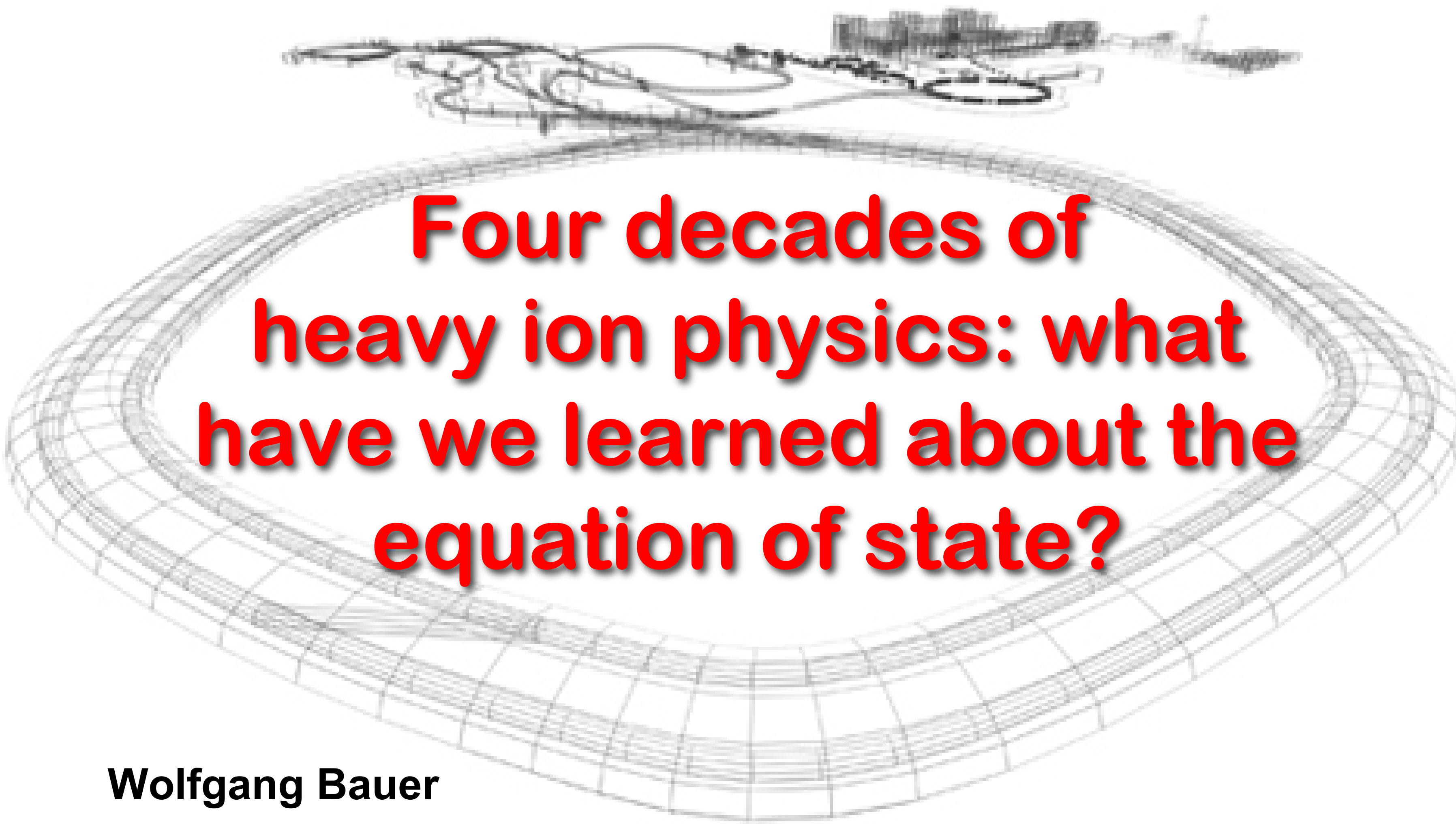


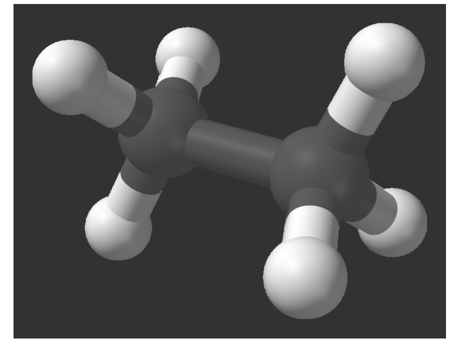
Four decades of heavy ion physics: what have we learned about the equation of state?

Wolfgang Bauer
Michigan State University

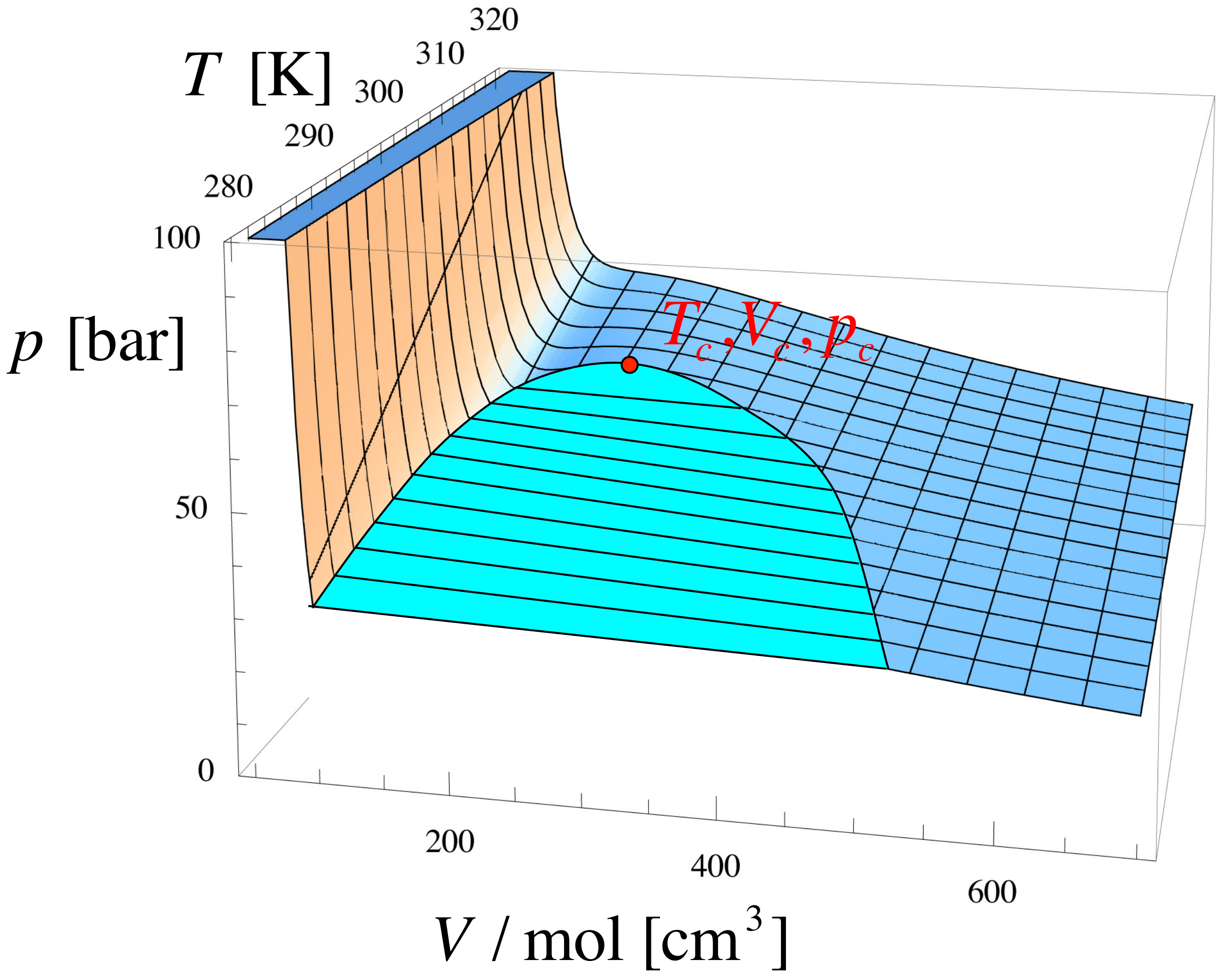
What is an "equation of state"?

- **State variables: pressure, temperature , density (internal energy, chemical potential, strangeness, …)**
- **Equation of state: relationship between state variables,** $f(p,T,V)=0$**.**
  - Thermodynamic equation describing state of matter under given physical conditions
  - Example: Ideal gas: $pV = nRT$
  - More realistic equations of state need to contain phase transitions, coexistence regions, critical points, …

EoS for C_2H_6 (Ethane)



- **Soave Redlich Kwong EoS,** $p(T,V)$



Critical Point

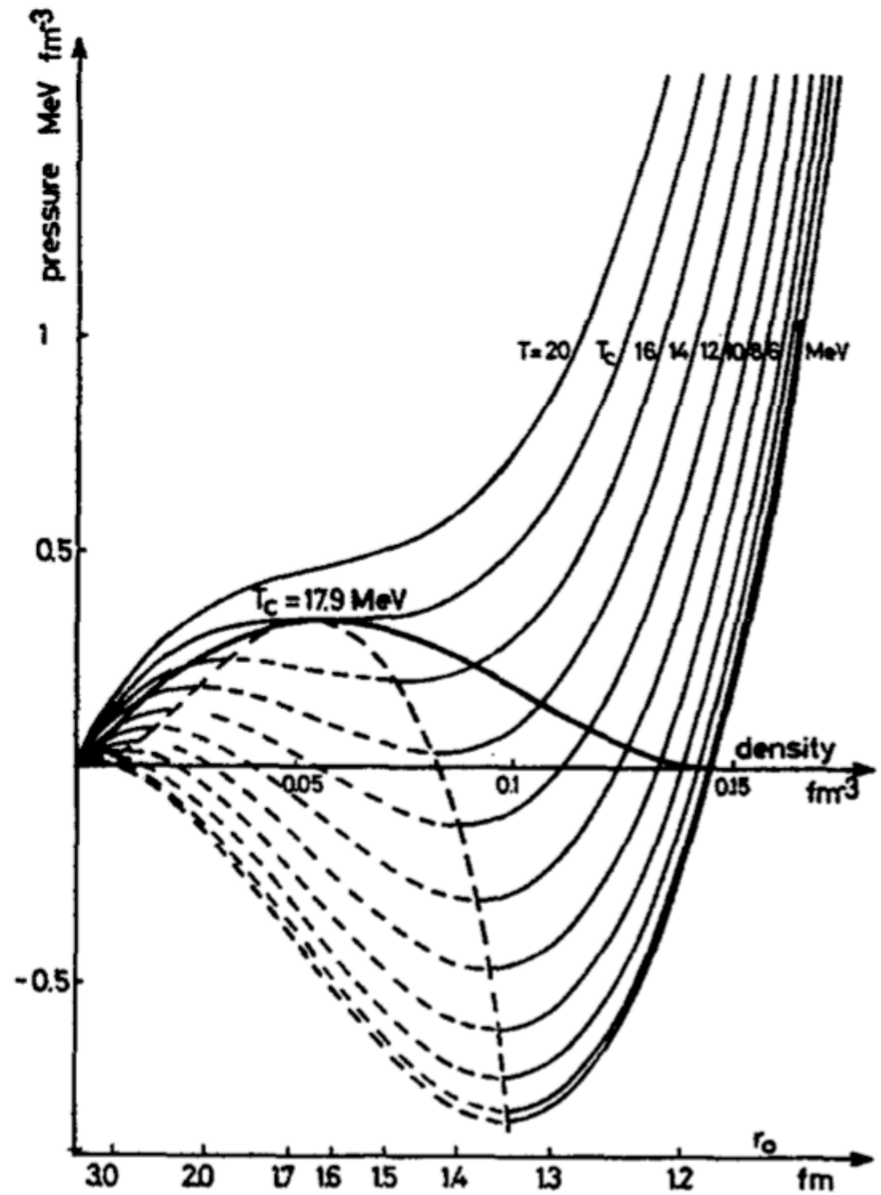
"Maxwell" construction

1st order

Nuclear EoS

- **Can be computed, if you know nuclear force**
- **Here: Skyrme**
  - Note:
    Coexistence region, critical point.

(Sauer, Chandra, Mosel, NPA 264 (1976))

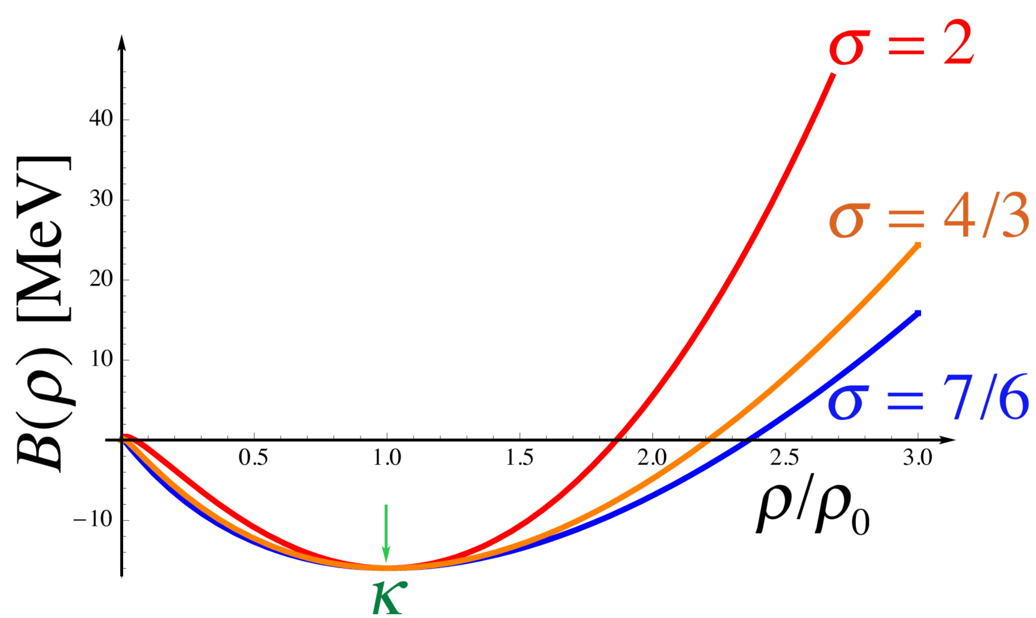


Nuclear Matter Compressibility

- **Curvature at minimum of binding energy (= at nuclear matter density, for temperature 0)**

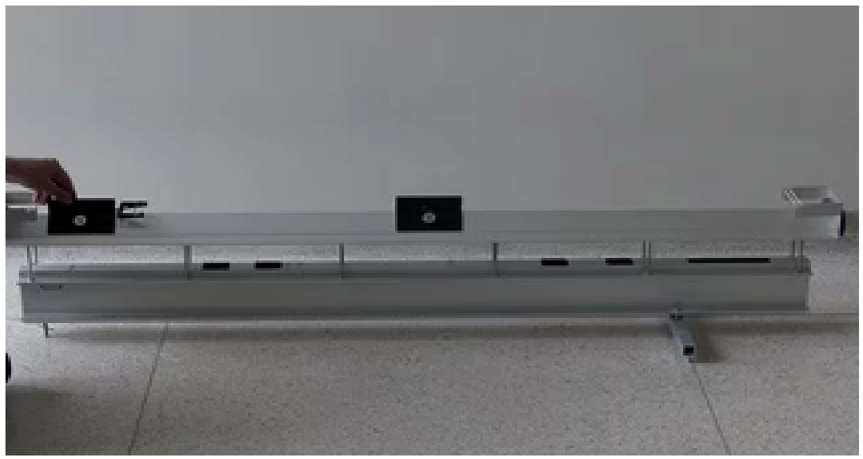
$$\kappa = k_F^2 \frac{d^2}{dk_F^2} B(\rho)\Big|_{\rho=\rho_0}$$

- **Astrophysical relevance**
- **Isospin dependence**



Compressibility from Nuclear Collisions

- **Not so easy!**



- **Final momenta determined by initial momenta and energy and momentum conservation**
- **Compressibility of spring between carts never entered!**

Compressibility from Pion Multiplicity

VOLUME 58, NUMBER 5 PHYSICAL REVIEW LETTERS 2 FEBRUARY 1987

Pion Production in High-Energy Nucleus-Nucleus Collisions

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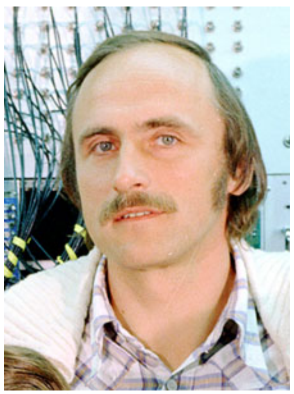
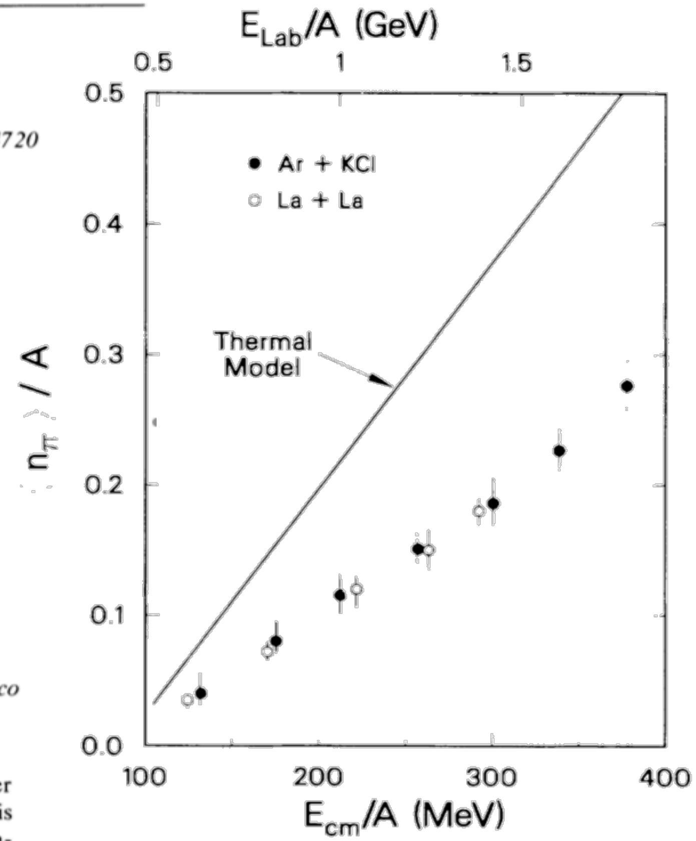
(Received 18 August 1986)



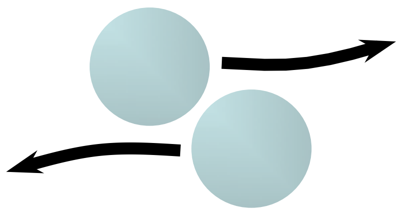
Stock 1976

Negative-pion multiplicity $\langle n_\pi \rangle$ was measured over the range of participant nucleon number $80 \leq A \leq 270$ for incident energies from 530 to 1350 MeV/nucleon in the La+La system. The $\langle n_\pi \rangle$ is proportional to A and increases linearly with the c.m. energy. Thermal and potential energies, and temperatures of the maximum-density phase of the collision are extracted from the data. The results require a stiff nuclear-matter equation of state.

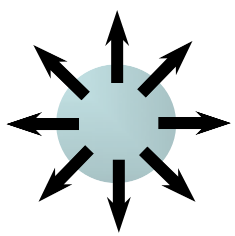


Compressibility from Flow

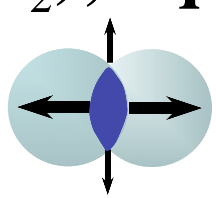
- **Directed (sideways in reaction plane)**



- **Radial (like monopole)**

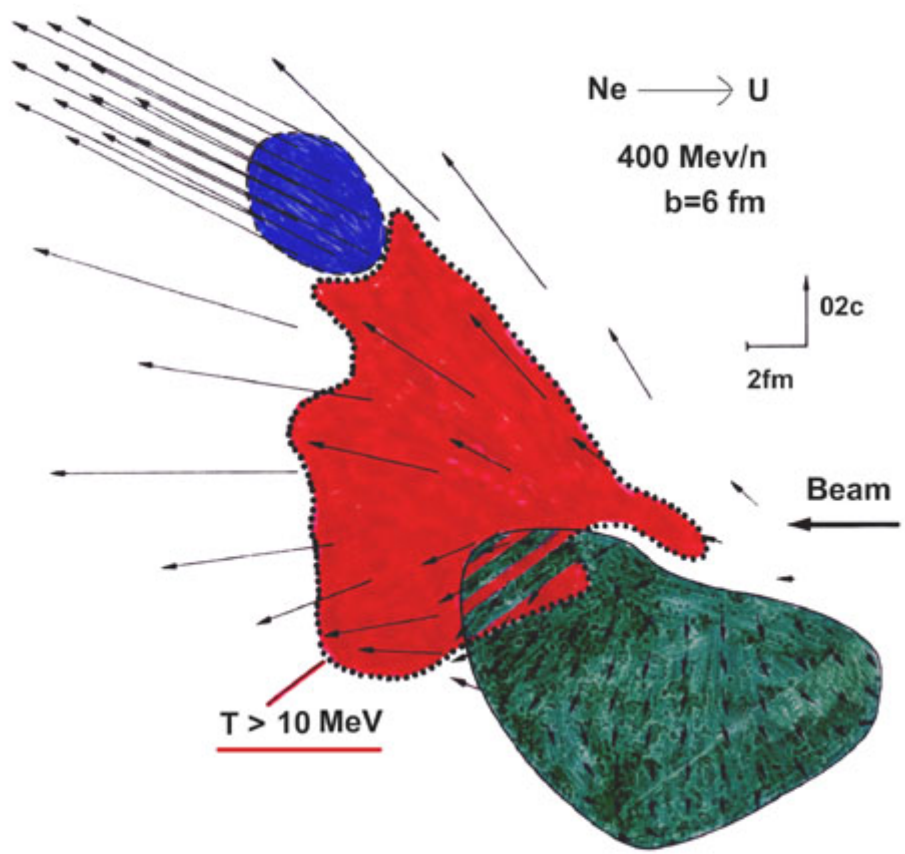


- **Elliptic** (v_2), **Squeezeout**



- **Main idea: see differential changes in collective motion from different amounts of transient nuclear compression**
  - Hydro / viscosity
  - EoS sensitivity
  - Momentum dependence

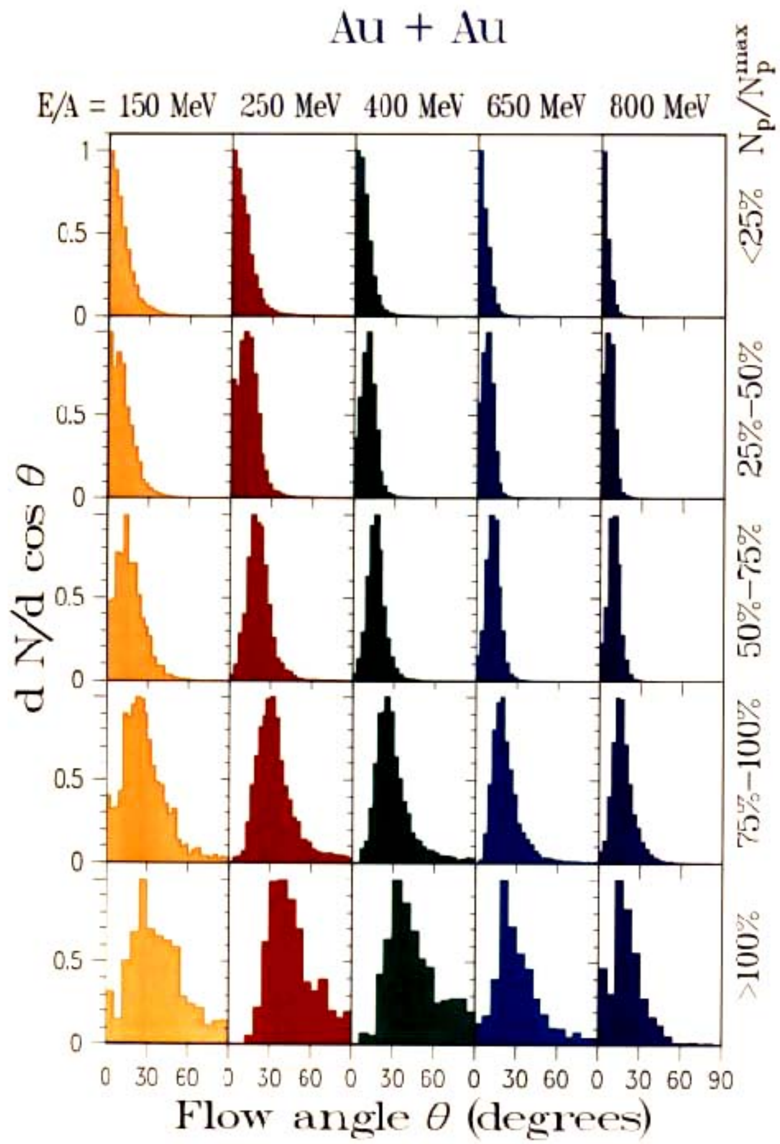
Directed Flow: Hydro Motivation



H. Stöcker, J.A. Maruhn, and W. Greiner, PRL **44**, 725 (1980)

Directed Flow: Plastic Ball Results

Gutbrod 1985



Plastic Ball, H.G. Ritter et al., Nucl. Phys. **A447,** 3c (1985)

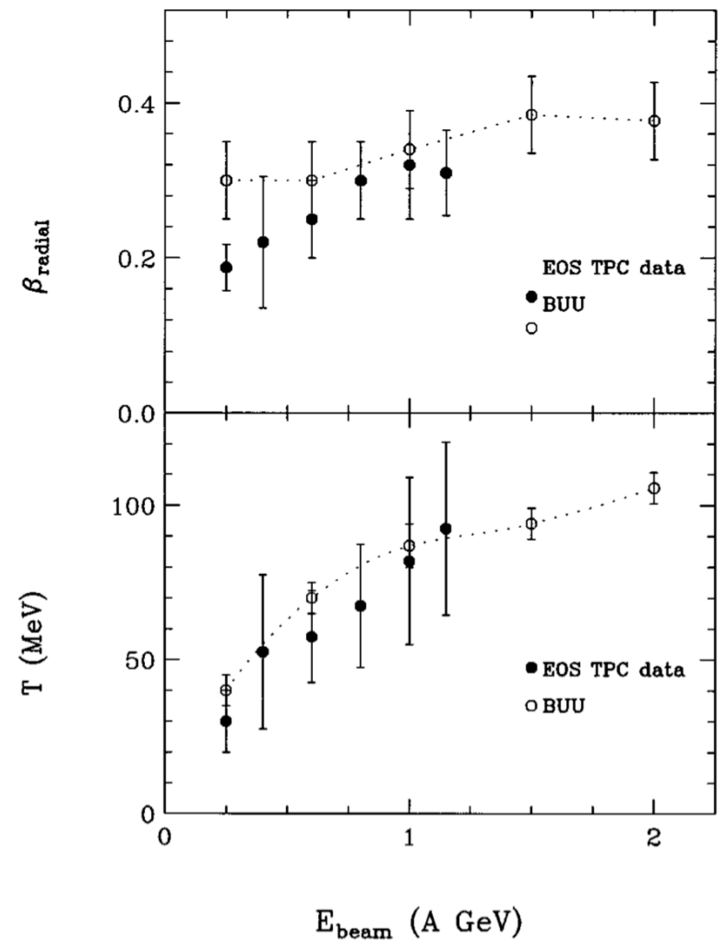
Radial Flow

- **Mean field** $$U = A\left(\frac{\rho}{\rho_0}\right) + B\left(\frac{\rho}{\rho_0}\right)^{\sigma} + \frac{C}{\rho_0}\left\{\int \frac{f(\vec{r},\vec{p}\,')d\vec{p}\,'}{1+[(\vec{p}-\langle\vec{p}\,'\rangle)/\Lambda]^2} + \frac{\rho}{1+[(\vec{p}-\langle\vec{p}\rangle)/\Lambda]^2}\right\}$$

- **In-medium NN cross section**

$$\sigma_{nn} = \sigma_{nn}^{\text{free}}\left(1+\alpha\frac{\rho}{\rho_0}\right)$$

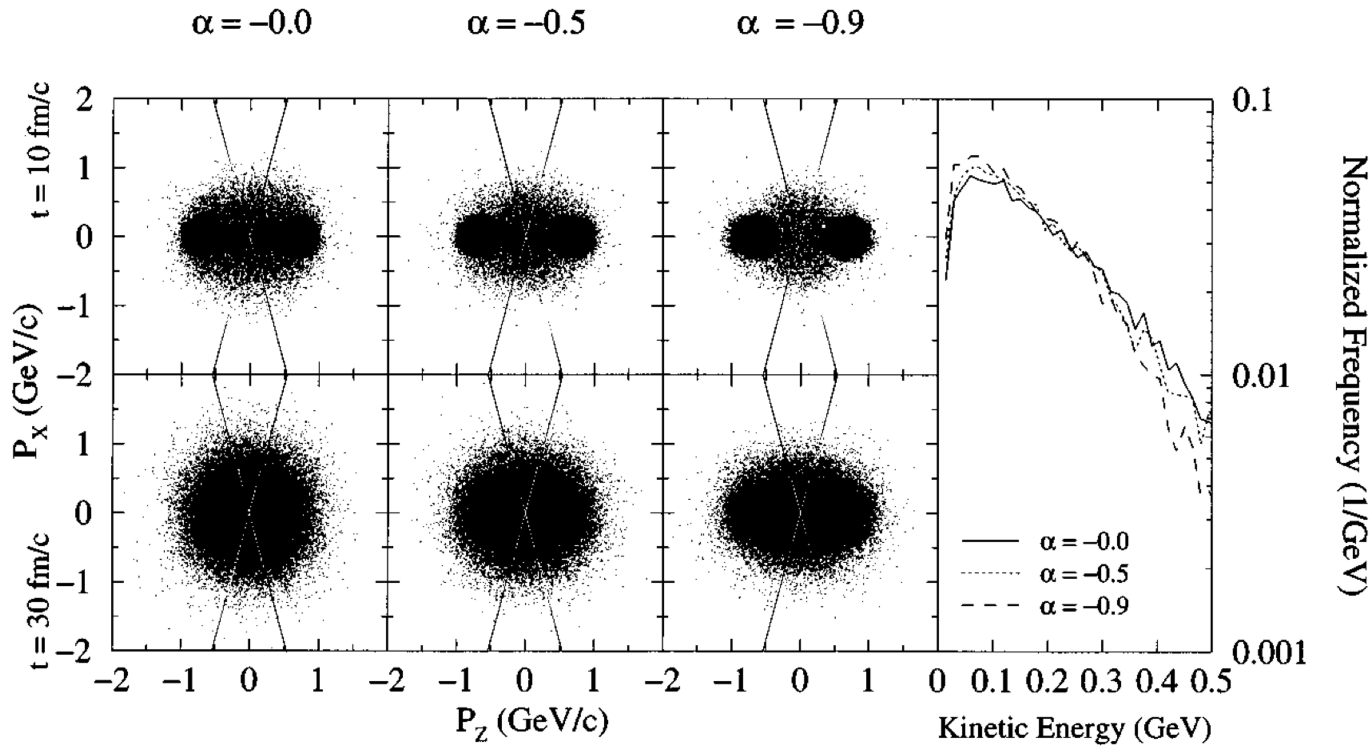
  - Effective form deduced from Brueckner T-matrix calculations (Alm et al., NPA587 (1995))



Daffin, Haglin, WB, PRC 54 (1996)

Thermalization

- **Depends a lot on in-medium NN cross section**

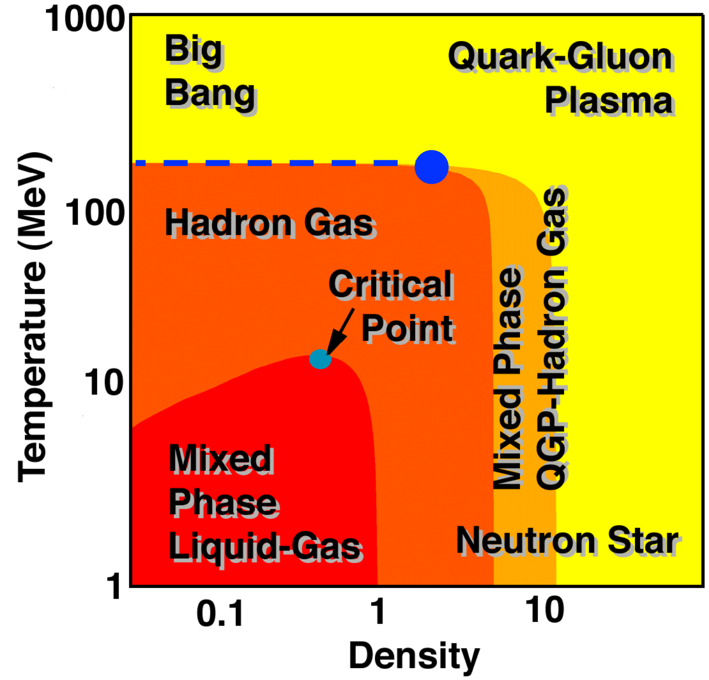


- **Depends very little on mean field parameters**

Daffin, Haglin, WB, PRC 54 (1996)

Nuclear Matter Phase Diagram

- **Two (at least) phase transitions in nuclear matter:**
  - "Liquid Gas"
  - Hadron gas->QGP / chiral restoration
- **Problems/ Opportunities:**
  - Finite size effects (finite size scaling! ✔)
  - Is there equilibrium? (✖)
  - Measurement of state variables (*r, T, S, p, …*✔)
  - Migration of nuclear system through phase diagram (non-equilibrium processes)
  - Near critical point(s): Critical slowing down! Not sufficient time for equilibrium phase transition!



Source: NUCLEAR SCIENCE, A Teacher's Guide to the Nuclear Science Wall Chart, Figure 9-2

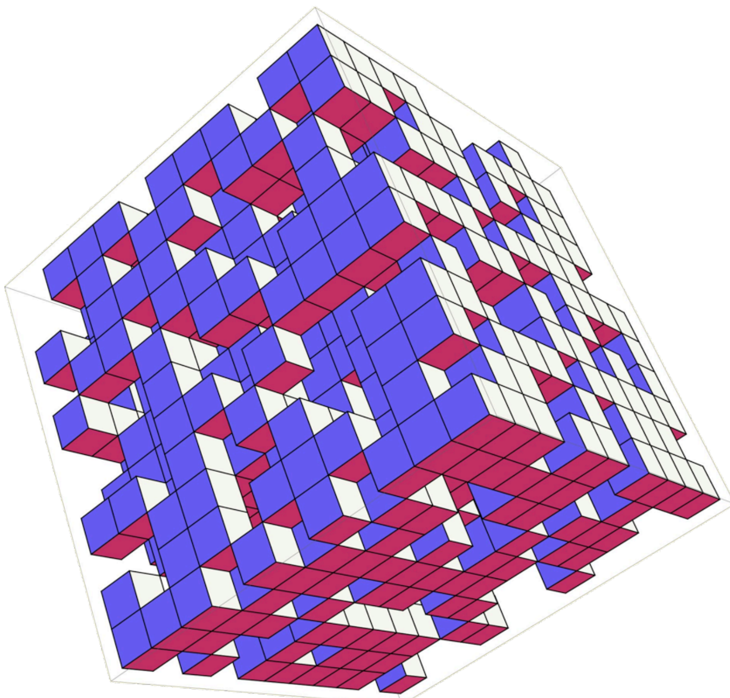
Critical Slowing Down

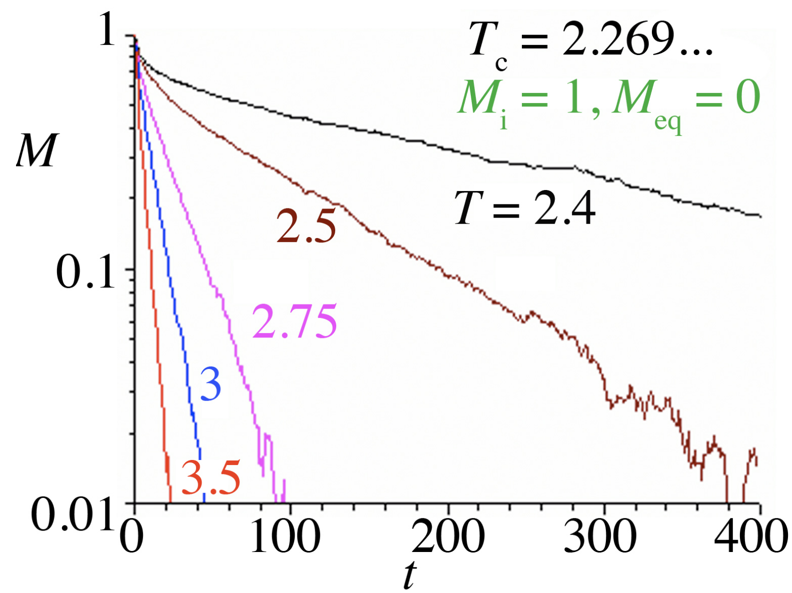
- Near critical point, $|T/T_c| \ll 1$, it takes longer and longer to re-establish equilibrium after changing the temperature
- Example: Ising Model,

$$M(t) \sim \exp(-t/t_r),$$

with

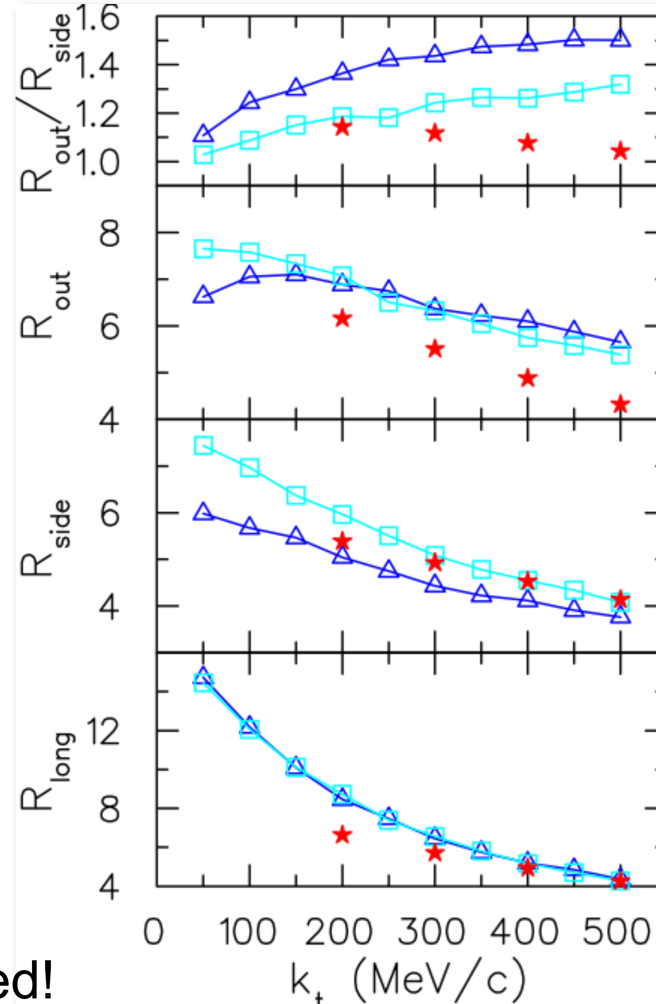
$$t_r = 4.5\,(T-T_c)^{-1.85}, \text{ for } T > T_c.$$





… but there is not enough time!

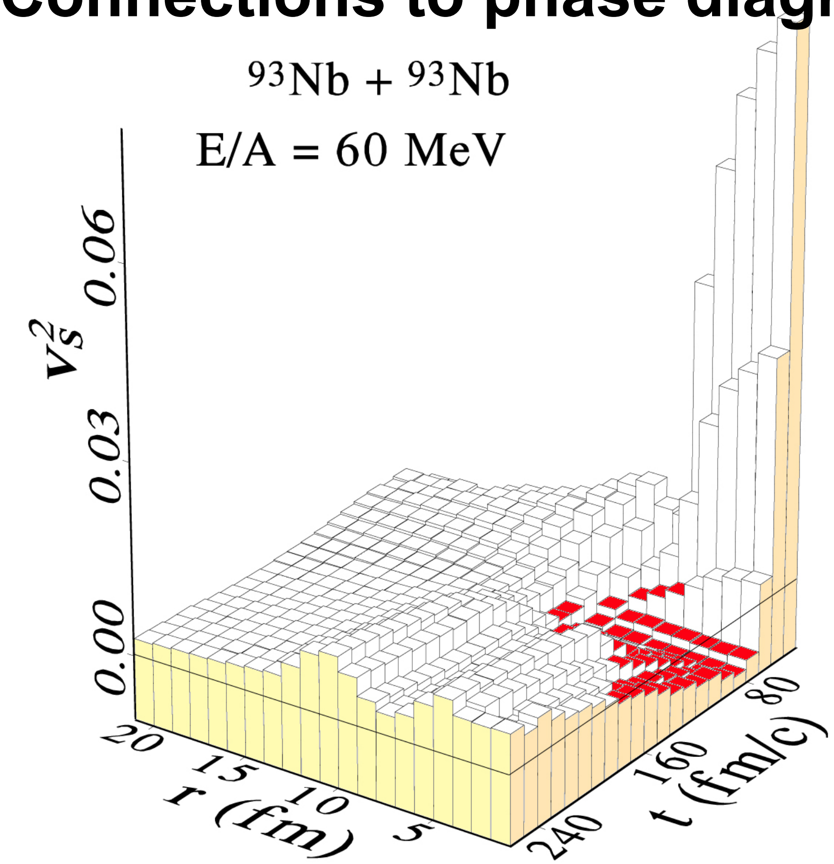
- **HBT puzzle**
- **Theoretical expectation**
  - Change of # of degrees of freedom in transition from quarks and gluons to hadrons
  - Large time delay
  - Expect $R_{out} >> R_{side}$
- **Not seen by experiment!**
- **Equilibrium thermodynamic phase transition may not be possible**
  - … but non-equilibrium transition not excluded!



S. Pratt

Dynamics

- **Thermal equilibrium assumptions not (always) valid**
- **Need transport theory**
- **Various event class averages (ensemble vs. thermal!)**
- **Connections to phase diagram poorly understood**



- **Transient formation of non-compact structures**
  - Sheet instabilities
    Moretto et al., PRL 69
  - Bubble and ring formation
    WB, Schulz, Bertsch, PRL 69
  - Imaginary sound velocity causes exponential growth in fluctuations; non-equilibrium in origin
  - Similar effect now postulated for RHIC collisions (Pratt 2008)

Non-Equilibrium Phase Transitions

- **Conventional thermodynamics**
  - Write down partition function from (known) Hamiltonian

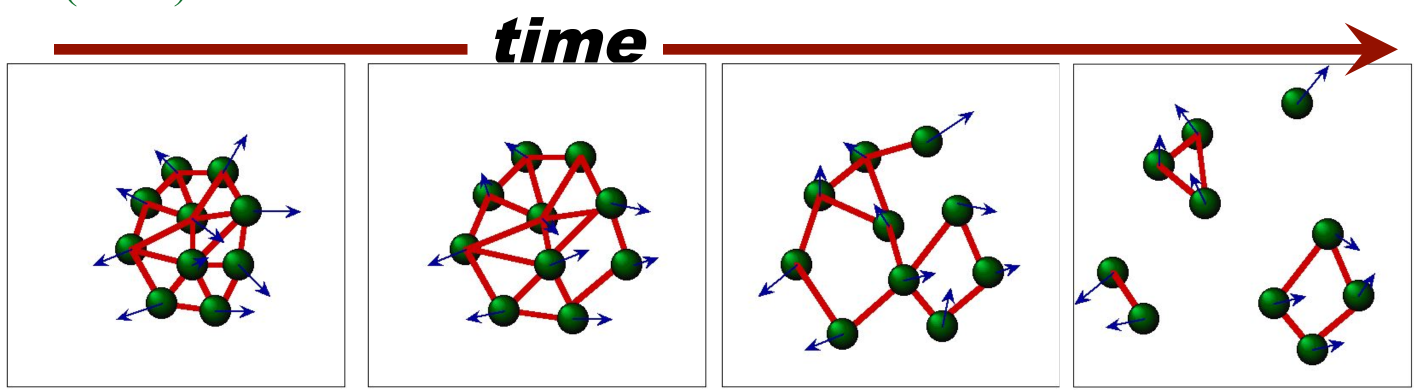
$$Z = tr(e^{-\beta H})$$

  - Take partial derivatives to obtain state variables
  - Static solution; equilibrium; no changes in time
- **Non-equilibrium Phase Transition**
  - Dynamics; time dependence
  - No thermal averages
  - Transitions between un/meta/bi-stable states
- **Are similar universality classes possible?**
  - Critical exponents can be obtained

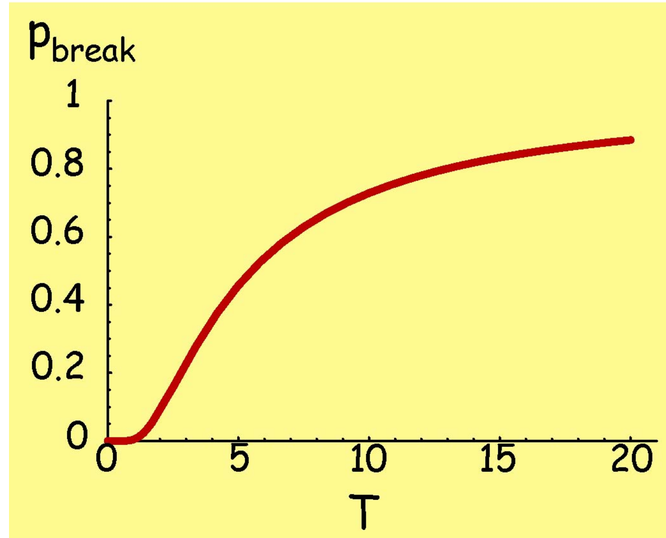
Non-Equilibrium Percolation in Nuclear Fragmentation

- **Short-range NN force: nucleons in contact with nearest neighbors**
- **Expansion (thermal, compression driven, dynamical, …)**
- **Bonds between nucleons rupture**
- **Remaining bonds bind nucleons into fragments**
- **One** *control parameter*: **bond breaking probability**

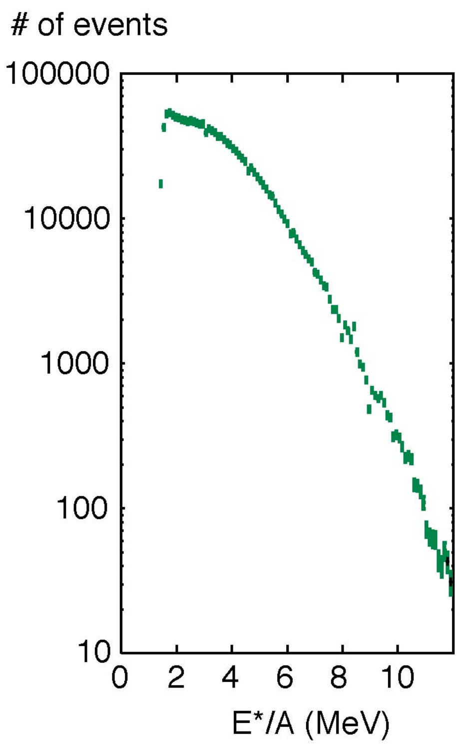
WB *et al.*, PLB **150**, 53 (1985)



Breaking Probability



- Determined by the excitation energy deposited
- **Infinite simple cubic lattice:**
  - 3 bonds/nucleon
  - It takes 5.25 MeV to break a bond



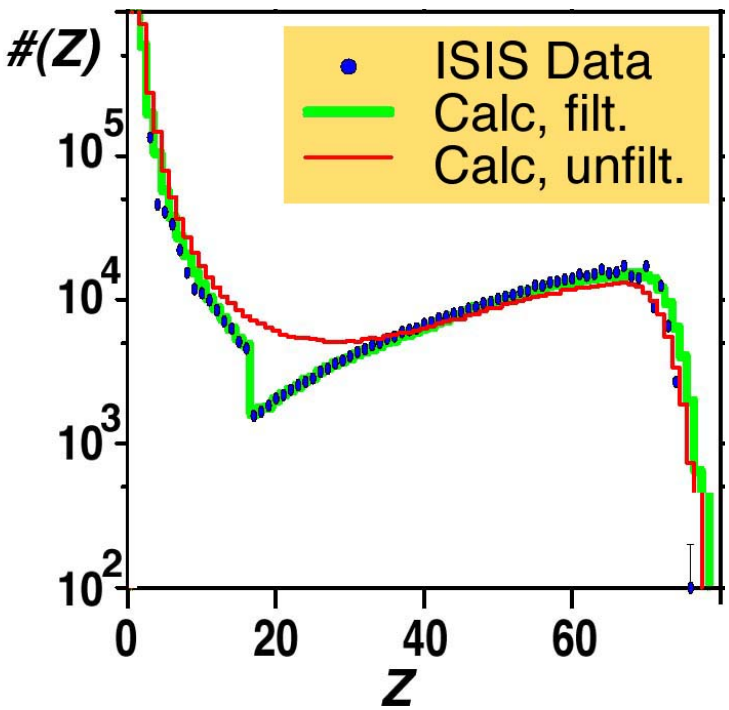
- **p,p induced: Glauber theory**
  - p_{break} proportional to path length through matter
- **General relation between p_{break} and T:**

$$p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}} \Gamma\left[\frac{3}{2}, 0, \frac{B}{T}\right]$$

- ***G* = generalized incomplete gamma function, *B* = binding energy per nucleon**
  T. Li et al., PRL 70
  (generalization of Coniglio-Klein for Fermi systems)
- **Obtain E^* or T from other model or directly from experiment**

Comparison to Experiment

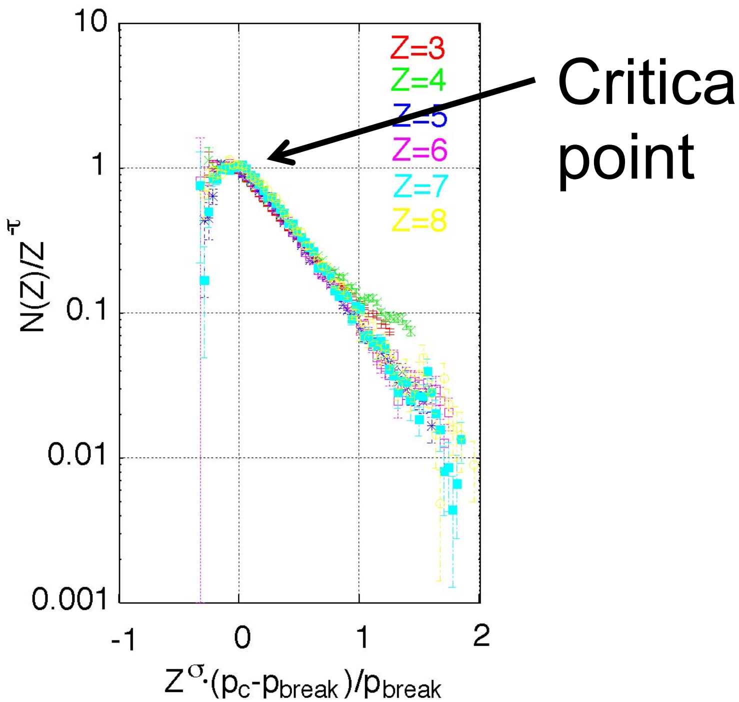
ISiS data set
10.8 GeV p, p+Au @AGS



- Idea (Elliott et al.): If data follow scaling function

$$N(Z,T) = Z^{-\tau} f\left[\frac{T - T_c}{T_c} Z^{\sigma}\right]$$

with $f(0) = 1$ (think "exponential"), then we can use scaling plot to see if data cross the point [0,1] -> critical events



Scaling of ISIS Data

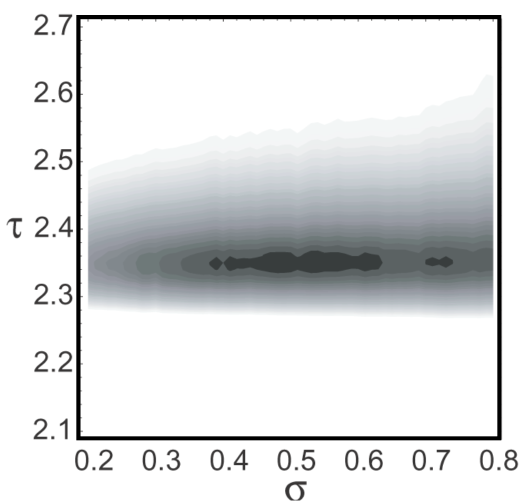
- **Most important: critical region and explosive events probed in experiment**
- **Possibility to narrow window of critical parameters**
  - tau: vertical dispersion
  - sigma: horizontal dispersion
  - T_c: horizontal shift
- **chi^2 Analysis to find critical exponents and temperature**

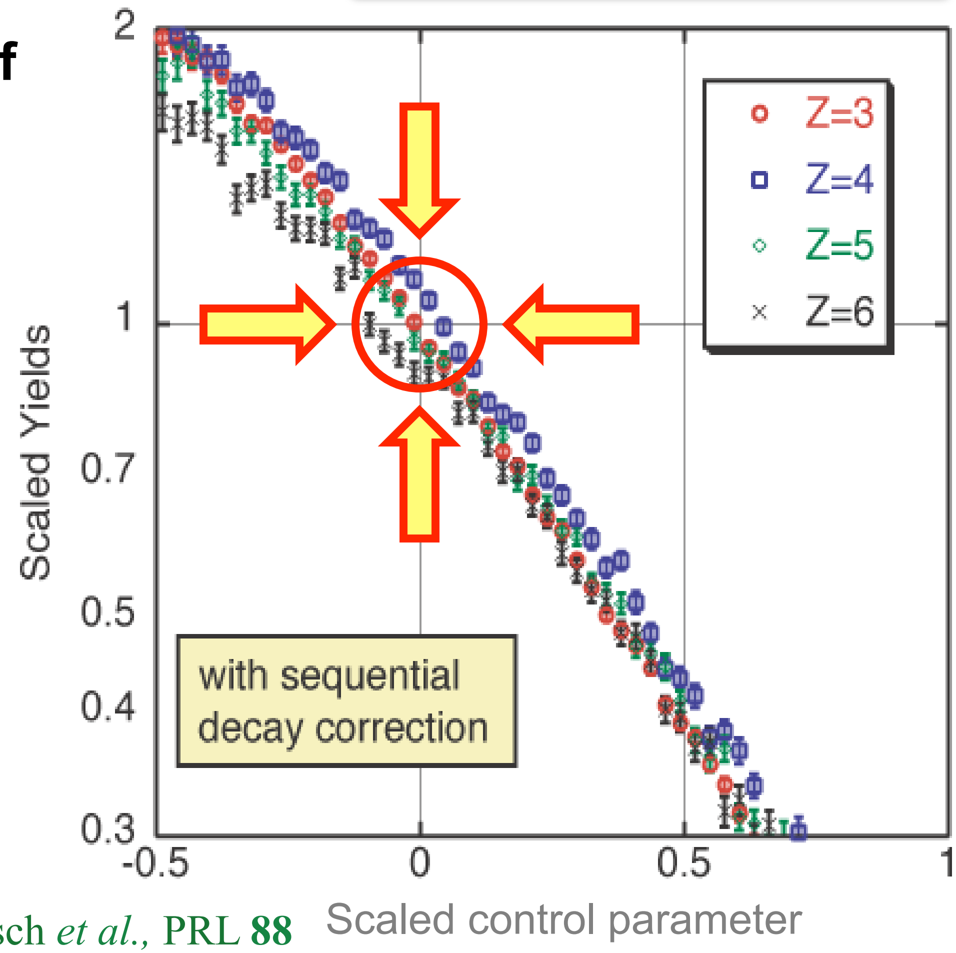
- Result:

  beta = 0.5 +- 0.1

  gamma = 2.35 +- 0.05

  T_c = 8.3 +- 0.2 MeV





M. Kleine Berkenbusch *et al.*, PRL **88**

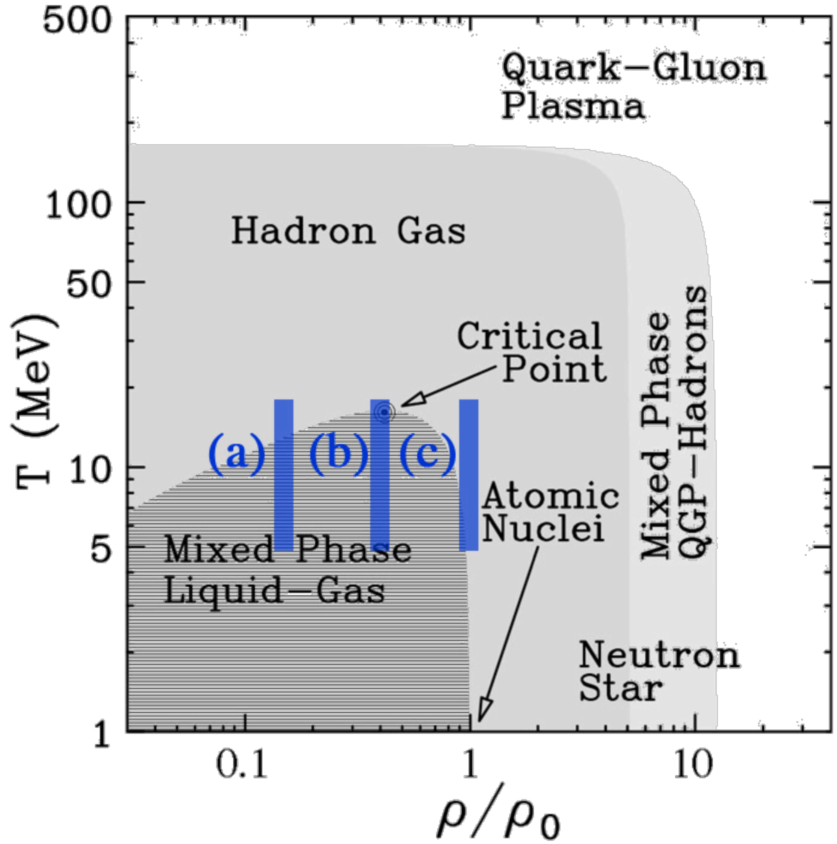
Freeze-Out Density

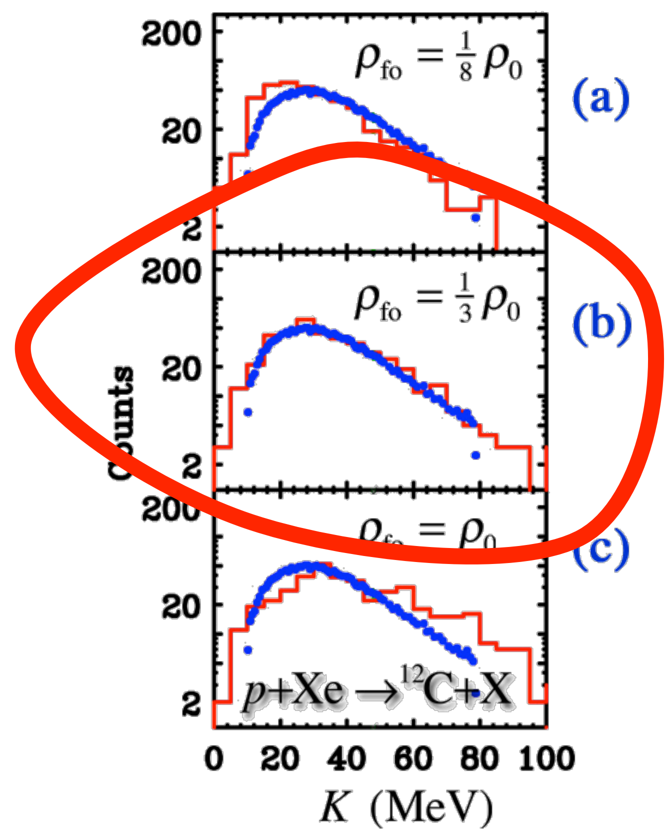
- **Percolation model only depends on breaking probability, which can be mapped into a temperature.**
- ***Q*: What determines the breakup moment?**
- ***A*: Density related to fragment energy spectra; Coulomb many-body expansion of pre-fragments**

WB, Alleman, Pratt, AIP conf.proc.884, 327 (2007)
WB, Nucl.Phys. A787, 595c (2007)

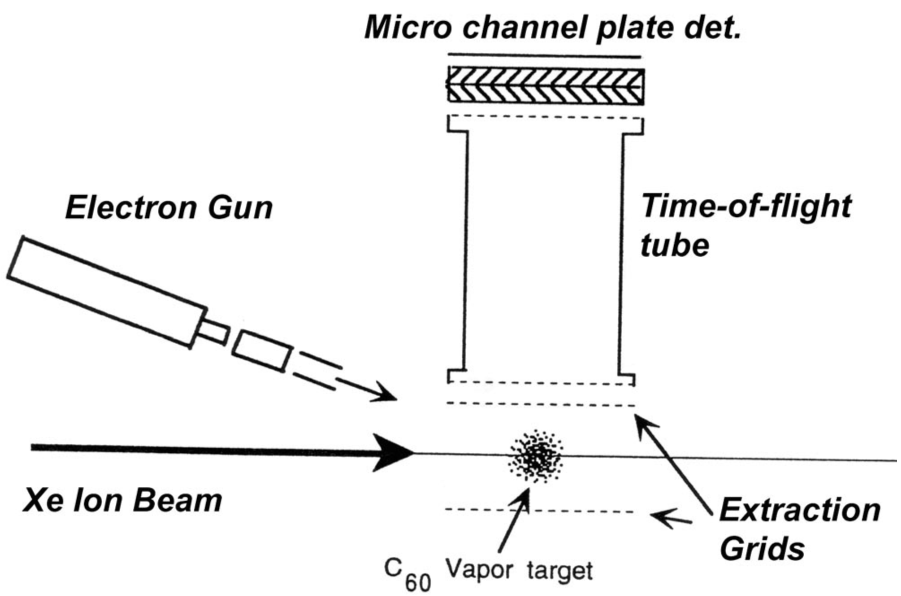
$$p_b = 1 - \frac{2}{\sqrt{\pi}}\Gamma(\tfrac{3}{2}, 0, B/T)$$

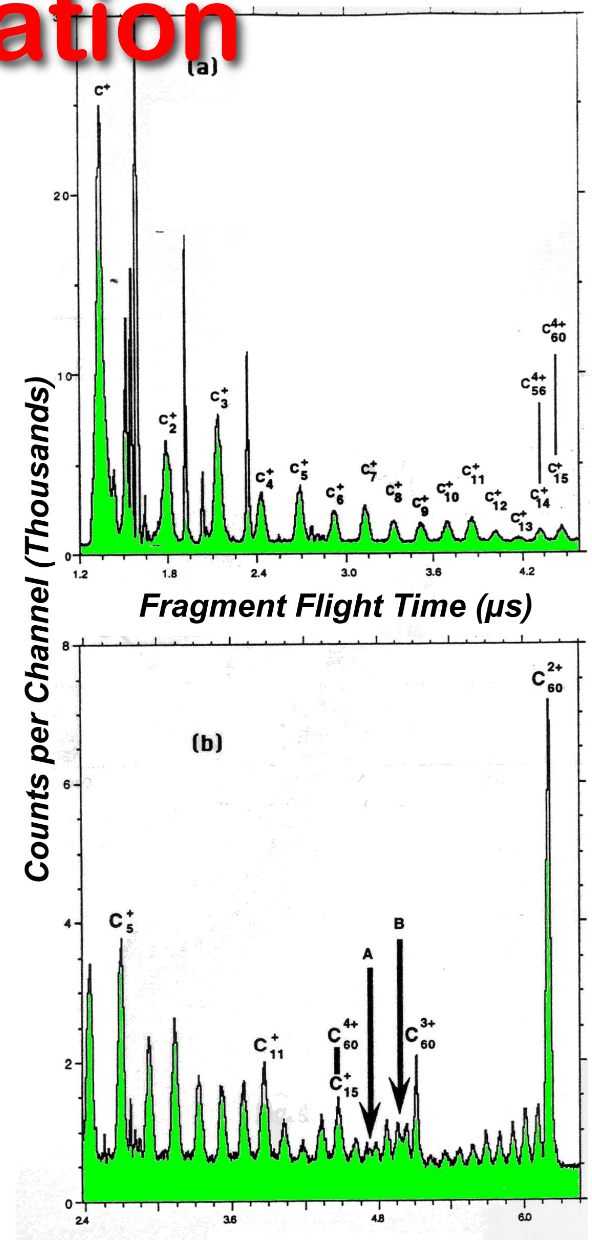
$$\rho_c = (0.35 \pm 0.1)\rho_0$$





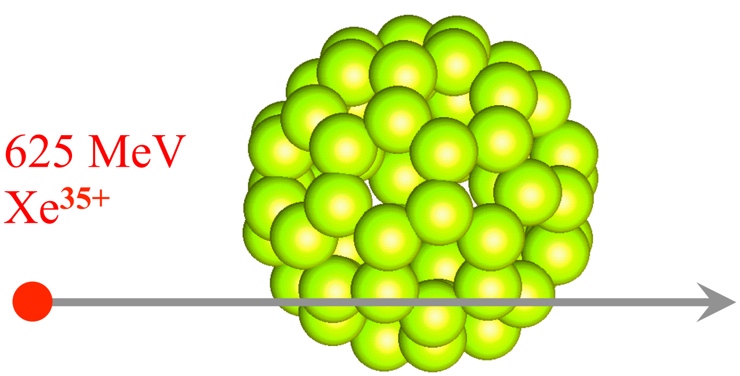
Buckyball Fragmentation





Binding energy of C_{60}:
420 eV

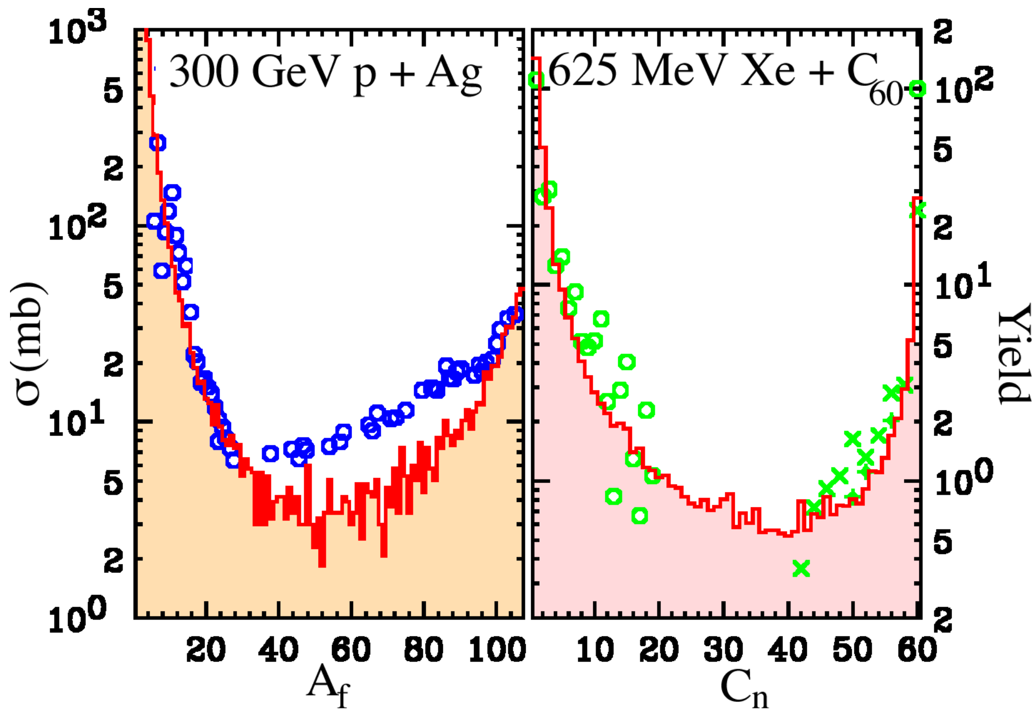
625 MeV
Xe^{35+}



Cheng et al., PRA 54

Cross-Disciplinary Comparison

- **Left: Nuclear Multifragmentation**
- **Right: Buckyball Fragmentation**
- **Histograms: Percolation Models**



- **Similarities:**
  - U - shape (b-integration)
  - Power-law for imf' s (1.3 vs. 2.6)
  - Binding energy effects provide fine structure

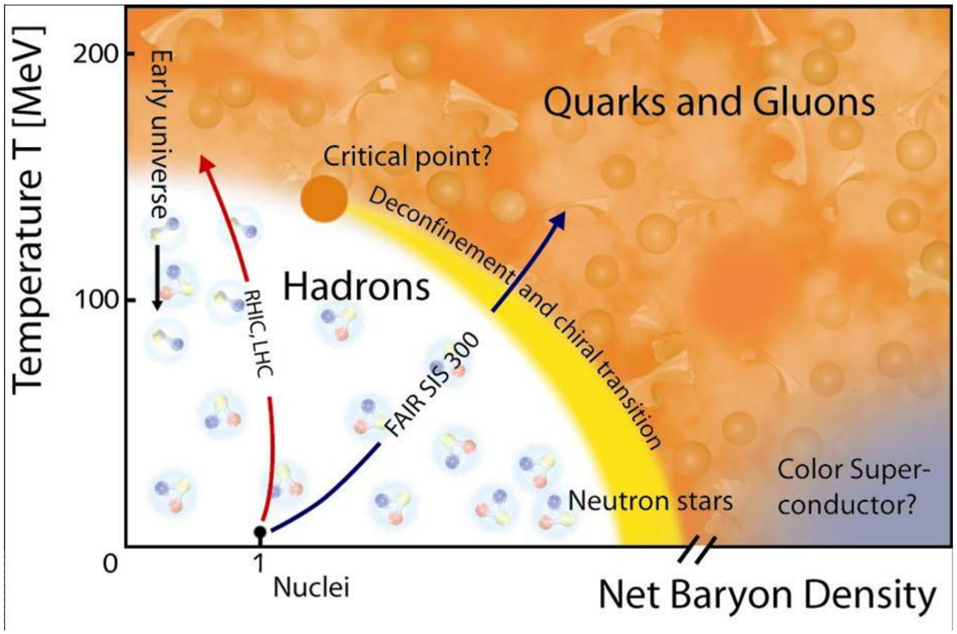
Data:
Bujak et al., PRC 32
LeBrun et al., PRL 72
Calc.:
WB, PRC 38
Cheng et al., PRA 54

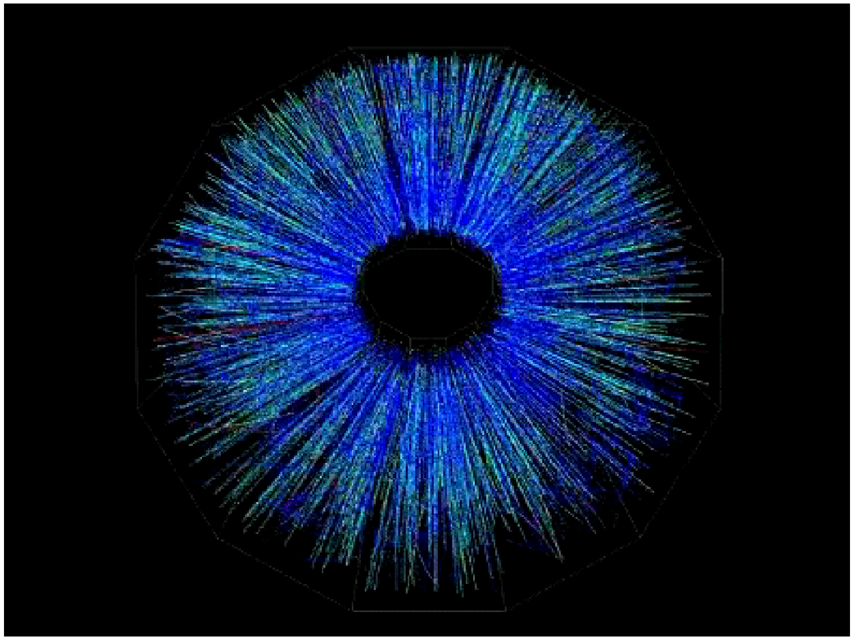
Conclusions

- **Simple picture of thermodynamic phase transitions in nuclear and quark-gluon matter likely wrong, because there is not enough time to approach the critical point**
- **More exciting possibility: non-equilibrium phase transition**
  - First instance of a non-equilibrium phase transition with non-mean-field exponents!
  - What is the connection between equilibrium phase transition theories of nuclear/quark-gluon matter and transition matrix elements between bi/un/meta-stable states?
  - Knowledge of elementary interaction is not enough (emergent phenomena in phase transitions)

QCD

- **Is this …. the same as this … ?**





Water

- **Is this ….** **the same as this … ?**