of type  $a$  reduces the net number of other fragments by one, making the existence of a second IMF less likely. The results of such a binning are displayed in Fig. 3. The binnings were performed by choosing  $p$  randomly between 0.4 and 1.0, then binning the event according to multiplicity, and finally using the average  $p$  of events with a given multiplicity as the horizontal axis. In this way, each point in Fig. 3 corresponds to a specific multiplicity but covers a range of  $p$  values. Again the variance is pushed into the sub-Poissonian range. This illustrates the general principal that any binning criteria that is autocorrelated with the number of IMFs pushes the IMF multiplicity distribution in sub-Poissonian direction.

We also investigate the effects of changing the mass range that defines an IMF. The choice of  $3 \leq Z \leq 20$  was motivated by a convention for the definition of IMFs in the analyses of nuclear experi-

ments. By raising the range to  $15 \leq Z \leq 20$ , we see in Fig. 4 that the multiplicity distribution becomes increasingly super-Poissonian. In fact, when binning by a fixed number of broken bonds or a by a fixed multiplicity, the multiplicity distributions can still be pushed into the super-Poissonian range by using a larger mass range for IMFs. This positive correlation is due to the fact a larger fragment offers more surface area for the creation of a second fragment. Since this positive correlation represents the signal for the fragmentative nature of the event, we suggest that analyzing more massive fragments may lead to a more insightful conclusion. However, one must be careful to steer away from fission-like correlations which set in when the mass range approaches half the overall lattice.

Finally, we study the consequences of smearing the binning criteria, e.g. the values of  $p$ , or some other binning variable, is chosen over a finite range rather than a discrete value. To understand the effects of a finite range, we consider a binning variable  $x$ , where a multiplicity distribution exists for any discrete value of  $x$  with moments  $\langle n \rangle(x)$  and  $\langle n^2 \rangle(x)$ . One can then consider the distribution over a finite range,  $x_{\min} \leq x \leq x_{\max}$ . Denoting quantities derived from distributions using the range of  $x$

values with overlines, we give expressions for the difference of the variance and the mean.

$$\overline{\sigma^2} - \overline{\langle n \rangle} = \frac{1}{x_{\max} - x_{\min}} \int_{x_{\min}}^{x_{\max}} dx \langle n^2 \rangle(x) - \left( \frac{1}{x_{\max} - x_{\min}} \int_{x_{\min}}^{x_{\max}} dx \langle n \rangle(x) \right)^2 - \frac{1}{x_{\max} - x_{\min}} \int_{x_{\min}}^{x_{\max}} dx \langle n \rangle(x). \quad (8)$$

Assuming that  $\langle n \rangle(x)$  varies linearly in the small range of  $x$ ,

$$\langle n \rangle(x) = \overline{\langle n \rangle} + \left( x - \frac{x_{\max} + x_{\min}}{2} \right) \times \frac{\langle n \rangle(x_{\max}) - \langle n \rangle(x_{\min})}{x_{\max} - x_{\min}}, \quad (9)$$

one can show that the difference of the variance and mean becomes,

$$\overline{\sigma^2} - \overline{\langle n \rangle} = \frac{1}{x_{\max} - x_{\min}} \int_{x_{\min}}^{x_{\max}} dx \sigma^2(x) - \langle n \rangle(x) + \frac{1}{12} (\langle n \rangle(x_{\max}) - \langle n \rangle(x_{\min}))^2. \quad (10)$$

Thus if the distribution is averaged over a region where the average multiplicity has changed, the resulting distribution is pushed into the super-Poissonian direction by the last term in Eq. (10).

Smearing the distribution over a range of  $x$  is not necessarily controllable in a nuclear fragmentation experiment. Due to the inability of an experiment to gate on a precise type of event such as a central collision, all binnings effectively cover a finite range of excitation energies. The widening of the multiplicity distribution is also most affected for regions where the average IMF multiplicity is rapidly changing as a function of  $x$ . Clearly, experiments where all outgoing particles have been measured offer the best chance of precisely characterizing events and minimizing this effect.

The behavior observed by Moretto [12] is clearly contrary to the behavior shown in Fig. 2, as the results remained clearly in the sub-Poissonian range, and were more strongly sub-Poissonian at lower excitation. However, conclusive statements can not be made without fully understanding the details of the measurement and analysis. In fact, we are cur-

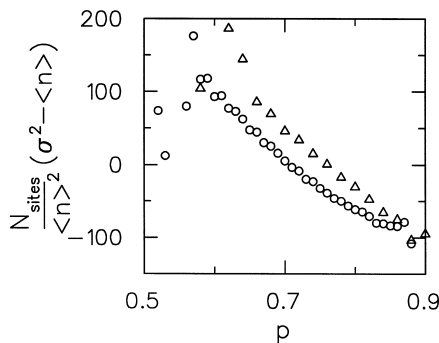


Fig. 4. The super-Poissonian nature of the multiplicity distribution is magnified by restricting the IMF mass range to heavier particles,  $15 \leq Z \leq 20$ . The 123-site case (circles) and the 4169-site case (triangles) behave similarly.

rently investigating systematic effects, such as the variation of reaction geometry and system size as a function of excitation energy.

We conclude by answering the four questions posed at the beginning of the manuscript. First, the multiplicity distributions as predicted from percolation models can be summarized by two parameters, the binomial parameters for the cases where the multiplicity distributions are sub-Poissonian and the negative-binomial parameters for the super-Poissonian cases.

Secondly, the variance of the multiplicity distribution can not be explained with simple conservation laws. In fact, the most important conclusion from this study is that measurement of IMF distributions yield an important insight into the process of fragmentation. If nuclear multifragmentation is truly fragmentative as in a percolation description, one should expect an increased variance of the multiplicity distribution, perhaps yielding super-Poissonian distributions. Furthermore, this variance should increase if one confines the analysis to larger fragments. Sequential models and thermal models are expected to behave in the opposite manner, as the presence of a fragment is anticorrelated to a second fragment due to particle and energy conservation, and this anticorrelation is increasingly strong for increasingly large fragments.

The third question centered on the role of fluctuations at the critical point. The multiplicity distribution at a fixed value of  $p$  is not governed by critical phenomena. The crossover from super-Poissonian to sub-Poissonian behavior shown in Fig. 2 arises from the dissolution of fragments larger than the IMF size as a function of  $p$ . Although the fraction of sites that are part of larger fragments is a rapidly changing function, there is no associated divergence.

The final question regarding binning resulted in a number of valuable lessons. Binning by observables such as multiplicity which are correlated to the number of IMFs narrows the IMF multiplicity distribution, while binning over a range of excitations effectively broadens the distributions. Thus the fact that IMF multiplicity distributions are super-Poissonian or sub-Poissonian is not decisive in itself. But, careful analysis, along with the study of the behavior as a function of the IMF mass range, should allow one to make conclusive statements regarding the nature of nuclear multifragmentation.

During the last 20 years a large number of studies have investigated mass yields in the region of nuclear multifragmentation, but the shape of the mass yields has been reproduced by a variety of disparate theoretical descriptions. The recent development of several full acceptance detector systems makes the analysis of multiplicity distributions tenable. Using the percolation model, we have demonstrated that the qualitative nature of fragmentation might be understood through the analysis of multiplicity distributions as sequential and percolative descriptions predict qualitatively different behaviors.

### Acknowledgements

This work was supported by the National Science Foundation, grant 96-05207.

### References

- [1] D.H.E. Gross, Rep. Prog. Phys. 53 (1990) 605; J.P. Bondorf, A.S. Botvina, A.S. Iijinov, I.N. Mishustin, K. Sneppen, Physics Reports 257 (1995) 133.
- [2] J. Randrup, S. Koonin, Nucl. Phys. A 356 (1981) 321.
- [3] P. Chomaz, Ann. Phys. France 21 (1996) 669; G.F. Burgio, Ph. Chomaz, J. Randrup, Phys. Rev. Lett. 69 (1992) 885.
- [4] H. Feldmeier, J. Schnack, Prog. Particle Nucl. Phys. 39 (1997) 393.
- [5] A. Ohnishi, J. Randrup, Phys. Lett. B 394, 260.
- [6] D. Kiderlen, P. Danielewicz, Nucl. Phys. A 620 (1997) 346.
- [7] W. Bauer, Phys. Lett. B 150 (1985) 53; W. Bauer et al., Nucl. Phys. A 452 (1986) 699; W. Bauer, Phys. Rev. C 38 (1986) 1297.
- [8] X. Campi, J. Phys. A 19 (1986) L917; Phys. Lett. B 208 (1988) 351.
- [9] D. Stauffer, A. Harony, Introduction to Percolation Theory, Taylor and Francis, London, 1985.
- [10] W. Bauer, A. Botvina, Phys. Rev. C 52 (1995) R1760.
- [11] S. Cheng, H.G. Berry, R.W. Dunford, H. Esbensen, D.S. Gemmel, E.P. Kanter, T. LeBrun, W. Bauer, Phys. Rev. A 54 (1986) 3182.
- [12] L.G. Moretto et al., Phys. Rev. Lett. 74 (1995) 1530; L.G. Moretto et al., Phys. Rep. 287 (1997) 249.
- [13] J. Toke, D.K. Agnihotri, B. Djerroud, W. Skulski, W.U. Schroeder, Phys. Rev. C 56 (1997) R1686.
- [14] M.B. Tsang, P. Danielewicz, Phys. Rev. Lett. 80 (1998) 1178.
- [15] L.G. Moretto, L. Bealieu, L. Phair, G.J. Wozniak, Los Alamos report, nucl-ex/9709001, 1997.
- [16] T. Li et al., Phys. Rev. C 49 (1994) 1630.
- [17] X. Campi, H. Krivine, Nucl. Phys. A 620 (1997) 46.
- [18] A. Coniglio, W. Klein, J. Phys. A 13 (1980) 2775.