Supernova Dynamics via Kinetic Theory

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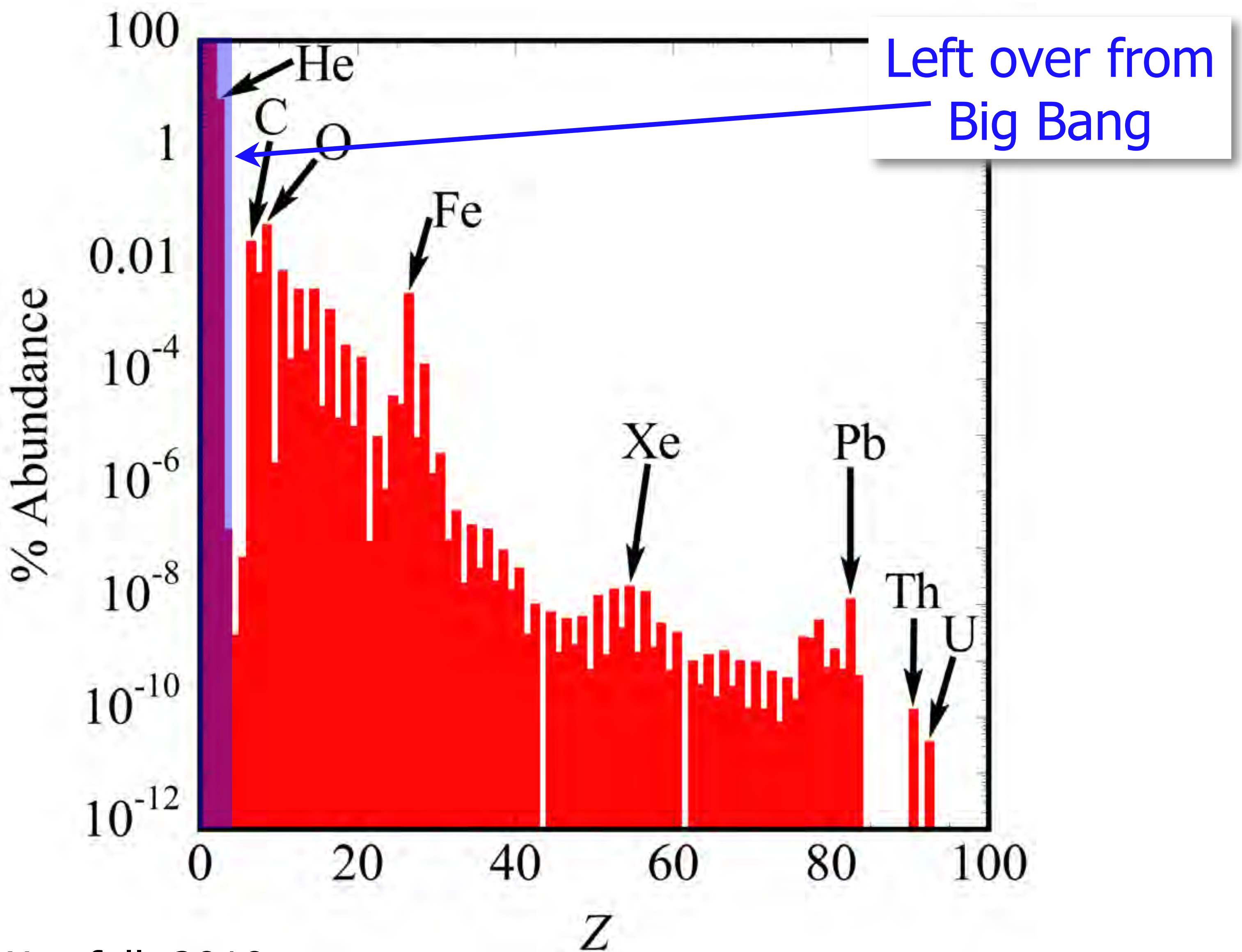
PASI 2010


East Lansing
Michigan

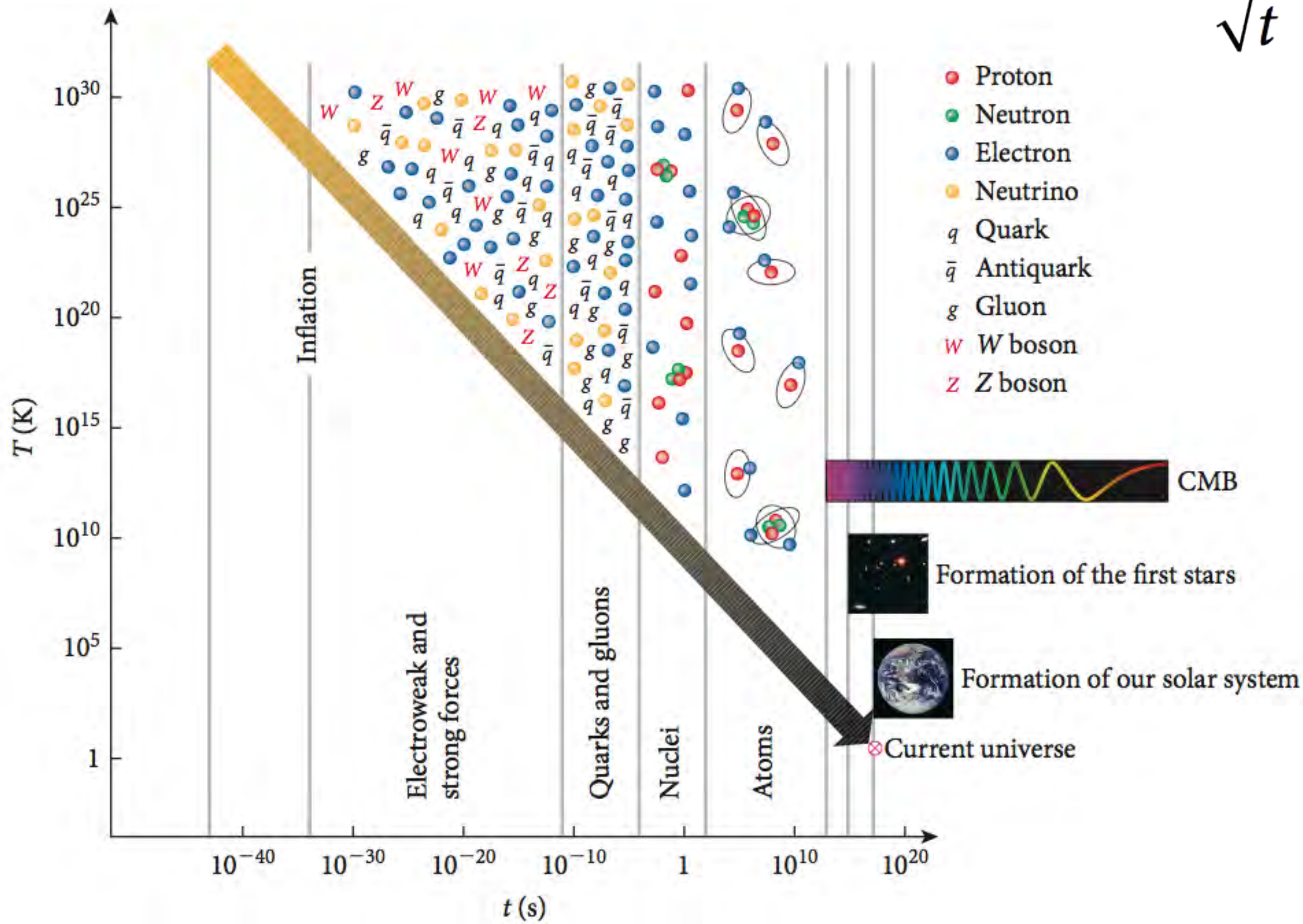
Element Abundances



Bauer & Westfall, 2010

Big Bang

$$T(t) = \frac{1.5 \cdot 10^{10}\ \mathrm{K\,s}^{1/2}}{\sqrt{t}}$$



He/H ratio from Big Bang

- Coming out of QGP, T ~ 10^{11} K (~10 MeV)
- p and n in equilibrium

$$n + \nu_e \rightleftarrows p + e^-$$

$$n + e^+ \rightleftarrows p + \bar{\nu}_e$$

- Number of p and n determined by Boltzmann factors

$$\frac{n_\text{n}}{n_\text{p}} = e^{(m_\text{n}c^2 - m_\text{p}c^2)/k_\text{B}T}$$

- T~0.86 MeV: weak reactions too slow to maintain equilibrium
- Ratio freezes out at $e^{-1.293/0.86} = 0.222$

He/H ratio from Big Bang

- So far (t = 1 s): 22 n for every 100 p, T still too high for nuclei to form
- t = 100 s: $T \sim 10^9$ K, nuclei (alpha particles) can from
  - Due to beta decay (half life 15 min), only ~16 n are left for every 100 p
- All free n can get trapped in α's:
  - 16 n and 16 p can form 8 α
  - Mass fraction of alphas is then 2*16/(100+16) = 27%
  - Close to observed value of 23%
  - Big success for early Big Bang cosmology!

Compartilhar Denunciar abuso Próximo blog»

PASI 2010

PAN AMERICAN STUDY INSTITUTE ON THE PHYSICS AND ASTROPHYSICS OF RARE ISOTOPES.

WEDNESDAY, AUGUST 4, 2010

Big Bang: He/H = 23%
OK, got that!
Turns out lecture is not a *complete* waste of time …

CONTRIBUTORS

Ana Becerril, Marcelo DelSanto, Alfredo Estrade, Meredith Howard, Ernesto Mane, Zach Meisel, William Newton

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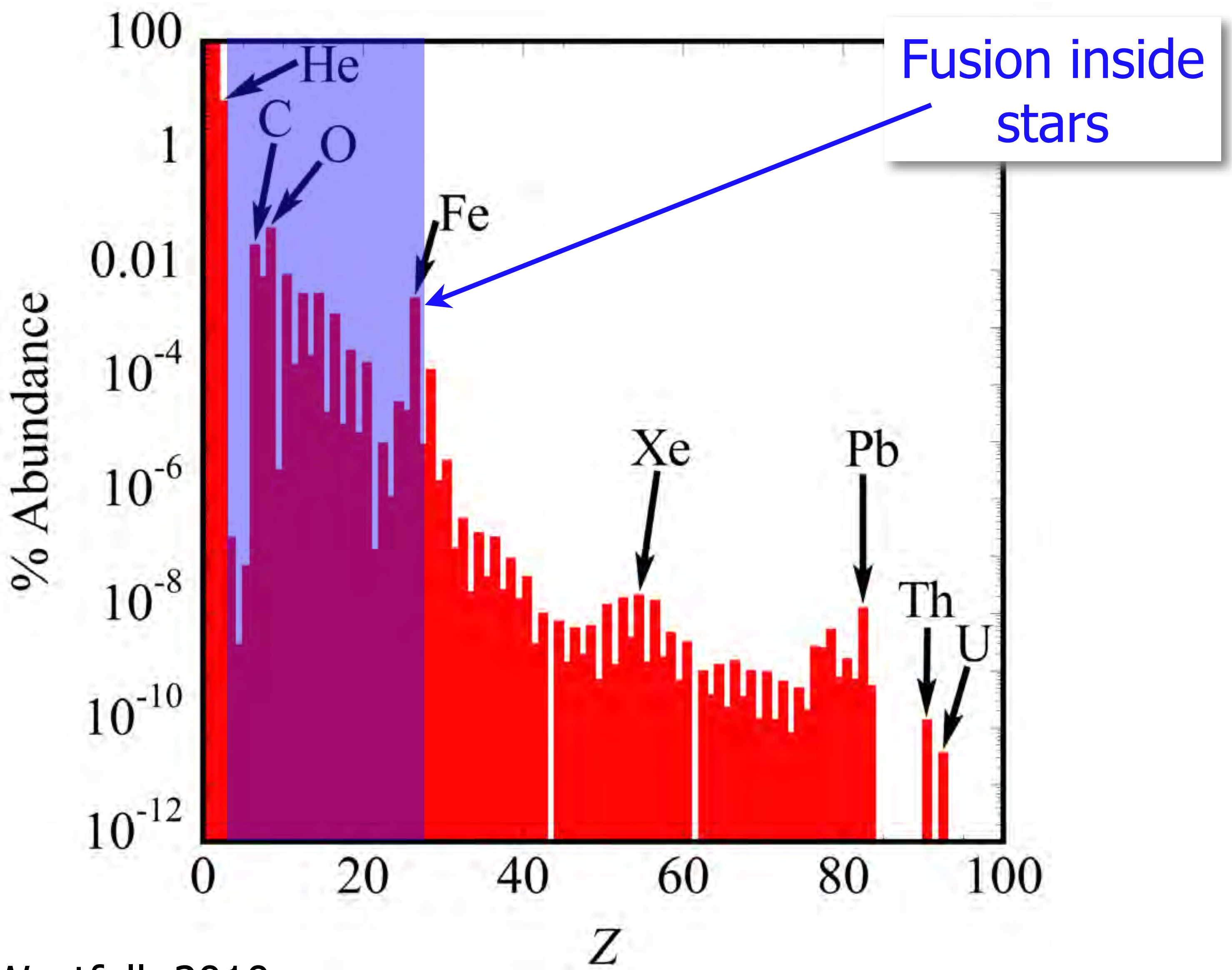
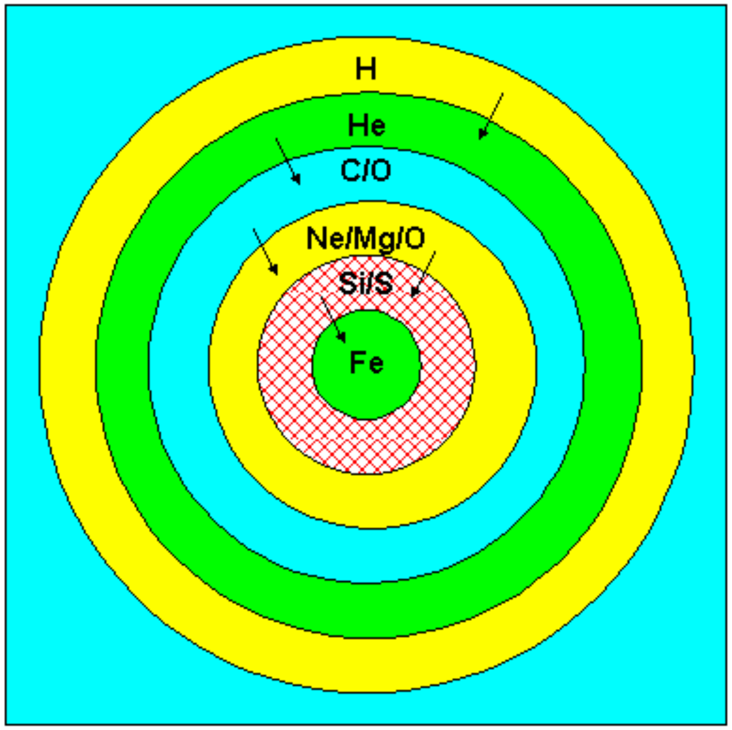
Element Abundances



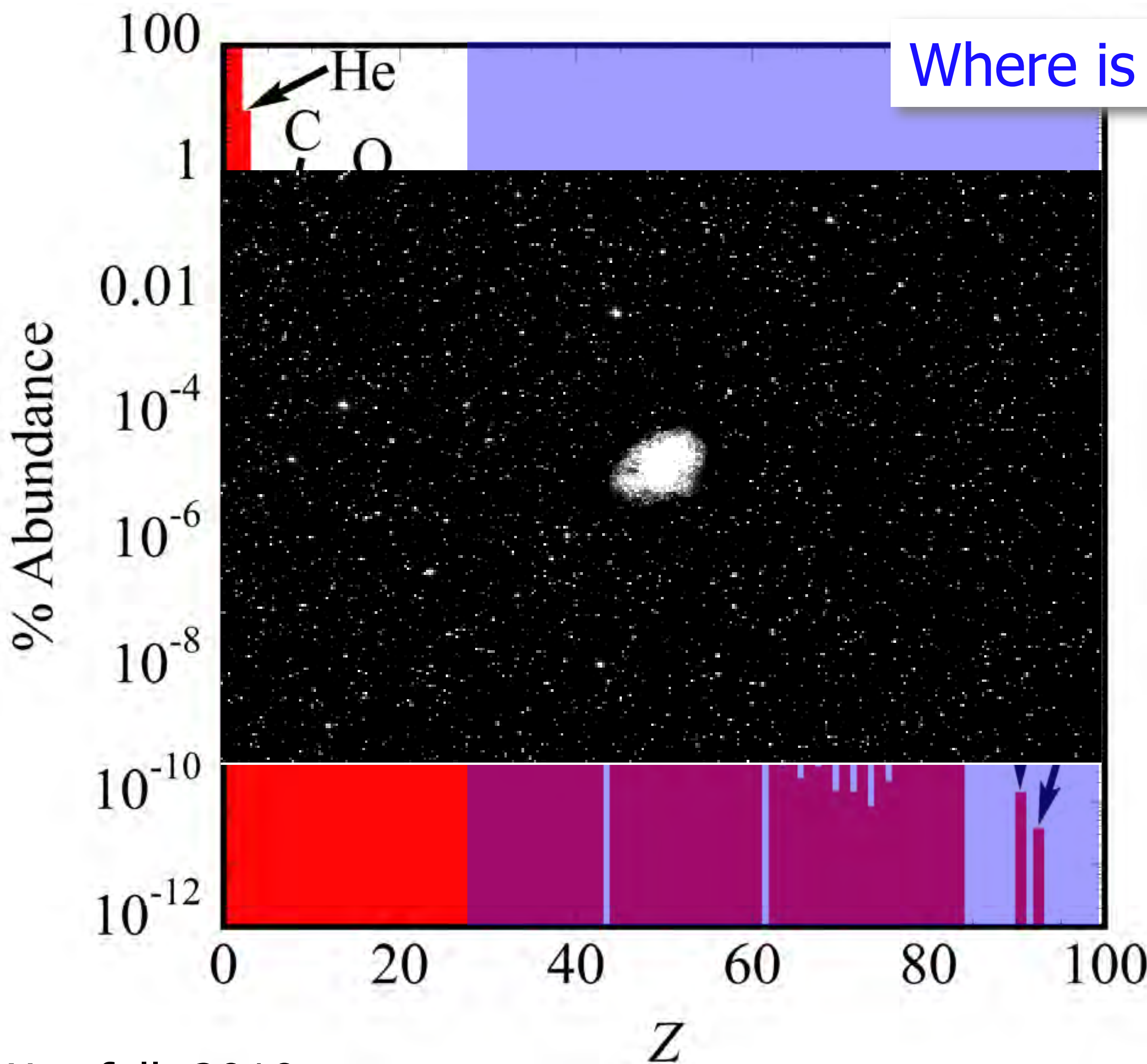
Bauer & Westfall, 2010

Nuclear Fusion in Stars

- Nuclear fusion $\Rightarrow$ hydrostatic equilibrium
- Burn to central fuel exhaustion
- Contraction
- Ignite next burning phase
- Onion skin layers



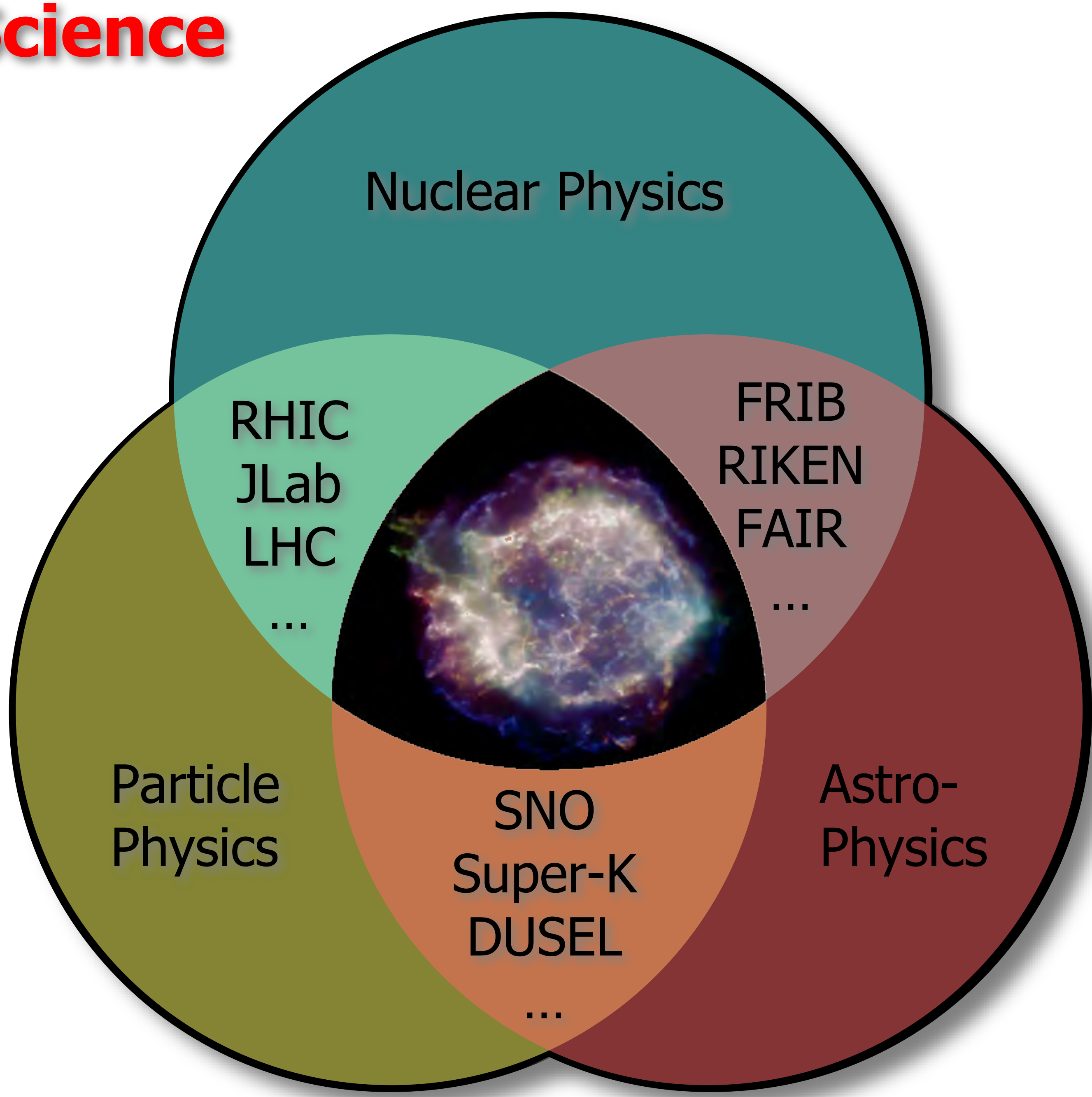
Element Abundances



Where is this from?

Bauer & Westfall, 2010

Our Science


Nuclear Physics
RHIC
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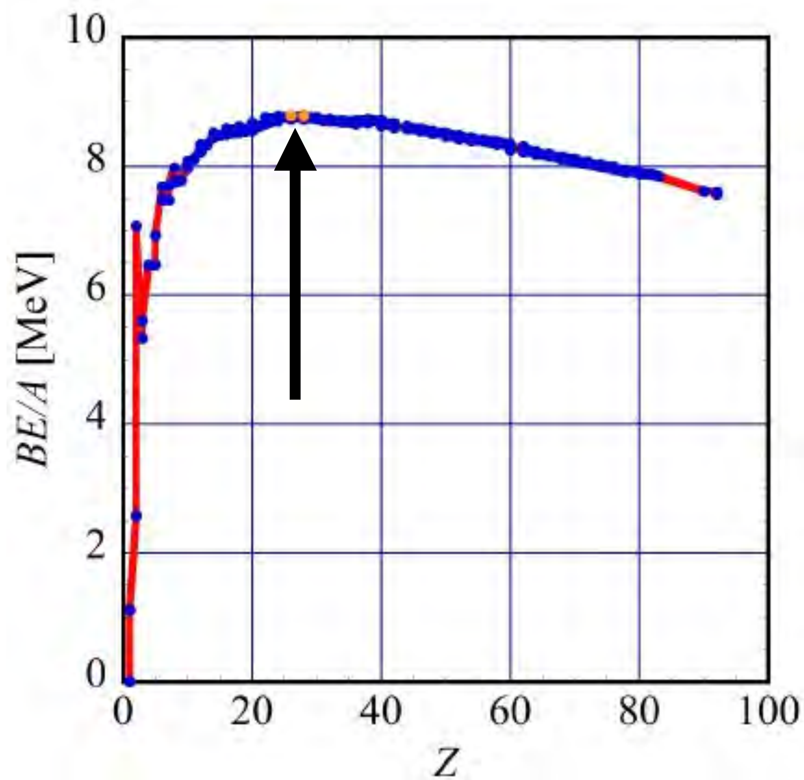
...
FRIB
RIKEN
FAIR
...
Particle Physics
SNO
Super-K
DUSEL
...
Astro-Physics

Pre-Collapse Fe Core

- Iron-mass range nuclei A ~ 45-65
- Fusion has fizzled



Pre-Collapse Fe Core

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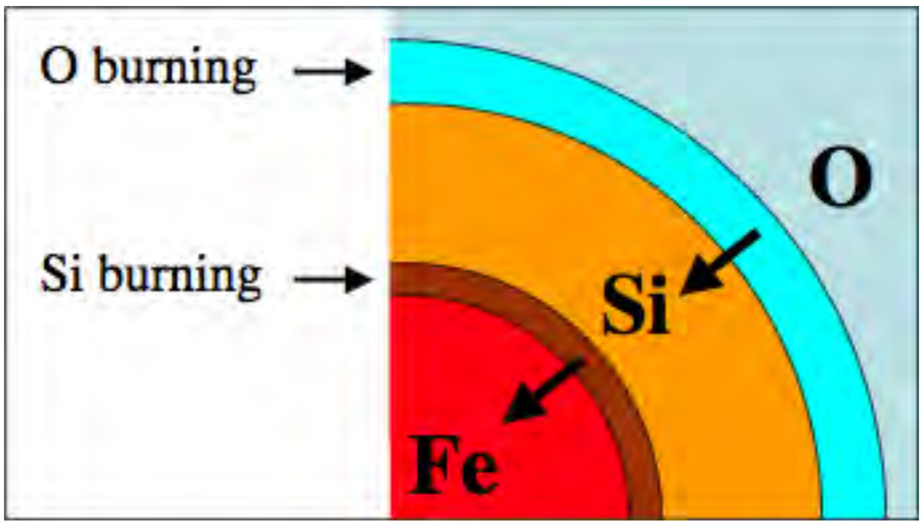
  - Some gravitational collapse
  - Electron gas becomes degenerate

Pre-Collapse Fe Core

- Iron-mass range nuclei A ~ 45-65
- Fusion has fizzled
  - Some gravitational collapse
  - Electron gas becomes degenerate
- (Only) electron degeneracy pressure stabilizes core

Onset of Collapse

– Silicon burns in shell around Fe core adding to its mass



Onset of Collapse

- Silicon burns in shell around Fe core adding to its mass
- Electrons are captured by nuclei

$$e^- + (Z,A) \rightarrow (Z-1,A) + \nu_e$$

  - endothermic
  - reduces η_e

Onset of Collapse

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$$e^- + (Z, A) \rightarrow (Z-1, A) + \nu_e$$

- Stability requires:

$$M_{Core} < M_{Ch} = 1.44(2\eta_e)^2 M_\odot$$

Two ways to trigger a catastrophe:
Make left side bigger; make right side smaller

Onset of Collapse

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Onset of Collapse

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$$e^- + (Z, A) \rightarrow (Z-1, A) + \nu_e$$

- Stability requires:

runaway instability

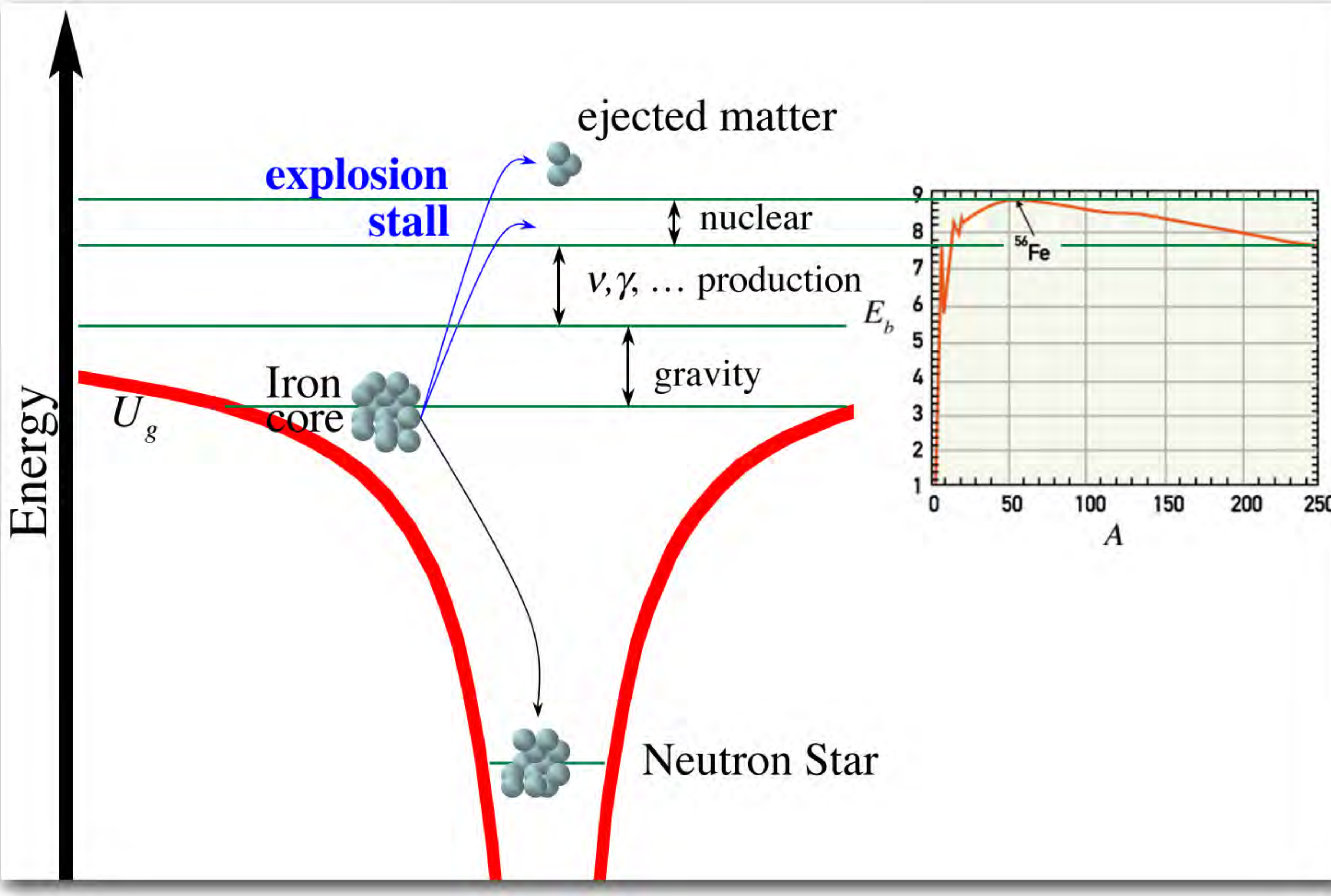
$$M_{Core} < M_{Ch} = 1.44(2\eta_e)^2 M_\odot$$

lower electron fraction => higher density => higher Fermi energy => higher capture rates => lower electron fraction …

Bounce Dynamics

- Accepted picture:
  - Approach free fall
  - Weak and electromagnetic interactions breakup heavy nuclei ⇒ create free neutrons
  - Collapse ensues until neutron pressure stops it
    - $\rho_c \sim 3\rho_0$
    - $\eta_e \sim 0.3$
    - $t_{bounce} \sim 0.1$ s
  - Large amount of gravitational energy released
    - Couple 1% of this energy to infall

Energy considerations



Traditional Approach: Hydro

- Tough problem for hydro based calculations
  - Multiple fluids
    - Track p, n, α, average heavy nucleus
    - Simplifying assumptions about neutrino flow
      - Optically thick and thin regions
      - MGFLD

can "average out" structure effects

trouble between trapping and free steaming

Traditional Approach: Hydro

- Tough problem for hydro based calculations
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  - Very large number of time steps
  - Special relativity, causality
  - Realistic rotation difficult to include in 1-2 D

Traditional Approach: Hydro

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  - Multiple fluids
    - Track p, n, α, average heavy nucleus
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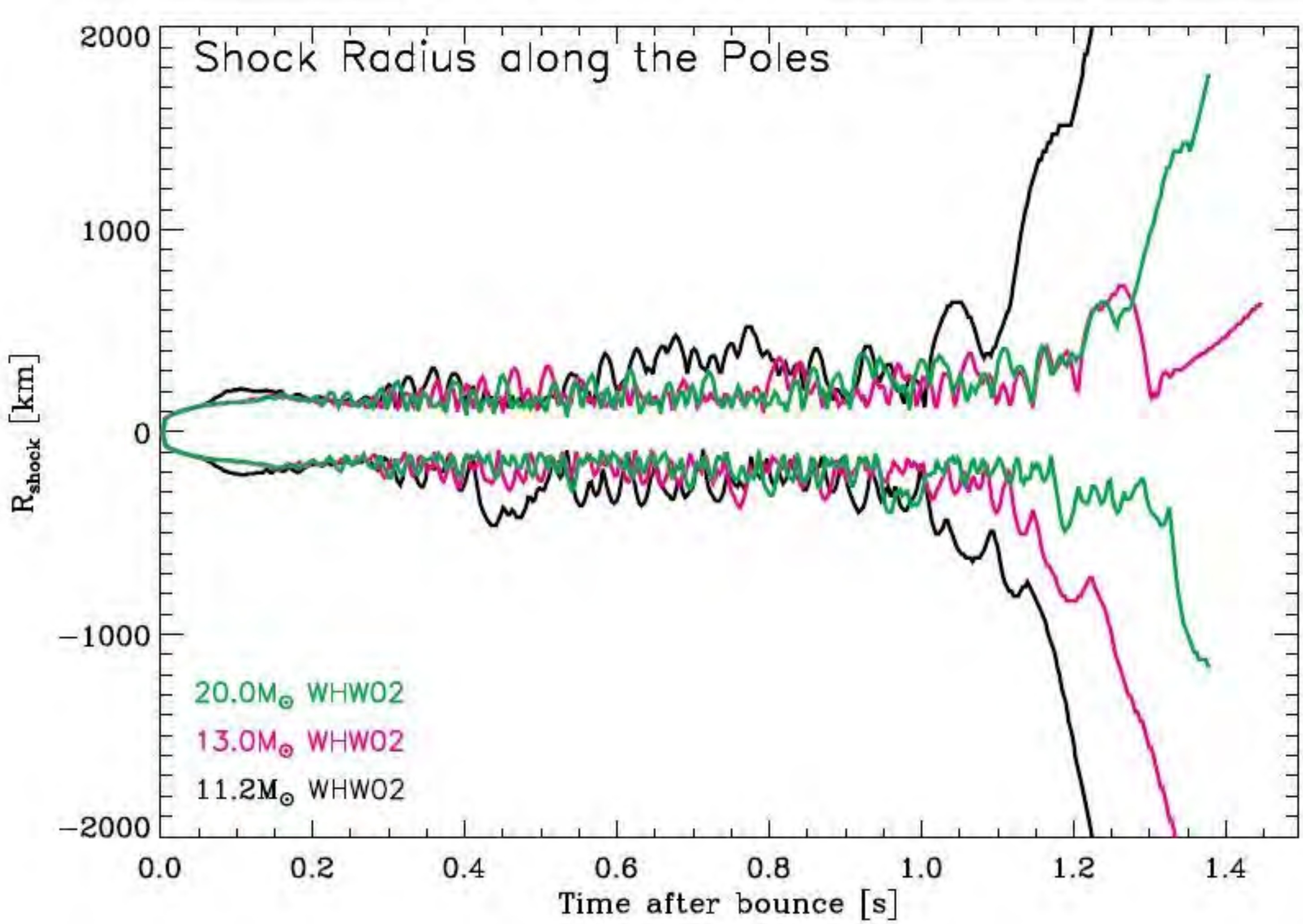
  - Very large number of time steps
  - Special relativity, causality
  - Realistic rotation difficult to include in 1-2 D
- Limited success
  - Burrows et al. (2D) & Fryer et al. (3D)

different explosion mechanisms

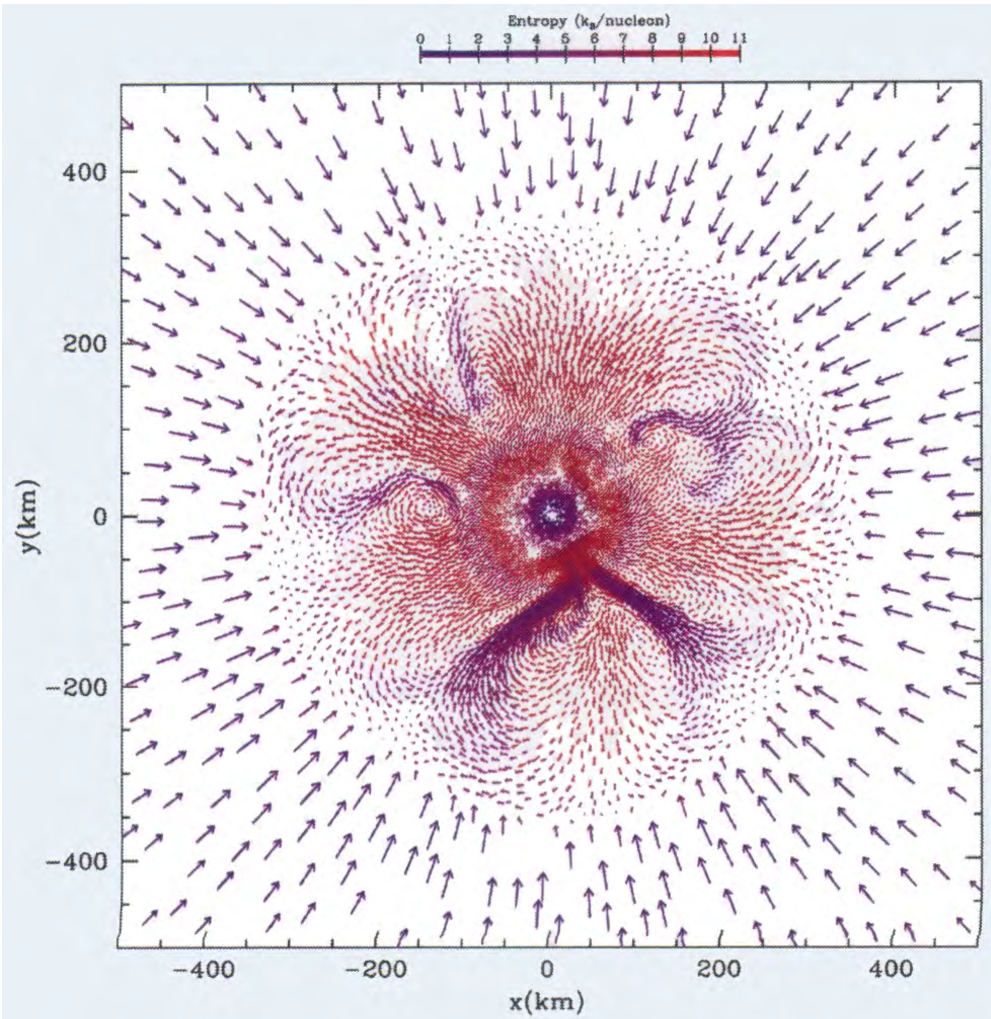
Burrows et al. Acoustic Mechanism

A. Burrows et al., ApJ 2006



Fryer et al. Convection

C. Fryer & P. Young, ApJ 2007



- Similar to their 2D convection
- Convection mechanism drives explosion
- No core oscillations
- 150-200 km/s kick

Supernovae Simulation Wish List

- Explicitly model propagation of full ensemble of nuclei
  - Full reaction network
  - Retain sensitivity to structure
- Explicitly model propagation of neutrinos in a general way
  - No simplifying assumptions
  - No problems between trapping and free streaming
- Do this in 3D

Too soon for hydro!

Simulations of Supernovae and Nuclear Collisions

- Similarities: Must simulate
  - particle production
    - neutrinos for supernovae
    - pions for nuclear collisions
  - shock wave formation
  - interplay between regular and chaotic collective dynamics

Simulations of Nuclear Collisions

- Characterize system with 6-D phase space density f
- Need to numerically solve transport equations

$$
\begin{aligned}
\frac{\partial}{\partial t} f(\vec{r},\vec{p},t) \quad &+ \quad \frac{\vec{p}}{m}\vec{\nabla}_r\, f(\vec{r},\vec{p},t) - \vec{\nabla}_r U\, \vec{\nabla}_p f(\vec{r},\vec{p},t) \\
&= \quad \frac{g}{2\pi^3\, m^2} \int d^3q_{1'}\, d^3q_2\, d^3q_{2'} \\
& \qquad \delta\left(\frac{1}{2m}(p^2 + q_2^2 - q_{1'}^2 - q_{2'}^2)\right) \cdot \delta^3(\vec{p} + \vec{q}_2 - \vec{q}_{1'} - \vec{q}_{2'}) \cdot \frac{d\sigma}{d\Omega} \\
& \qquad \cdot \Big\{ f(\vec{r},\vec{q}_{1'},t)\, f(\vec{r},\vec{q}_{2'},t) \Big(1 - f(\vec{r},\vec{p},t)\Big) \Big(1 - f(\vec{r},\vec{q}_2,t)\Big) \\
& \qquad - f(\vec{r},\vec{p},t)\, f(\vec{r},\vec{q}_2,t) \Big(1 - f(\vec{r},\vec{q}_{1'},t)\Big) \Big(1 - f(\vec{r},\vec{q}_{2'},t)\Big) \Big\}
\end{aligned}
$$

Wait, where does this equation come from?

Many-Body Theory in a Nut Shell

- Schrödinger eq. for n-body wave function, $H\psi = E\psi$.
- Form density matrix $\psi\psi^*_n$, which obeys von Neumann eq. of motion
- Define n-1 body density matrix $\psi\psi^*_{n-1}$ via integration over n-body density matrix $\psi\psi^*_n$.
  - Eq. of motion: $d_t\psi\psi^*_{n-1} = F(\psi\psi^*_{n-1},\psi\psi^*_n)$
  - Truncate at some level by neglecting hire correlation
  - Lowest level (no 2-body correlations: mean field): TDHF
  - Second-lowest (no 3-body correlations: mena field + two-body collision: correlation dynamics
- Wigner transform: $f(r,p,t)$
  - Lowest level: Vlasov equation
  - Second lowest: BUU equation (shown 2 pages ago)

Simulations of Nuclear Collisions

- Numerical approach
  - Fully discritize relevant phase space
  - Course grid ~ 10^9 lattice sites ← **Too big!**
  - Alternative:
    - Only follow initially occupied phase space cells in time and represent them by imaginary test particles
    - One-body mean-field potentials (ρ,p,τ)
    - Scatter via realistic cross sections
    - Coupled equations for many species no problem
    - 100 -1000 test particles/nucleon
    - ~10 -100 k test particles total ← sufficient

Test Particle Approach

- Formally approximate f by a sum of delta functions (test particles)

$$f(\vec{r},\vec{p},t) = \int d^3r_0\, d^3p_0\, \delta^3(\vec{r}-\vec{R}(\vec{r}_0,\vec{p}_0,t_0))\, \delta^3(\vec{p}-\vec{P}(\vec{r}_0,\vec{p}_0,t_0))\, f(\vec{r}_0,\vec{p}_0,t_0)$$

- Insert this into integral transport equation to obtain equations of motion for 6 coordinates of each test particle

Test Particle Approach

$$\frac{d}{dt}\vec{p}_i = -\vec{\nabla}U_{EOS}(\vec{r}_i) - \vec{\nabla}U_C(q_i, \vec{r}_i) + \vec{C}(\vec{p}_i)$$

$$\frac{d}{dt}\vec{r}_i = \frac{\vec{p}_i}{m_i}$$

$$i = 1, \ldots, N$$

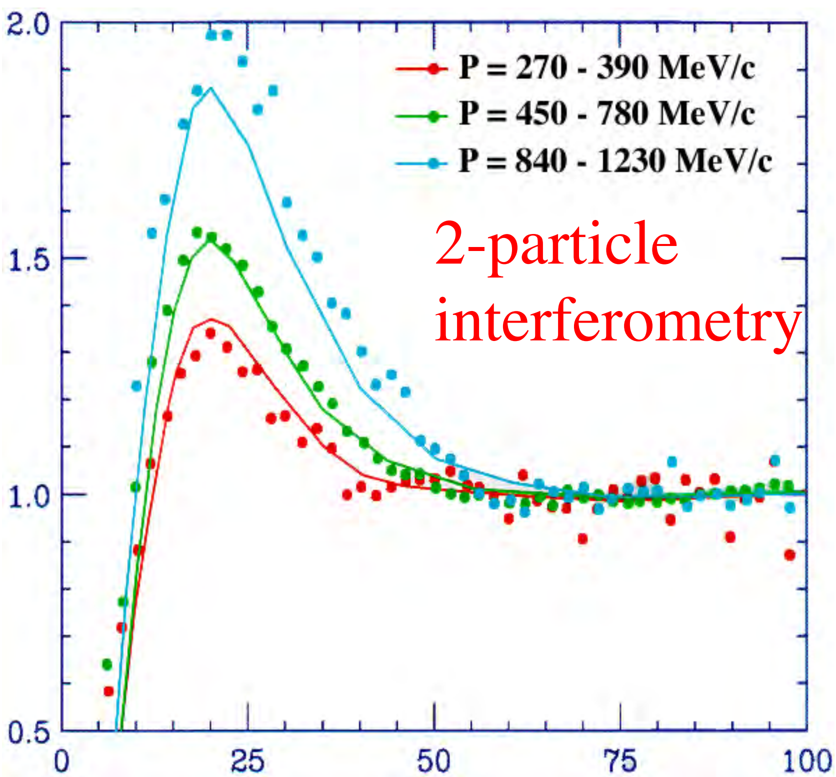
Coulomb

Nuclear EOS

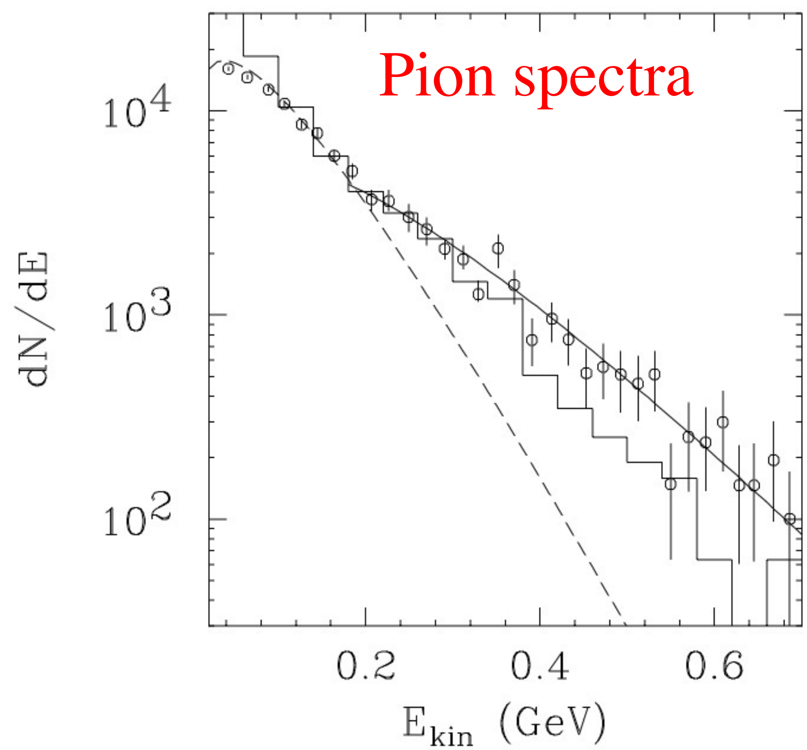
2-body scattering

Test Particle Approach

- Reproduces experiments (lots of them)



W.G. Gong et al., PRL (1990)



B.A. Li & WB, PLB (1991)

Test Particle Approach Applicability

- Require number of test particles large enough to accommodate phase space complexity (important details)
- Whether

$$\frac{\text{number of physical particles}}{\text{number of test particles}} \ll 1$$

microscopic

or

macroscopic

$$\frac{\text{number of physical particles}}{\text{number of test particles}} \gg 1$$

Apply This To Supernovae

- $\sim$1 $M_\odot$ iron core $\sim 10^{57}$ baryons & leptons
- $\sim 10^{7}$ matter test particles $\Rightarrow$ $\sim 10^{50}$ baryons per matter test particle
- Matter test particles interact via one-body potentials $(\rho, \eta_e, \vec{r})$ and 2-body scattering
- Neutrino test particles can be created and destroyed

Coupled transport equations

(includes relativity; otherwise very similar derivation to BUU eq.)

$$\frac{\partial f_b(xp)}{\partial t} + \frac{\Pi^i}{E_b^*(p)}\nabla_i^x f_b(xp) - \frac{\Pi^\mu}{E_b^*(p)}\nabla_i^x U_\mu(x)\nabla_p^i f_b(xp) + \frac{M_b^*}{E_b^*(p)}\nabla_i^x U_s \nabla_p^i f_b(xp)$$

$$= I_{bb}^b(xp) + I_{b\nu}^b(xp)$$

$$\frac{\partial f_\nu(xk)}{\partial t} + \frac{k\cdot\nabla^x}{E_\nu(k)} f_\nu(xk) = I_{b\nu}^\nu(xk)$$

- 2-body collision terms structurally identical to BUU source term
  - Couples transport equations
  - Essential input: neutrino-nucleus cross sections (Nakamura et al, ApJ 1999; K. Sumiyoshi et al, NPA 2001, Fröhlich et al, PRL 2006, B.A. Brown, …)

Selecting Scattering Pairs

- Matter test particles
  - Baryonic matter in hydro limit
  - Time and distance between collisions is small
  - Organize matter test particles in 3D grid
  - Randomize COM momenta

$t_{CPU} \sim N \log N$
Main operation:
Database sort

Algorithm: stochastic Direct Simulation Monte Carlo (DSMC)
[Nuclear physics: Kortemeyer et al., PLB 374, 25 (1996)]

Selecting Scattering Pairs

- Neutrinos *not* generally in hydro limit
  - Some free-streaming, some trapped, some in between
  - Use beam attenuation arguments

$$P_{int} = 1 - exp\left\{-\sum_j \int_{\vec{x}_i}^{\vec{x}_f} \overline{\sigma}_j(\vec{x}) n_j(\vec{x}) d\vec{x}\right\}$$

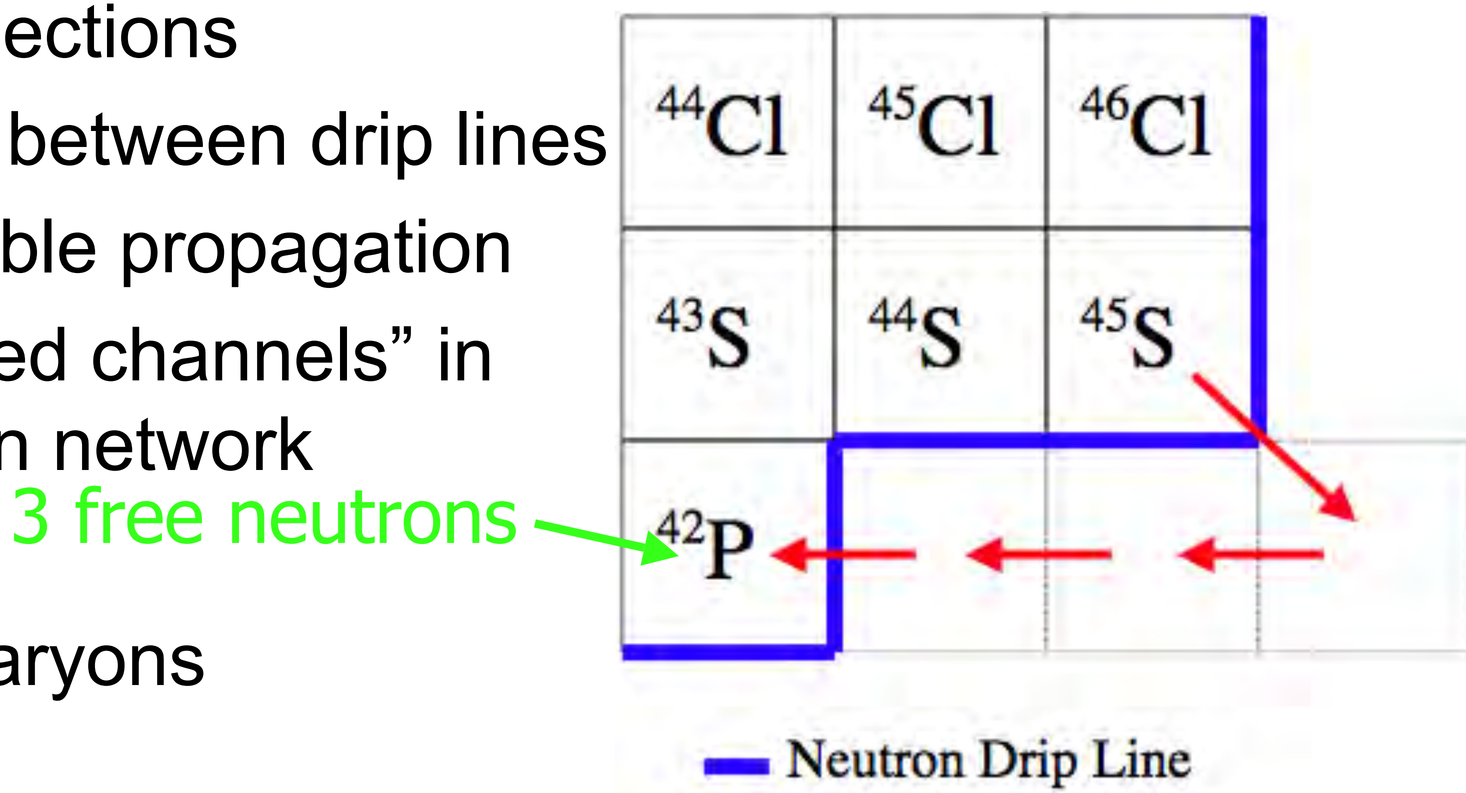
  - Construct relative probabilities

$$P(i) = \frac{n_i \overline{\sigma}_i}{\sum_j n_j \overline{\sigma}_j}$$

APPLICALBE EVERYWHERE AT ALL TIMES!

Matter Test Particle Properties

- Explicitly represent all nuclei
  - Many hundreds of isotopes
  - Lots of work: reaction network, weak interaction cross sections
  - All Z,A between drip lines
  - Ensemble propagation
  - "Coupled channels" in reaction network
  - Free baryons

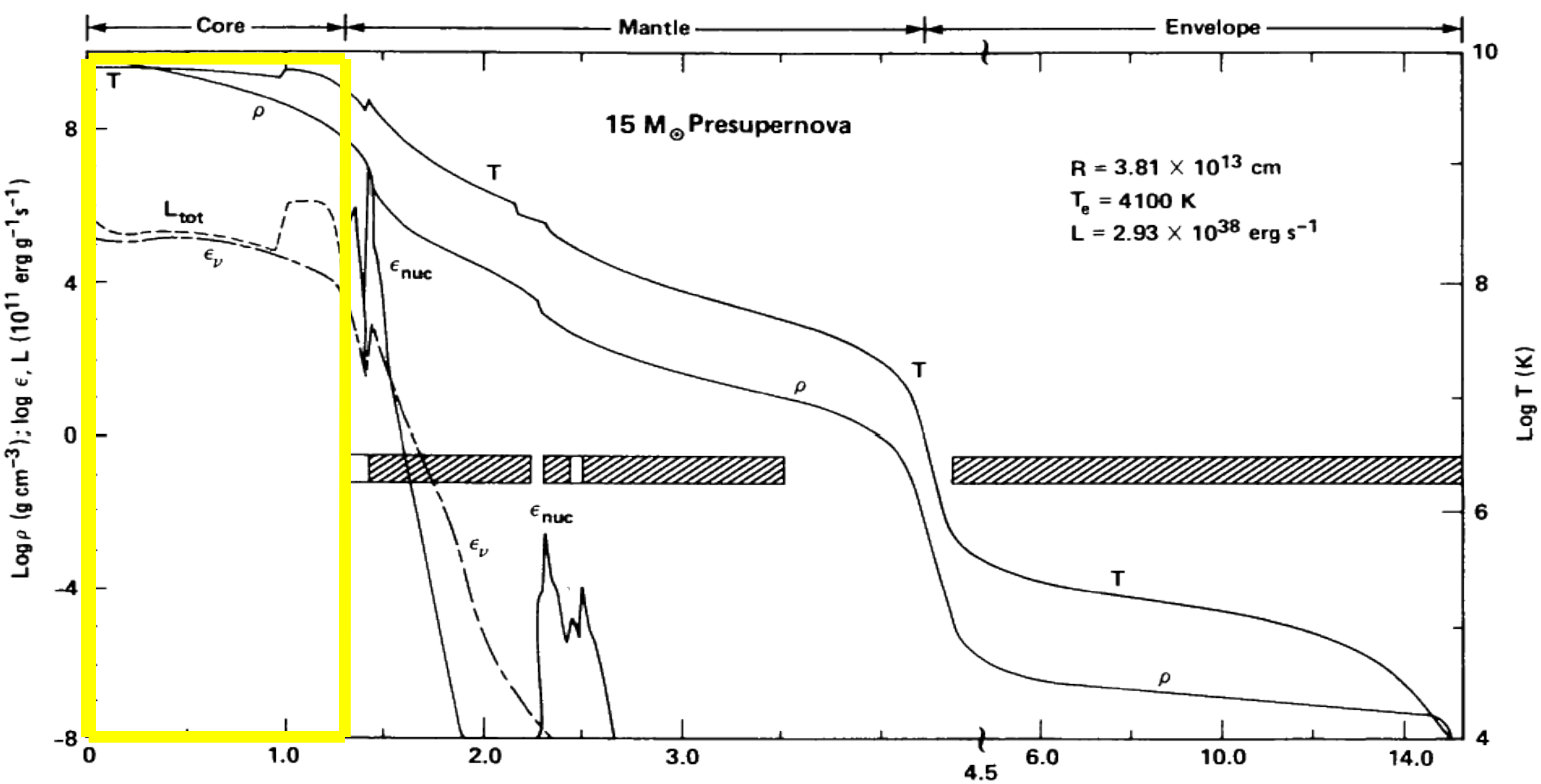


Initial Conditions

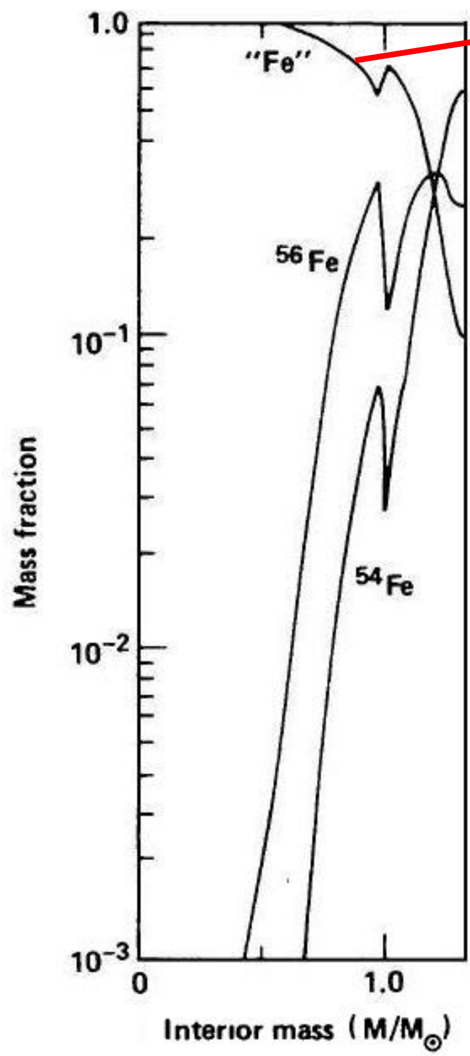
Use for the first 10^7 years

Concentrate on last 0.3 seconds

- Start with Woosley & Weaver's 15 $M_\odot$ progenitor



Core Modeled: Initial Conditions



"Fe" ⇒ $A \in [48,65]$ $Z/A < 26/56$

- $M_{core} = 1.33\ M_{\odot}$
- Spherically symmetric
- Radius ≈ 1000 km

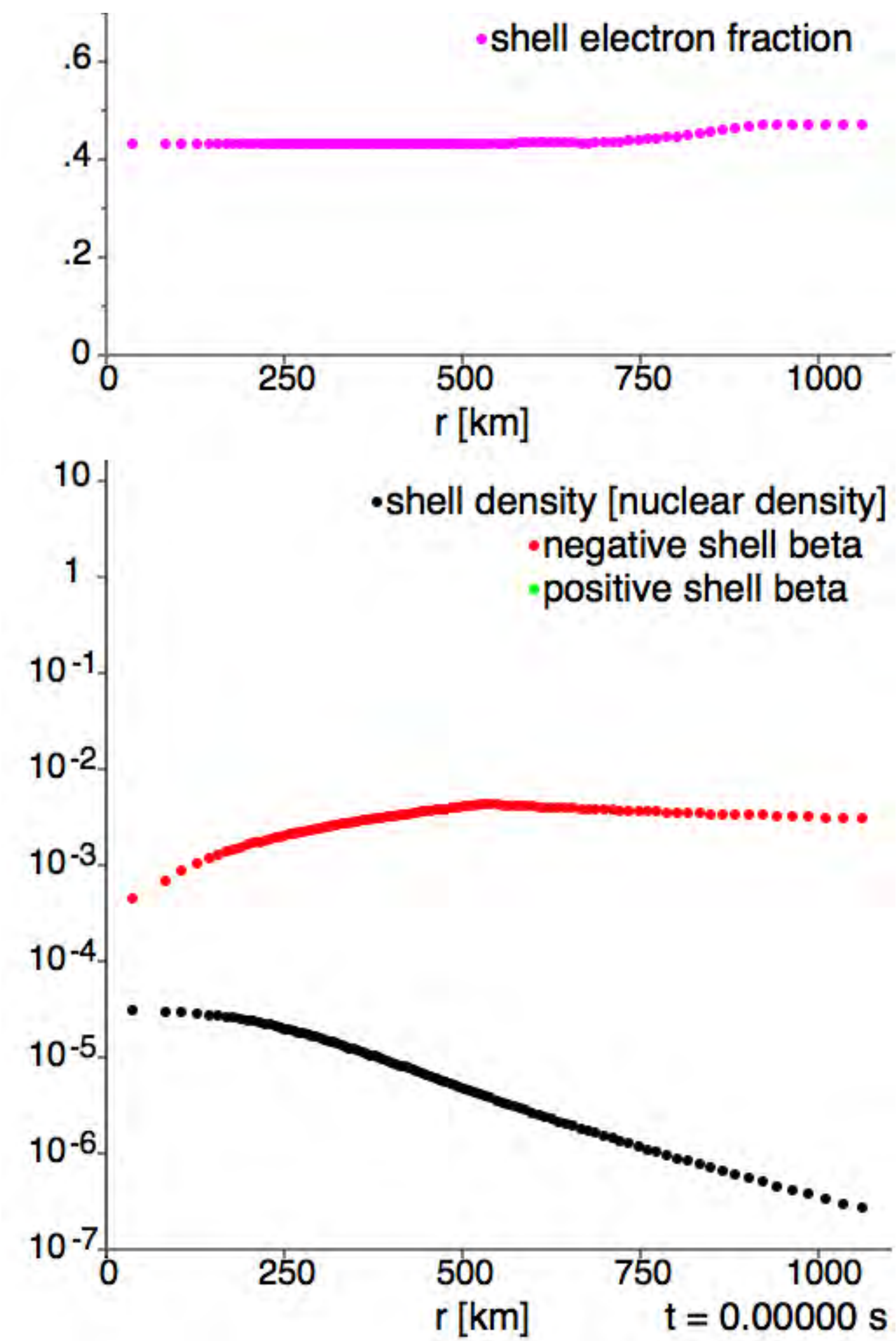
Some Results

- Single processor (spherical symmetry)
- 1 million matter test particles
  - 385 nuclei + free baryons
- Cold soft BKD nuclear EOS
- Weak interaction network
  - Electron capture (reduced FFN rates)
  - Neutrino-matter interactions
  - Neutrino oscillations a la “MSW”
- No fusion or photo-disintegration channels included

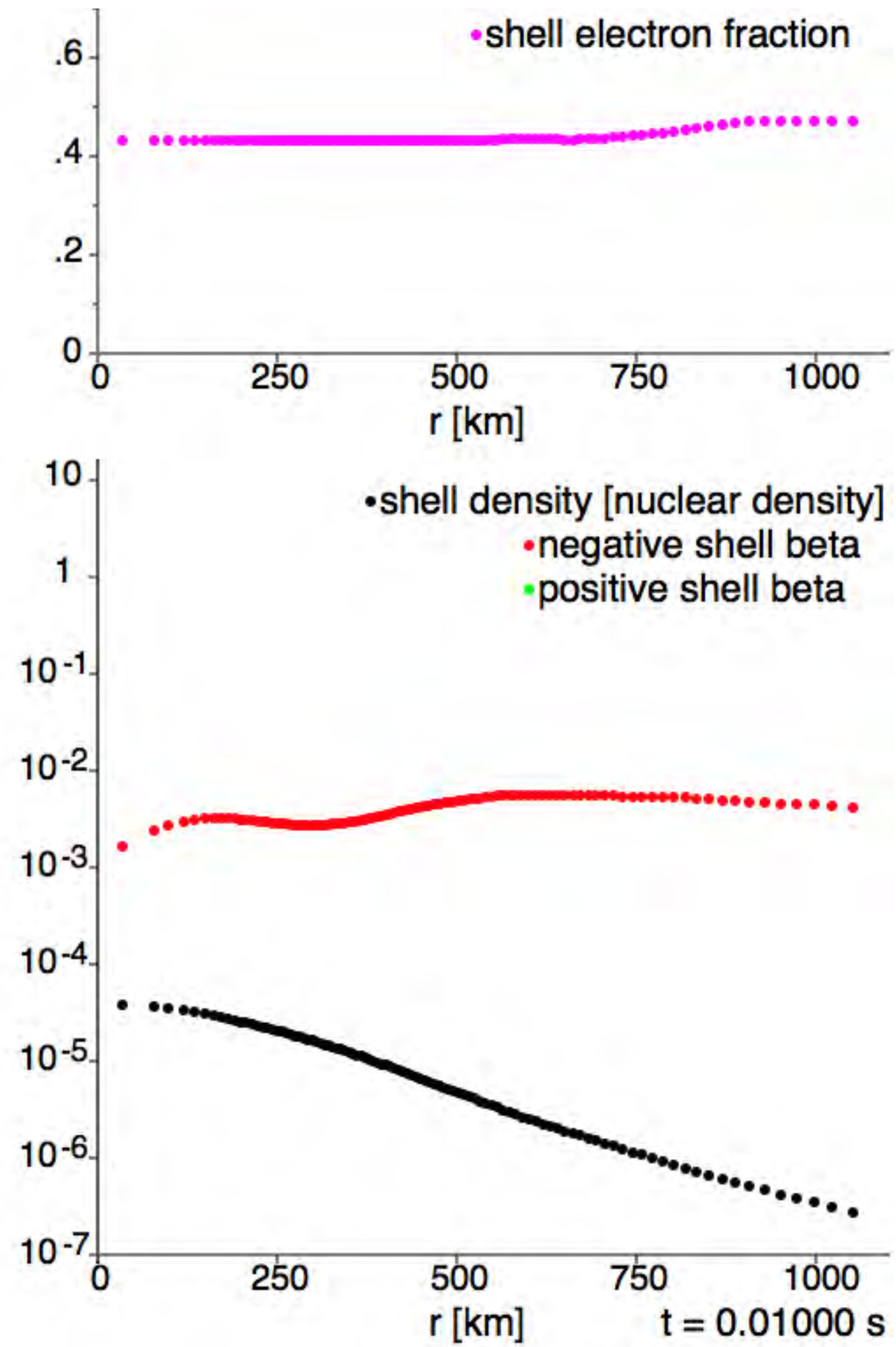
Time evolution

Interplay of macro- and micro-scales forces very large number of comparatively small time steps
($c \sim 1$ ft/ns)
$\Delta t = 10^{-5}$ s

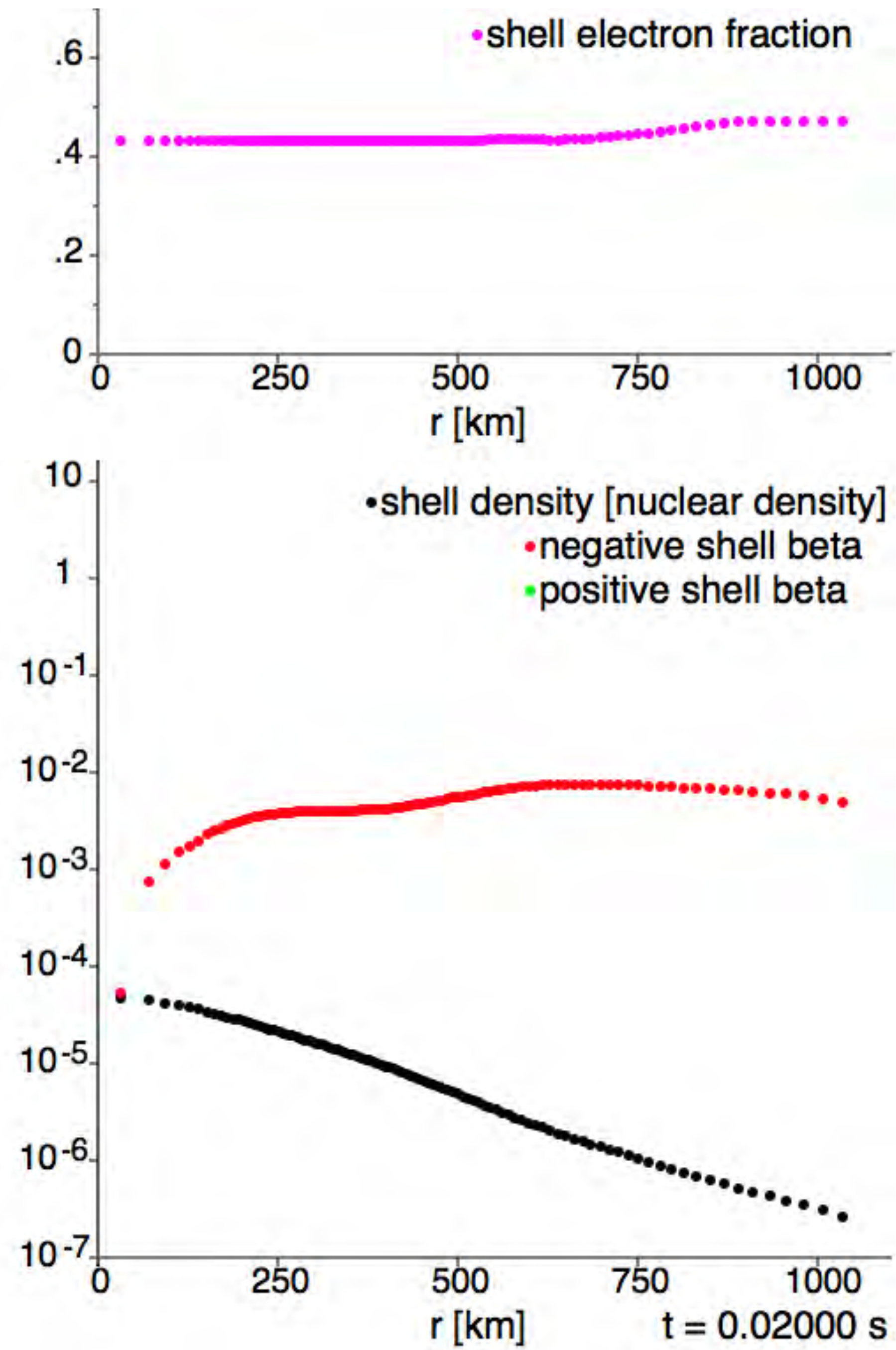
=> Mostly boring initial time evolution (take 1000 steps between frames)



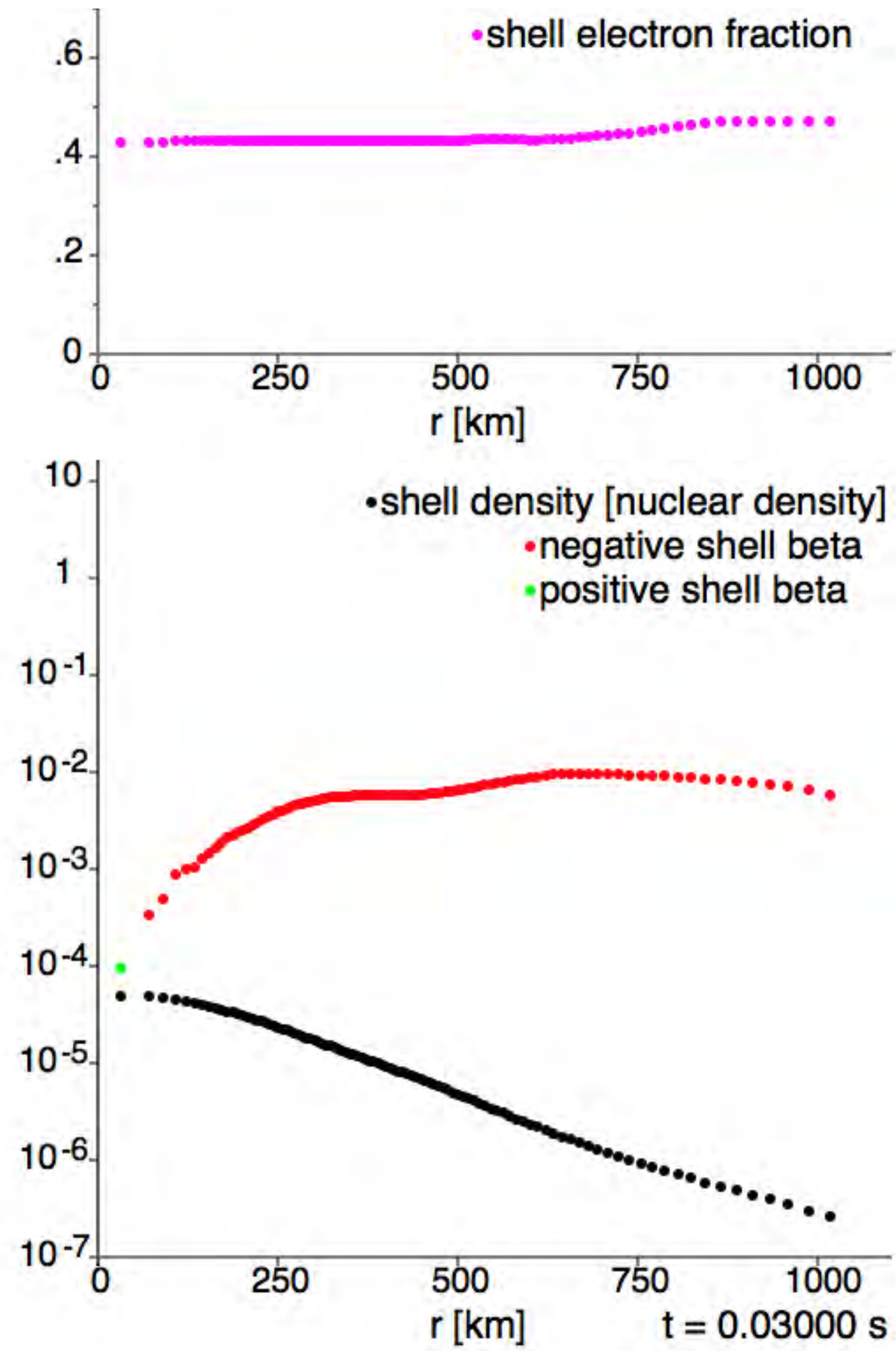
Time evolution



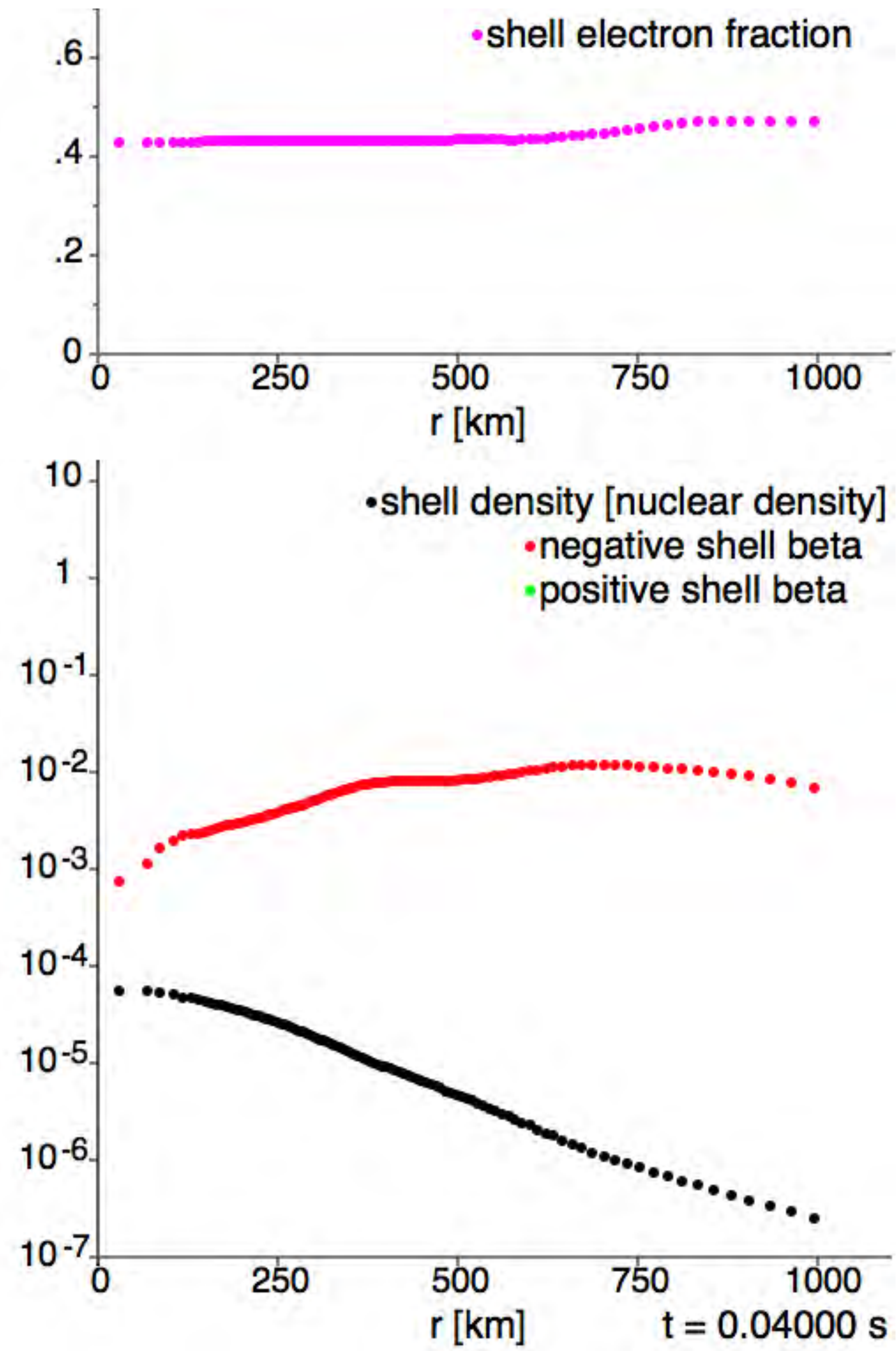
Time evolution



Time evolution



Time evolution



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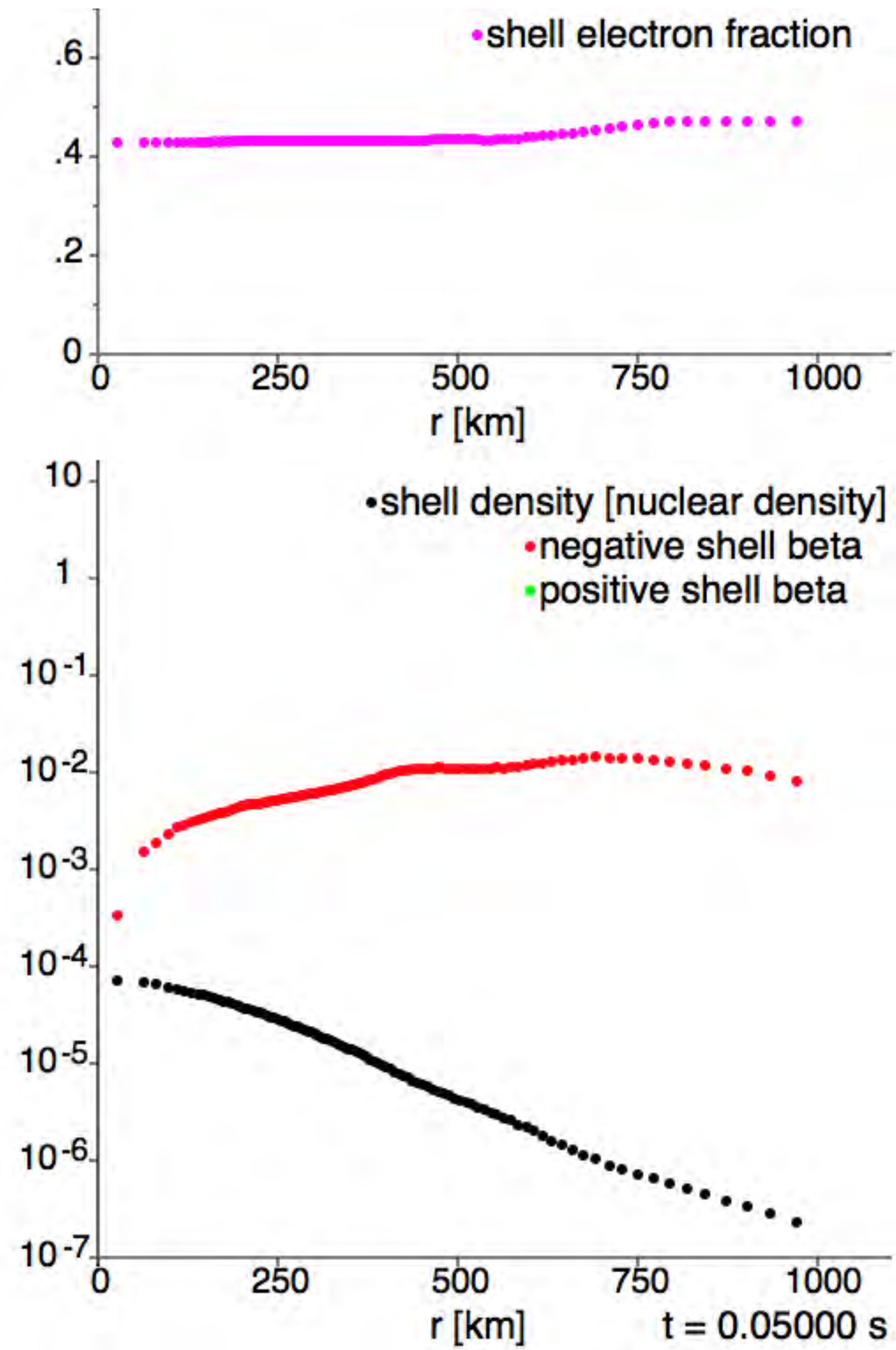

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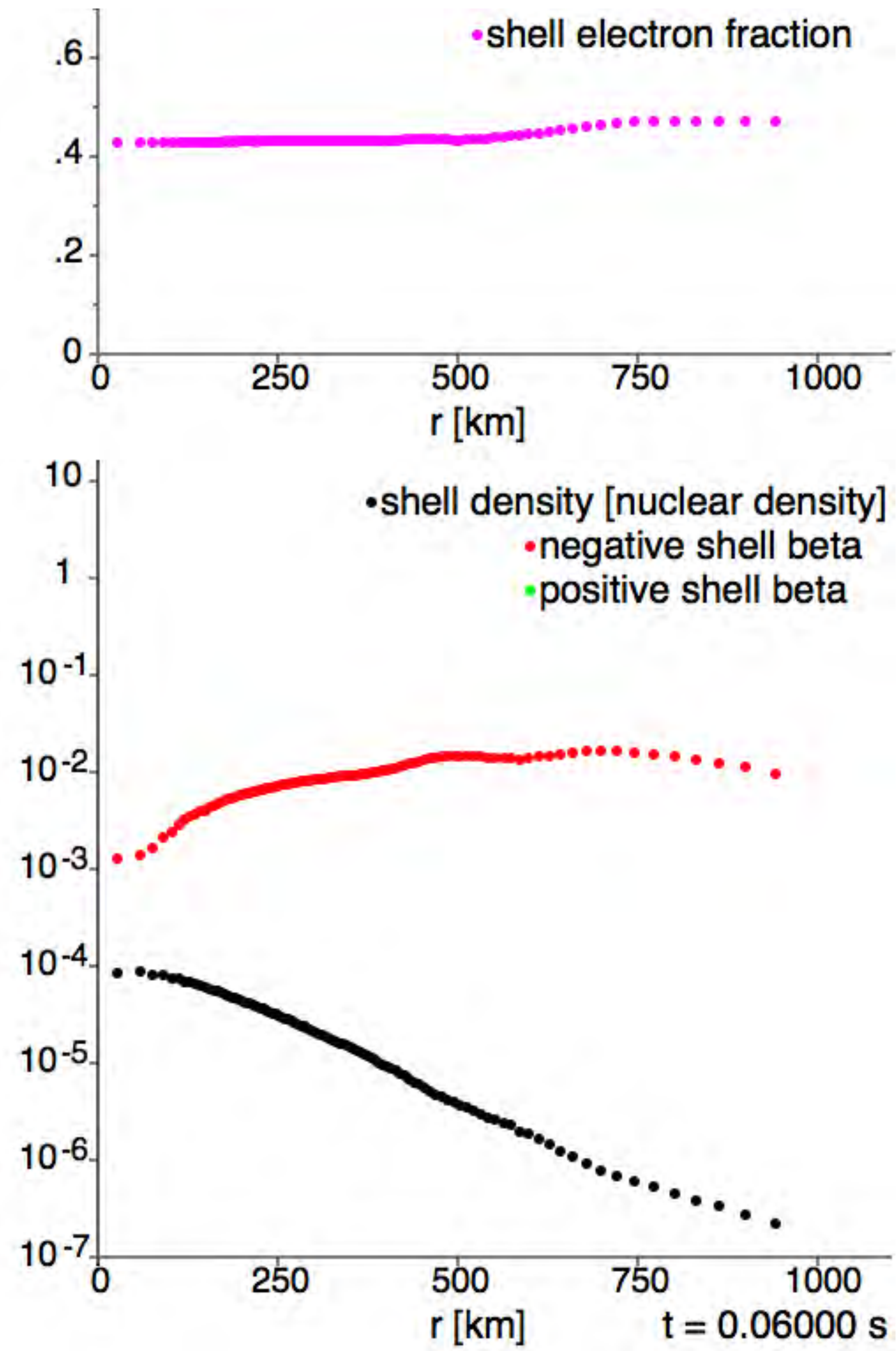

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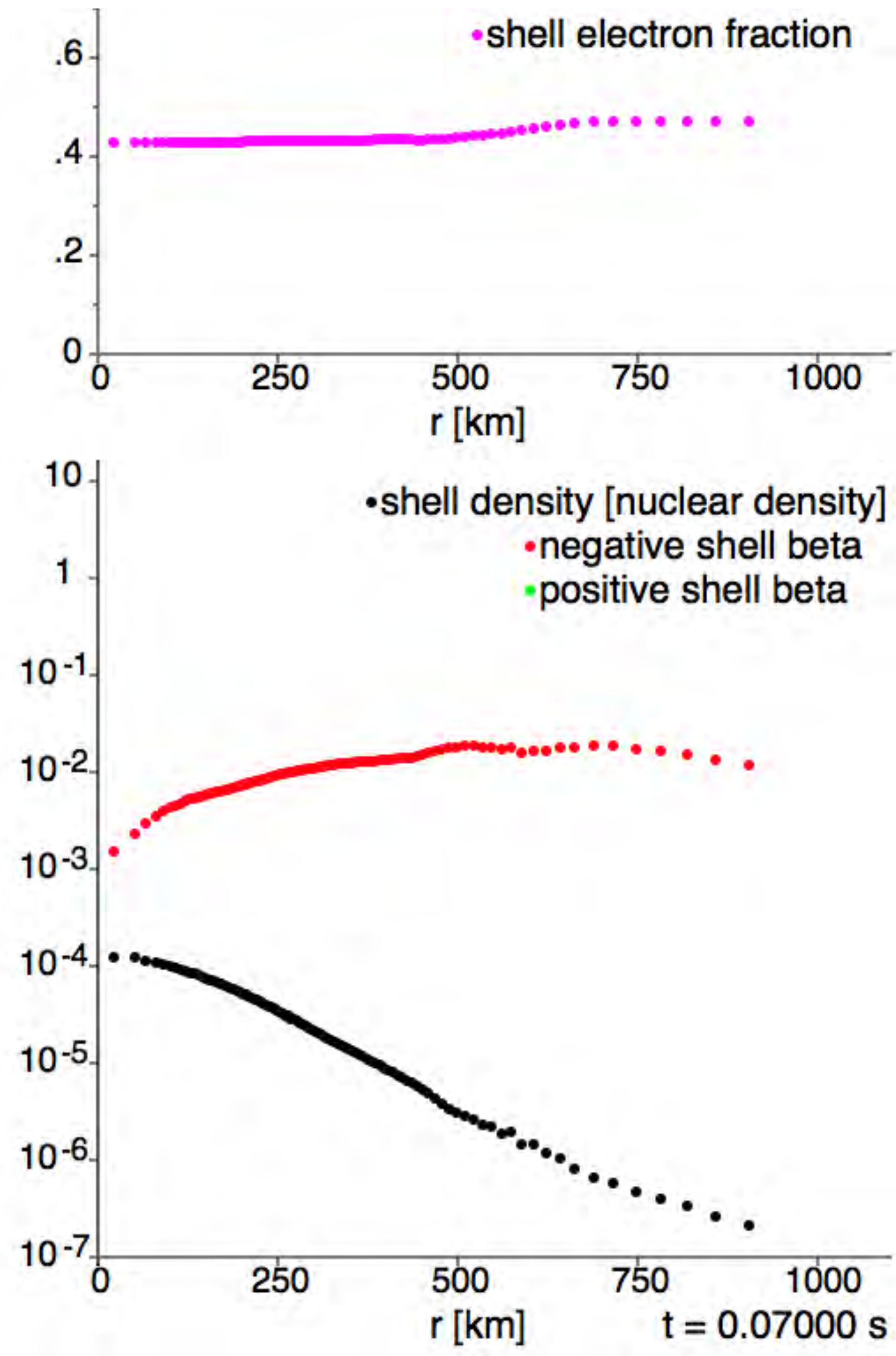
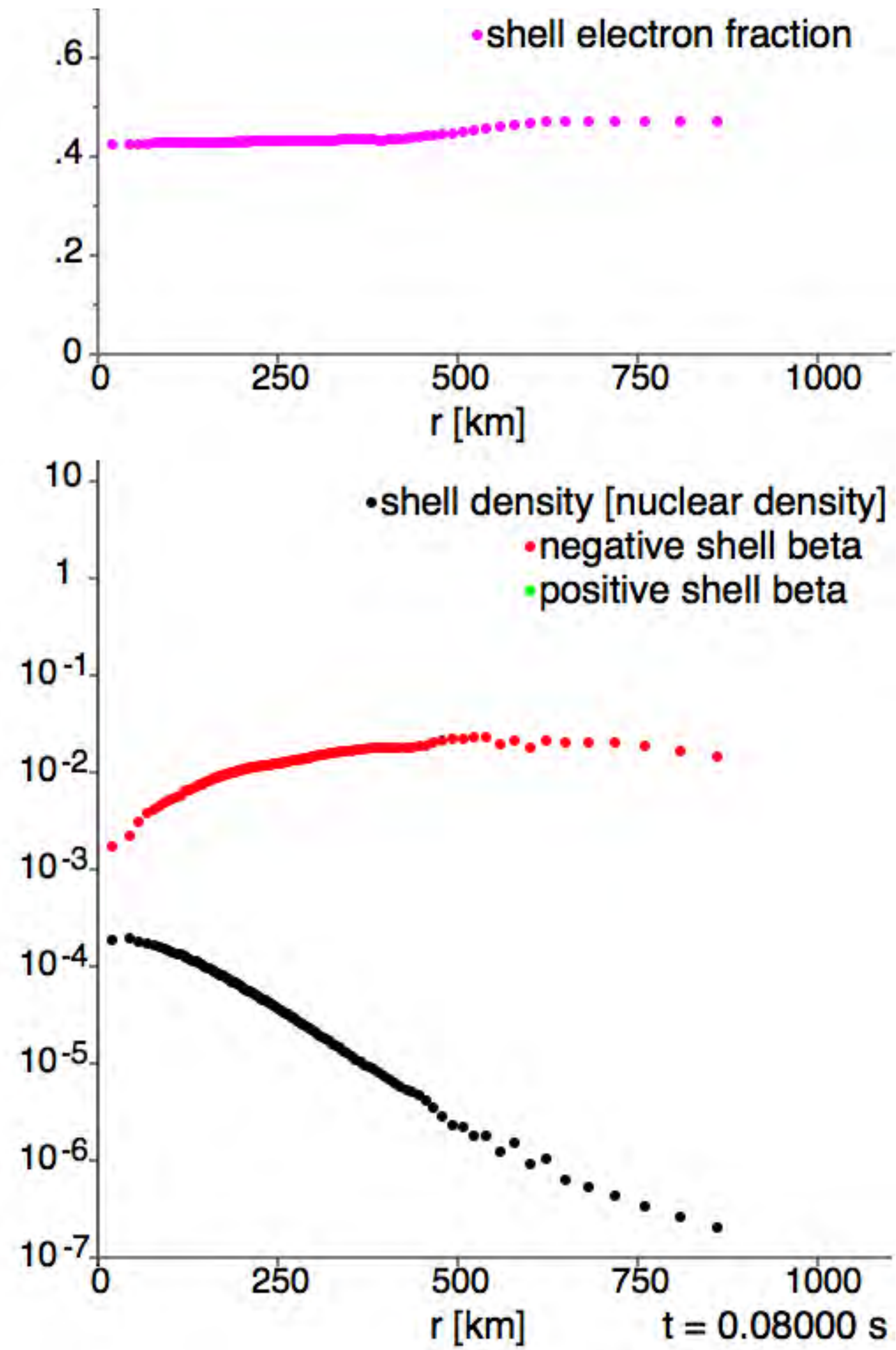


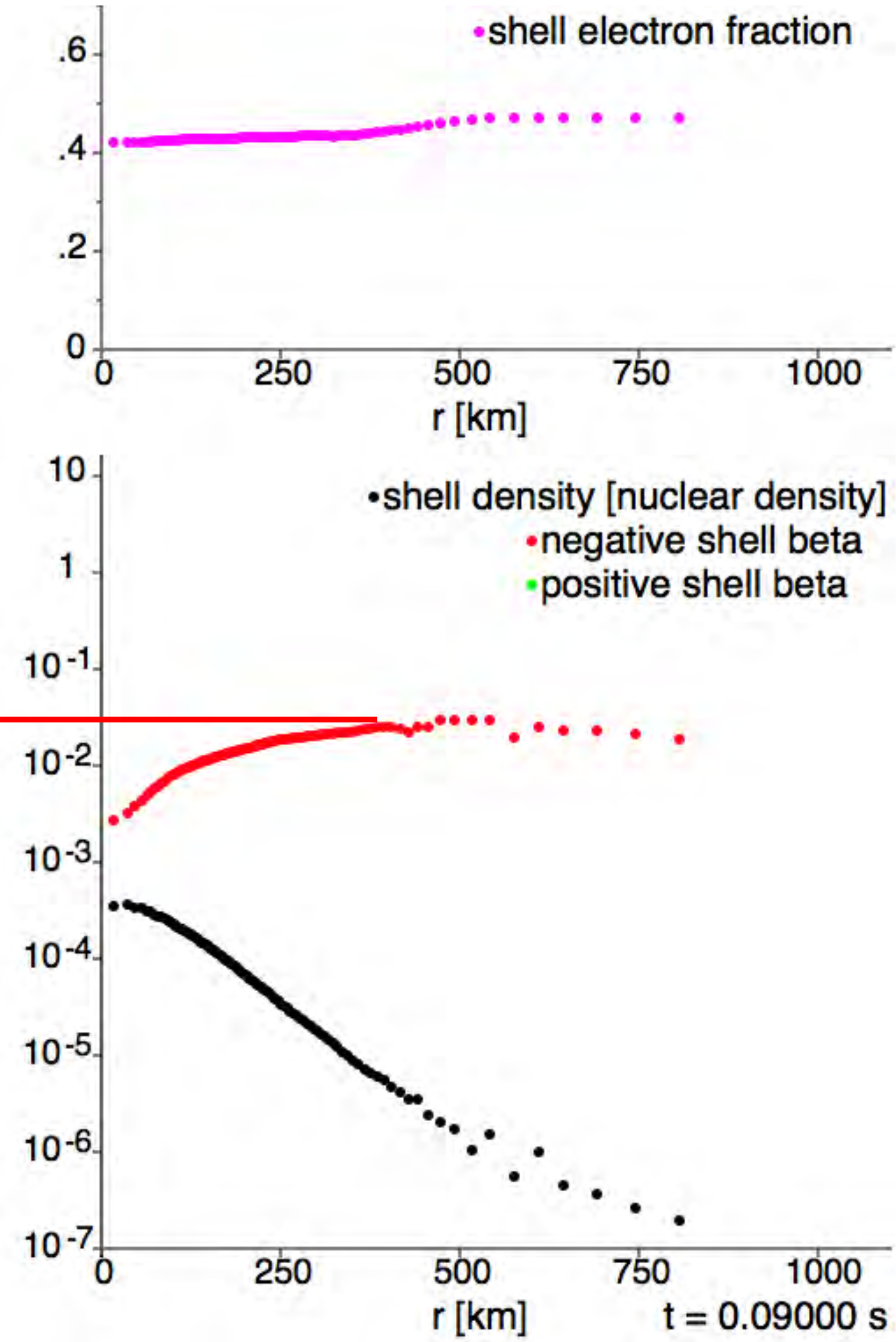
Time evolution



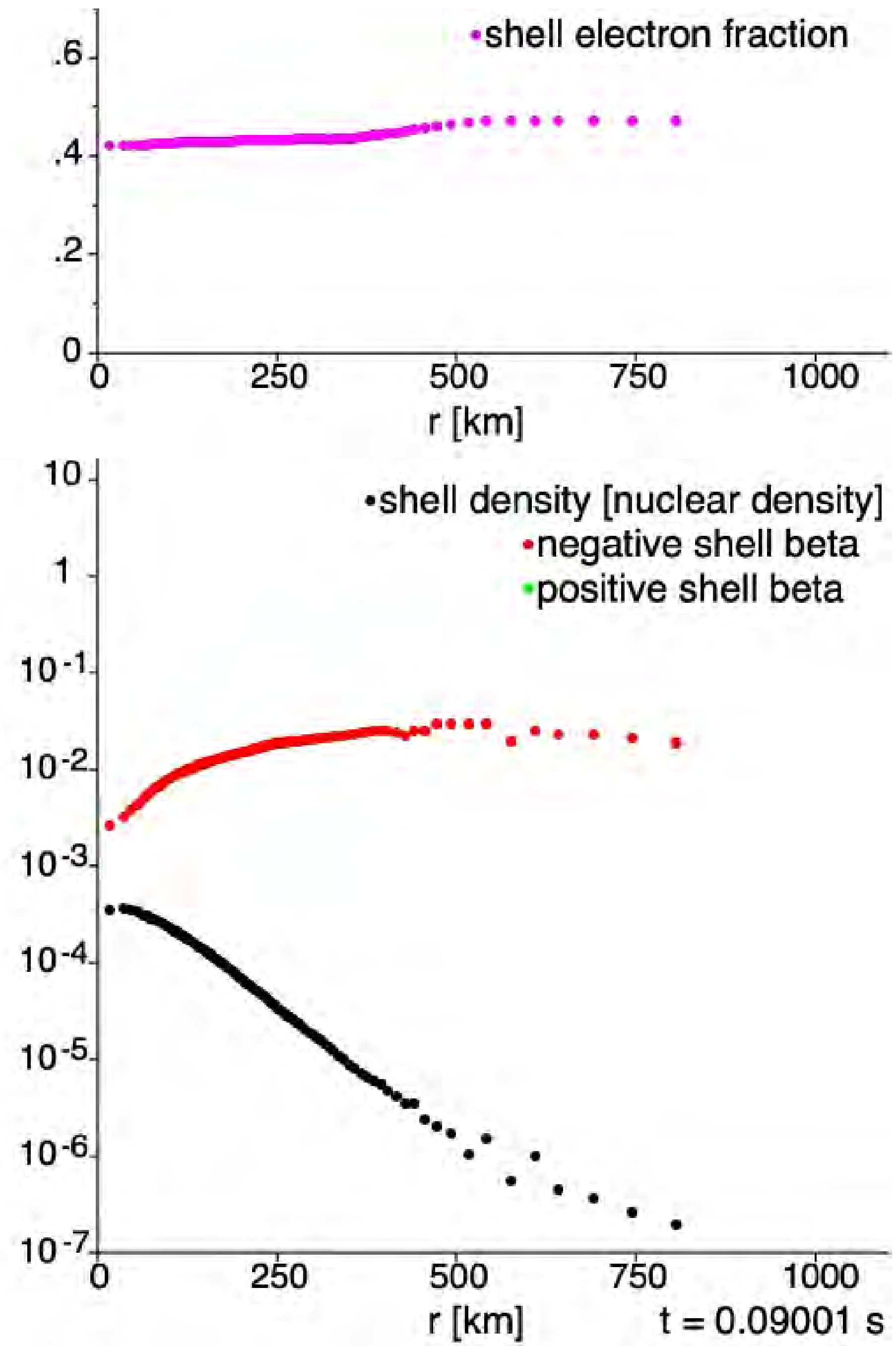
Time evolution

3% *c*

Boring first 9000
time steps are done
Now: Movie



Time evolution



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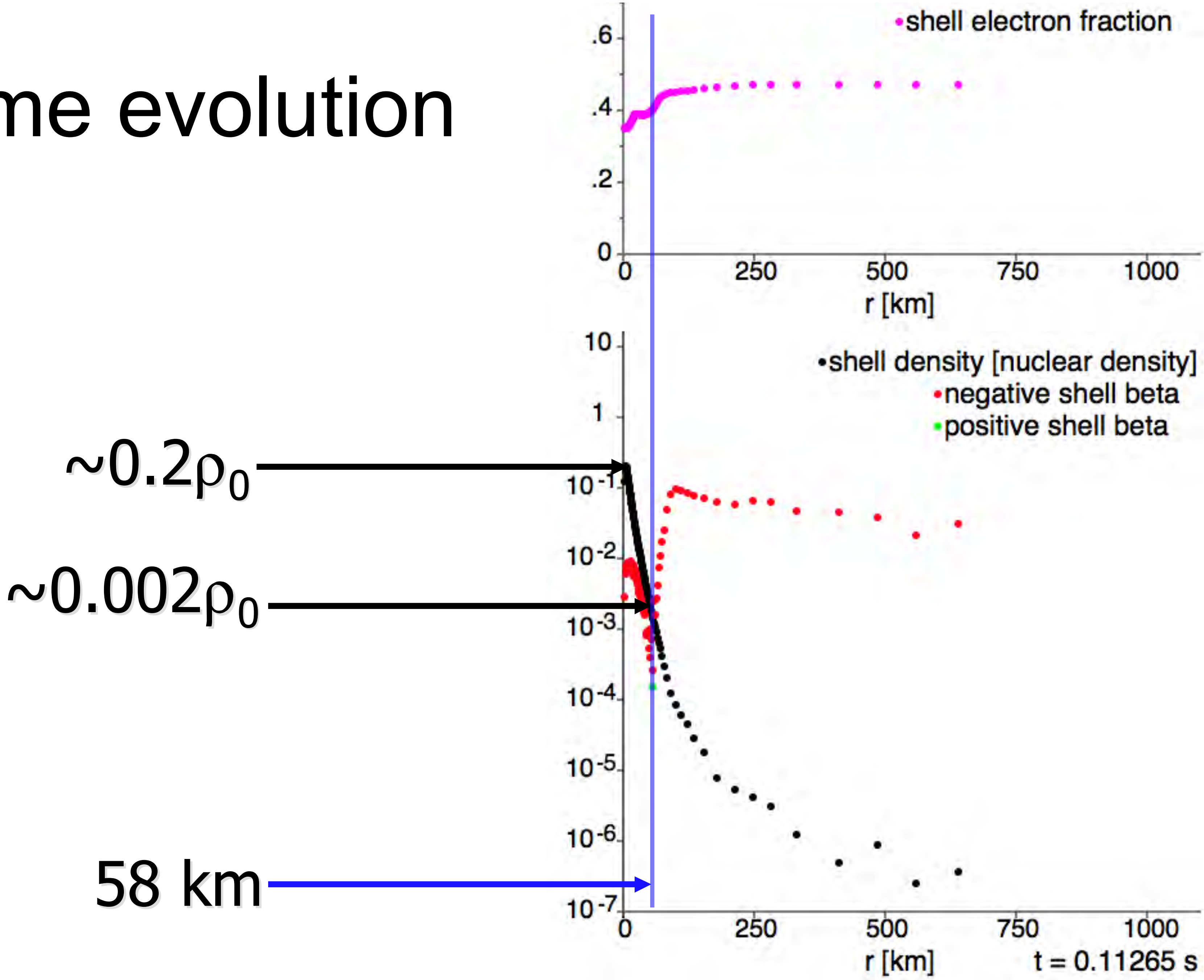
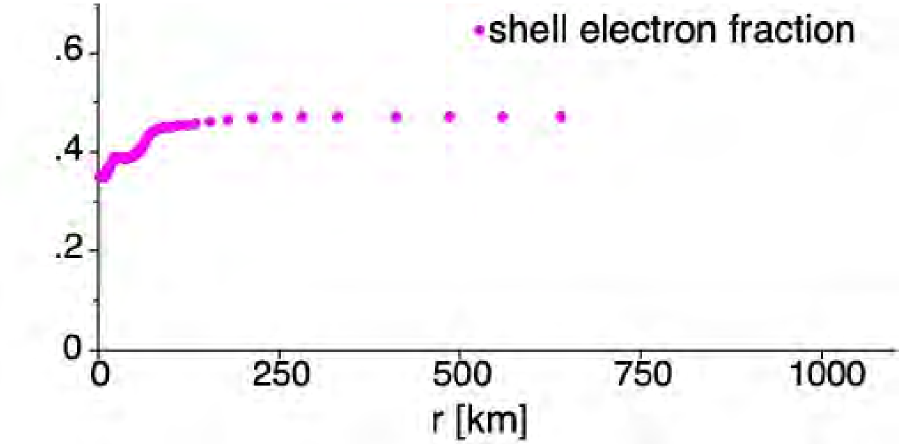
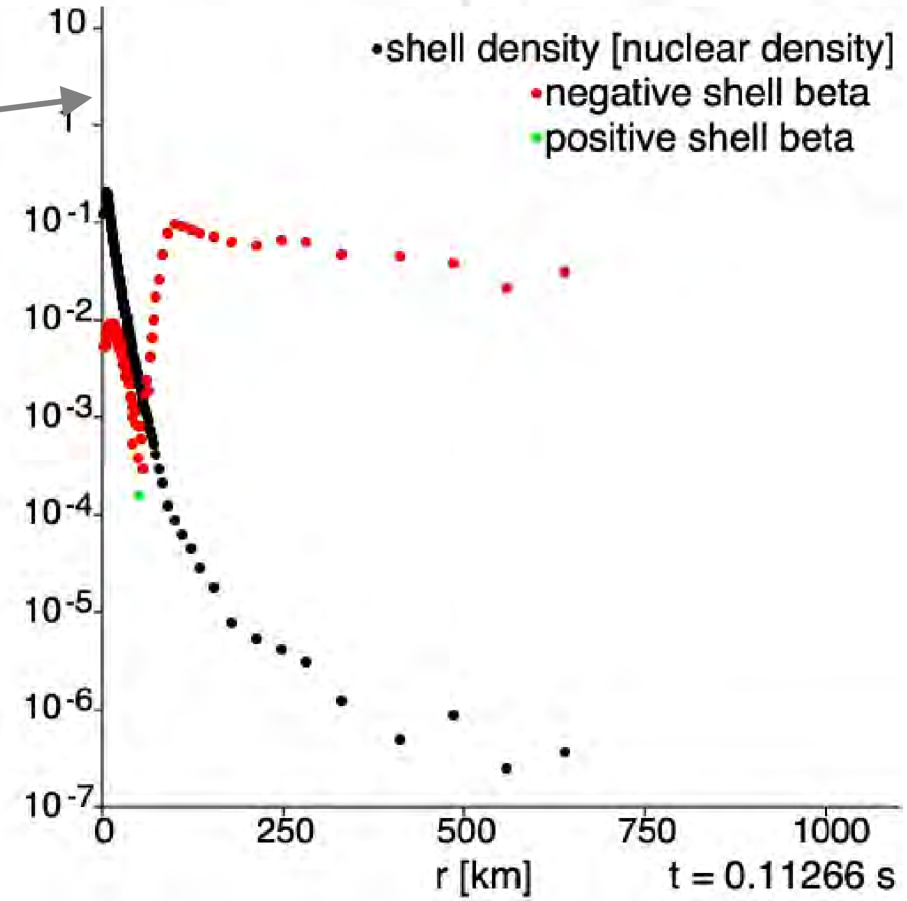


Time evolution



Remnant

- M=0.25 $M_\odot$
- R=7.3 km
- ρ_c=1.7ρ_0
- η_c=0.27

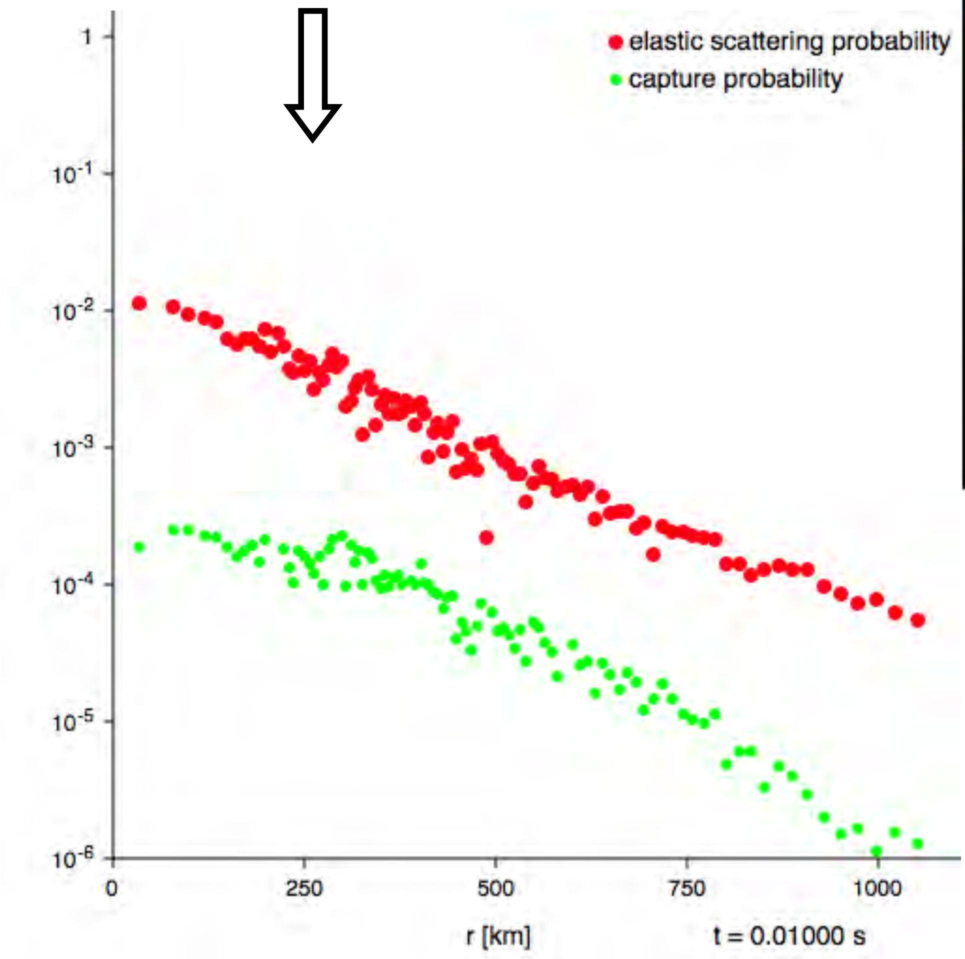


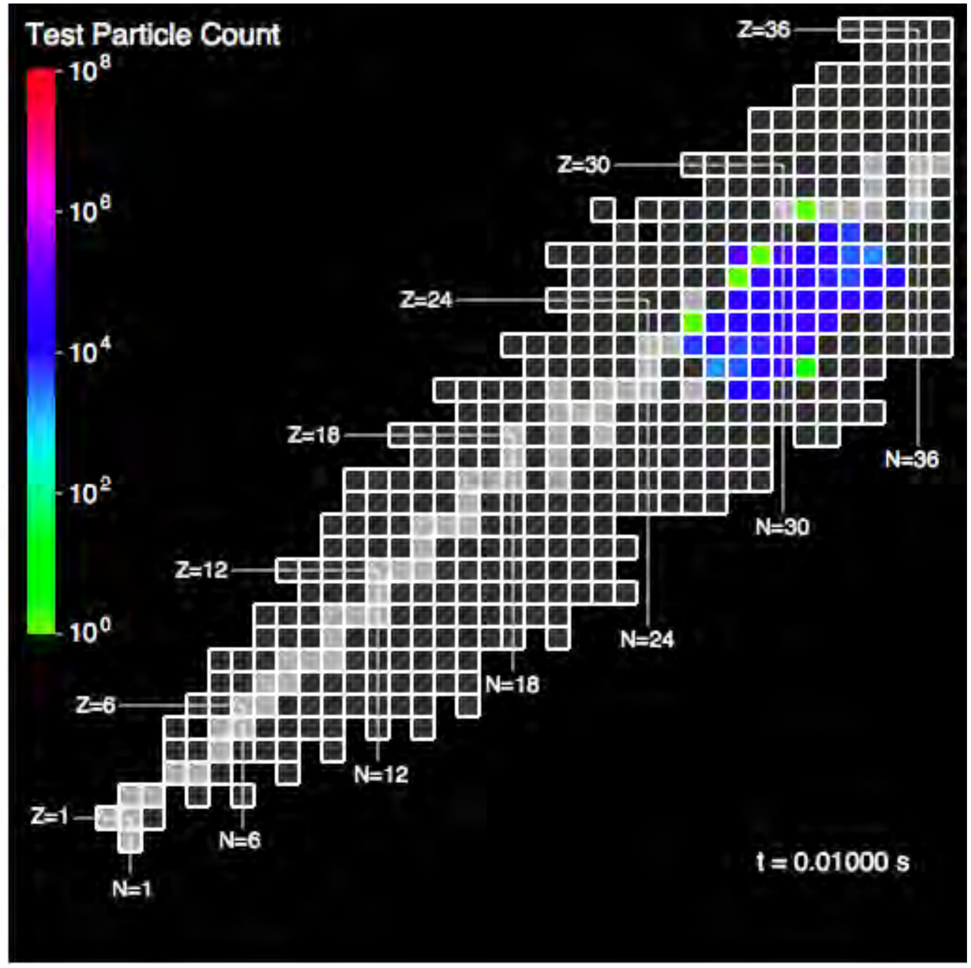
Why?

- Electron fraction spike "cuts" the core in two
  - Proto-remnant "gently" assumes ideal configuration
  - Role of nuclear EOS totally different
- How does the spike form?
  - $\rho(r_{exp}) \sim 0.002\rho_0$
  - Study neutrino-matter interaction probabilities
    - Nuclear structure
    - Relativistic electron gas statistical mechanics
    - Essential input: neutrino cross sections & nuclear structure (weak neutral current ~ A^2)

t = 0.01000 s

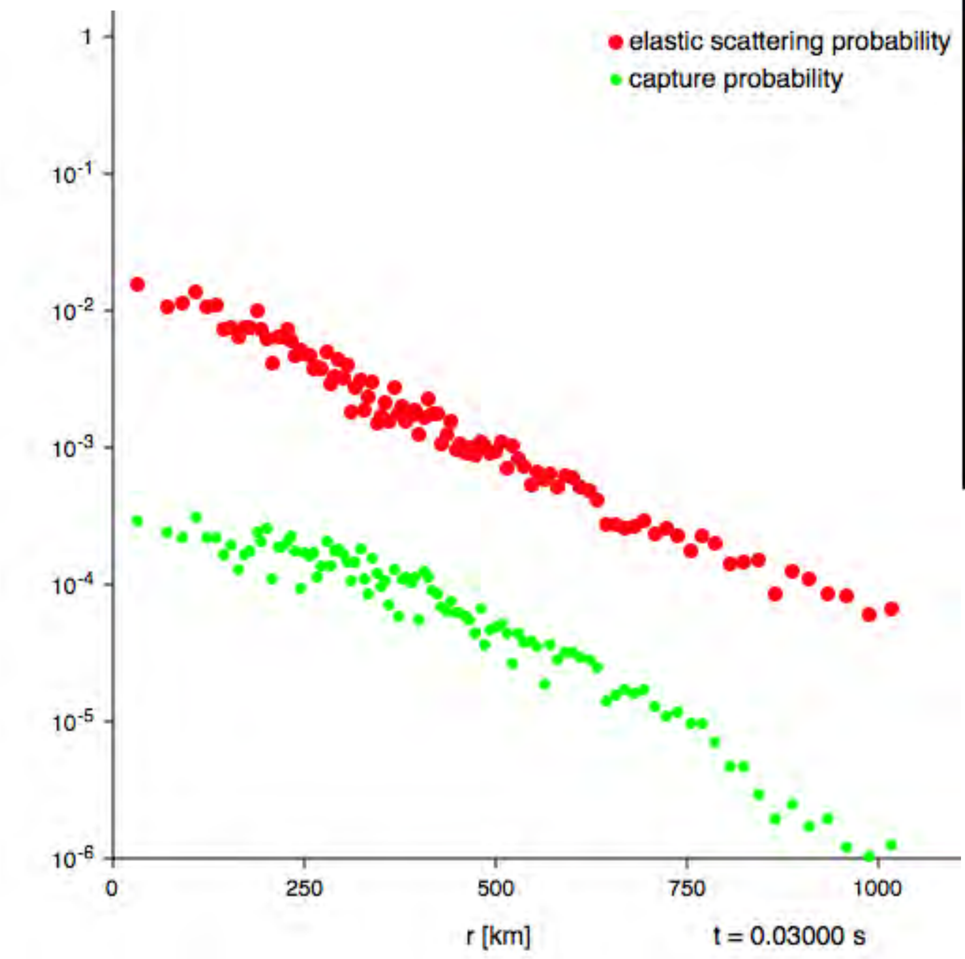
Average neutrino interaction probability

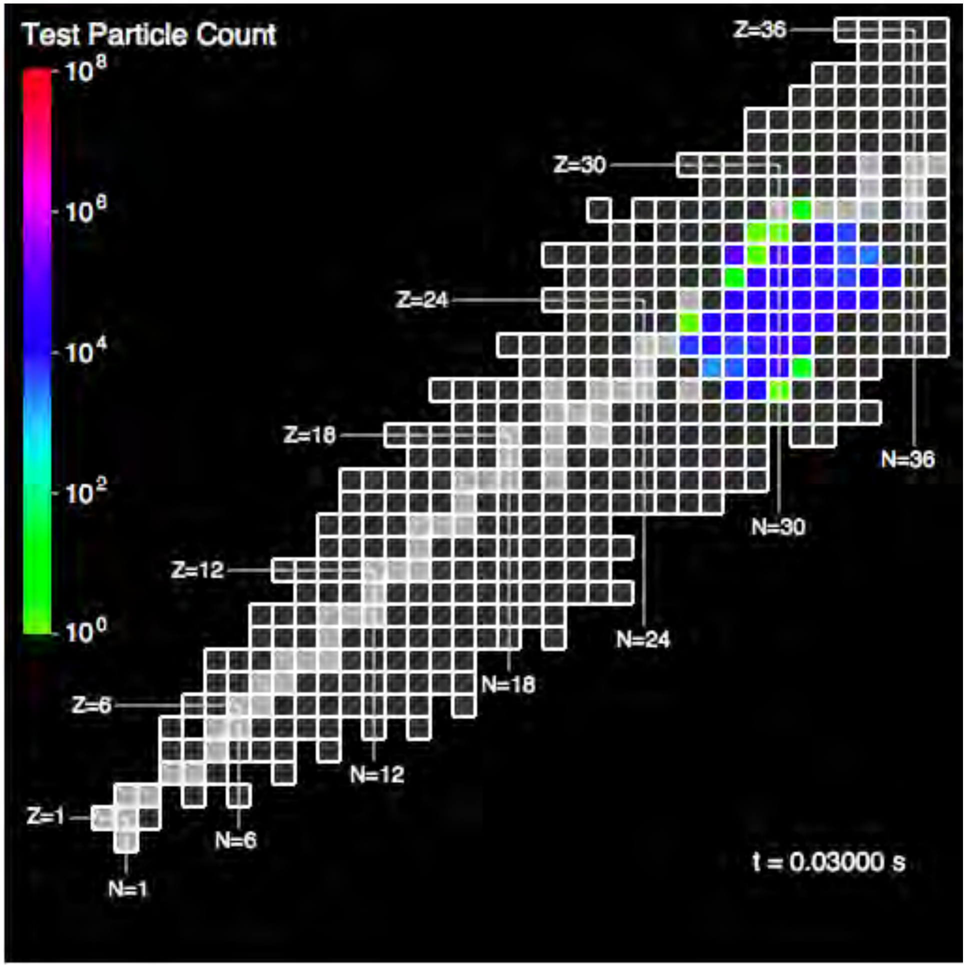




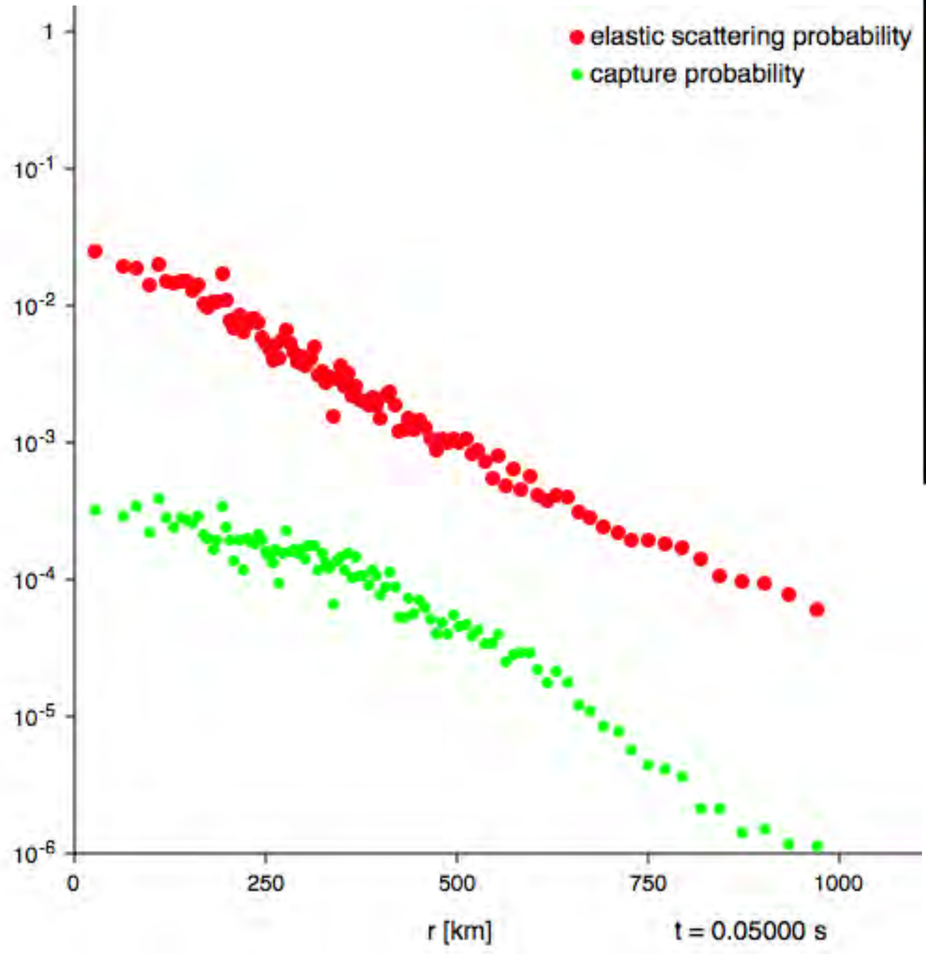
Isotope composition

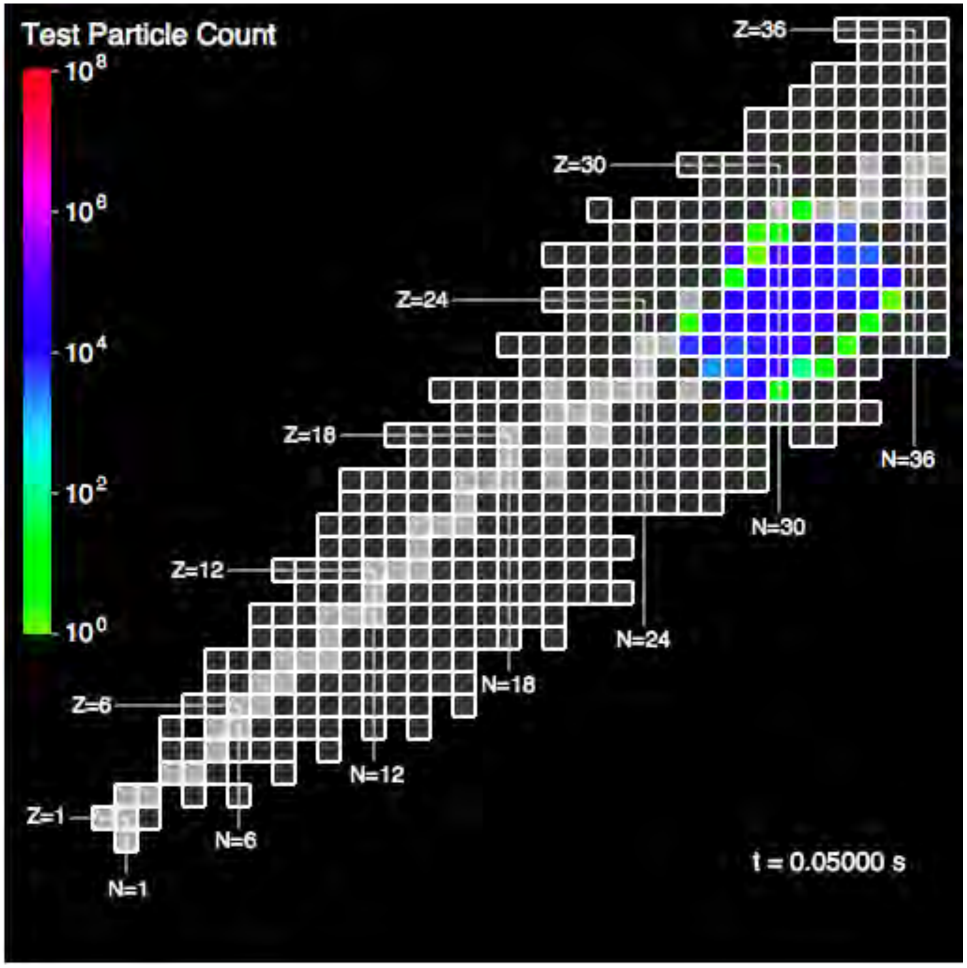
t = 0.03000 s


elastic scattering probability
capture probability
1
10-1
10-2
10-3
10-4
10-5
10-6
0
250
500
750
1000
r [km]
t = 0.03000 s

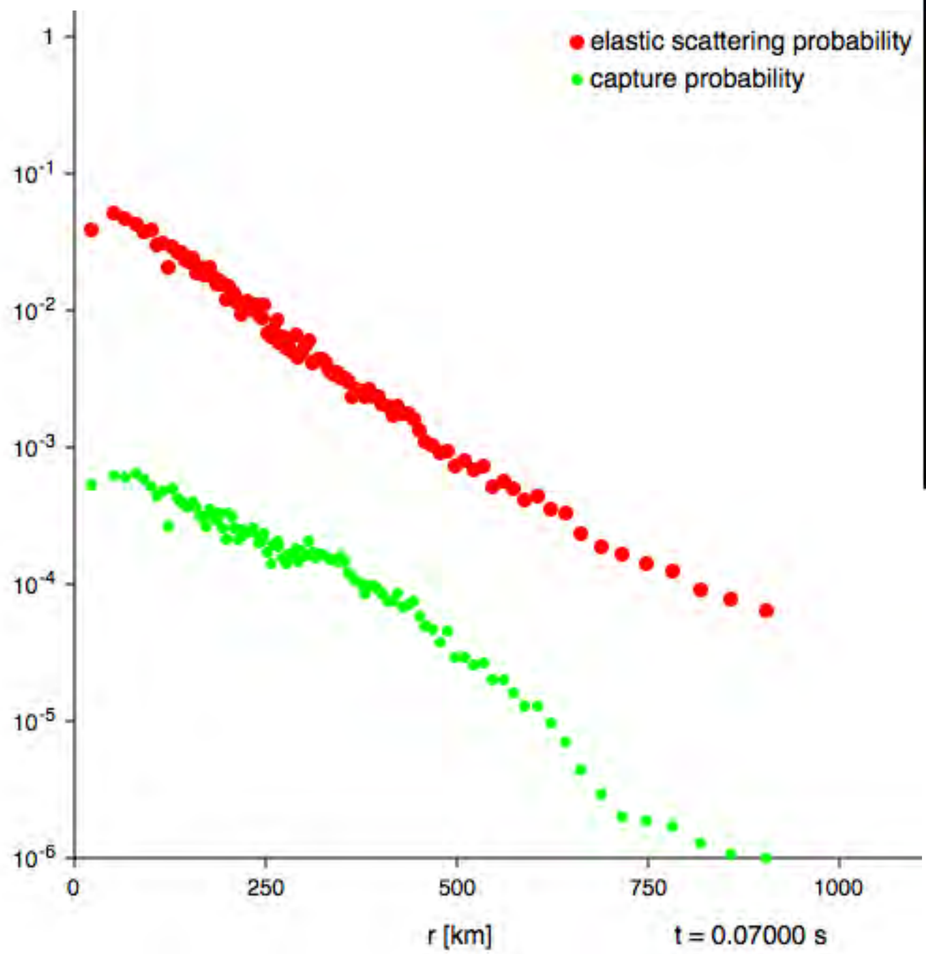

Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.03000 s

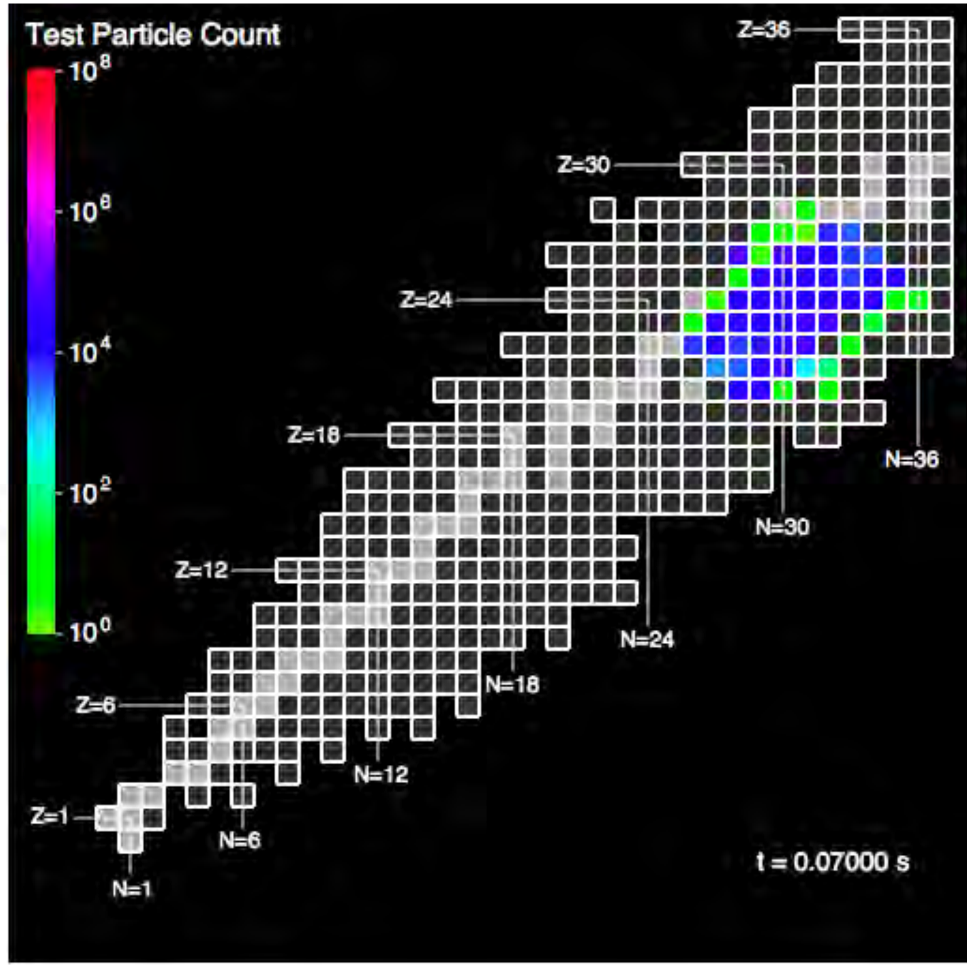
$t = 0.05000$ s


elastic scattering probability
capture probability
1
10-1
10-2
10-3
10-4
10-5
10-6
0
250
500
750
1000
r [km]
t = 0.05000 s

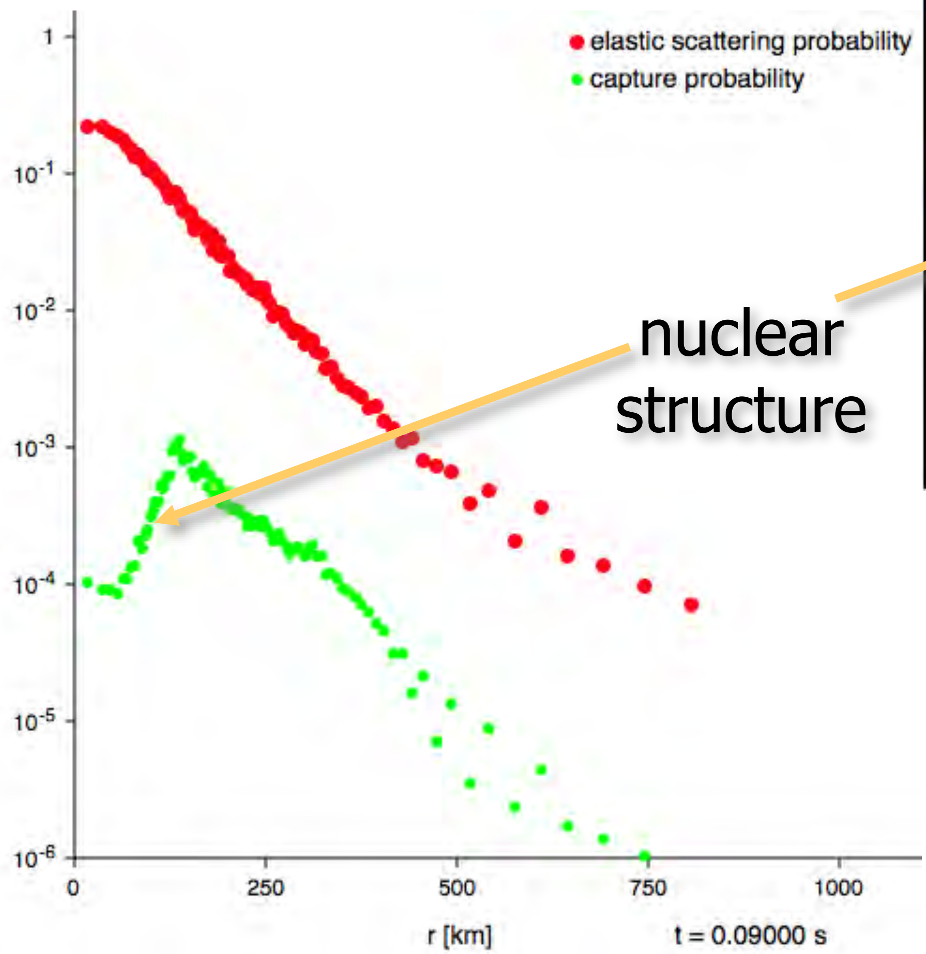

Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.05000 s

t = 0.07000 s

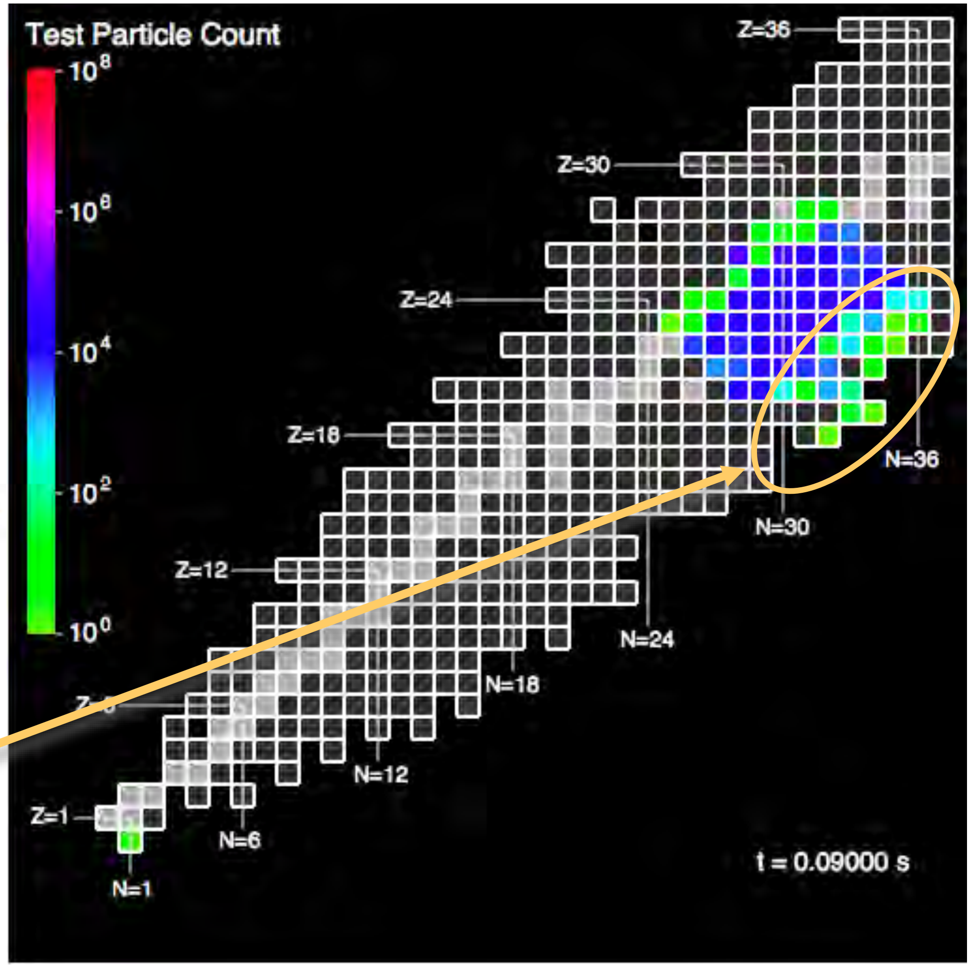

elastic scattering probability
capture probability
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
250
500
750
1000
r [km]
t = 0.07000 s


Test Particle Count
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.07000 s

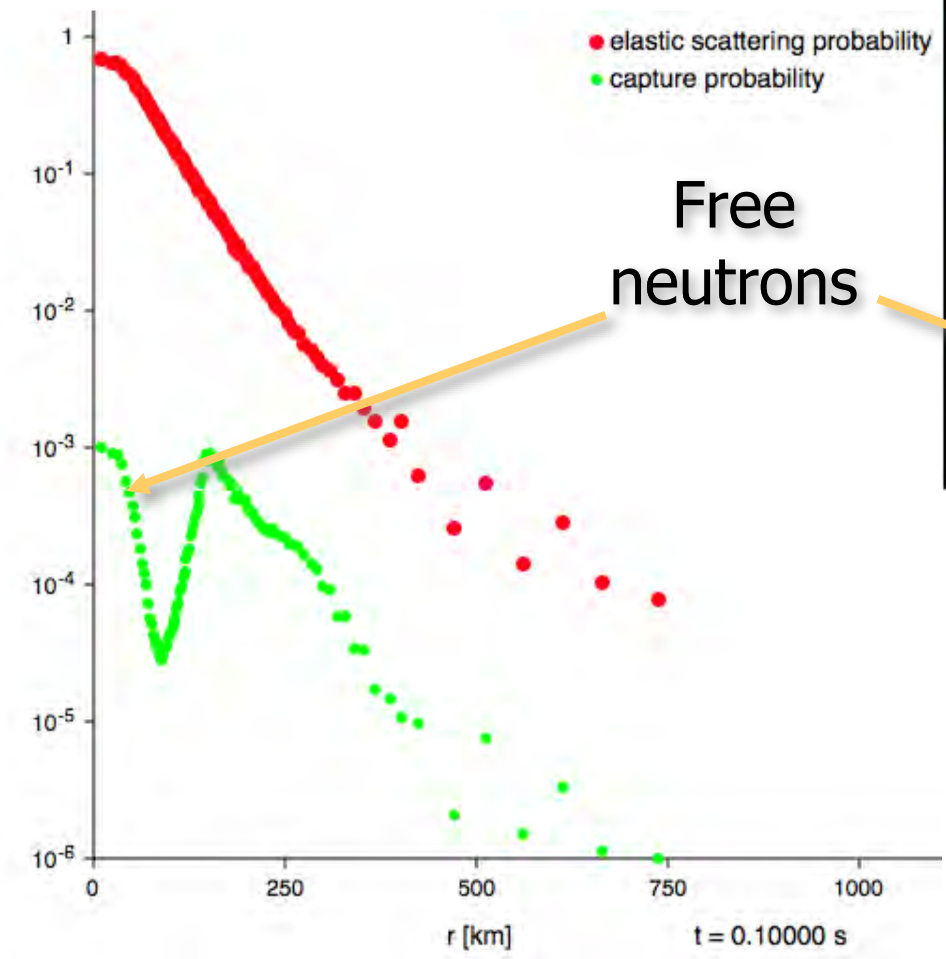
t = 0.09000 s



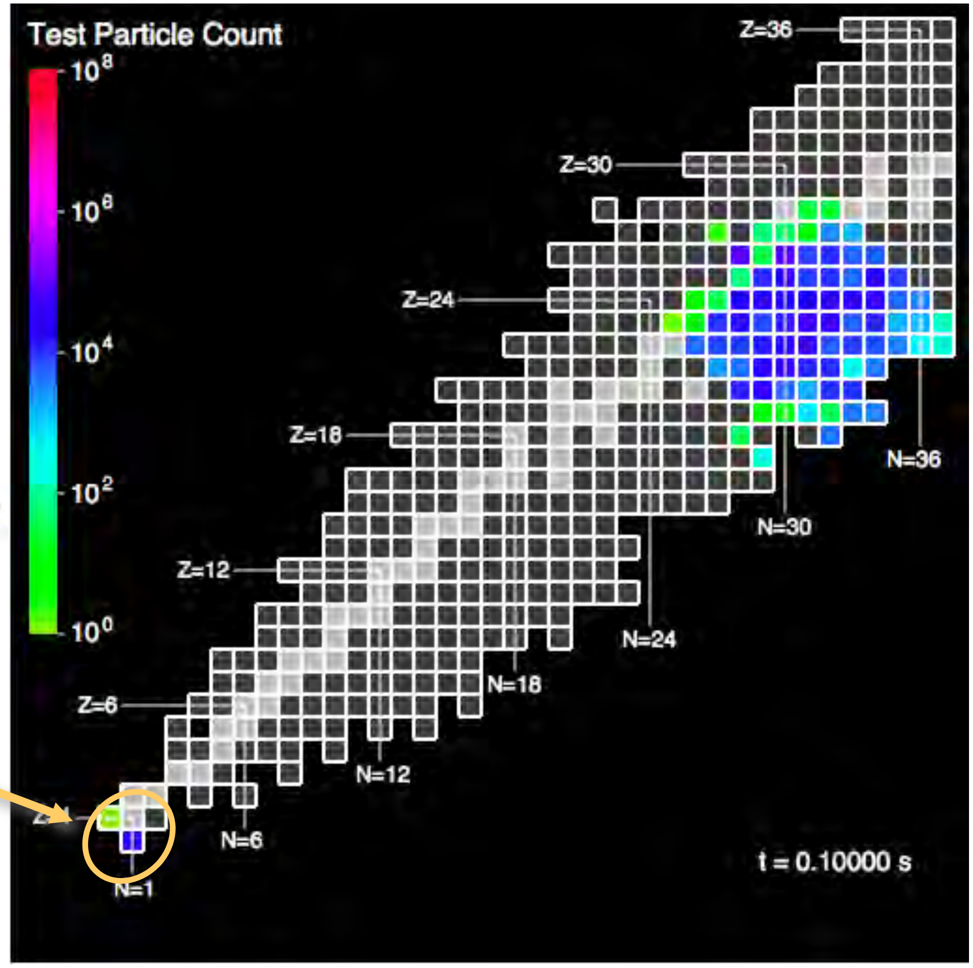
nuclear structure



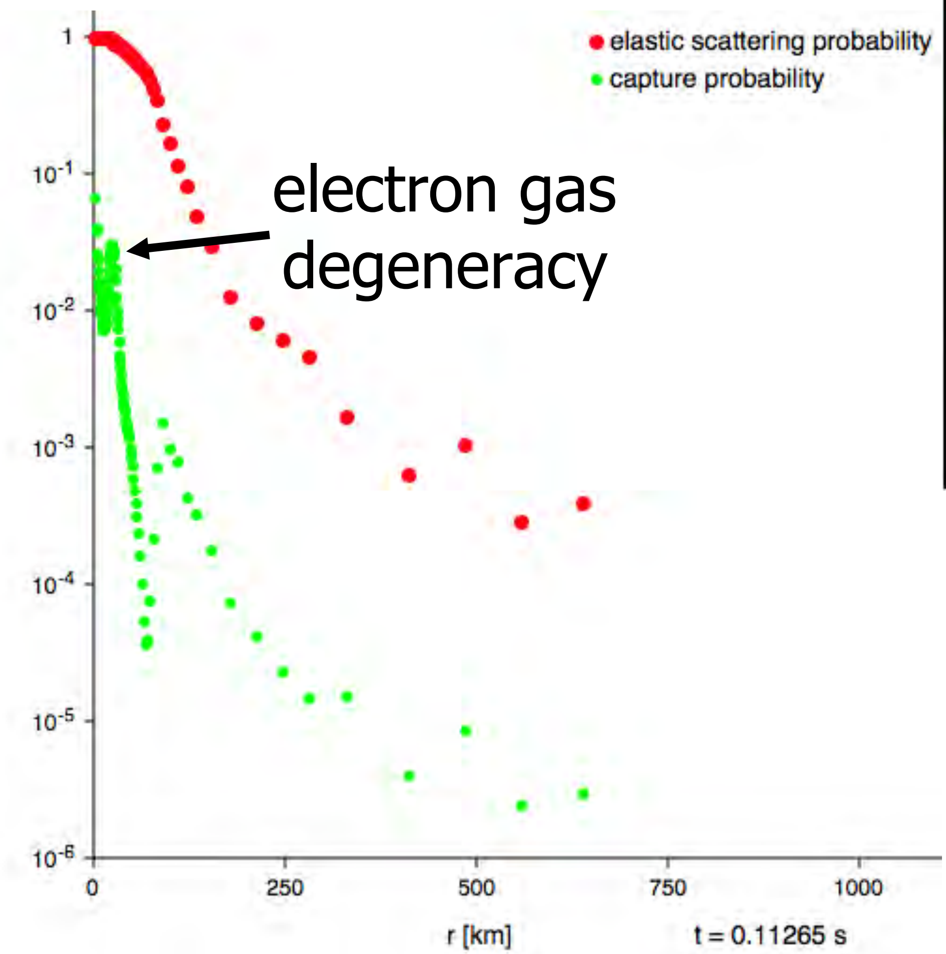
t = 0.10000 s



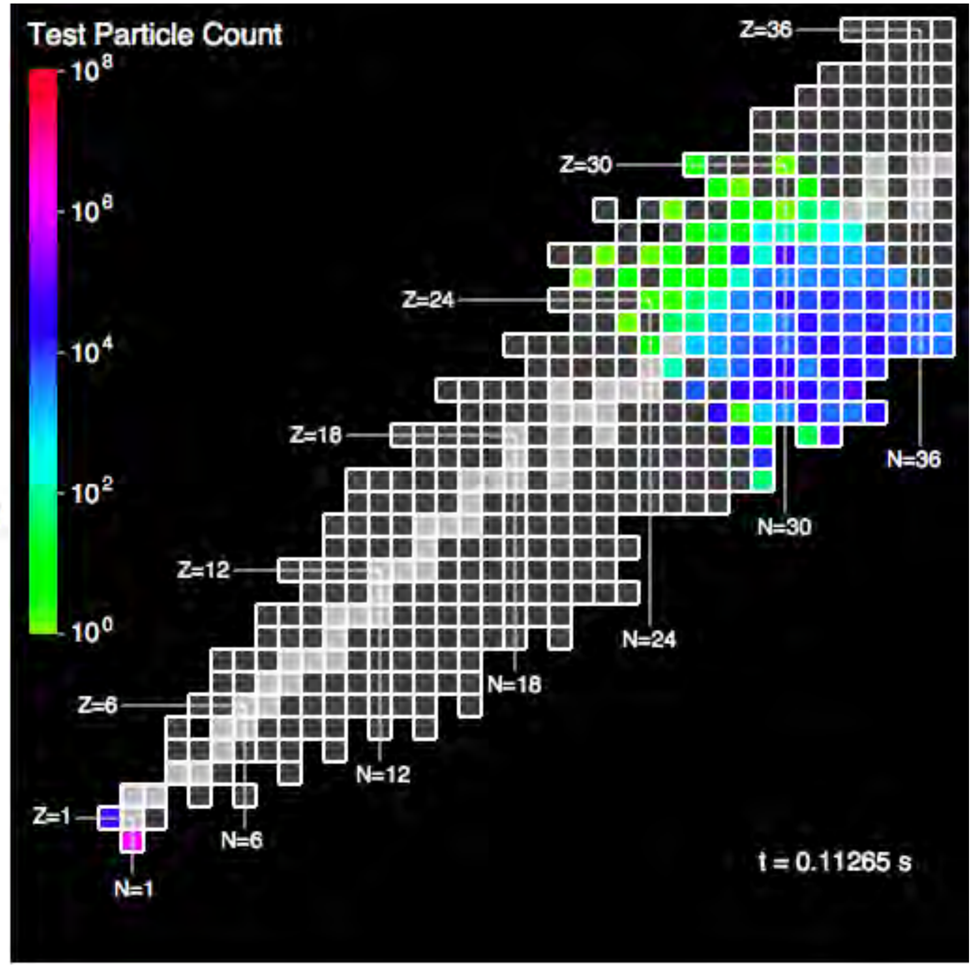
Free neutrons



t = 0.11265 s

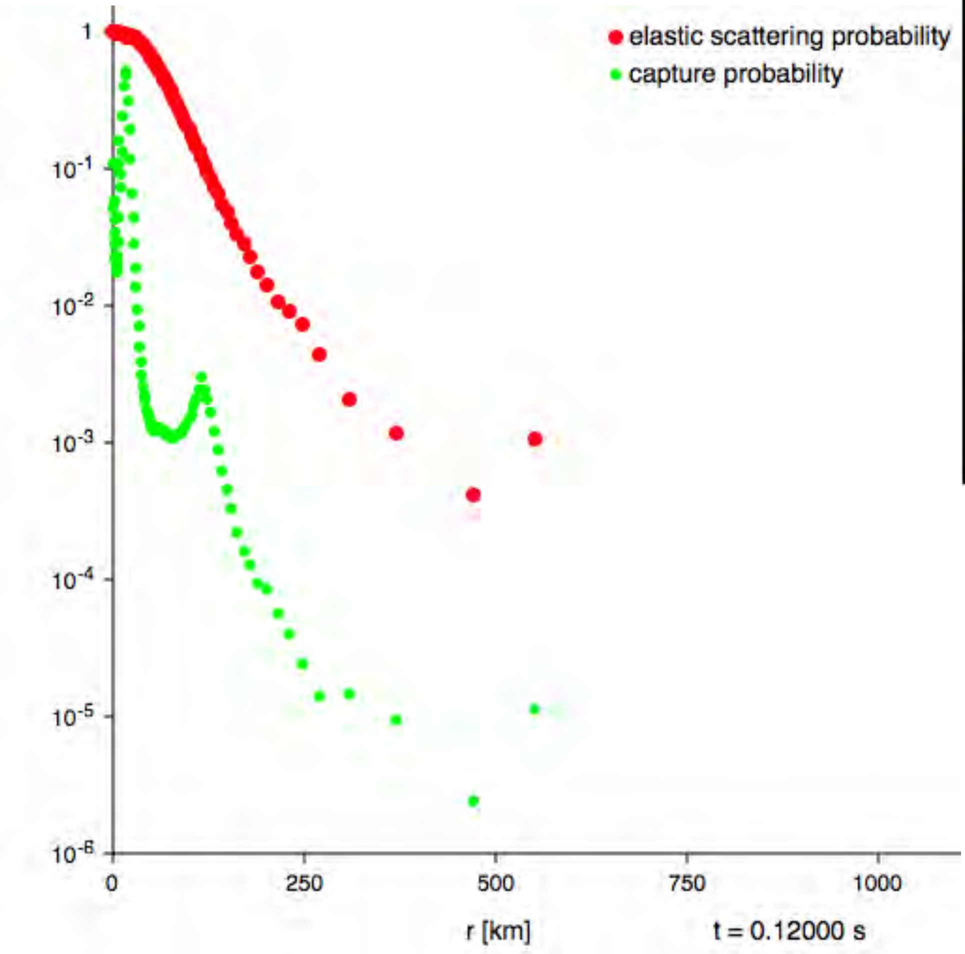


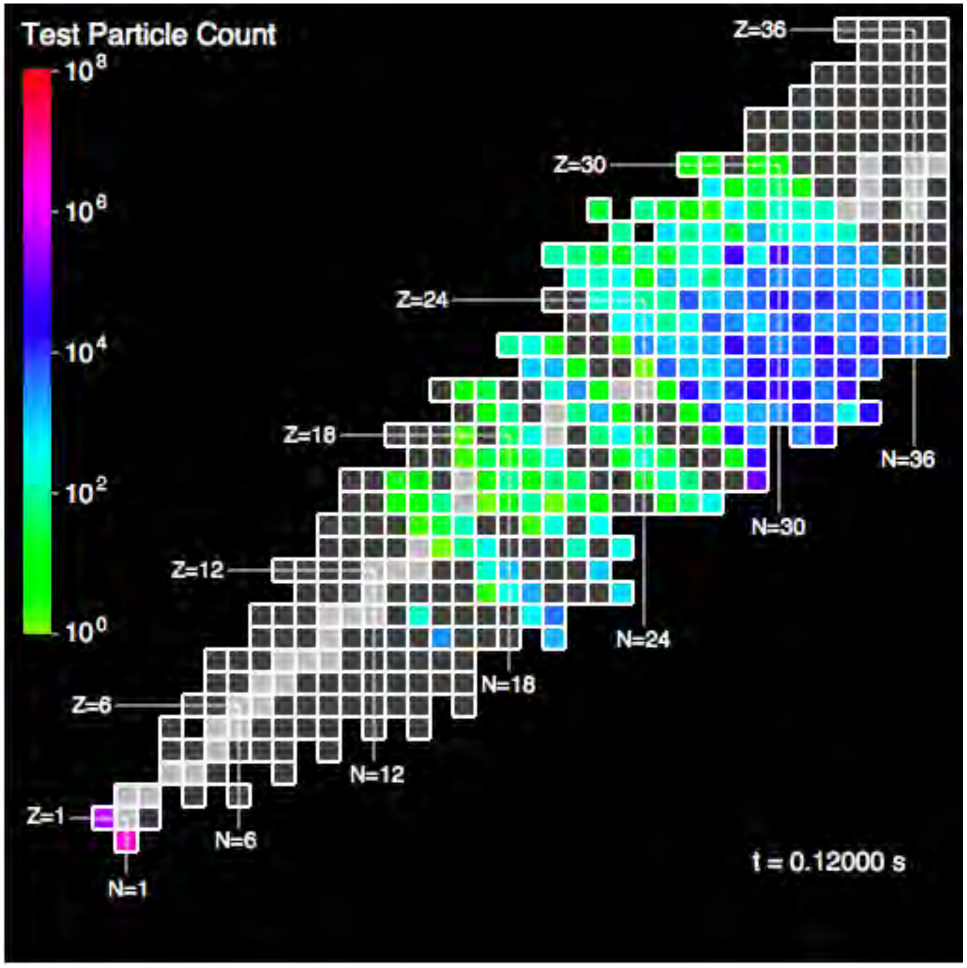
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
250
500
750
1000
r [km]
t = 0.11265 s
elastic scattering probability
capture probability
electron gas degeneracy



Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.11265 s

t = 0.12000 s


1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
250
500
750
1000
r [km]
t = 0.12000 s
elastic scattering probability
capture probability


Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.12000 s

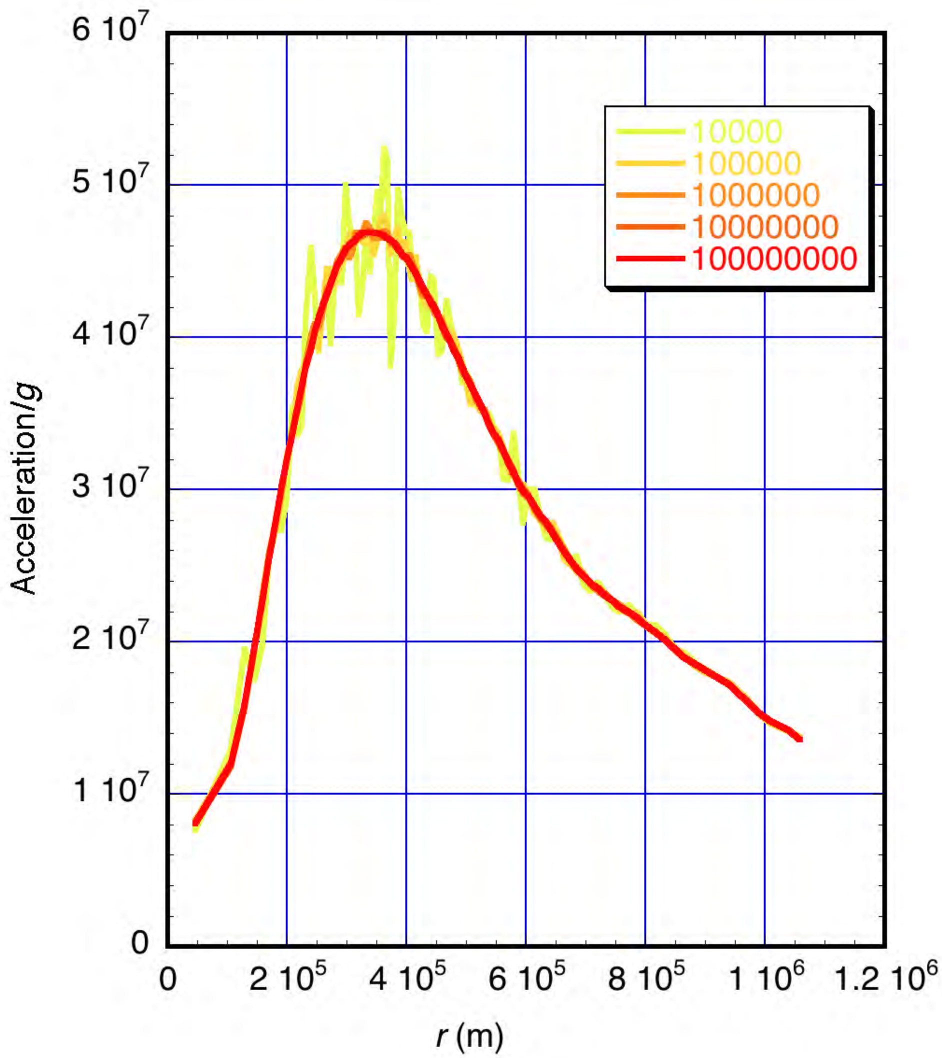
Summary

- New solution method for supernova dynamics
  - Test particle method
  - Link between nuclear dynamics and astrophysics
- New explosion mechanism
  - Shockwave originates ~ 50 km above neutron star surface
  - Due to neutrino heating / opacity change
  - VERY dependent on nuclear structure and neutrino cross sections

Next

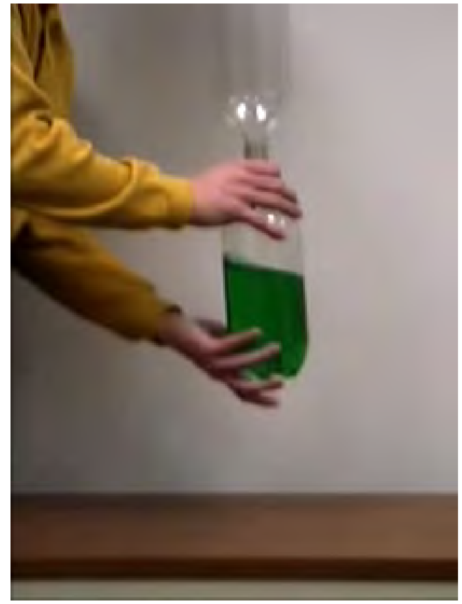
- Need more test particles to test for convergence
  - Shown today: results for 10^6 test particles
  - Perhaps 10^8 needed
- Use parallel processor installations

Convergence



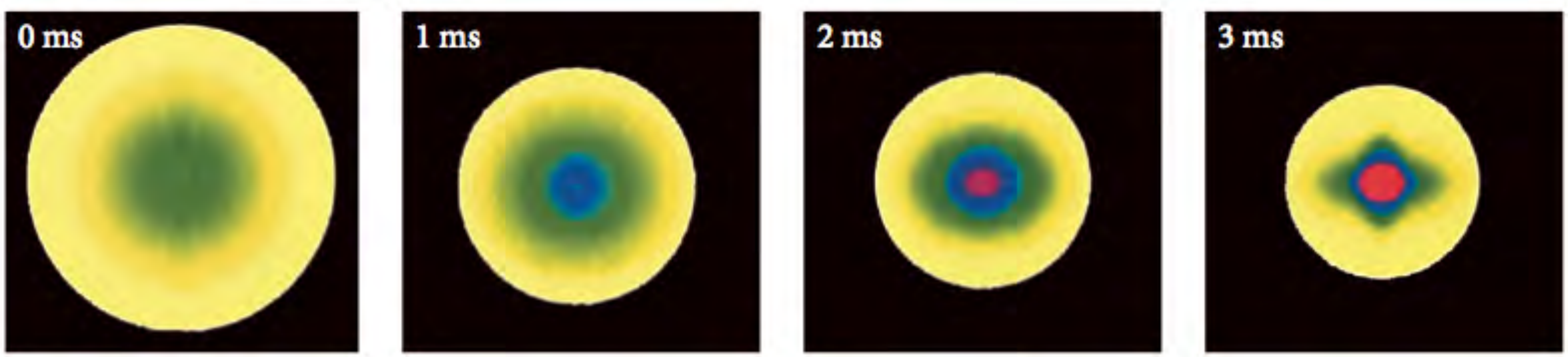
With 10^8 test particles interesting 3d effects can be probed

Effect of Rapid Rotation



$\vec{L}$ conserved (no external torques) during collapse

I reduced by $\sim 10^4 \Rightarrow \omega$ increases by same factor



- Collaborators
  - Terrance Strother (Ph.D. thesis)
  - Tobias Bollenbach (M.S. thesis)
  - Dirk Colbry