Four decades of heavy ion physics: what have we learned about the equation of state?

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What is an “equation of state”?

- **State variables**: pressure, temperature, density (internal energy, chemical potential, strangeness, …)
- **Equation of state**: relationship between state variables, \( f(p,T,V) = 0 \).
  - Thermodynamic equation describing state of matter under given physical conditions
  - Example: Ideal gas: \( pV = nRT \)
  - More realistic equations of state need to contain phase transitions, coexistence regions, critical points, …
EoS for $\text{C}_2\text{H}_6$ (Ethane)

- Soave Redlich Kwong EoS, $p(T,V)$

Critical Point

“Maxwell” construction

1st order
Nuclear EoS

• Can be computed, if you know nuclear force
• Here: Skyrme
  - Note: Coexistence region, critical point.

(Sauer, Chandra, Mosel, NPA 264 (1976))
Nuclear Matter Compressibility

- Curvature at minimum of binding energy (= at nuclear matter density, for temperature 0)

\[ \kappa = k_F^2 \frac{d^2}{dk_F^2} B(\rho) \bigg|_{\rho=\rho_0} \]

- Astrophysical relevance
- Isospin dependence
Compressibility from Nuclear Collisions

• Not so easy!

• Final momenta determined by initial momenta and energy and momentum conservation

• Compressibility of spring between carts never entered!
Pion Production in High-Energy Nucleus-Nucleus Collisions

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Negative-pion multiplicity \( \langle n_p \rangle \) was measured over the range of participant nucleon number \( 80 \leq A \leq 270 \) for incident energies from 530 to 1350 MeV/nucleon in the La+La system. The \( \langle n_p \rangle \) is proportional to \( A \) and increases linearly with the c.m. energy. Thermal and potential energies, and temperatures of the maximum-density phase of the collision are extracted from the data. The results require a stiff nuclear-matter equation of state.
Compressibility from Flow

- Directed (sideways in reaction plane)

- Radial (like monopole)

- Elliptic ($v_2$), Squeezeout

- Main idea: see differential changes in collective motion from different amounts of transient nuclear compression
  - Hydro / viscosity
  - EoS sensitivity
  - Momentum dependence
Directed Flow: Hydro Motivation

H. Stöcker, J.A. Maruhn, and W. Greiner, PRL 44, 725 (1980)
Directed Flow: Plastic Ball Results

Gutbrod 1985

Radial Flow

- **Mean field**
  \[ U = A \left( \frac{\rho}{\rho_0} \right) + B \left( \frac{\rho}{\rho_0} \right)^\sigma + \frac{C}{\rho_0} \left\{ \int \frac{f(\tilde{r}, \tilde{r}')}{1 + [(\tilde{p} - \langle \tilde{p} \rangle)/\Lambda]^2} d\tilde{p}' \right\} \]

- **In-medium NN cross section**
  \[ \sigma_{nn} = \sigma_{nn}^{\text{free}} \left( 1 + \alpha \frac{\rho}{\rho_0} \right) \]
  - Effective form deduced from Brueckner T-matrix calculations (Alm et al., NPA587 (1995))

Daffin, Haglin, WB, PRC 54 (1996)
Thermalization

- Depends **a lot** on in-medium NN cross section

\[ \alpha = -0.0 \quad \alpha = -0.5 \quad \alpha = -0.9 \]

- Depends **very little** on mean field parameters

Daffin, Haglin, WB, PRC 54 (1996)
Two (at least) phase transitions in nuclear matter:

- “Liquid Gas”
- Hadron gas->QGP / chiral restoration

Problems/Opportunities:

- Finite size effects (finite size scaling! ✔️)
- Is there equilibrium? (❌)
- Measurement of state variables (r, T, S, p, …✔️)
- Migration of nuclear system through phase diagram (non-equilibrium processes)
- Near critical point(s): Critical slowing down! Not sufficient time for equilibrium phase transition!

Source: NUCLEAR SCIENCE, A Teacher’s Guide to the Nuclear Science Wall Chart, Figure 9-2
Critical Slowing Down

- Near critical point, $|T/T_c| << 1$, it takes longer and longer to re-establish equilibrium after changing the temperature.
- Example: Ising Model,
  \[ M(t) \sim \exp(-t/t_r), \]
  with
  \[ t_r = 4.5 (T-T_c)^{-1.85}, \text{ for } T > T_c. \]
... but there is not enough time!

- HBT puzzle
- Theoretical expectation
  - Change of # of degrees of freedom in transition from quarks and gluons to hadrons
  - Large time delay
  - Expect $R_{out} \gg R_{side}$
- Not seen by experiment!
- Equilibrium thermodynamic phase transition may not be possible
  - … but non-equilibrium transition not excluded!
Dynamics

- Thermal equilibrium assumptions not (always) valid
- Need transport theory
- Various event class averages (ensemble vs. thermal!)
- Connections to phase diagram poorly understood

- Transient formation of non-compact structures
  - Sheet instabilities
    Moretto et al., PRL 69
  - Bubble and ring formation
    WB, Schulz, Bertsch, PRL 69
  - Imaginary sound velocity causes exponential growth in fluctuations; non-equilibrium in origin
  - Similar effect now postulated for RHIC collisions ([Pratt 2008])
Non-Equilibrium Phase Transitions

• Conventional thermodynamics
  ▪ Write down partition function from (known) Hamiltonian
    \[ Z = \text{tr}(e^{-\beta H}) \]
  ▪ Take partial derivatives to obtain state variables
  ▪ Static solution; equilibrium; no changes in time

• Non-equilibrium Phase Transition
  ▪ Dynamics; time dependence
  ▪ No thermal averages
  ▪ Transitions between un/meta/bi-stable states

• Are similar universality classes possible?
  ▪ Critical exponents can be obtained
Non-Equilibrium Percolation in Nuclear Fragmentation

- Short-range NN force: nucleons in contact with nearest neighbors
- Expansion (thermal, compression driven, dynamical, …)
- Bonds between nucleons rupture
- Remaining bonds bind nucleons into fragments
- One control parameter: bond breaking probability

WB et al., PLB 150, 53 (1985)
Breaking Probability

- Determined by the excitation energy deposited
- **Infinite simple cubic lattice:**
  - 3 bonds/nucleon
  - It takes 5.25 MeV to break a bond

- **$p, p$ induced: Glauber theory**
  - $p_{\text{break}}$ proportional to path length through matter

- **General relation between $p_{\text{break}}$ and $T$:**
  \[ p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}} \Gamma \left[ \frac{3}{2}, 0, \frac{B}{T} \right] \]

- $G =$ generalized incomplete gamma function, $B =$ binding energy per nucleon
  - T. Li et al., *PRL* 70 (generalization of Coniglio-Klein for Fermi systems)

- Obtain $E^*$ or $T$ from other model or directly from experiment
Comparison to Experiment

ISiS data set
10.8 GeV p, p+Au @AGS

- Idea (Elliott et al.): If data follow scaling function

\[ N(Z, T) = Z^{-\tau} f \left( \frac{T - T_c}{T_c} Z^\sigma \right) \]

with \( f(0) = 1 \) (think “exponential”), then we can use scaling plot to see if data cross the point [0,1] -> critical events
Scaling of ISIS Data

- Most important: critical region and explosive events probed in experiment
- Possibility to narrow window of critical parameters
  - $\tau$: vertical dispersion
  - $\sigma$: horizontal dispersion
  - $T_c$: horizontal shift
- $\chi^2$ Analysis to find critical exponents and temperature

- Result:
  - $\beta = 0.5 \pm 0.1$
  - $\gamma = 2.35 \pm 0.05$
  - $T_c = 8.3 \pm 0.2$ MeV

M. Kleine Berkenbusch et al., PRL 88
Freeze-Out Density

- Percolation model only depends on breaking probability, which can be mapped into a temperature.
- Q: What determines the breakup moment?
- A: Density related to fragment energy spectra; Coulomb many-body expansion of pre-fragments

WB, Alleman, Pratt, AIP conf.proc.884, 327 (2007)

\[ p_b = 1 - \frac{2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, 0, B/T\right) \]

\[ \rho_c = (0.35 \pm 0.1)\rho_0 \]
Buckyball Fragmentation

Binding energy of C$_{60}$:
420 eV

625 MeV Xe$^{35+}$

Cheng et al., PRA 54
Cross-Disciplinary Comparison

- Left: Nuclear Multifragmentation
- Right: Buckyball Fragmentation
- Histograms: Percolation Models

Similarities:
- U - shape (b-integration)
- Power-law for imf’s (1.3 vs. 2.6)
- Binding energy effects provide fine structure

Data:
- Bujak et al., PRC 32
- LeBrun et al., PRL 72
- Calc.: WB, PRC 38
- Cheng et al., PRA 54
Conclusions

• Simple picture of thermodynamic phase transitions in nuclear and quark-gluon matter likely wrong, because there is not enough time to approach the critical point

• More exciting possibility: non-equilibrium phase transition
  - First instance of a non-equilibrium phase transition with non-mean-field exponents!
  - What is the connection between equilibrium phase transition theories of nuclear/quark-gluon matter and transition matrix elements between bi/un/meta-stable states?
  - Knowledge of elementary interaction is not enough (emergent phenomena in phase transitions)
QCD

- Is this ....

the same as this ... ?
Water

• Is this .... the same as this ... ?