Non-Linear Adventures in Taking Things Apart: Fragmentation of Nuclei and Molecules

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Power of 100,000

$10^{-15}$ m scale: Nuclei

$10^{-10}$ m scale: Atoms

$10^{-5}$ m scale: Cells

$10^1$ m scale: People

$10^5$ m scale: City

$10^{10}$ m scale: Earth

$10^{10}$ m scale: City
Nuclear Physics: the micro-nano science

- Nuclear size \(\sim 10^{-14} \text{ m}\) (10,000 times too small for direct observation)
- System of \(\sim 10^2\) fermions => finite size effects extremely important, just like quantum dots (only smaller …)
- Mesoscopic physics
- Low-energy excitations (rotation, vibration): gamma emission, keV
- Other nuclear decays: beta, alpha, fission
- Higher energies: disintegration of entire nucleus
- Time scales \(\sim 10^{-22} \text{ s}\) (1,000,000 times too fast for direct measurements)
- Energy scales 10-100 MeV
Favorite Isotope Decays

$N = \text{Number of neutrons}$

$Z = \text{Number of protons}$
Nuclear Fragmentation

300 GeV p + Ag

$\sigma$ (mb)

$A_f$

NOLPA2011

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W. Bauer
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What is an “equation of state”? 

- State variables: pressure, temperature, density (internal energy, chemical potential, strangeness, …)

- Equation of state: relationship between state variables, 
  \[ f(p, T, \rho) = 0. \]
  - Thermodynamic equation describing state of matter under given physical conditions
  - Example: Ideal gas: \( pV = nRT \)
  - Example: Ultra-relativistic fluid: \( p = c_s^2 \varepsilon \)
  - More realistic equations of state need to contain phase transitions, coexistence regions, critical points, …
Two (at least) phase transitions in nuclear matter:
- “Liquid Gas”
- Hadron gas $\rightarrow$ QGP / chiral restoration

Problems/
Opportunities:
- Finite size effects (finite size scaling!)
- Is there equilibrium? (☐)
- Measurement of state variables ($\rho$, $T$, $S$, $p$, …)
- Migration of nuclear system through phase diagram (non-equilibrium processes)
- Near critical point(s): Critical slowing down! Not sufficient time for equilibrium phase transition!
Volume

- Information from interferometry:
- Two-particle correlation are sensitive to space-time extension of emitting source

\[ C(P,q) = \int d^3x \, F_p(r) \, |\phi(r,q)|^2 \]

Relative Momentum, q

Last interaction modifies correlation function
Interaction = f(Δr, Δp, Δt)

Expanding & cooling heavy ion reaction zone
Temperature

- Measure nuclear temperature indirectly via:
  - Slopes of charged particle spectra
  - Bound-state populations
  - Unbound states
  - Fragment isotopic yields

“He-Li” thermometer
Albergo et al., Nuovo C. A89

\[ T = 13.3 \text{ MeV} / \ln \left( 2.18 \frac{Y(6Li)Y(4He)}{Y(7Li)Y(3He)} \right) \]

Central question: At which time do we measure the temperature with each thermometer?
Temperature from Fragment Spectra

- Nucleon momentum distribution at temp. $T$:
  \[ \rho(p) = \frac{1}{1 + \exp\left[\frac{(p^2/2m - \mu)/T_{in}}{T_{in}}\right]} \]

- Fragment momentum = sum of momenta of nucleons in it

- Problem equivalent to solving Pearson random walk in momentum space

- Limiting distribution:
  \[ \rho(E_f) = 2 \sqrt{\frac{E_f}{\pi T_{eff}^3}} \exp \left( -\frac{E_f}{T_{eff}} \right) \]
  (Boltzmann with $T_{eff} = \sigma^2/M_N$)

- Fragment slope “temperature”, $T_{eff}$, is not equal to $T$, but is a monotonous function of it
  \[ \Rightarrow \text{Nuclear Thermometer} \]

- Approximation:
  \[ T_{eff} \approx \left(\frac{2}{5}\right)\epsilon_f \left(1 + 5\pi^2 \frac{T^2}{\epsilon_f^2} + \ldots\right) \]

WB, Phys. Rev. C 51
Isospin: FRIB Reaction Physics

- Exploration of the drip lines below charge ~40 via projectile fragmentation reactions
- Determination of the isospin degree of freedom in the nuclear equation of state
- Astrophysical relevance (origin of heavy elements!)
- Review: Li, Ko, WB, Int. J. Mod. Phys. E 7
Width of Isotope Distribution, Sequential Decays

- Predictions for width of isotope distribution are sensitive to isospin term in nuclear EoS.
- Complication: Sequential decay almost totally dominates experimentally observable fragment yields.

Pratt, WB, Morling, Underhill, PRC 63
First-Order Phase Transition

- Coexistence of two phases (e.g. ice+water, water+steam)
- Addition of heat does not change temperature
  
  Latent heat
  
  \( \text{H}_2\text{O}: L_f = 80 \text{ kcal/kg}, \quad L_v = 540 \text{ kcal/kg} \)

- Different specific heat capacities in the different phases -> different slopes \( T \) vs. \( Q \)
- Pressure kept constant!
Observation of First-Order Phase Transition?

- Low $E^*$: Liquid-like
  $T \sim E^{*1/2}$
- High $E^*$: Gas-like
  $T \sim E^*$
- 1st order transition: Liquid-gas coexistence
  Temperature does not change in phase mixture while liquid is converted to vapor.
- Analogy: Boiling of water
- But what about “constant pressure”?

J. Pochodzalla et al. (ALADIN), PRL 75
Buckyballs-Melting

- $C_{60}$ Cluster
- Soccer Ball Geometry

Molecular dynamics calculations
Hoover-Nose heat bath

S.G. Kim & D. Tomanek, PRL 72
Continuous Phase Transition

- Near critical point, we expect scaling behavior: all physical quantities have power-law dependencies on the control parameter.
- No characteristic scales in observables.
- Critical exponents of power-laws are main quantities of interest:
  - Cluster size: \( n_s(T_c) \sim s^{-\tau} \)
  - Order parameter: \( P \sim (T - T_c)^\beta \)
  - Divergence of \( s \): \( \sim |T - T_c|^\gamma \)
- Hyper-scaling assumption:
  \[
  2 - \alpha = \frac{\tau - 1}{\sigma} = 2\beta + \gamma
  \]
  (Determine 2 critical exponents sufficient)
Finite Size Scaling

- Phase transitions strictly only defined for (almost) infinite systems
- Lattice calculations work on finite lattices and extrapolate to infinite lattices (hardest part!)
- Finite size scaling exponent, $\nu$
  - Modify control parameter by $L^{1/\nu}$
  - Modify order parameter by $L^{\beta/\nu}$
- Opportunity for nuclear physics: Learn about extreme finite size scaling in real systems
Self-Organized Criticality

- How to achieve scale-invariance?
  - Vicinity of critical point: power-laws
  - Very careful tuning of control parameter(s) required

- Another possibility: SOC
  - Sequence of avalanches between metastable states
  - Continually driven to criticality
  - No external tuning required
  - Example: Bak’s sand pile

Finite-size scaling

Held et al., PRL 65
Critical Slowing Down

- Near critical point, $|T/T_c| << 1$, it takes longer and longer to re-establish equilibrium after changing the temperature.
- Example: Ising Model,
  
  $M(t) \sim \exp(-t/t_r),$

  with

  $t_r = 4.5 \ (T-T_c)^{-1.85}$, for $T > T_c$. 

$$T_c = 2.269... \quad M_i = 1, M_{eq} = 0$$
... but there is not enough time!

- HBT puzzle
- Theoretical expectation
  - Change of # of degrees of freedom in transition from quarks and gluons to hadrons
  - Large time delay
  - Expect $R_{out} >> R_{side}$
- Not seen by experiment!

- Equilibrium thermodynamic phase transition may not be possible
  - … but percolation-type transition not excluded!
Dynamics

- Thermal equilibrium assumptions not (always) valid
- Need transport theory
- Various event class averages (event vs. thermal!)
- Connections to underlying phase diagram poorly understood

- Transient formation of non-compact structures
  - Sheet instabilities
    Moretto et al., PRL 69
  - Bubble and ring formation
    WB, Schulz, Bertsch, PRL 69
  - Imaginary sound velocity causes exponential growth in fluctuations; non-equilibrium in origin
  - Similar effect now postulated for RHIC collisions (Pratt 2008)

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Non-Equilibrium Phase Transitions

- Conventional thermodynamics
  - Write down partition function from (known) Hamiltonian
    \[ Z = tr(e^{-\beta H}) \]
  - Take partial derivatives to obtain state variables
  - Static solution; equilibrium; no changes in time

- Non-equilibrium Phase Transition
  - Dynamics; time dependence
  - No thermal averages
  - Transitions between un/meta/bi-stable states

- Are similar universality classes possible?
  - Critical exponents can be obtained
Multi-Component Systems

- What happens when physically different components are in the system undergoing phase transition (protons+neutrons, different flavor quark&gluons, …)?

- Possible:
  - Change of character of phase transition
    Müller&Serot, PRC 52
  - Shift in critical value of control parameter, same critical exponents
    Harreis&WB, PRB 62
Event-by-Event

- Near critical point, information on fluctuations is essential; averaging destroys it
- Promising candidates: E-by-E moment analyses
  \[ M_k(e) = \sum_i n_e(i) \cdot i^k \]
  
  \( e = \) event, \( n_e(i) = \# \) of times \( i \) is contained in \( e \)
- E-by-E for different observables can generate \( N \)-dimensional scatter plots
- Big question: How to sort events into classes?
- Natural choice: If you know control parameter, use it!
  (easy for theory, impossible for experiment)
- Closest choice: observable that is \( \sim \)linear in control parameter.
- Attempt: use charged particle multiplicity, \( m \).
Determining Critical Exponents?

- EOS-TPC: Gilkes et al., PRL 73
- Complete reconstruction of events: all charges recovered

- Assume charged-particle multiplicity is proportional to control parameter
- Find critical value, $m_c$; extract critical exponents $\beta$ and $\gamma$:
  $$\gamma = 1.4, \quad \beta = 0.29$$
- Assuming validity of hyper-scaling: universality class of transition is completely determined

Interesting data; incorrect interpretation
Percolation

- Short-range NN force: nucleons in contact with nearest neighbors
- Expansion (thermal, compression driven, dynamical, …)
- Bonds between nucleons rupture
- Remaining bonds bind nucleons into fragments
- One control parameter: bond breaking probability

WB et al., PLB 150, 53 (1985)
WB et al., NPA 452, 699 (1986)
X.Campi, JPA 19, L917 (1986)
T. Biro et al., NPA 459, 692 (1986)
J. Nemeth et al., ZPA 325, 347 (1986)
…
Breaking Probability

- Determined by the excitation energy deposited
- Infinite simple cubic lattice:
  - 3 bonds/nucleon
  - It takes 5.25 MeV to break a bond

- $p, \pi$-induced: Glauber theory
  - $p_{\text{break}}$ proportional to path length through matter

- General relation between $p_{\text{break}}$ and $T$:
  \[
  p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}} \Gamma \left( \frac{3}{2}, 0, \frac{B}{T} \right)
  \]

  $\Gamma =$ generalized incomplete gamma function, $B =$ binding energy per nucleon
  T. Li et al., PRL 70
  (generalization of Coniglio-Klein for Fermi systems)

- Obtain $E^*$ or $T$ from other model or directly from experiment
AA Collisions: Hybrid Model

- **First stage**: Intra-nuclear cascade (or other transport model)
  - Produces distribution of residue sizes and $E^*$
  - Convert $E^*$ into temperature and percolation breaking probability

$$p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}} \Gamma \left[ \frac{3}{2}, 0, \frac{B}{T} \right]$$

- **Second stage**: Percolation model with lattice size = charge of residue
  - Produces fragments

- Total multiplicity = INC pre-equilibrium + percolation output
ISiS BNL Experiment

- 10.8 GeV p or $\pi$ + Au
- Indiana Silicon Strip Array
- Experiment performed at AGS accelerator of Brookhaven National Laboratory
ISIS Data Analysis

- Marko Kleine Berkenbusch
- Collaboration w. Viola group

- Reaction: \( p, \pi + Au @ AGS \)
- Very good statistics (~\(10^6\) complete events)
- Philosophy: Don’t deal with energy deposition models, but take this information from experiment!
- Detector acceptance effects crucial
  - filtered calculations, instead of corrected data
- Parameter-free calculations
Comparison

- Charge yield spectrum
- Second moments
- Very good agreement between theory and data

- Filter very important
- Sequential decay corrections huge

Figures show two graphs comparing the charge yield spectrum for ISIS Data, calculated with and without filtering. The graphs indicate a close agreement between theory and data, with filtering being particularly important for accurate results.
Scaling Analysis

- Idea (Elliott et al.): If data follow scaling function

\[ N(Z, T) = Z^{-\gamma} f \left( \frac{T - T_c}{T_c} Z^\sigma \right) \]

with \( f(0) = 1 \) (think “exponential”), then we can use scaling plot to see if data cross the point \([0,1] \rightarrow \) critical events

- Note:
  - Critical events present, \( p > p_c \)
  - Critical value of \( p \) was corrected for finite size of system
Scaling of ISIS Data

- Most important: critical region and explosive events probed in experiment
- Possibility to narrow window of critical parameters
  - $\tau$: vertical dispersion
  - $\sigma$: horizontal dispersion
  - $T_c$: horizontal shift

- $\chi^2$ Analysis to find critical exponents and temperature

Result:
- $\sigma = 0.5 \pm 0.1$
- $\tau = 2.35 \pm 0.05$
- $T_c = 8.3 \pm 0.2$ MeV

M. Kleine Berkenbusch et al., PRL 88
Scaling of ISIS Data (2)

- Note: This only works because of very careful correction for sequential decays!
- Best-fit scenarios for both cases:
  - Scaling collapse only when sequential decay correction is performed
  - Technique fails without it

![Scaling of ISIS Data](image_url)
Freeze-Out Density

- Percolation model only depends on breaking probability, which can be mapped into a temperature.
- **Q:** How to map a 2-dimensional phase diagram?
- **A:** Density related to fragment energy spectra; Coulomb many-body expansion of pre-fragments
  
  WB, Alleman, Pratt, AIP conf.proc.884, 327 (2007)

\[ p_b = 1 - \frac{2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, 0, B/T\right) \]

\[ \rho_c = (0.35 \pm 0.1) \rho_0 \]
Buckyball Fragmentation

Binding energy of $C_{60}$: 420 eV

625 MeV $Xe^{35+}$

Cheng et al., PRA 54
Cross-Disciplinary Comparison

- Left: Nuclear Multifragmentation
- Right: Buckyball Fragmentation
- Histograms: Percolation Models

Similarities:
- \( U \)-shape (\( b \)-integration)
- Power-law for imf’s (1.3 vs. 2.6)
- Binding energy effects provide fine structure
How to find the QCD Critical Point

- There are no large fragments!
- What clusters? What fluctuates?
- What is the order parameter that can be measured experimentally?
- What should CBM look for?
Liquid at the Critical Point

Cyclohexane-Aniline

Liquid-vapor coexistence

Critical Opalescence: Light scattering off fluctuations

Ferrell (1968)
Stanley (1971)
Fluctuations

WMAP CMB

2.7248 K

2.7250 K

2.7252 K

June 1992 Earth Temperatures [K]

210 260 310
Multipole Analysis

Angular Scale

$CL_{l}/2\pi$ (\mu K^2)

Multipole moment ($l$)

TT Power Spectrum

- WMAP Data
The “CMB” of CBM

- Photons emitted from early collision stages scatter off the fluctuations
  - critical opalescence
- Similar effect for pions
  - “critical pion opacity”
- Cluster analysis in momentum space for pions and photons
- FAIR chance that the signal of the critical point survives the later stages of final state interaction
- Finite size constraints
  - Do not expect a sharp peak!
  - A bump is all that you will get at best
  - Unambiguous experimental signals are hard to come by
  - Lots of modeling needed to interpret results
Zipf’s Law: Probabilities (1)

- Probability that cluster of size $A$ is the largest one = probability that at least one cluster of size $A$ is present times probability that there are 0 clusters of size $>A$

$$P_{1st}(A) = p_{\geq 1}(A) \cdot p_0(> A)$$

$$= [1 - p_0(A)] \cdot p_0(> A)$$

- $N(A) = \text{average yield of size } A: N(A) = aA^{-\tau}$

- $N(>A) = \text{average yield of size }>A: (V = \text{event size})$

$$N(> A) = \sum_{i=A+1}^{V} N(i) = \sum_{i=A+1}^{V} ai^{-\tau} = a\zeta(\tau, 1+A) - a\zeta(\tau, 1+V)$$

- Normalization constant $\alpha$ from condition: $\sum_{A=1}^{V} A \cdot N(A) = V$

$$\alpha = V / \sum_{A=1}^{V} A^{1-\tau} = V / H_V^{(1-\tau)}$$
Zipf’s Law: Probabilities (2)

- Use Poisson statistics for individual probabilities:

\[ p_n(i) = \frac{\langle N(i) \rangle^n e^{-\langle N(i) \rangle}}{n!} \]

\[ p_0(i) = e^{-\langle N(i) \rangle}, \quad p_1(i) = \langle N(i) \rangle p_0(i), \quad p_2(i) = \frac{1}{2} \langle N(i) \rangle p_1(i) \ldots \]

- Put it all together:

\[
P_{1st}(A) = [1 - p_0(A)] \cdot p_0(> A) \\
= [1 - e^{-N(A)}] \cdot e^{-[a(\tau,1+A) - a(\tau,1+V)]}
\]

- Average size of biggest cluster

\[
\langle A_{1st} \rangle = \sum_{A=1}^{V} A \cdot P_{1st}(A) \quad \text{(Exact expression!)}
\]
Zipf’s Law: Probabilities (3)

- Probability for given $A$ to be 2\textsuperscript{nd} biggest cluster:
  \[
P_{2nd}(A) = p_{\geq 2}(A) \cdot p_0(>A) + p_{\geq 1}(A) \cdot p_1(>A)
  = [1 - p_0(A) - p_1(A)] \cdot p_0(>A) + [1 - p_0(A)] \cdot p_1(>A)
  \]

- Average size of 2\textsuperscript{nd} biggest cluster:
  \[
  \langle A_{2nd} \rangle = \sum_{A=1}^{V} A \cdot P_{2nd}(A)
  \]

- And so on …
- Recursion relations!

$V = 8,000$
$\tau = 2.18$
Zipf-Mandelbrot

- Limiting distributions for cluster size vs. rank
  \[ \langle A_{r_{th}} \rangle = \frac{c}{(r + k)^\lambda} \]
  with exponent \[ \lambda \sim \frac{1}{\tau - 1} \]

- Proof for infinite system in continuum limit with \( \tau = 2 \): Paech, WB, Pratt, PRC 76, 2007
Summary

- Common threads in phase transitions of molecules, nuclei, and hadronic matter
- Non-equilibrium effects make extraction of EoS information hard
- View of multifragmentation as a critical phenomenon is on solid footing
- Critical slowing down not important, because it is a non-equilibrium phase transition
- Finite-size corrections can be dealt with rather effectively; opportunity for us to contribute to larger science community
- Self-organized criticality, Zipf’s Law, …: nuclear fragmentation continues to be a rich playground for testing out nonlinear physics concepts.
Colleagues

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