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Non-Linear Adventures in Taking Things Apart: Fragmentation of Nuclei and Molecules

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Supported by:
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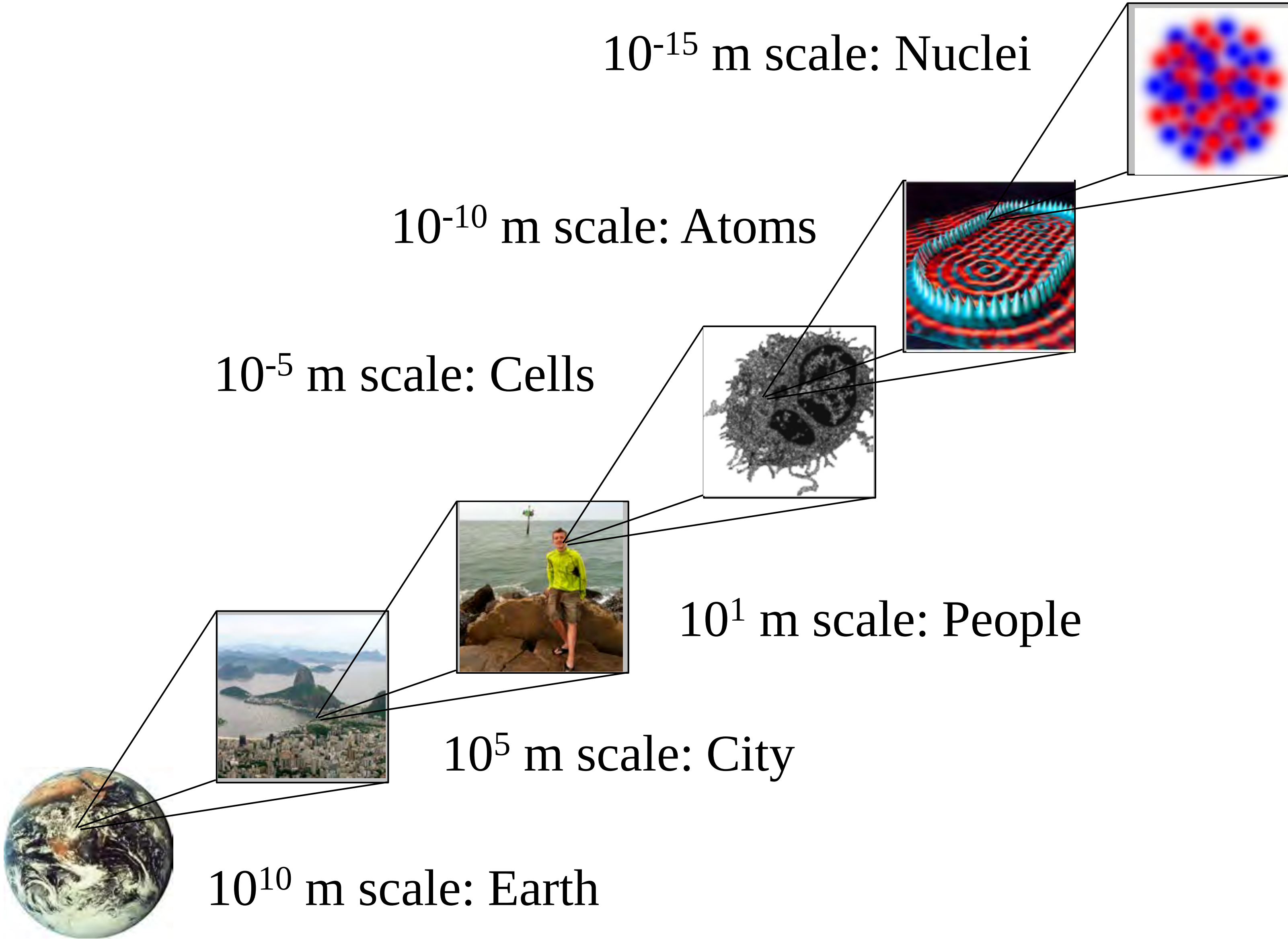

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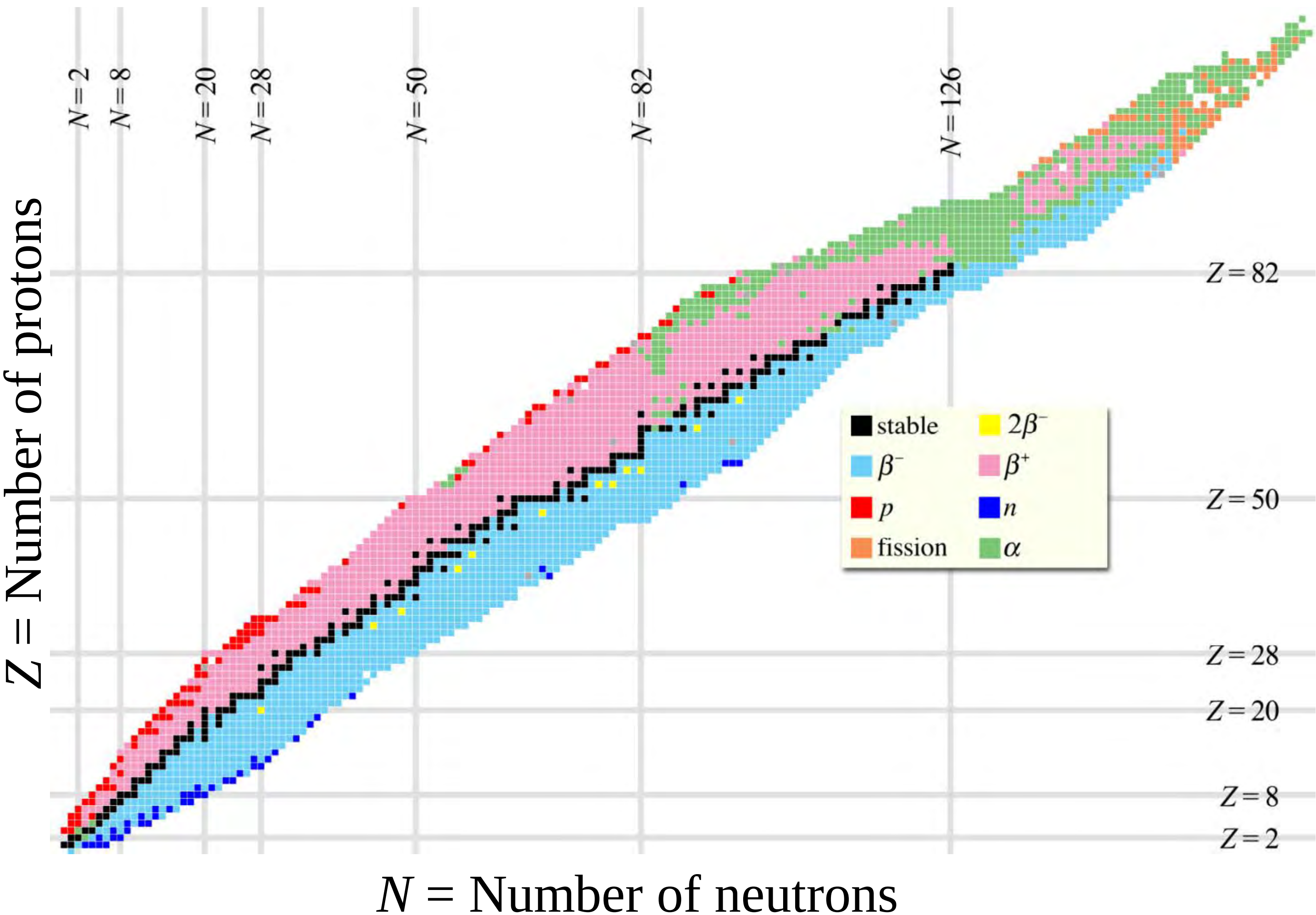
Nuclear Physics: the micro-nano science

- Nuclear size ~10^{-14} m (10,000 times too small for direct observation)
- System of ~10^{2} fermions => finite size effects extremely important, just like quantum dots (only smaller …)
- Mesoscopic physics
- Low-energy excitations (rotation, vibration): gamma emission, keV
- Other nuclear decays: beta, alpha, fission
- Higher energies: disintegration of entire nucleus
- Time scales ~10^{-22} s (1,000,000 times too fast for direct measurements)
- Energy scales 10-100 MeV

Favorite Isotope Decays



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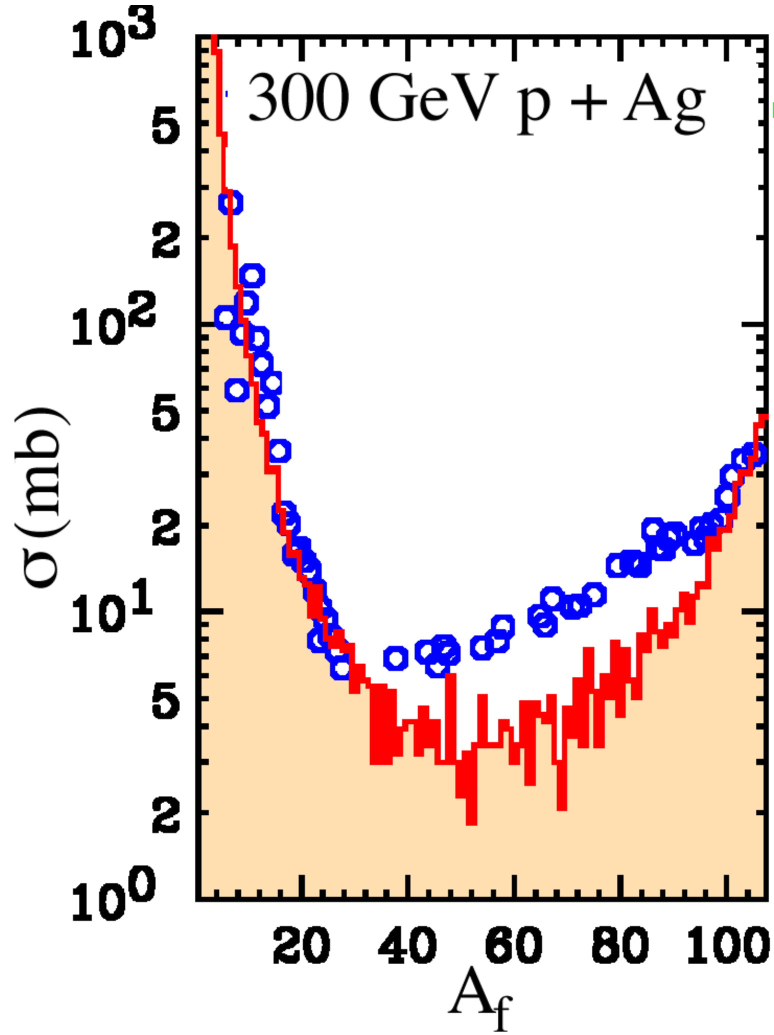
Nuclear Fragmentation



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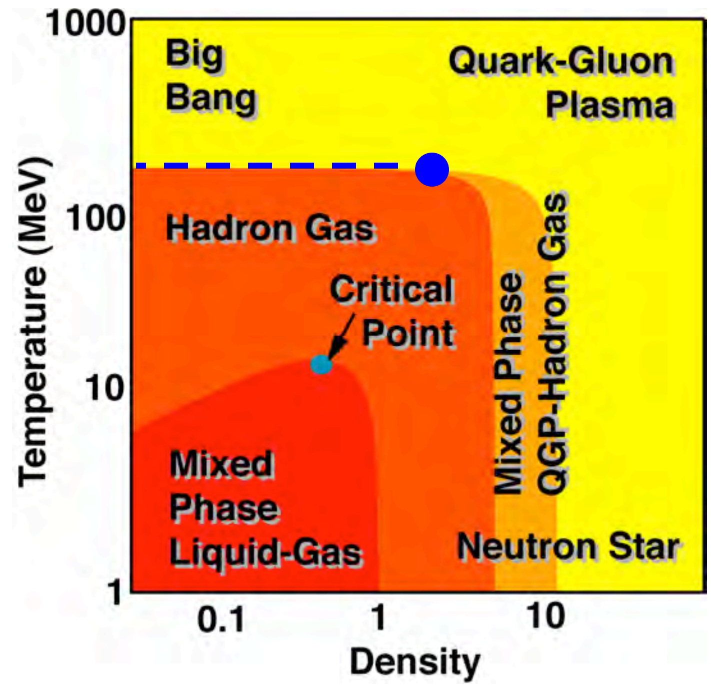
What is an "equation of state"?

- State variables: pressure, temperature, density (internal energy, chemical potential, strangeness, …)
- Equation of state: relationship between state variables, $f(p,T,\rho)=0$.
  - *Thermodynamic equation describing state of matter under given physical conditions*
  - *Example: Ideal gas:* $pV = nRT$
  - *Example: Ultra-relativistic fluid:* $p = c_s^2 \varepsilon$
  - *More realistic equations of state need to contain phase transitions, coexistence regions, critical points, …*

Nuclear Matter Phase Diagram

- Two (at least) phase transitions in nuclear matter:
  - *"Liquid Gas"*
  - *Hadron gas ➔QGP / chiral restoration*
- Problems/ Opportunities:
  - *Finite size effects (finite size scaling! □)*
  - *Is there equilibrium? (□)*
  - *Measurement of state variables (ρ , T, S, p, …□)*
  - *Migration of nuclear system through phase diagram (non-equilibrium processes)*
  - *Near critical point(s): Critical slowing down! Not sufficient time for equilibrium phase transition!*



Source: NUCLEAR SCIENCE, A Teacher's Guide to the Nuclear Science Wall Chart, Figure 9-2

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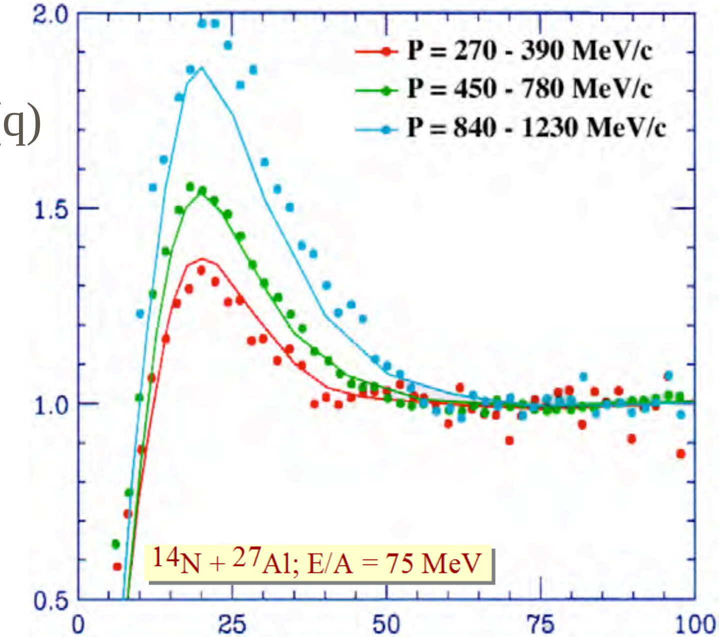
Volume

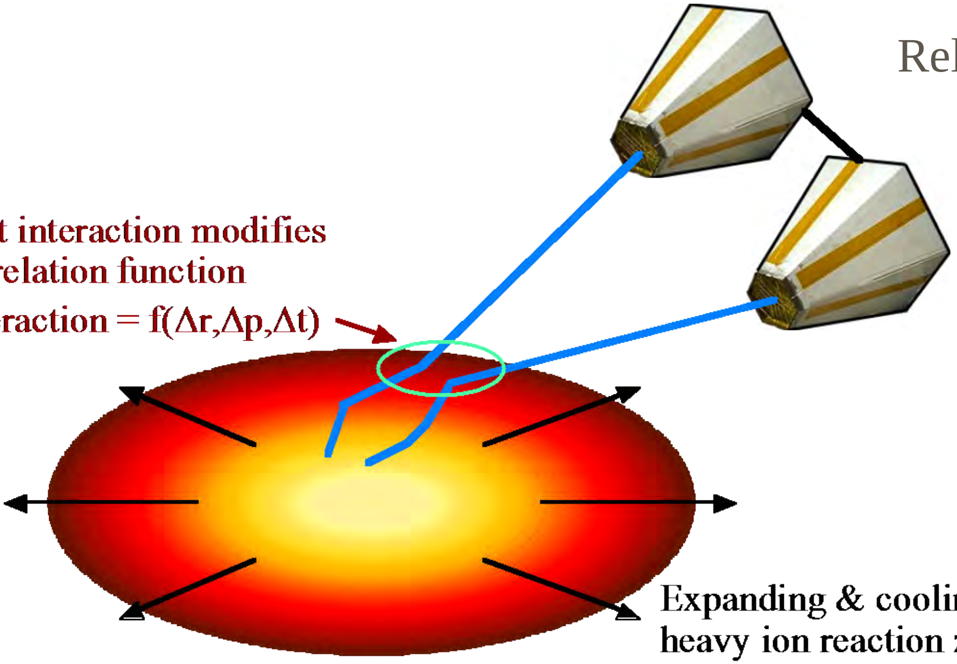
- Information from interferometry:
- Two-particle correlation are sensitive to space-time extension of emitting source

$$C(P,q)=\int d^3x\, F_p(r)\, |\phi(r,q)|^2$$

C(q)



Relative Momentum, q



Review: W.B., Gelbke, Pratt, Annu.Rev.Nuc.Part.Sci. 42

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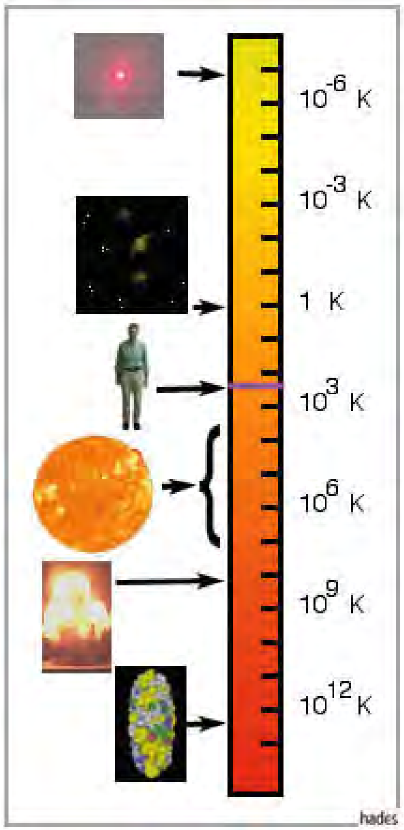
Temperature

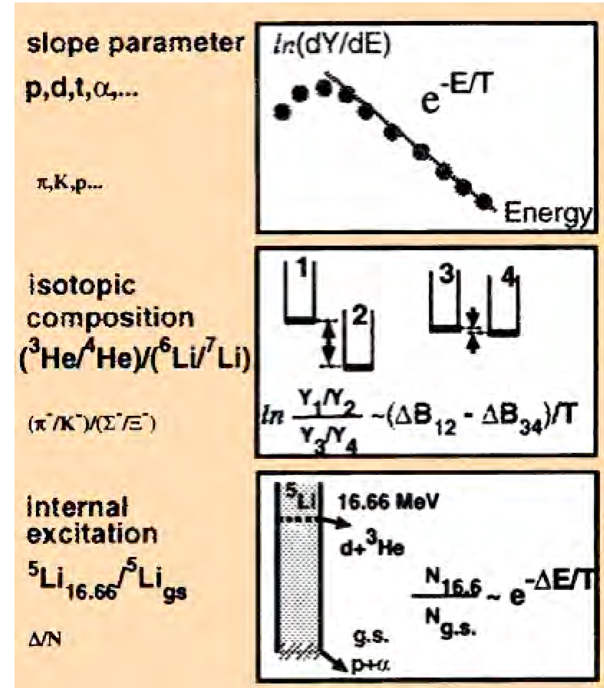
J. Pochodzalla, CRIS '96

- Measure nuclear temperature indirectly via:



- *Slopes of charged particle spectra*
- *Bound-state populations*
- *Unbound states*
- *Fragment isotopic yields*

"He-Li" thermometer
Albergo et al., Nuovo C. A89



$$T = 13.3\ \mathrm{MeV}/\ln\left(2.18\frac{Y(^6Li)\,Y(^4He)}{Y(^7Li)\,Y(^3He)}\right)$$

Central question: At which ***time*** do we measure the temperature with each thermometer?

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Temperature from Fragment Spectra

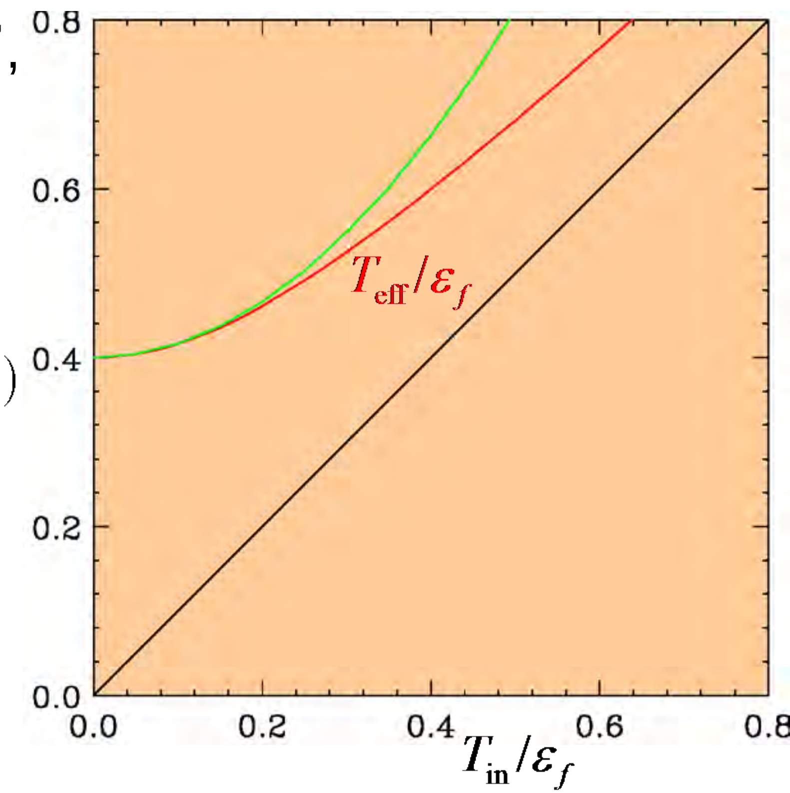
- Nucleon momentum distribution at temp. *T*: $$\rho(p) = \frac{1}{1+\exp[(p^2/2m-\mu)/T_{\rm in}]}$$
- Fragment momentum = sum of momenta of nucleons in it
- Problem equivalent to solving Pearson random walk in momentum space
- Limiting distribution: $$\rho(E_{\rm f}) = 2\sqrt{\frac{E_{\rm f}}{\pi\, T_{\rm eff}^3}}\exp\left(-\frac{E_{\rm f}}{T_{\rm eff}}\right)$$

  (Boltzmann with $T_{\rm eff} = \sigma^2/M_N$)
- Fragment slope "temperature", $T_{\rm eff}$, is not equal to *T*, but is a monotonous function of it

  ➔ *Nuclear Thermometer*
- Approximation:

  $$T_{\rm eff} \approx (2/5)\epsilon_f\,(1+5\pi^2\,T^2/\epsilon_f^2+\ldots)$$

  WB, Phys. Rev. C **51**



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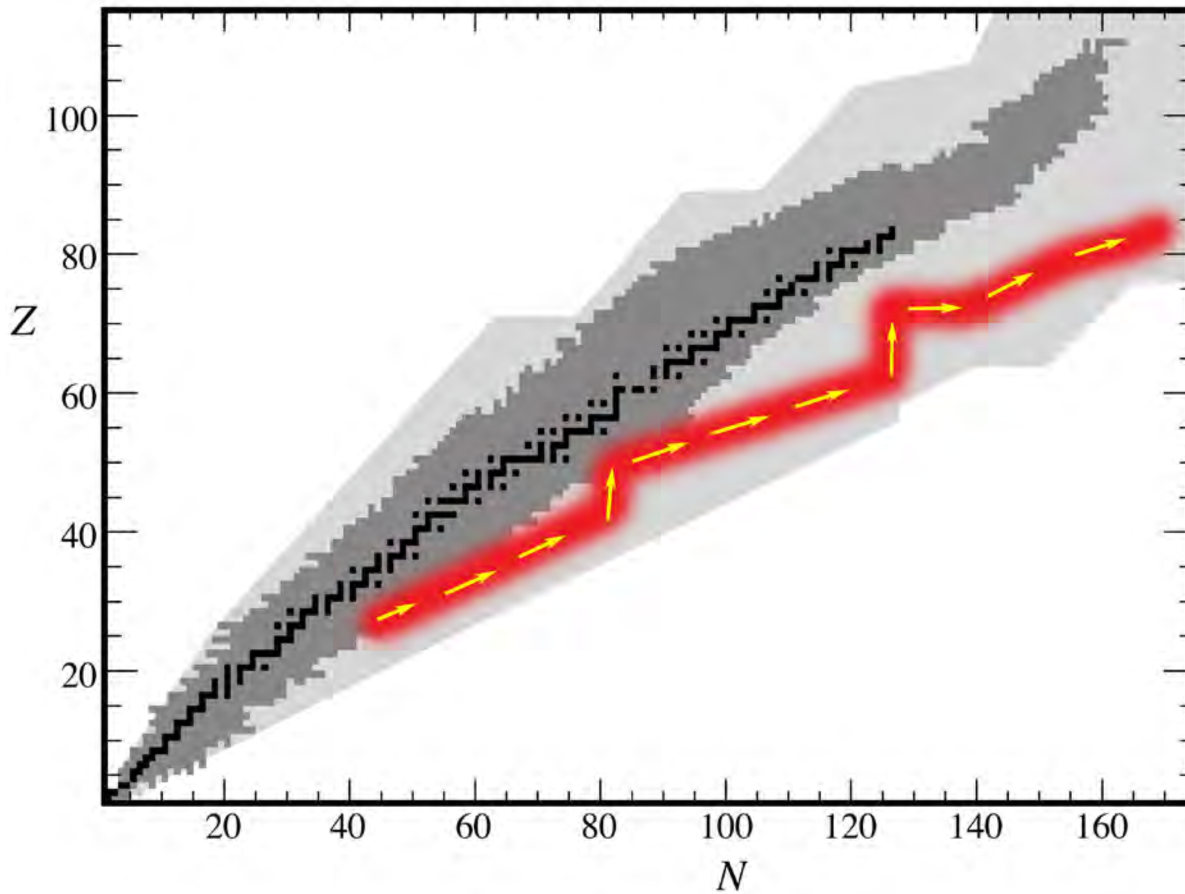
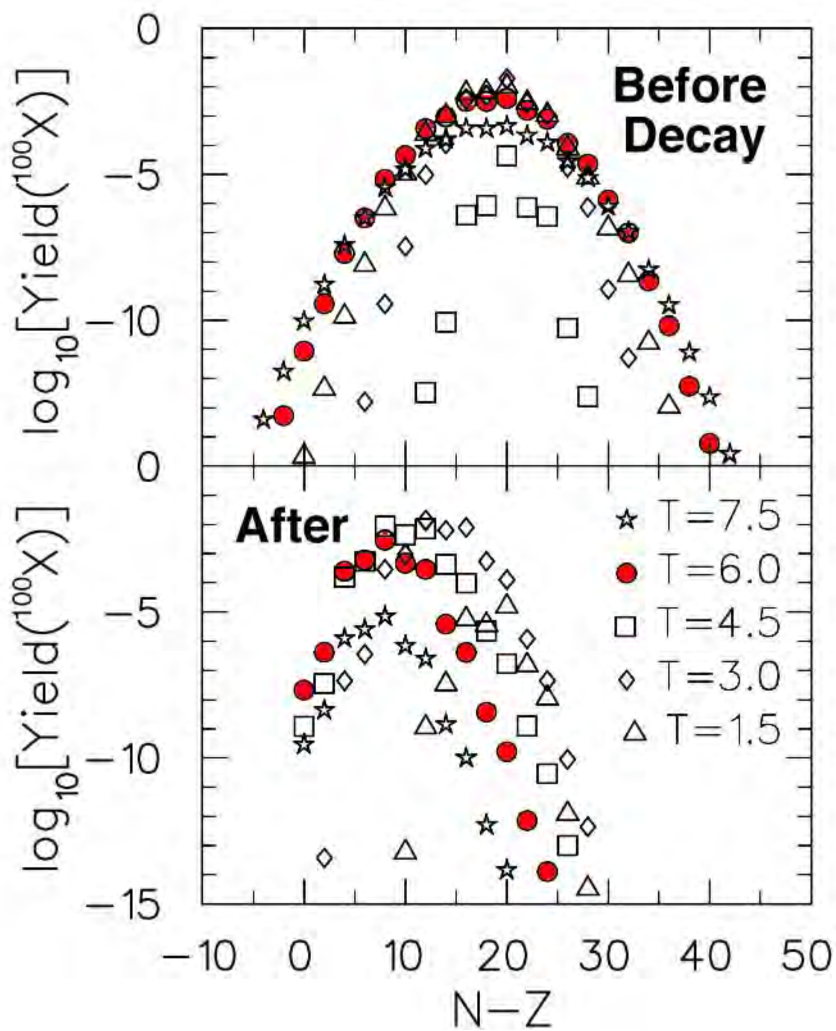
Isospin: FRIB Reaction Physics

- Exploration of the drip lines below charge ~40 via projectile fragmentation reactions
- Determination of the isospin degree of freedom in the nuclear equation of state
- Astrophysical relevance (origin of heavy elements!)
- Review: Li, Ko, WB, Int. J. Mod. Phys. E **7**



Width of Isotope Distribution, Sequential Decays



- Predictions for width of isotope distribution are sensitive to isospin term in nuclear EoS
- Complication: Sequential decay almost totally dominates experimentally observable fragment yields
  Pratt, WB, Morling, Underhill, PRC **63**

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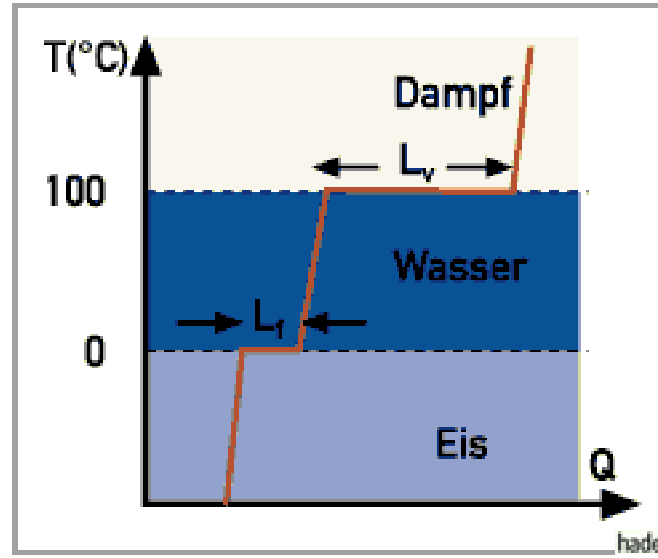
First-Order Phase Transition

- Coexistence of two phases (e.g. ice+water, water+steam)
- Addition of heat does not change temperature
  Latent heat
  (H_2O: L_f = 80 kcal/kg,
  L_v = 540 kcal/kg)
- Different *specific heat capacities* in the different phases -> different slopes T vs. Q
- *Pressure kept constant!*



Bauer
Benenson
Westfall
cliXX Physik

Verlag Harri Deutsch, Frankfurt 1999

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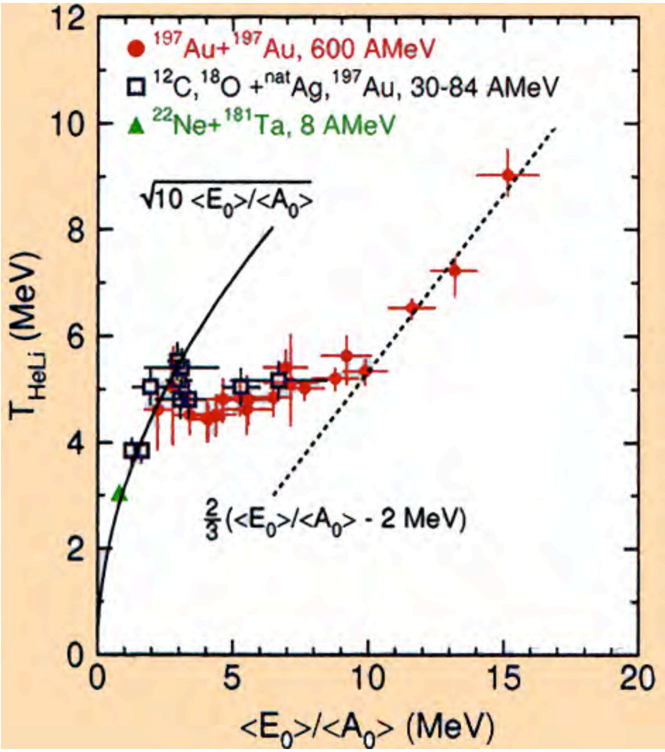
Observation of First-Order Phase Transition?

- Low E^*: Liquid-like
  $T \sim E^{*1/2}$
- High E^*: Gas-like
  $T \sim E^*$
- 1st order transition:
  Liquid-gas coexistence
  Temperature does not change in phase mixture while liquid is converted to vapor.
- Analogy: Boiling of water
- But what about "constant pressure"?



J.Pochodzalla et al. (ALADIN), PRL **75**

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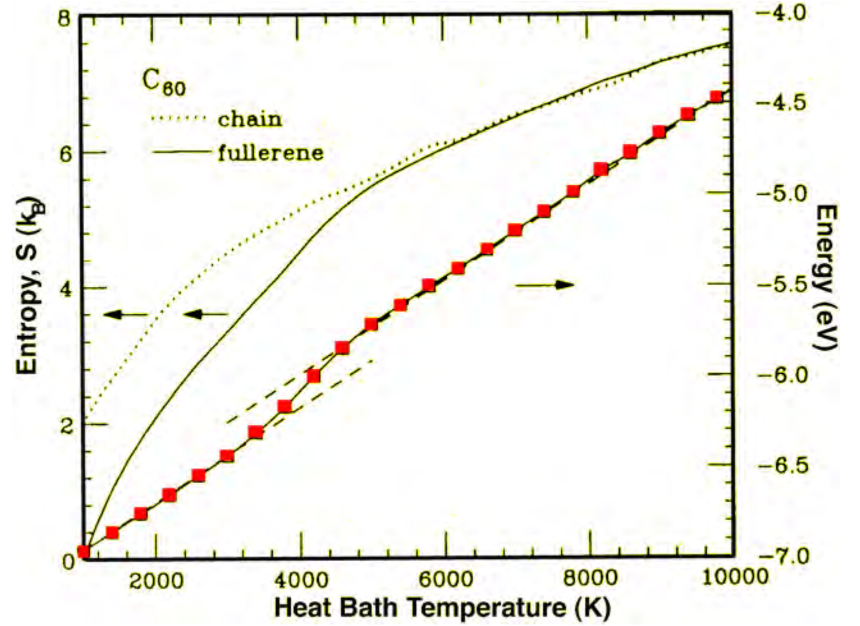
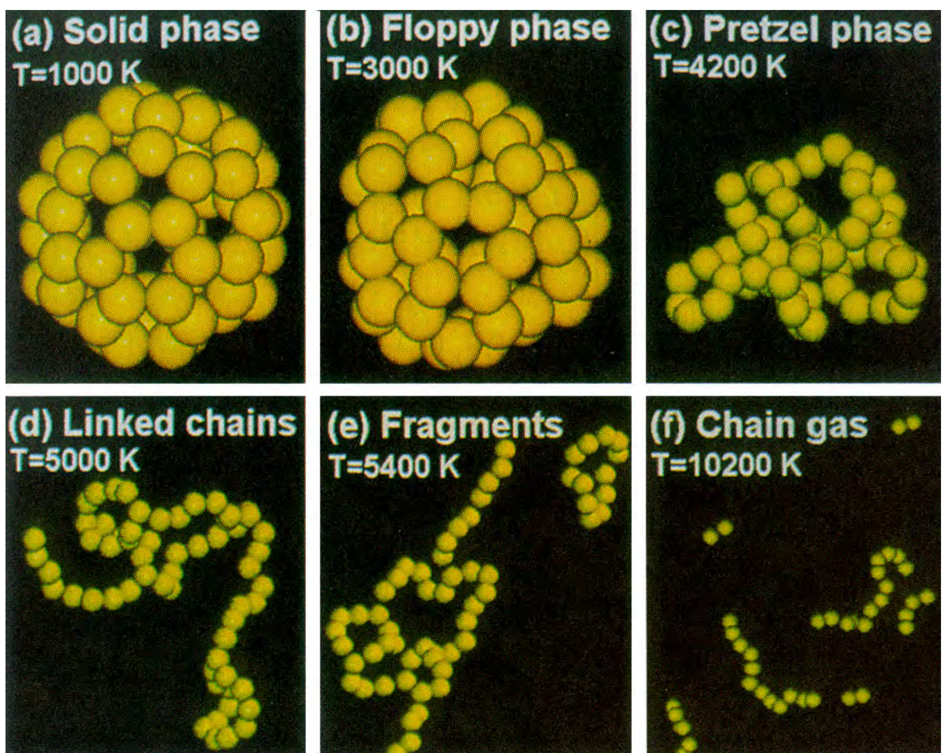
Buckyballs-Melting



- C_{60} Cluster
- Soccer Ball Geometry



- Molecular dynamics calculations
- Hoover-Nose heat bath

S.G. Kim & D. Tomanek, PRL 72

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Continuous Phase Transition

- Near critical point, we expect scaling behavior: all physical quantities have power-law dependencies on the control parameter
- No characteristic scales in observables
- Critical exponents of power-laws are main quantities of interest
  - ç τ , *Cluster size:* $n_s(T_c) \sim s^{-\tau}$
  - ç β , *Order parameter:* $P \sim (T - T_c)^{\beta}$
  - ç γ , *Divergence of* s: $\sim |T - T_c|^{\gamma}$
- Hyper-scaling assumption

$$2 - \alpha = \frac{\tau - 1}{\sigma} = 2\beta + \gamma$$

(Determine 2 critical exponents sufficient)

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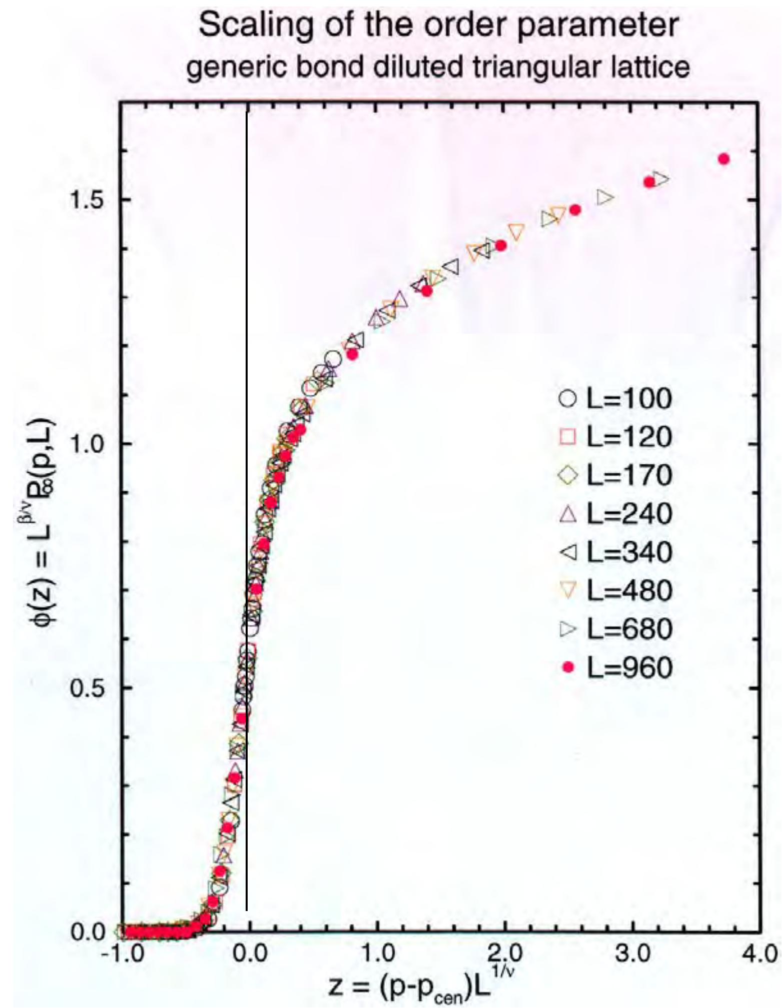
Finite Size Scaling

- Phase transitions strictly only defined for (almost) infinite systems
- Lattice calculations work on finite lattices and extrapolate to infinite lattices (hardest part!)
- Finite size scaling exponent, ν
    - *Modify control parameter by* $L^{1/\nu}$
    - *Modify order parameter by* $L^{\beta/\nu}$
- Opportunity for nuclear physics: **Learn about extreme finite size scaling in real systems**

M. Thorpe, MSU



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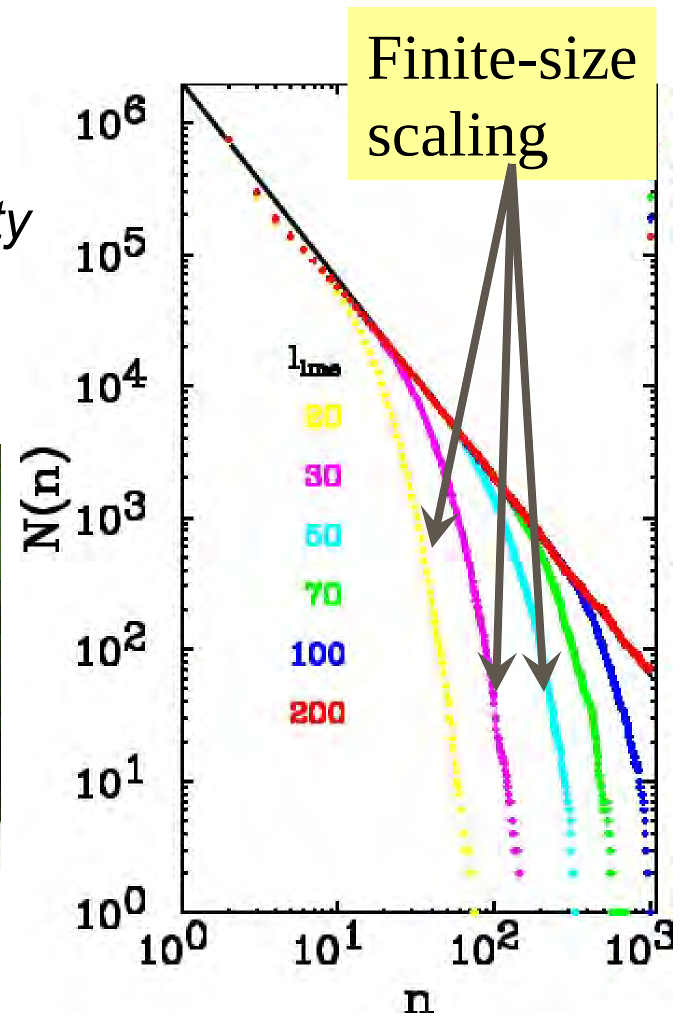
Self-Organized Criticality

- How to achieve scale-invariance?
  - *Vicinity of critical point: power-laws*
  - *Very careful tuning of control parameter(s) required*
- Another possibility: **SOC**
  - *Sequence of avalanches between metastable states*
  - *Continually driven to criticality*
  - *No external tuning required*
  - *Example: Bak's sand pile*

Held et al., PRL 65



Critical Slowing Down

- Near critical point, $|T/T_c| \ll 1$, it takes longer and longer to re-establish equilibrium after changing the temperature
- Example: Ising Model,

$$M(t) \sim \exp(-\, t/t_r),$$

with

$$t_r = 4.5\,(T - T_c)^{-1.85}, \text{ for } T > T_c.$$

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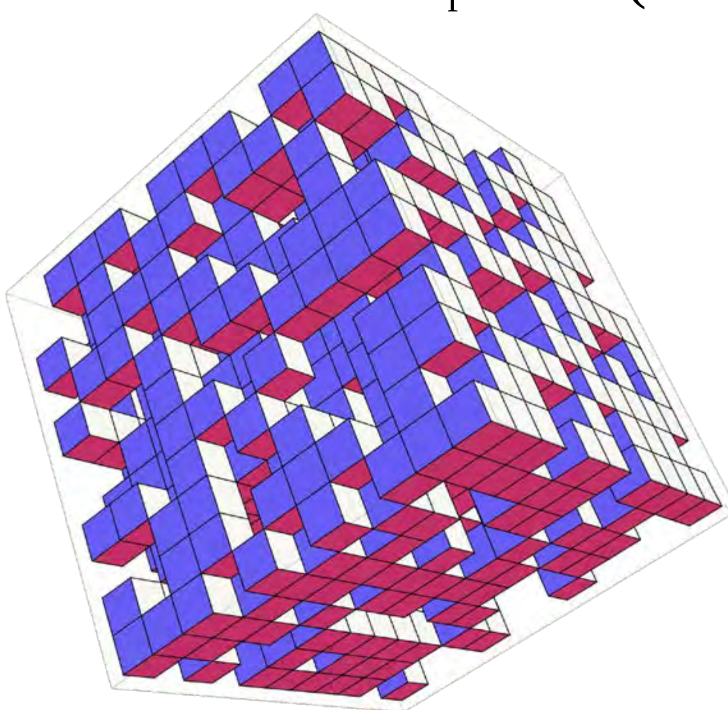
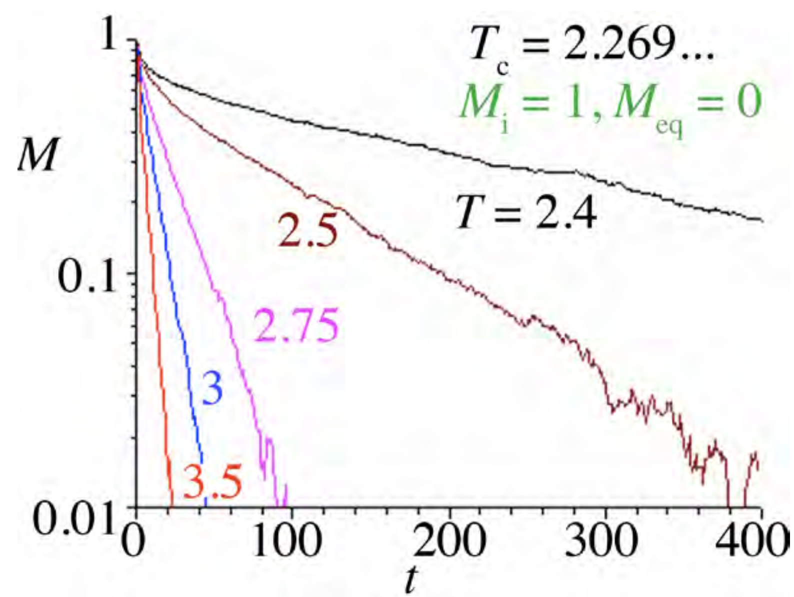
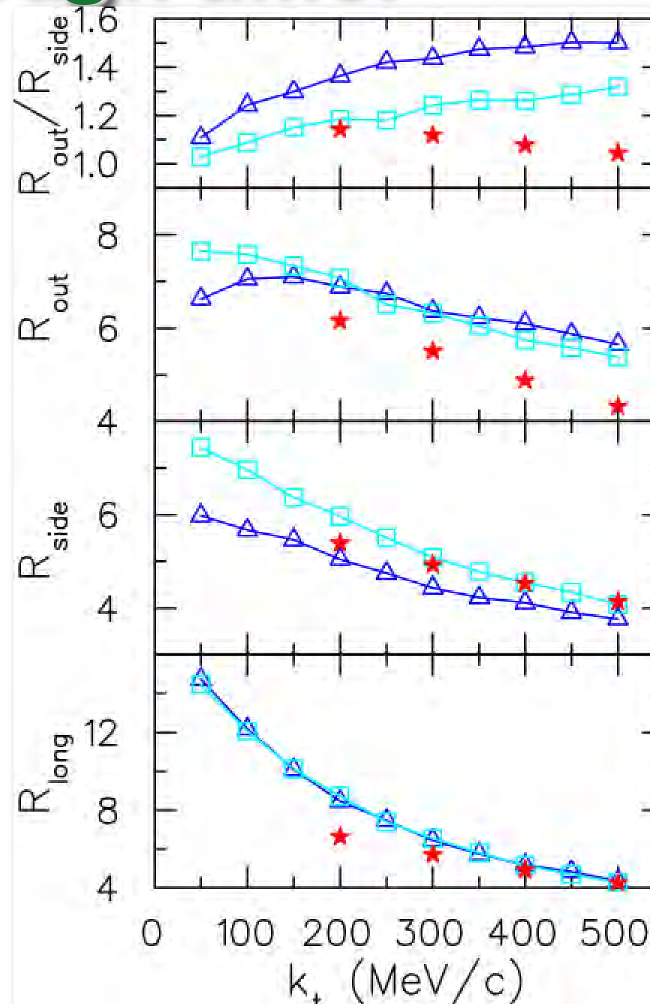




… but there is not enough time!

- HBT puzzle
- Theoretical expectation
  - *Change of # of degrees of freedom in transition from quarks and gluons to hadrons*
  - *Large time delay*
  - *Expect $R_{out} >> R_{side}$*
- Not seen by experiment!



- Equilibrium thermodynamic phase transition may not be possible
  - *… but percolation-type transition not excluded!*

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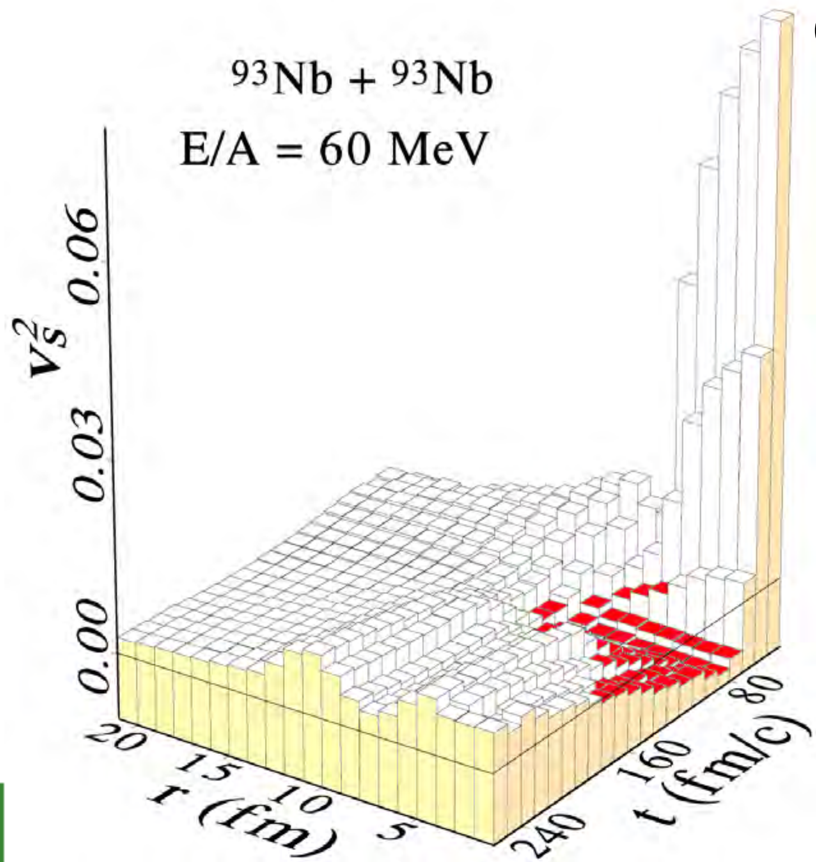
Dynamics

- Thermal equilibrium assumptions not (always) valid
- Need transport theory
- Various event class averages (event vs. thermal!)
- Connections to underlying phase diagram poorly understood
- Transient formation of non-compact structures
  - *Sheet instabilities*
    Moretto et al., PRL 69
  - *Bubble and ring formation*
    WB, Schulz, Bertsch, PRL 69
  - *Imaginary sound velocity causes exponential growth in fluctuations; non-equilibrium in origin*
  - *Similar effect now postulated for RHIC collisions* (*Pratt 2008*)



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Non-Equilibrium Phase Transitions

- Conventional thermodynamics
  - *Write down partition function from (known) Hamiltonian*

$$Z = tr(e^{-\beta H})$$

  - *Take partial derivatives to obtain state variables*
  - *Static solution; equilibrium; no changes in time*
- Non-equilibrium Phase Transition
  - *Dynamics; time dependence*
  - *No thermal averages*
  - *Transitions between un/meta/bi-stable states*
- Are similar universality classes possible?
  - *Critical exponents can be obtained*

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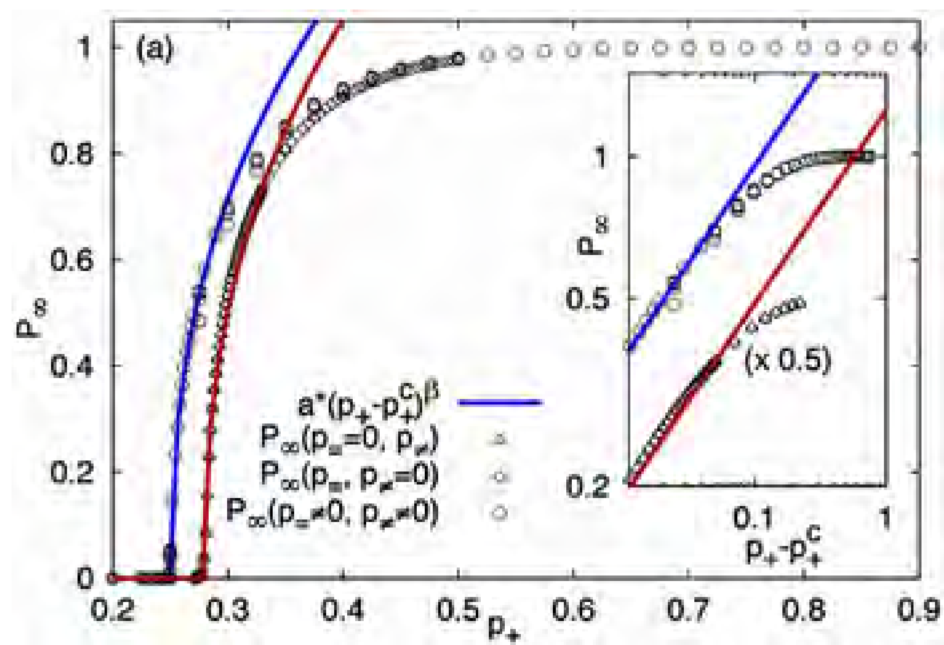
Multi-Component Systems

- What happens when physically different components are in the system undergoing phase transition (protons+neutrons, different flavor quark&gluons, …)?
- Possible:
  - *Change of character of phase transition*
    Müller&Serot, PRC ***52***
  - *Shift in critical value of control parameter, same critical exponents*
    Harreis&WB, PRB ***62***



Event-by-Event

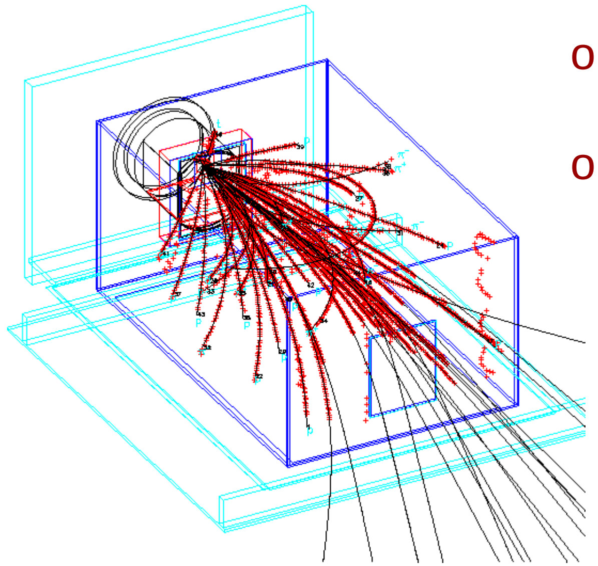
- Near critical point, information on fluctuations is essential; averaging destroys it
- Promising candidates: E-by-E moment analyses

$$M_k(e) = \sum_i n_e(i)\, i^k$$

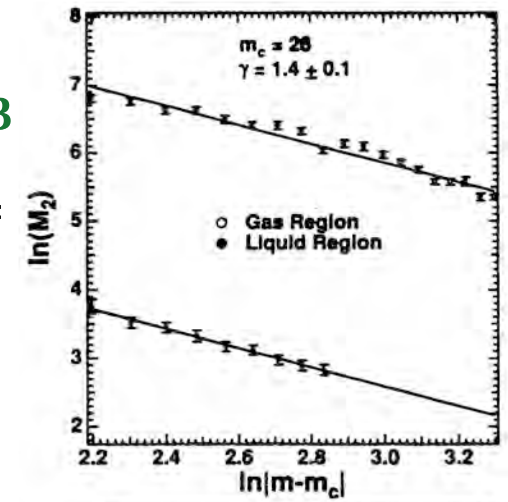
e = event, $n_e(i)$ = # of times i is contained in e

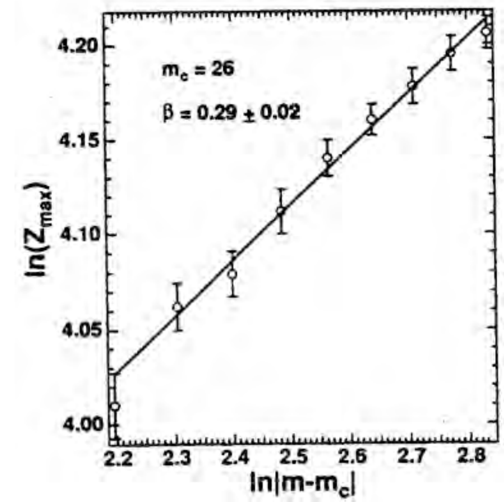
- E-by-E for different observables can generate N-dimensional scatter plots
- Big question: How to sort events into classes?
- Natural choice: If you know control parameter, use it!
(easy for theory, impossible for experiment)
- Closest choice: observable that is ~linear in control parameter.
- Attempt: use charged particle multiplicity, m.

Determining Critical Exponents?



- EOS-TPC: Gilkes *et al.*, PRL **73**
- Complete reconstruction of events: all charges recovered





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- Assume charged-particle multiplicity is proportional to control parameter
- Find critical value, m_c; extract critical exponents β and γ :

$$\gamma = 1.4, \quad \beta = 0.29$$

- Assuming validity of hyper-scaling: universality class of transition is completely determined

Interesting data; incorrect interpretation

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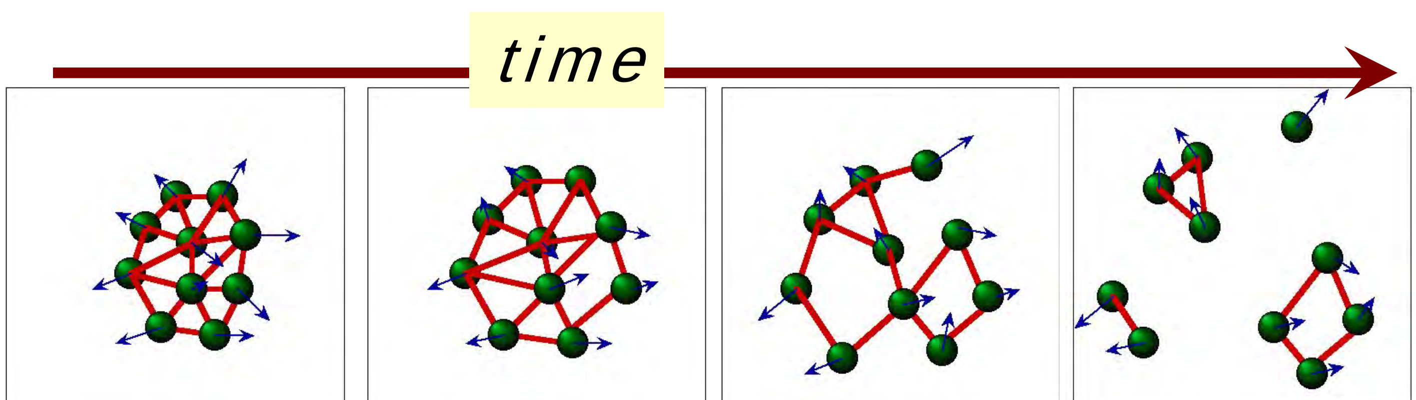
Percolation

WB *et al.*, PLB **150**, 53 (1985)
WB *et al.*, NPA **452**, 699 (1986)
X.Campi, JPA **19**, L917 (1986)
T. Biro *et al.*, NPA **459**, 692 (1986)
J. Nemeth *et al.*, ZPA **325**, 347 (1986)
…

- Short-range NN force: nucleons in contact with nearest neighbors
- Expansion (thermal, compression driven, dynamical, …)
- Bonds between nucleons rupture
- Remaining bonds bind nucleons into fragments
- One ***control parameter***: bond breaking probability



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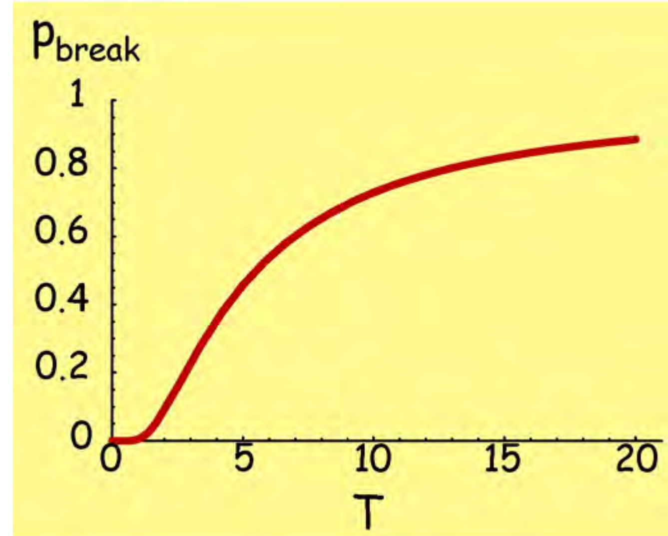
Breaking Probability

- Determined by the excitation energy deposited
- Infinite simple cubic lattice:
  - *3 bonds/nucleon*
  - *It takes 5.25 MeV to break a bond*

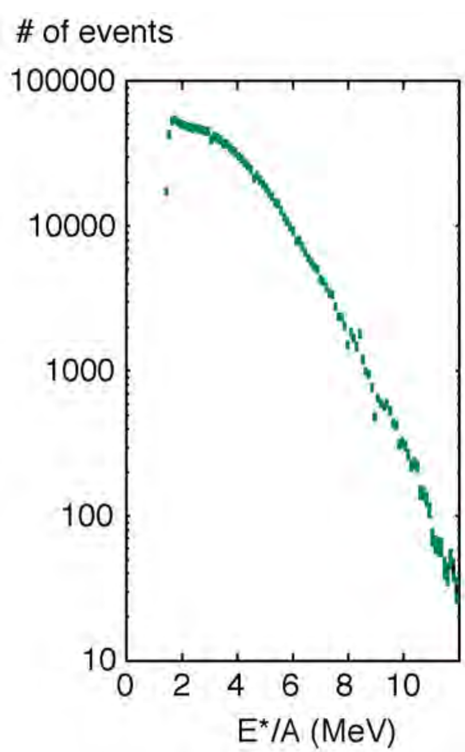


- p, π-induced: Glauber theory
  - *p_{break} proportional to path length through matter*
- General relation between p_{break} and T:

$$p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}}\,\Gamma\left[\frac{3}{2}, 0, \frac{B}{T}\right]$$

Γ = generalized incomplete gamma function, B = binding energy per nucleon
T. Li et al., PRL **70**
(generalization of Coniglio-Klein for Fermi systems)

- Obtain E^* or T from other model or directly from experiment



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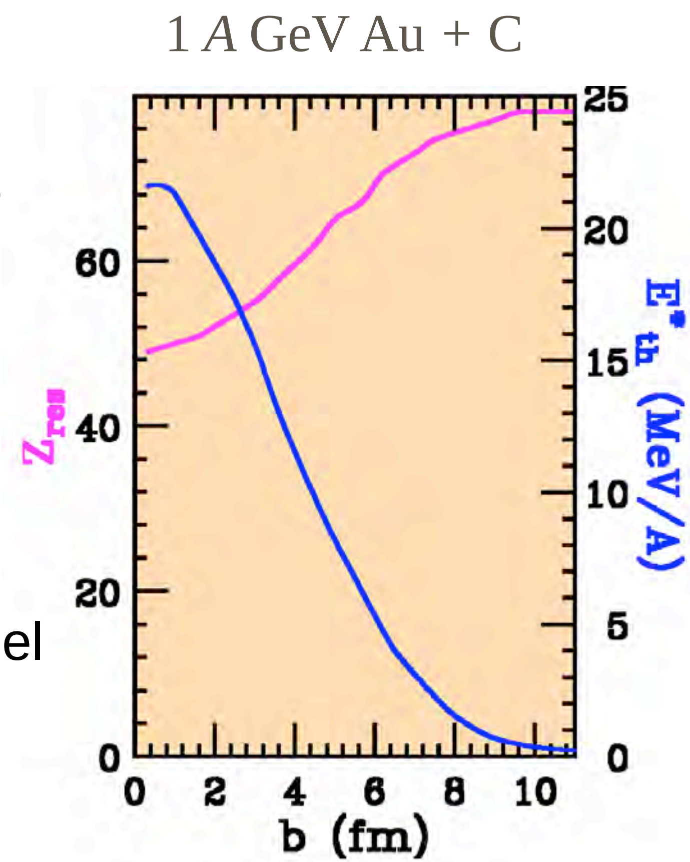
AA Collisions: Hybrid Model

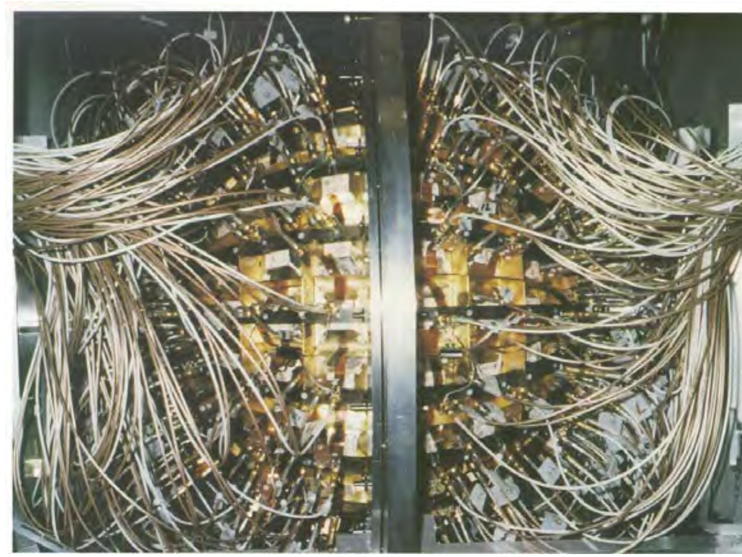
- First stage: Intra-nuclear cascade (or other transport model)
  - *Produces distribution of residue sizes and* E^*
  - *Convert* E^* *into temperature and percolation breaking probability*

$$p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}}\,\Gamma\left[\frac{3}{2}, 0, \frac{B}{T}\right]$$

- Second stage: Percolation model with lattice size = charge of residue
  - *Produces fragments*
- Total multiplicity = INC pre-equilibrium + percolation output



ISiS BNL Experiment

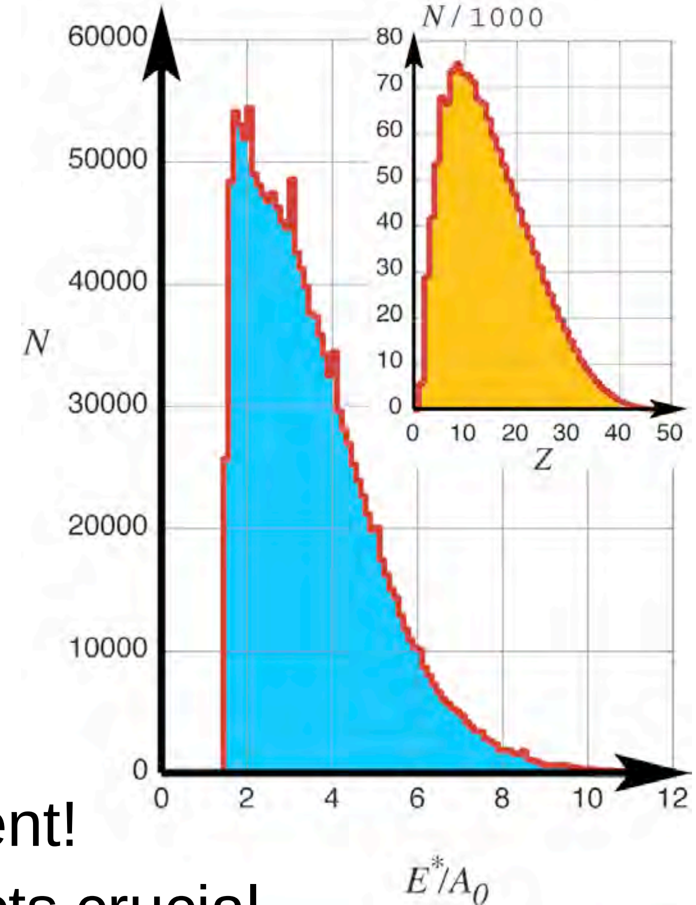


- 10.8 GeV p or π + Au
- Indiana Silicon Strip Array
- Experiment performed at AGS accelerator of Brookhaven National Laboratory

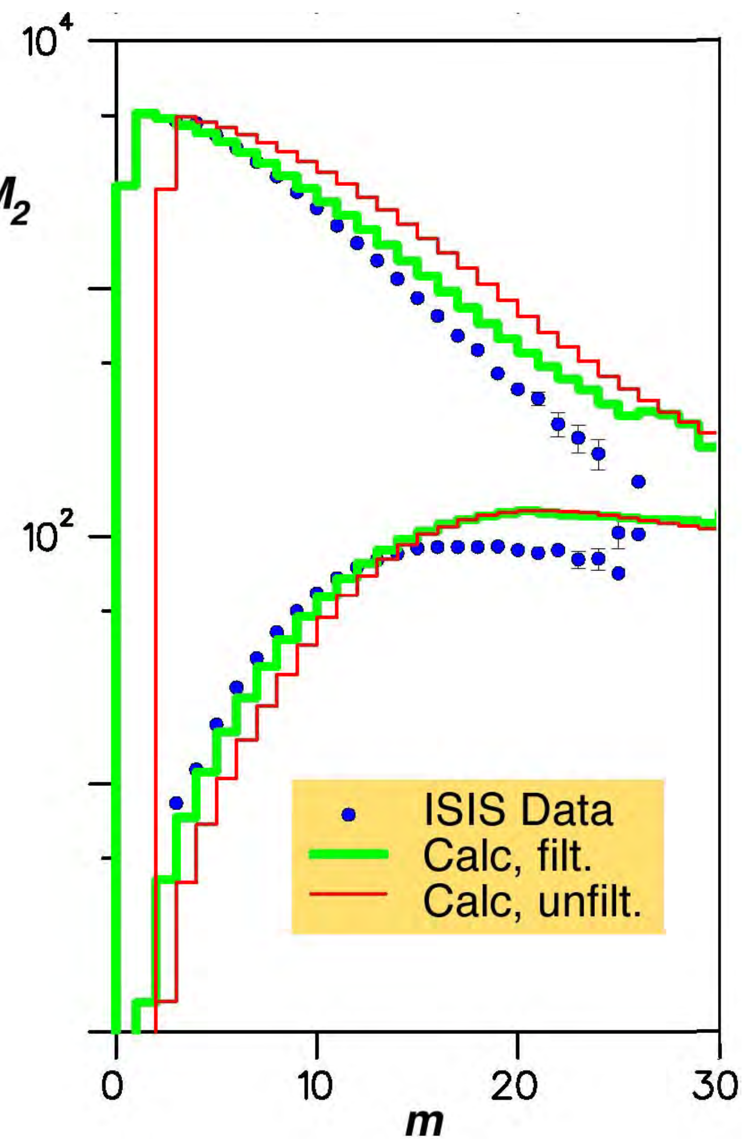
ISIS Data Analysis

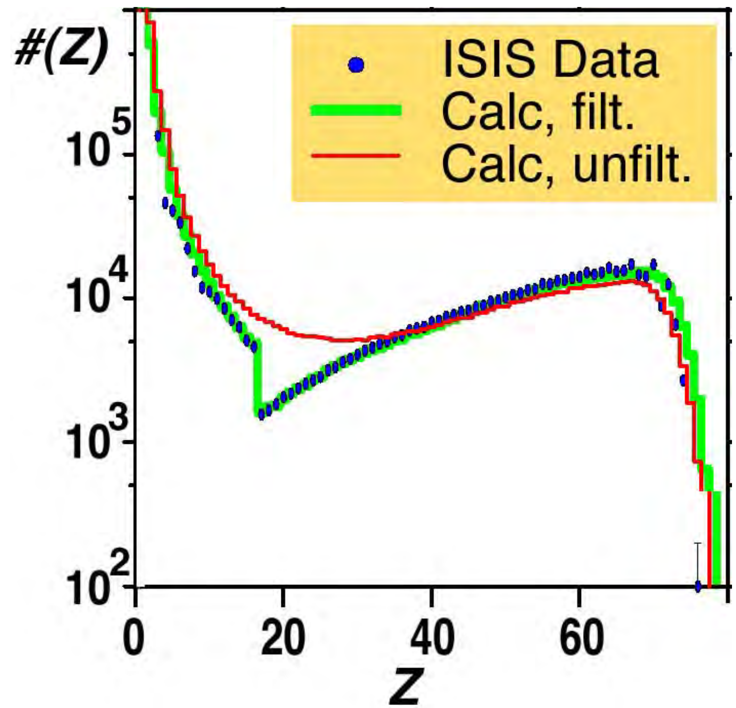
- Marko Kleine Berkenbusch
- Collaboration w. Viola group

- Reaction:
  p, π+Au @AGS
- Very good statistics
  (~10^6 complete events)
- Philosophy: Don't deal with energy deposition models, but take this information from experiment!
- Detector acceptance effects crucial
  - *filtered calculations, instead of corrected data*
- **Parameter-free** calculations



Comparison





- Charge yield spectrum
- Second moments
- **Very good agreement** between theory and data
  - *Filter very important*
  - *Sequential decay corrections huge*

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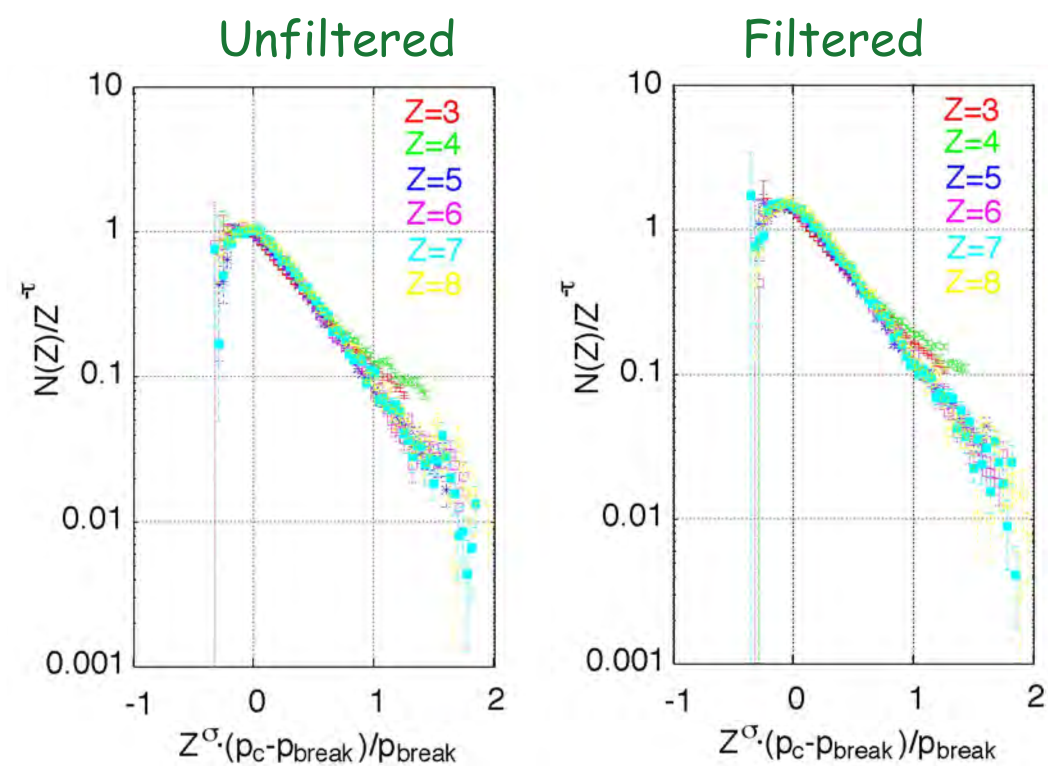
Scaling Analysis

- Idea (Elliott et al.): If data follow scaling function

$$N(Z,T) = Z^{-\tau} f\left[\frac{T-T_c}{T_c} Z^{\sigma}\right]$$

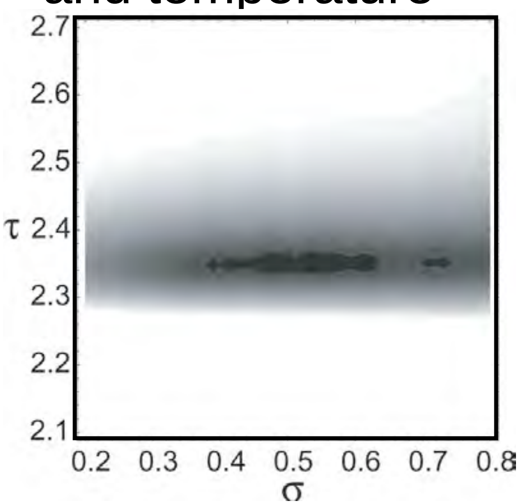
with $f(0) = 1$ (think "exponential"), then we can use scaling plot to see if data cross the point [0,1] -> critical events



- Note:
- Critical events present, $p > p_c$
- Critical value of p was corrected for finite size of system

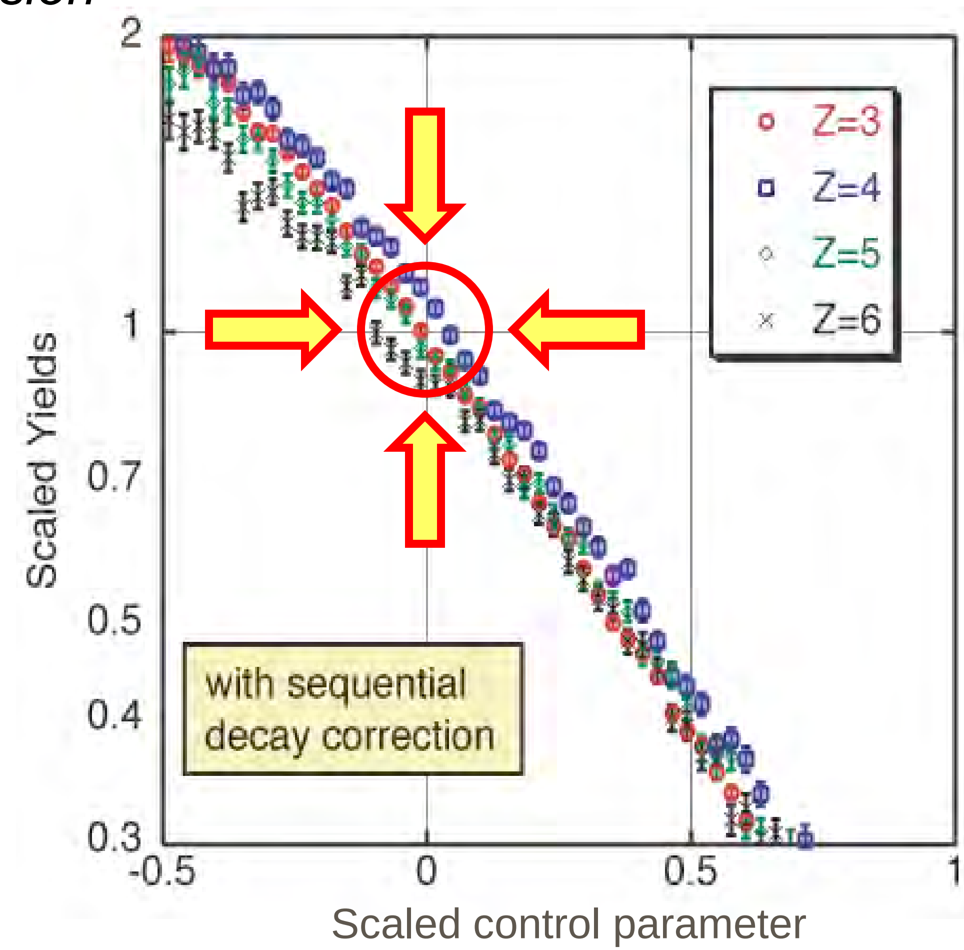
Scaling of ISIS Data

- Most important: critical region and explosive events probed in experiment
- Possibility to narrow window of critical parameters
  - τ : vertical dispersion
  - σ : horizontal dispersion
  - T_c: horizontal shift
- χ^2 Analysis to find critical exponents and temperature



- Result:
  σ = 0.5 +- 0.1
  τ = 2.35 +- 0.05
  T_c = 8.3 +- 0.2 MeV

M. Kleine Berkenbusch *et al.*, PRL **88**



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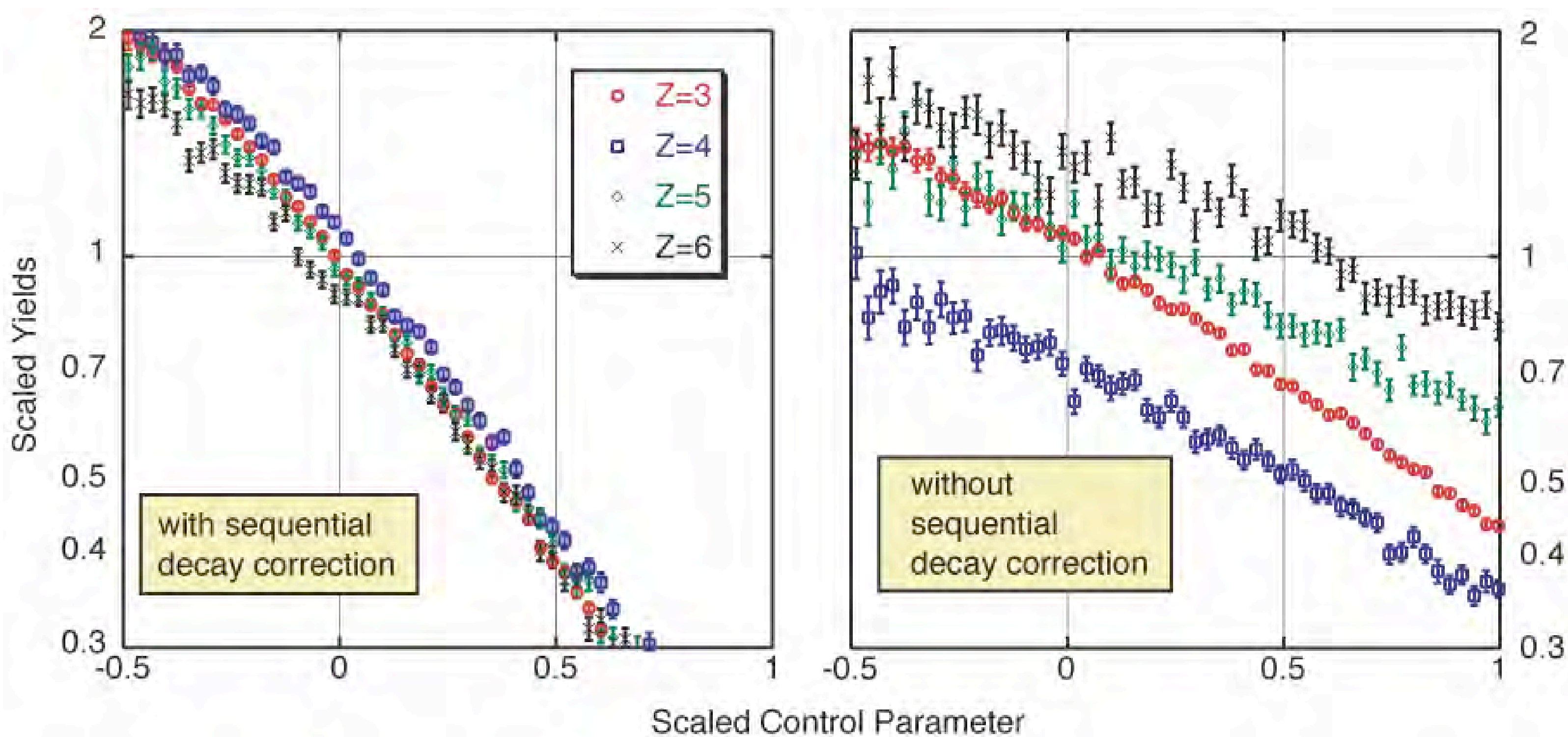
Scaling of ISIS Data (2)

- Note: This only works because of very careful correction for sequential decays!
- Best-fit scenarios for both cases:
  - *Scaling collapse only when sequential decay correction is performed*
  - *Technique fails without it*



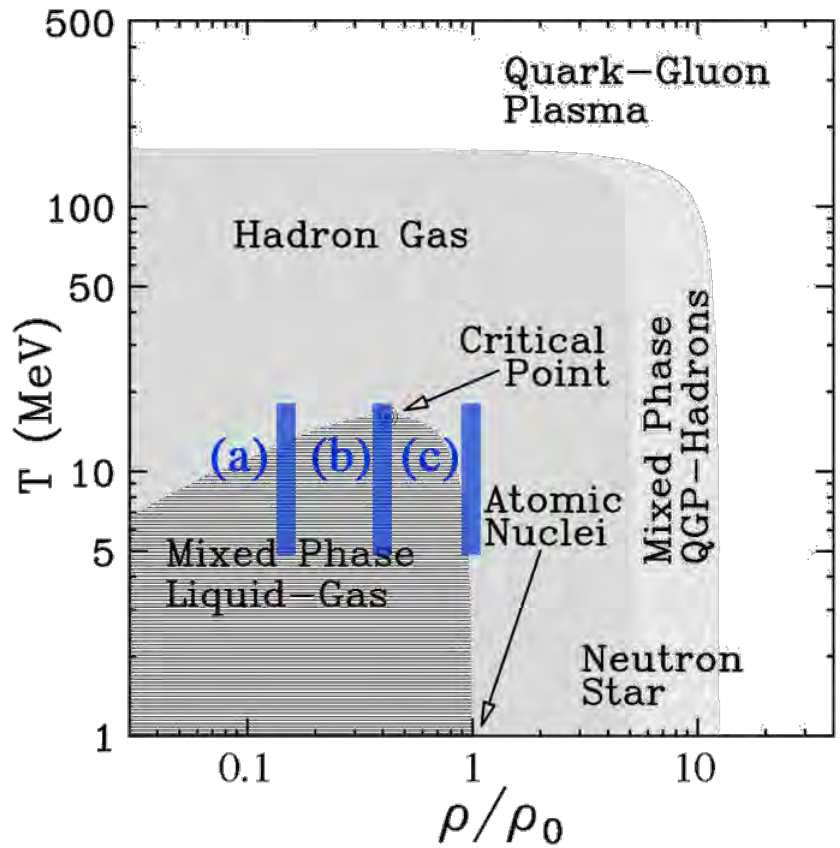
Freeze-Out Density

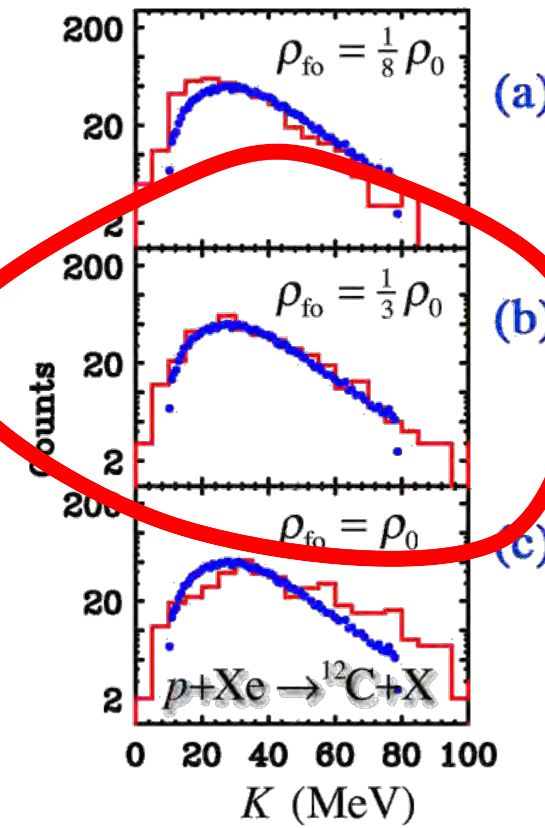
$$p_b = 1 - \frac{2}{\sqrt{\pi}}\Gamma(\tfrac{3}{2}, 0, B/T)$$

- Percolation model only depends on breaking probability, which can be mapped into a temperature.
- **Q**: How to map a 2-dimensional phase diagram?
- **A**: Density related to fragment energy spectra; Coulomb many-body expansion of pre-fragments

WB, Alleman, Pratt, AIP conf.proc.884, 327 (2007)
WB, Nucl.Phys. A**787**, 595c (2007)

$$\rho_c = (0.35 \pm 0.1)\rho_0$$





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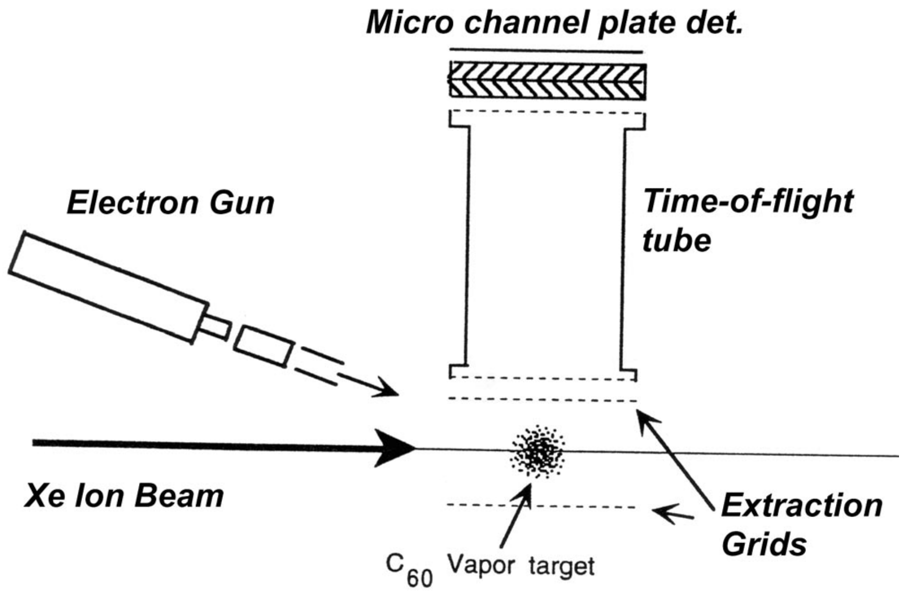
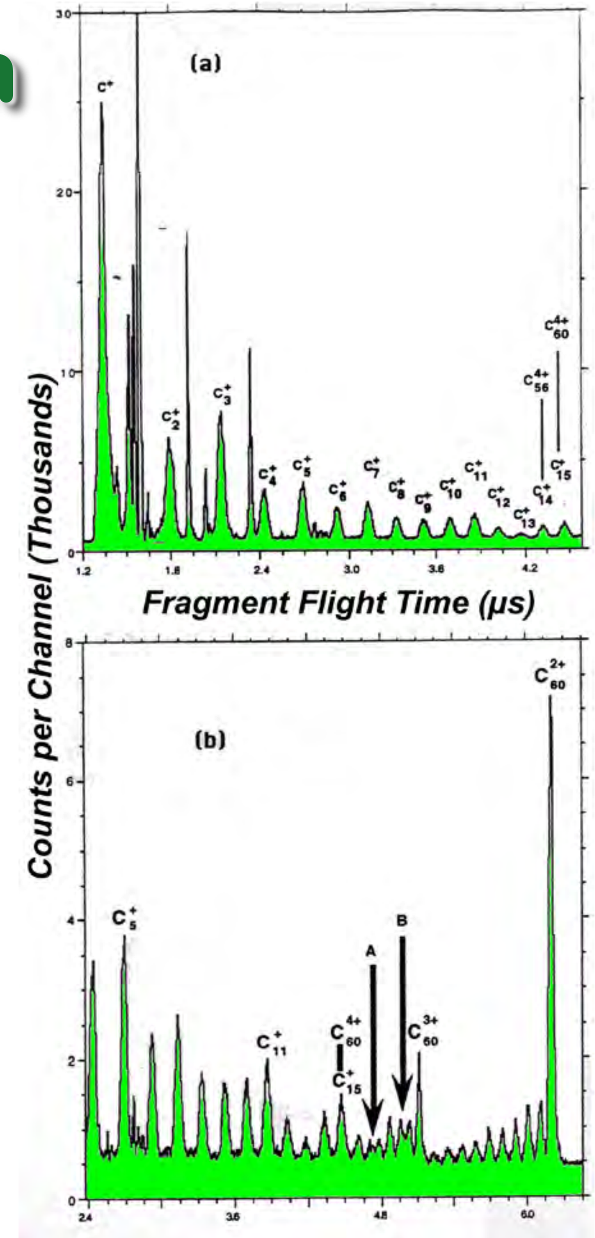
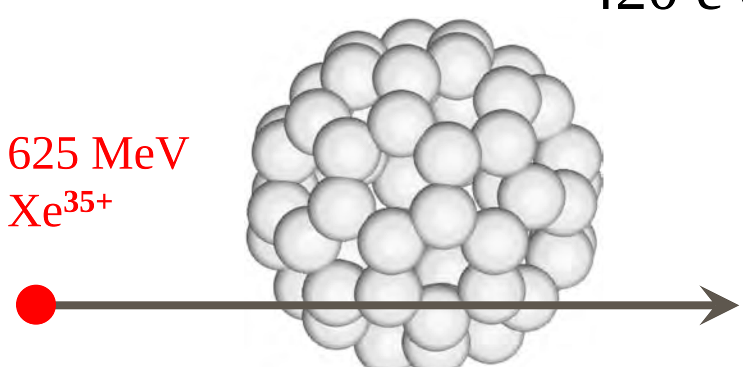
Buckyball Fragmentation





Binding energy of C_{60}:
420 eV



Cheng et al., PRA 54

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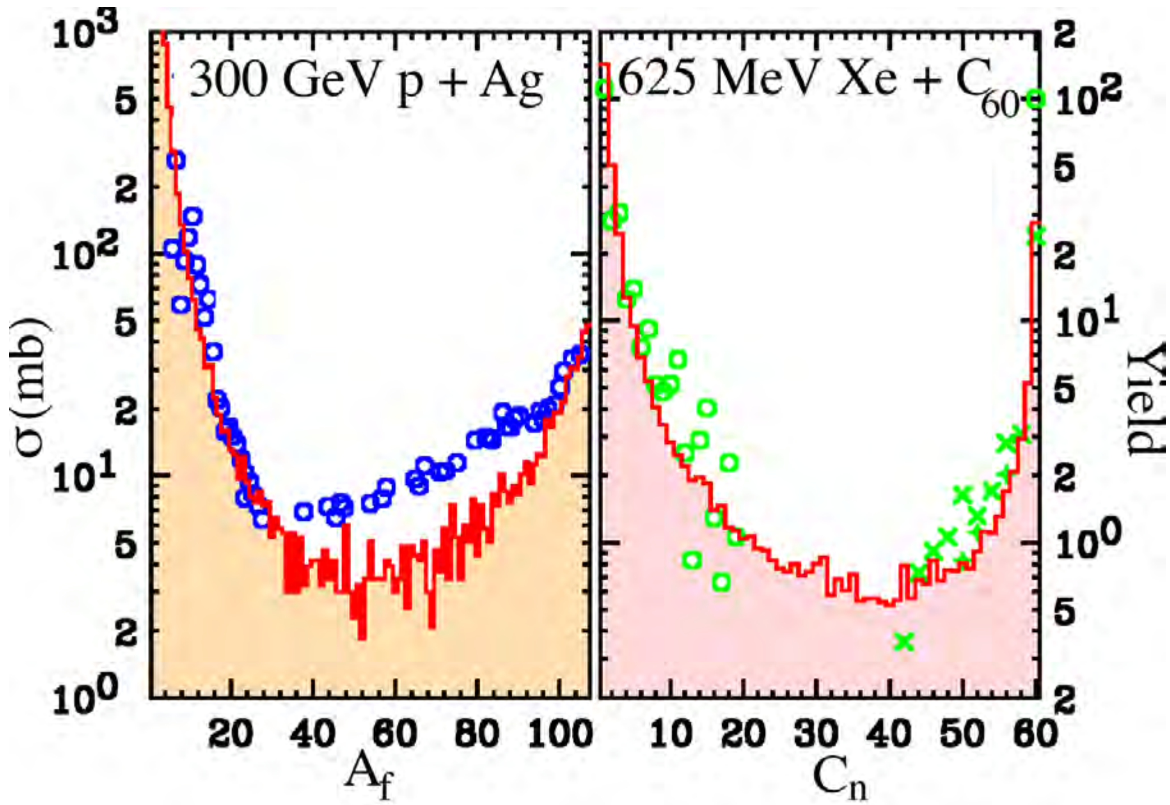
Cross-Disciplinary Comparison

Data:
Bujak et al., PRC 32
LeBrun et al., PRL 72
Calc.:
WB, PRC 38
Cheng et al., PRA 54

- Left: Nuclear Multifragmentation
- Right: Buckyball Fragmentation
- Histograms: Percolation Models



- Similarities:
  - ç *U - shape (b-integration)*
  - ç *Power-law for imf's (1.3 vs. 2.6)*
  - ç *Binding energy effects provide fine structure*

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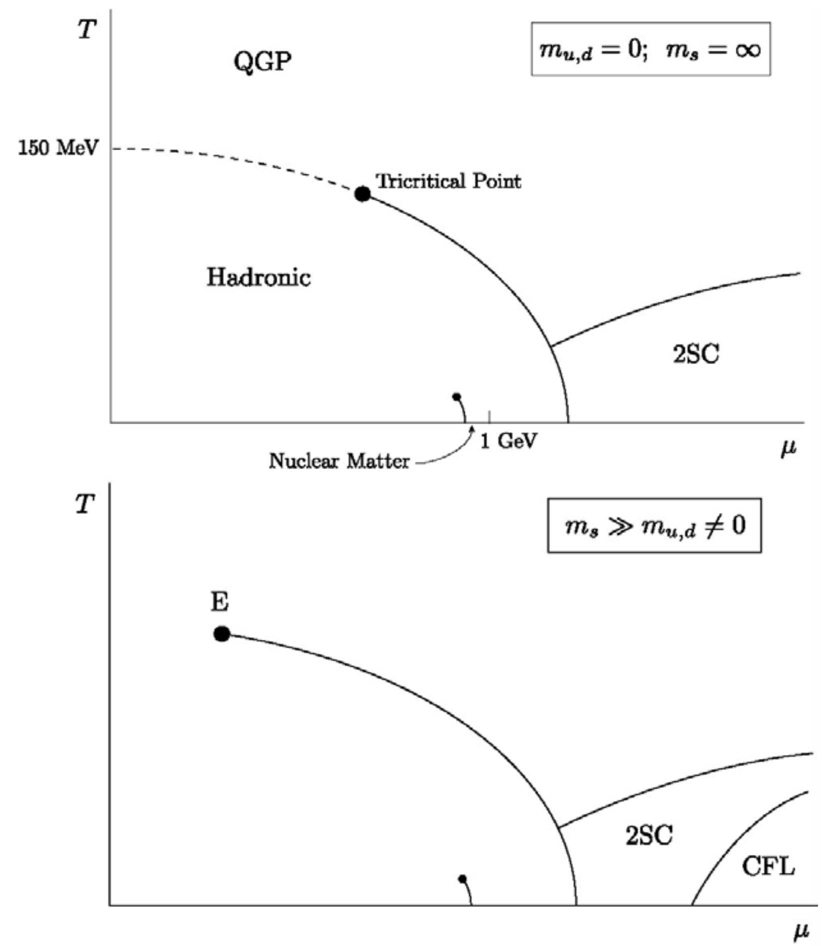
How to find the QCD Critical Point

- There are no large fragments!
- What clusters? What fluctuates?
- What is the order parameter that can be measured experimentally?
- What should CBM look for?



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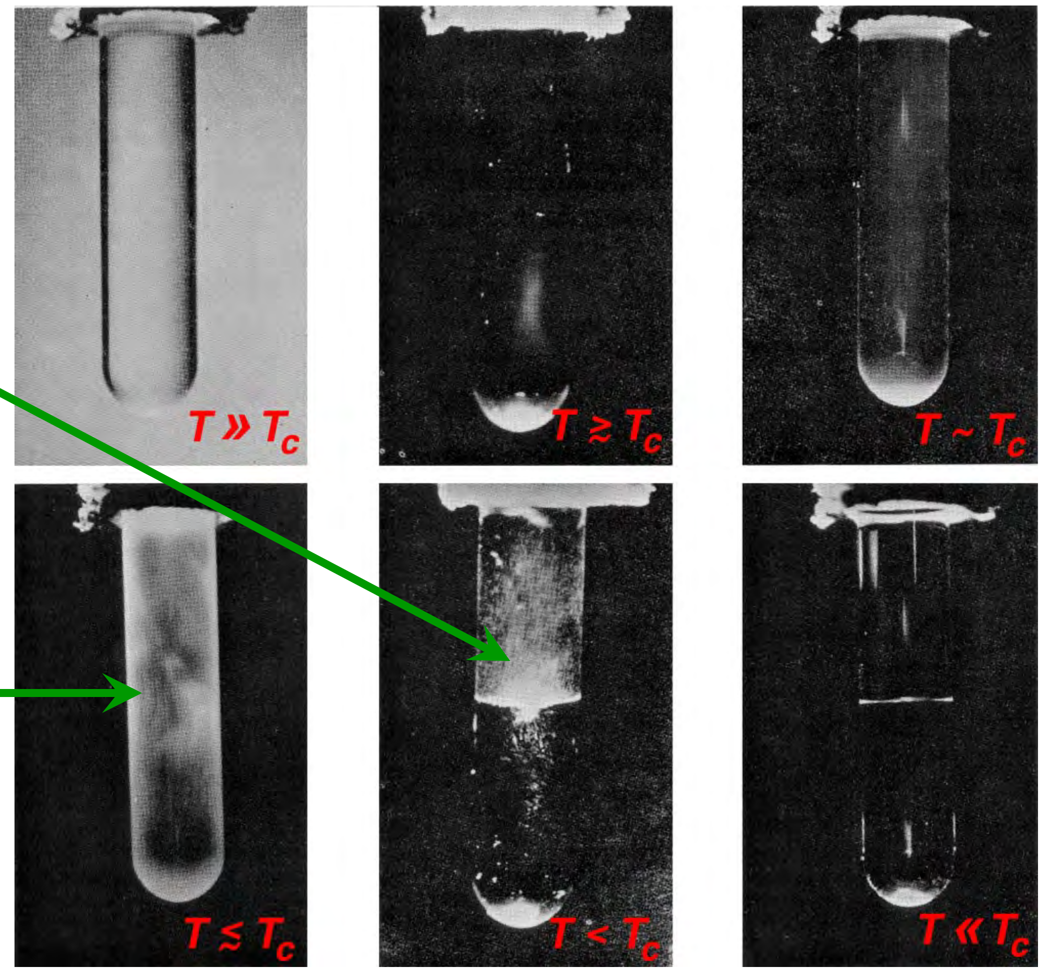
Liquid at the Critical Point

Cyclohexane-Aniline

Liquid-vapor coexistence

Critical Opalescence: Light scattering off fluctuations



Ferrell (1968)
Stanley (1971)

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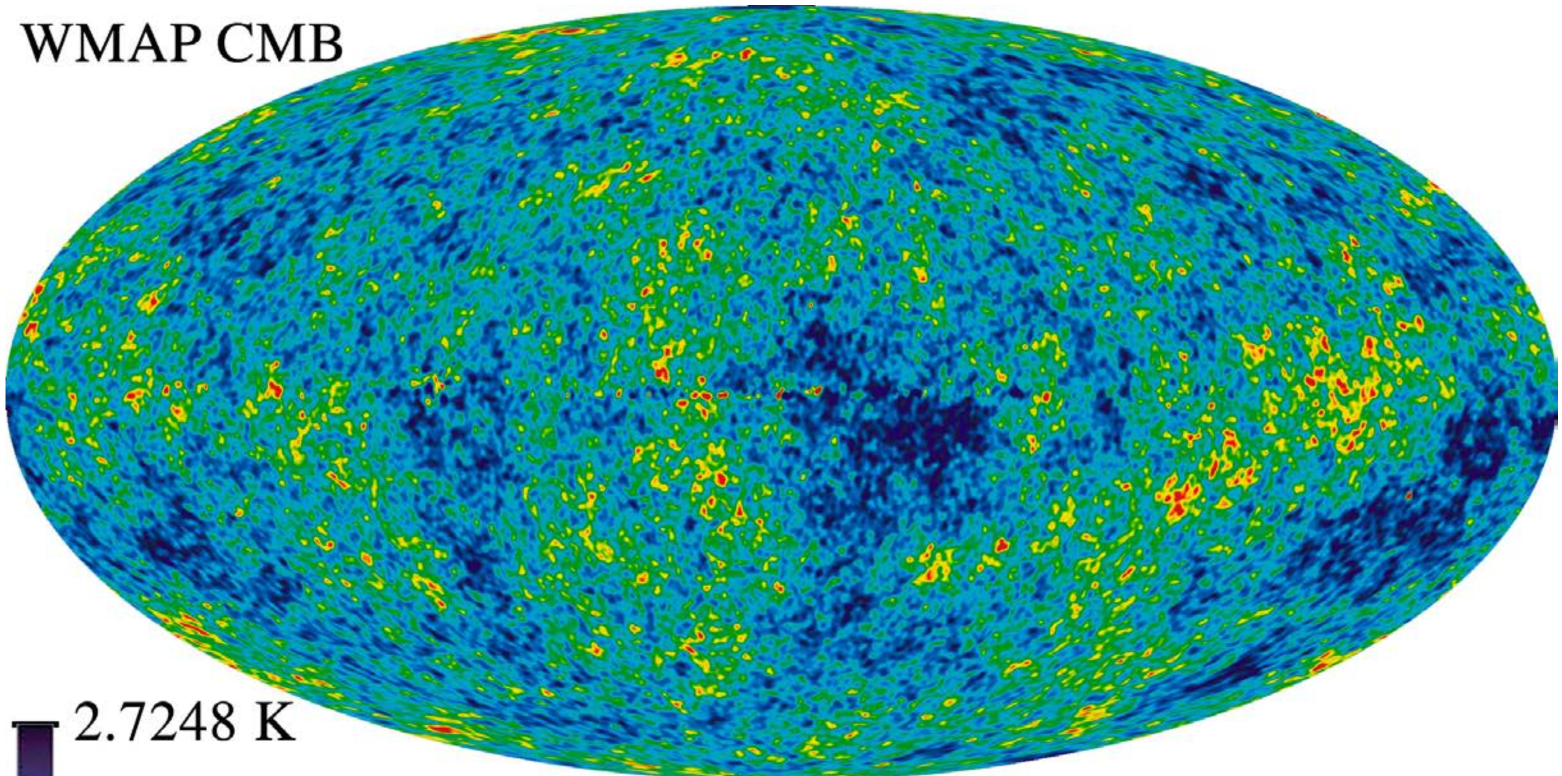
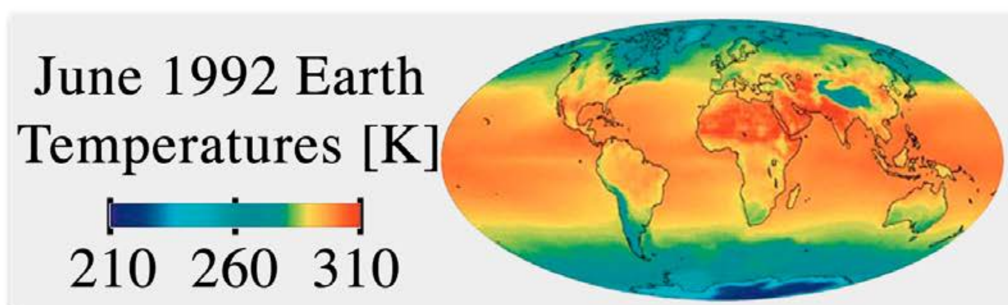
Fluctuations





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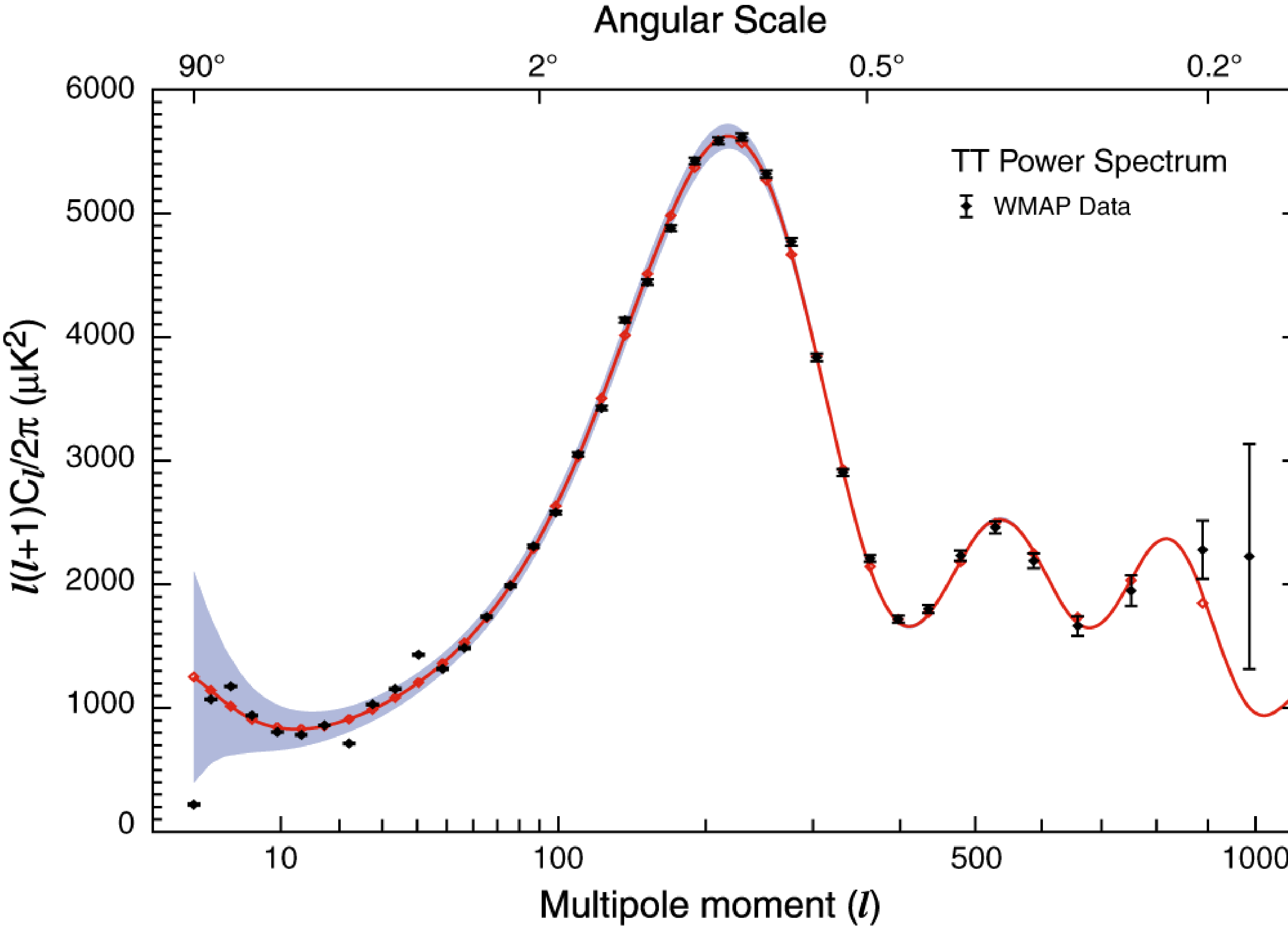
Multipole Analysis



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The "CMB" of CBM

- Photons emitted from early collision stages scatter off the fluctuations
  - *critical opalescence*
- Similar effect for pions
  - *"critical pion opacity"*
- Cluster analysis in momentum space for pions and photons
- FAIR chance that the signal of the critical point survives the later stages of final state interaction
- Finite size constraints
  - *Do not expect a sharp peak!*
  - *A bump is all that you will get at best*
  - *Unambiguous experimental signals are hard to come by*
  - *Lots of modeling needed to interpret results*

Zipf's Law: Probabilities (1)

- Probability that cluster of size A is the largest one = probability that at least one cluster of size A is present times probability that there are 0 clusters of size $>A$

$$P_{1st}(A) = p_{\geq 1}(A) \cdot p_0(>A) = [1 - p_0(A)] \cdot p_0(>A)$$

- $N(A)$ = average yield of size A: $N(A) = aA^{-\tau}$
- $N(>A)$ = average yield of size $>A$: (V = event size)

$$N(>A) = \sum_{i=A+1}^{V} N(i) = \sum_{i=A+1}^{V} a i^{-\tau} = a\zeta(\tau, 1+A) - a\zeta(\tau, 1+V)$$

- Normalization constant a from condition: $\sum_{A=1}^{V} A \cdot N(A) = V$

$$a = V \Big/ \sum_{A=1}^{V} A^{1-\tau} = V / H_V^{(1-\tau)}$$

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Zipf's Law: Probabilities (2)

- Use Poisson statistics for individual probabilities:

$$p_n(i) = \frac{\langle N(i)\rangle^n e^{-\langle N(i)\rangle}}{n!}$$

$$p_0(i) = e^{-\langle N(i)\rangle}; p_1(i) = \langle N(i)\rangle p_0(i); p_2(i) = \tfrac{1}{2}\langle N(i)\rangle p_1(i)\ldots$$

- Put it all together:

$$\begin{aligned} P_{1st}(A) &= [1 - p_0(A)] \cdot p_0(> A) \\ &= [1 - e^{-N(A)}] \cdot e^{-[a\zeta(\tau,1+A) - a\zeta(\tau,1+V)]} \end{aligned}$$

- Average size of biggest cluster

$$\langle A_{1st}\rangle = \sum_{A=1}^{V} A \cdot P_{1st}(A)$$

(Exact expression!)

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Zipf's Law: Probabilities (3)

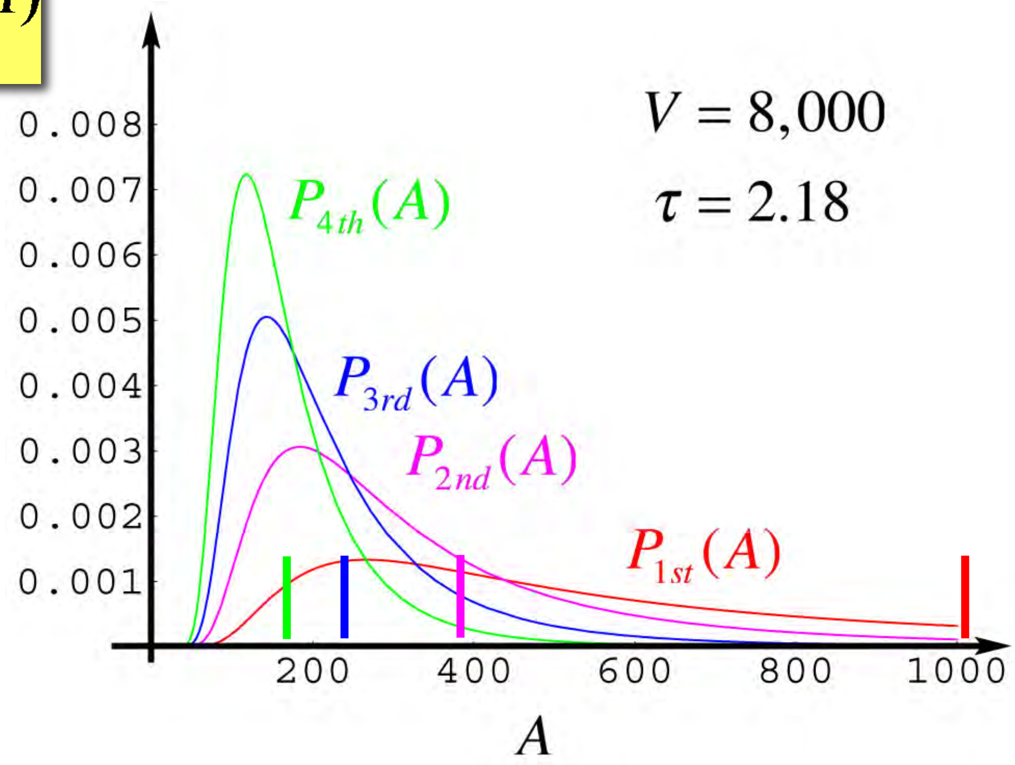
- Probability for given A to be 2nd biggest cluster:

$$P_{2nd}(A) = p_{\geq 2}(A) \cdot p_0(>A) + p_{\geq 1}(A) \cdot p_1(>A)$$
$$= [1 - p_0(A) - p_1(A)] \cdot p_0(>A) + [1 - p_0(A)] \cdot p_1(>A)$$

- Average size of 2nd biggest cluster:

$$\langle A_{2nd} \rangle = \sum_{A=1}^{V} A \cdot P_{2nd}(A)$$

- And so on …
- Recursion relations!



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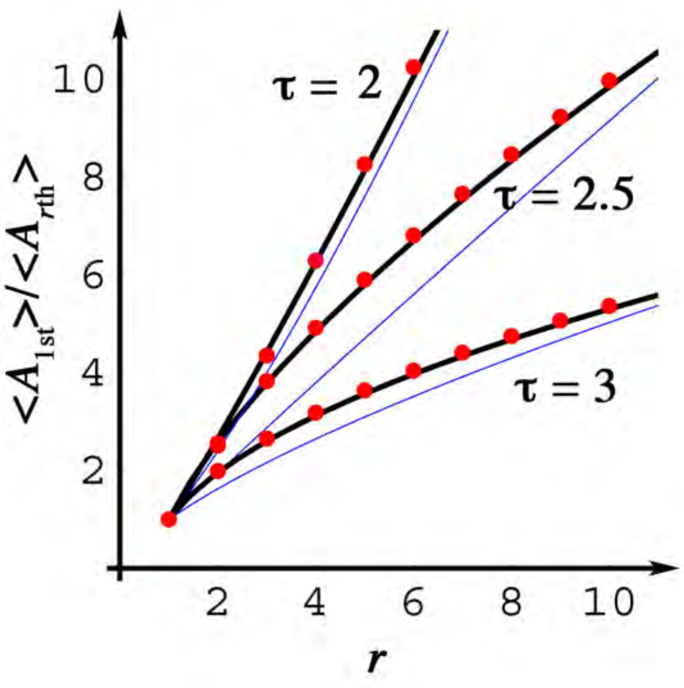
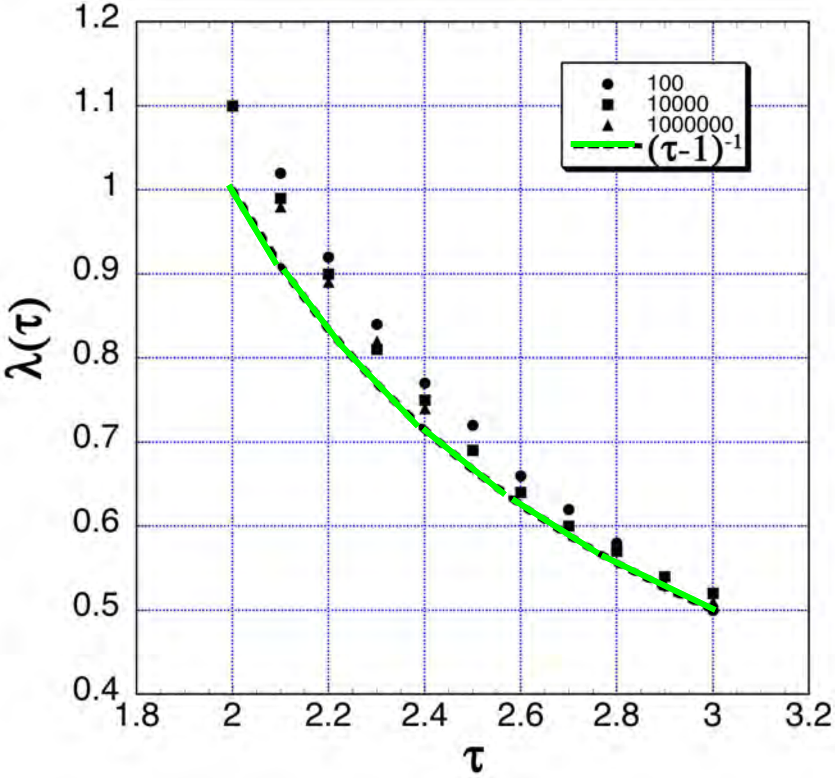
Zipf-Mandelbrot

- Limiting distributions for cluster size vs. rank

$$\langle A_{r\text{th}} \rangle = \frac{c}{(r+k)^{\lambda}}$$ with exponent $$\lambda \sim \frac{1}{\tau - 1}$$

- Proof for infinite system in continuum limit with τ =2: Paech, WB, Pratt, PRC 76, 2007





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Summary

- Common threads in phase transitions of molecules, nuclei, and hadronic matter
- Non-equilibrium effects make extraction of EoS information hard
- View of multifragmentation as a **critical phenomenon** is on solid footing
- Critical slowing down not important, because it is a non-equilibrium phase transition
- Finite-size corrections can be dealt with rather effectively; opportunity for us to contribute to larger science community
- Self-organized criticality, Zipf's Law, …: nuclear fragmentation continues to be a rich playground for testing out nonlinear physics concepts.

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