- we’re going to take a walk–“unspoiled”–on a tough course
- our walk will have only good lies, avoiding the rough
- we start in wide fairways, but dogleg to a difficult approach
- we’ll take drops, in order to stay in the fairway and avoid the rough
- we’ll use “gimmes” to quickly skip through material that Soper covered (he’s already holed out)
- we’ll occasionally be schematic - it’s the broad lay of the land that I want to get across as this is highly technical stuff - doesn’t naturally recommend itself in full glory to lectures …

The game is the Royal and Ancient:

W and Z Boson Production and Resummation

1. **a bit of history**
2. **Drell-Yan formalism**

   naïve DY - kinematics and cross section

   kinematics, corrected for finite p_T

   Compton and annihilation cross sections

3. **multiple gluon emission - Sudakov exponential**
4. **resummation: "CSS" formalism**
5. ***W/Z* boson production**
6. **global fitting of all Drell-Yan data - new**
7. **conclusions**

www.pa.msu.edu/ ~brock/cteq01.pdf

▷ Searching for the *W*

the idea was mature by 1960

- Heisenberg's notion of an exchange force, 1932
- Fermi's theory of β decay, 1934
- Yukawa's notion of an exchanged boson, 1935

Feynman and GellMann, 1958

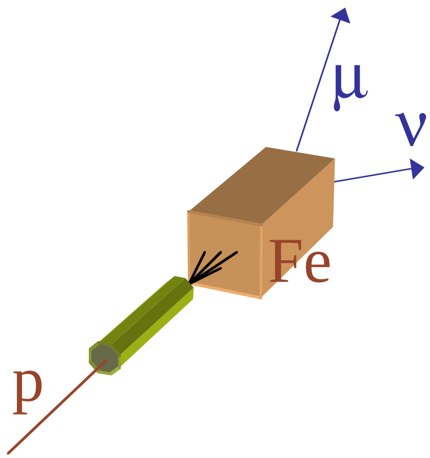
- an audacious paper - so enamored of the "Universal V-A" interaction that they introduced, that they concluded that a long-standing experiment on He was wrong: A not the measured T.

Lee and Yang

- didn't predict parity violation in 1957, but indicated that it hadn't been tested
- in 1960, fleshed out the properties of the *W* (W^0 & $W^{\pm}$) and proposed production scenarios

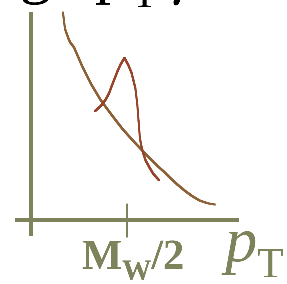
▷ Lee and Yang considered *W* production

they suggested
$$\nu N \to N\mu W(\to \ell\nu)$$
$$h N \to pW(\to \ell\nu)$$



K, π decays a serious low p_T bkgnd

but W should produce high p_T μ's

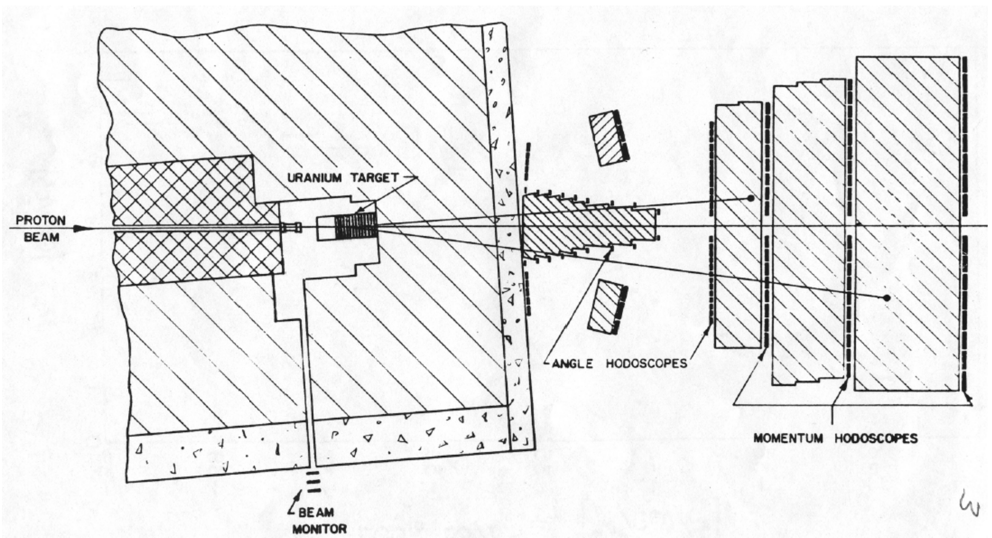


An experiment was mounted in 1970 at BNL with a 30 GeV/c proton beam.

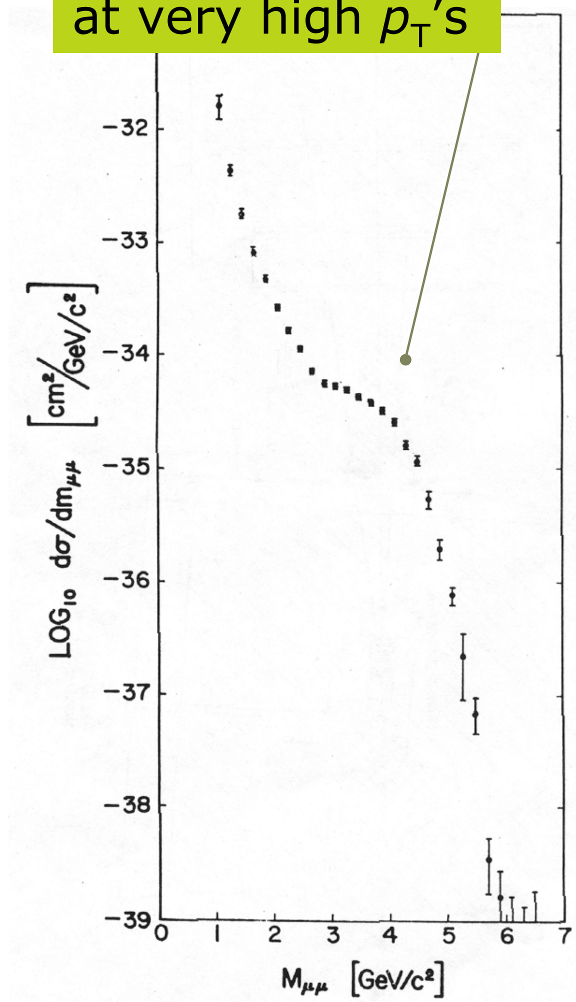
no evidence of W, but what Lederman and Pope found was a little unsettling:

- **unrelated to the W search, at a suggestion that there would be a γ continuum, they looked…**

- **investigated by forming mass-pairs of muons**
  - momenta from range…



at very high p_T's

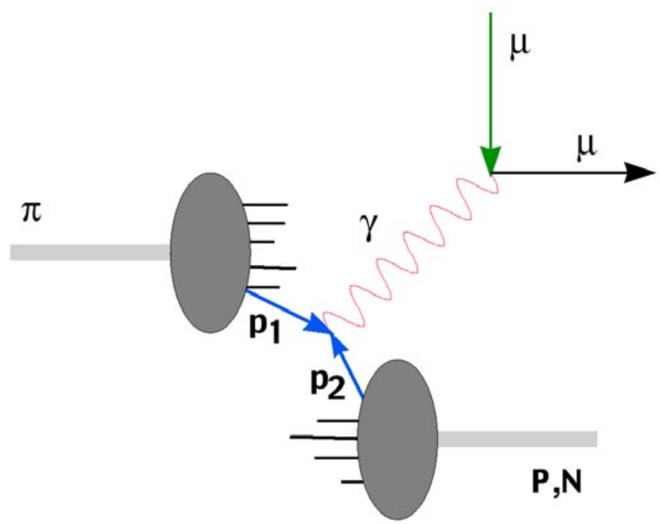


▷ it's not the *W*

the only known source was photons…but at such high p_T?

- much theoretical scurrying about
- one explanation stuck - that of Drell and Yan in 1971

they extrapolated from the (young, circa. 1970) parton model to suggest that the mechanism was:

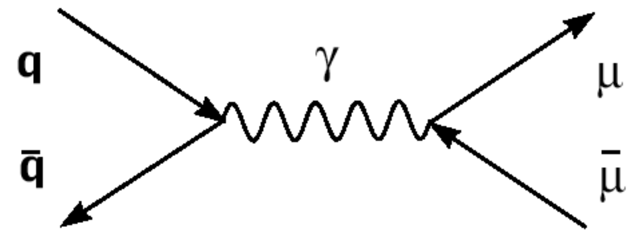


we can quickly reproduce their calculation

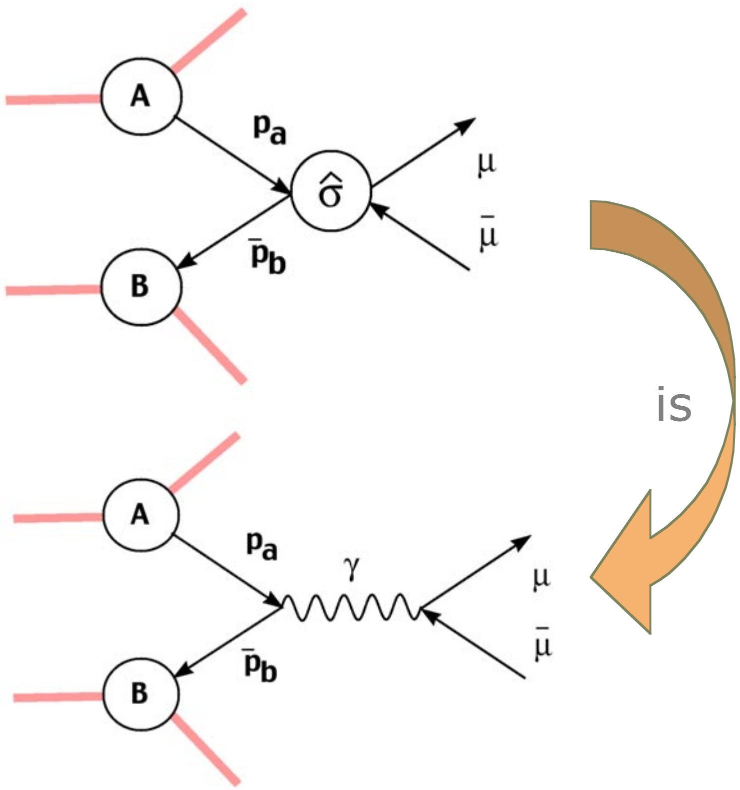
1. **work out kinematics for the naïve model**
2. **calculate the naïve Drell-Yan cross section**
   – find some measurables
   – check predictions
3. **un-naïve the calculation a bit**
   – especially work on the kinematics for finite p_T for the produced photon

▷ the Drell-Yan ansatz:

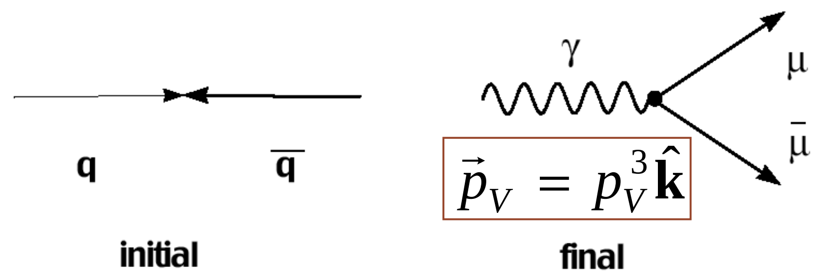
independence and incoherence
of the primary process:



i.e, buried inside



Let's calculate it:
first, kinematics.
we'll presume the simple CM:



hadron/parton/V kinematics

hadrons
$$\left\{\begin{aligned} P_A^\mu &= (P, \vec{0}, P) \\ P_B^\mu &= (P, \vec{0}, -P) \end{aligned}\right.$$

partons
$$\left\{\begin{aligned} p_A^\mu &= (E_a, \vec{0}, p_a^3) \\ p_B^\mu &= (E_b, \vec{0}, -p_b^3) \end{aligned}\right.$$

the IVB
$$\left\{\ p_V^\mu = (E_V, \vec{0}, p_V^3)\right.$$

parton-hadron connection:

$$\begin{aligned} p_A^\mu &= \xi_a P_A \\ p_B^\mu &= \xi_b P_B \end{aligned}$$

$$\begin{aligned} p_L &= p_a^3 - p_b^3 \\ &= \xi_a P_A^3 - \xi_b P_B^3 = (\xi_a - \xi_b) P \qquad (1) \\ x_F &= p_L / P = \xi_a - \xi_b \end{aligned}$$

$$
\begin{aligned}
p_V^\mu = p_a^\mu + p_b^\mu \quad &\rightarrow \quad p_V^3 = p_a^3 - p_b^3 = p_L \\
E_V \quad &= \quad E_a + E_b \\
&= \quad \xi_b P_A^0 + \xi_b P_B^0 = (\xi_a + \xi_b)P \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
p_V^2 \quad &= \quad M^2 \\
&= \quad E_V^2 - (p_V^3)^2 \\
&= \quad (E_V - p_V^3)(E_V + p_V^3) \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
\text{eq (1)} \pm \text{eq (2)} \quad \rightarrow \quad & E_V - p_V^3 = 2\xi_b P \\
& E_V + p_V^3 = 2\xi_a P
\end{aligned}
$$

▷ hadron invariants

$$s \equiv (P_A + P_B)^2 = 4P^2$$
$$t \equiv (p_V - P_A)^2 = M^2 - 2P(E_V - p_V^3) = M^2 - \xi_b s$$
$$u \equiv (p_V - P_B)^2 = M^2 - 2P(E_V + p_V^3) = M^2 - \xi_a s$$

▷ parton invariants

$$\hat{s} \equiv (p_a + p_b)^2 = \xi_a \xi_b s$$
$$\hat{t} \equiv (p_V - p_a)^2 = M^2 - \xi_a \xi_b s$$
$$\hat{u} \equiv (p_V - p_b)^2 = M^2 - \xi_a \xi_b s$$

from equation (3) $M^2 = (2\xi_b P)(2\xi_a P) = 4\xi_a \xi_b P = \xi_a \xi_b s$

- which allows for a definition: $\tau = M^2 / s$ here: $\tau = \xi_a \xi_b$

▷ **define:**

$$y_V \equiv \frac{1}{2}\ln\left(\frac{E_V + p_V}{E_V - p_V}\right)$$

the familiar "rapidity", additive under Lorentz boosts...

in this simple case: $= \frac{1}{2}\ln\left(\frac{\xi_a}{\xi_b}\right)$

using the above... $\xi_{a,b} = \sqrt{\tau}\, e^{\pm y}$

also, there are connections among invariants:

$$\left(\frac{\hat{t}}{s} - \tau\right) = \xi_a\left(\frac{t}{s} - \tau\right)$$

$$\left(\frac{\hat{u}}{s} - \tau\right) = \xi_b\left(\frac{u}{s} - \tau\right)$$

&

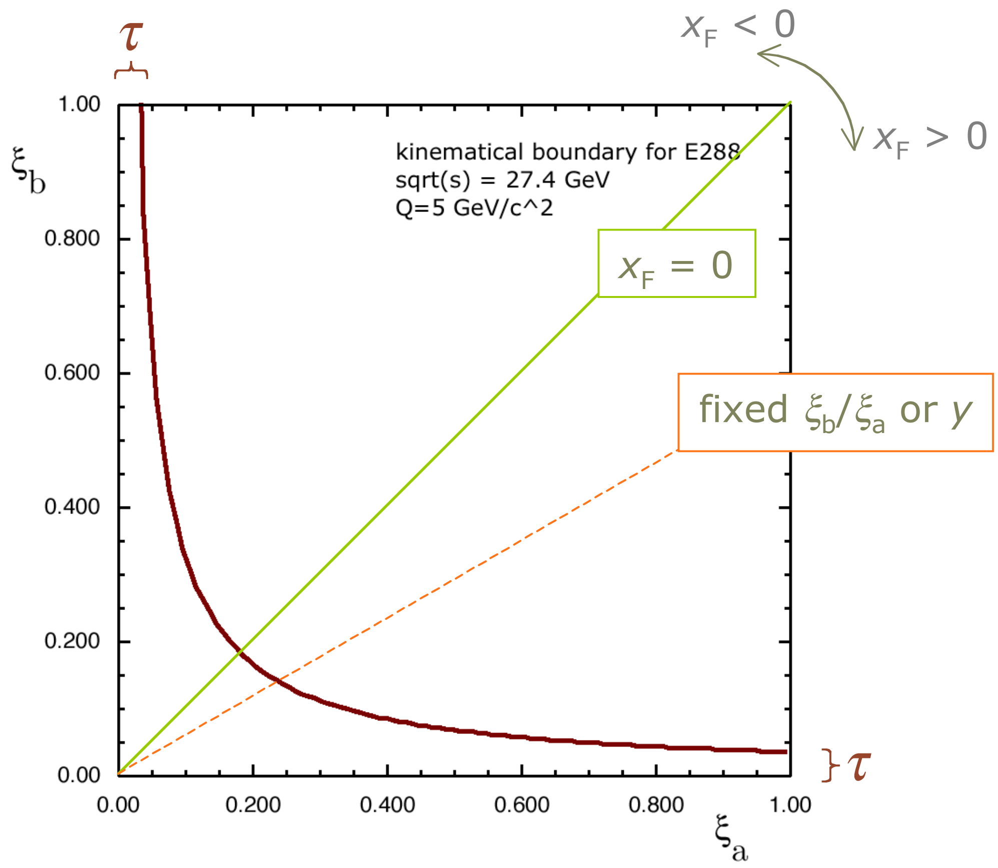
$$E_V - p_V^3 = -\left(\frac{t - M^2}{\sqrt{s}}\right)$$

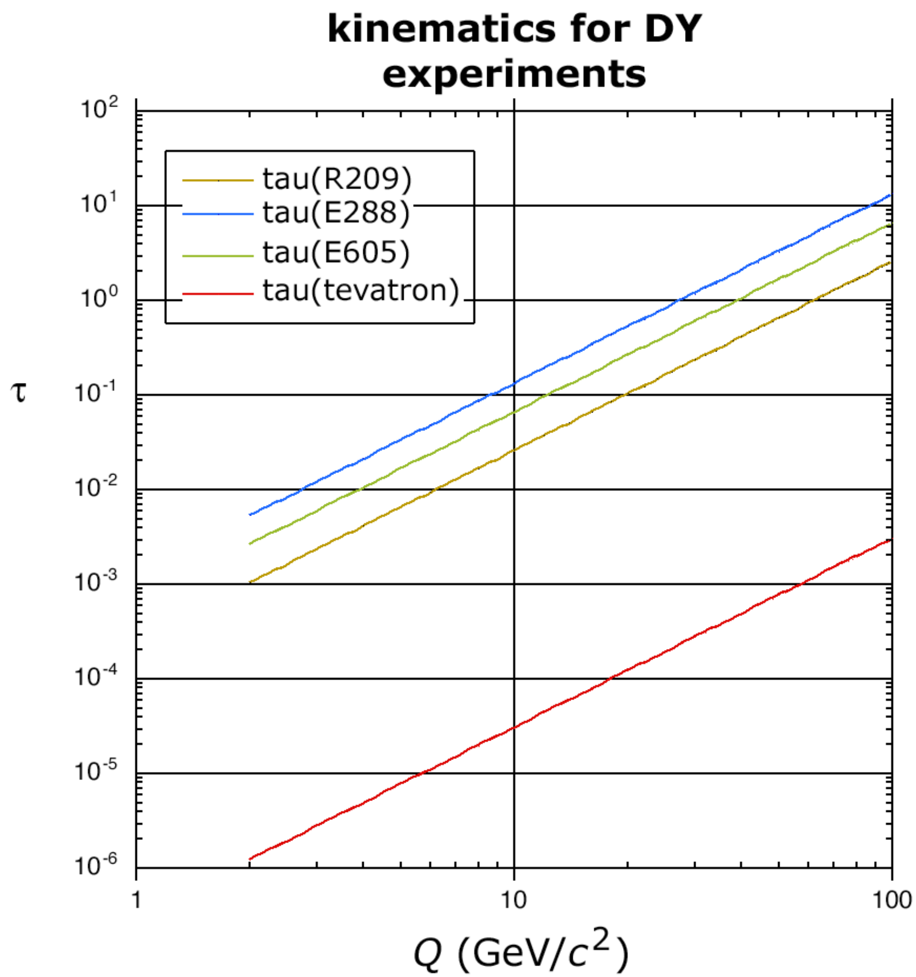
$$E_V + p_V^3 = -\left(\frac{u - M^2}{\sqrt{s}}\right)$$

so: $p_V^3 = \left(\frac{M^2 - u}{4P}\right) - \left(\frac{M^2 - t}{4P}\right) = (\xi_a - \xi_b)P$

for this simple case

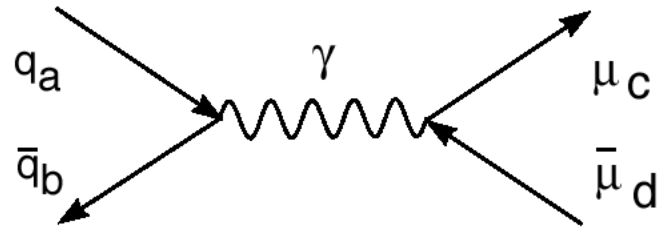
▷ for one early Drell-Yan experiment - E288





kinematics for DY experiments
tau(R209)
tau(E288)
tau(E605)
tau(tevatron)
τ
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1
10
100
Q (GeV/c^2)

▷ **simple Feynman diagram:**



with cross section

$$d\sigma = \frac{\overline{\sum_i} \sum_f |T|^2 d_2\rho}{(flux \cdot normalization)}$$

where

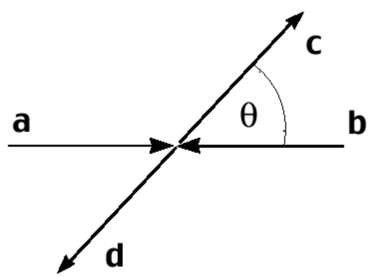
$$(flux \cdot normalization) = 4\sqrt{(p_a \cdot p_b)^2 - m_a m_b} = 4p\sqrt{s}$$

colinear beams

and

$$d_2\rho(\vec{P}_c, \vec{P}_d) = (2\pi)^4 \delta(p_a + p_b - p_c - p_d) d\vec{P}_c d\vec{P}_d$$

In CM of a+b:



integrate away “d”
detect angular distribution of “c”

▷ my definitions…

$$d_2\rho(\vec{p}c) = d_2\rho(\Omega_c, p_c) \equiv \int d_2\rho(\vec{p}_c, \vec{p}_d) d^3\vec{p}_d$$

$$d_2\rho(\vec{p}_3) = \frac{1}{(2\pi)^2} d\Omega_c \frac{p_c dE_c}{4} \left(\frac{p_c^2}{E_a p_c^2 - \vec{p}_a \cdot \vec{p}_c E_c^R} \right) \delta(E_c - E_c^R)$$

$$E_c^R = E_a - \sqrt{\left(\vec{p}_a - \vec{p}_c\right)^2 + m_d^2}$$

where $d_2\rho(\Omega_c) = \int d_2\rho(\vec{p}_c) dp_c \ldots$

and then $= \frac{p_c}{16\pi^2\sqrt{s}} d\Omega_c$

so, $d\sigma = \frac{1}{64\pi^2} \frac{1}{s} \frac{p_c}{P} \overline{\sum_i} \sum_f |T|^2 d\Omega$

▷ the whole enchilada

$$\bar{\sum_i}\sum_f |T|^2 = \frac{1}{4}\sum_i\sum_f |T|^2 = \frac{1}{4}\frac{e_q^2 e^4}{(p_a+p_b)^4}\sum_q\sum_{\bar q}\sum_\mu\sum_{\bar\mu} [\bar u(p_b)\gamma_\mu u(p_a)]\,[\bar v(p_b)\gamma_\nu u(p_a)] \cdot [\bar u(p_c)\gamma^\mu v(p_d)]\,[\bar u(p_c)\gamma^\nu v(p_d)]^\dagger$$

$$= \left(\frac{e_q^2 e^4}{4s^2}\right)\mathrm{Tr}\left[\not p_a\gamma_\mu \not p_b\gamma_\nu\right]\mathrm{Tr}\left[\not p_c\gamma^\mu \not p_d\gamma^\nu\right]$$

$$= \left(\frac{e_q^2 e^4}{4s^2}\right) 32\left[p_a\cdot p_c p_b\cdot p_d + p_a\cdot p_d p_b\cdot p_c\right]$$

with our frame choice:

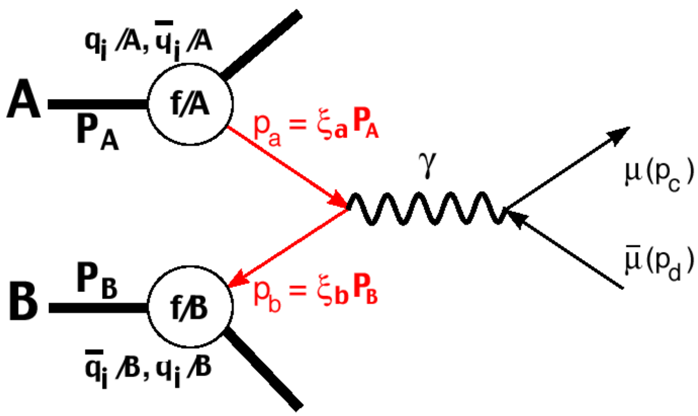
$$= \left(\frac{e_q^2 e^4}{4s^2}\right) 64 E_a^2 E_c^2\left(1+\cos^2\theta\right)$$

indicative of 1/2-1/2 scattering: a prediction

▷ the real inspiration:

a function which incoherently sums all quarks:

$$\mathcal{P}_{q\bar{q}}(\xi_a\xi_b) = \sum_{i=1}^{n_f} e_{q_i}^2 [f_{q_i/A}(\xi_a) f_{\bar{q}_i/B}(\xi_b) + f_{\bar{q}_i/A}(\xi_a) f_{q_i/B}(\xi_b)]$$



where $f_{q_i/A}(\xi_a)$ is the probability of finding an a-type parton in hadron A carrying fraction of P_A equal to ξ_a

$$\sigma(AB \rightarrow \gamma^\star \rightarrow \mu\mu) = \frac{4\alpha^2}{3\hat{s}} \frac{\pi}{3} \int \mathcal{P}_{q\bar{q}}(\xi_a\xi_b) d\xi_a d\xi_b$$

must average over colors - DY didn t know this

▷ **in order to get the differential cross section:**

$$\frac{d^2\sigma}{d\xi_a\xi_b} = \frac{4\alpha^2\pi}{9\hat{s}}\mathcal{P}_{q\bar{q}}(\xi_a,\xi_b)$$

use $\tau = \xi_a\xi_b$ (or $\hat{s} = \tau s$) plus $x_F = \xi_a - \xi_b$

which allows us to calculate the Jacobian to take

$$d\xi_a d\xi_b \to d\tau dx_F \to dQ^2$$

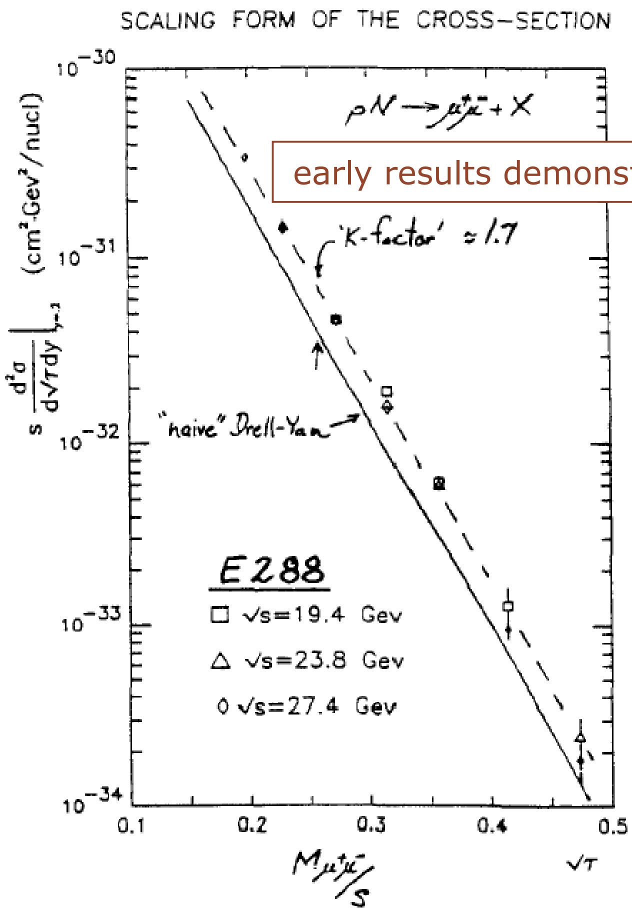
$$\frac{d\sigma}{d\tau dx_F} = \frac{1}{(\xi_a+\xi_b)}\frac{d^2\sigma}{d\xi_a\xi_b} \longrightarrow \frac{d\sigma}{d\tau dx_F} = \frac{4\alpha^2\pi}{9Q^2}\frac{\mathcal{P}_{q\bar{q}}(\xi_a,\xi_b)}{\sqrt{x_F^2+4\tau}}$$

$$\frac{d\sigma}{dQ^2} = \frac{4\alpha\pi}{9Q^4}\tau\int_\tau^1\frac{d\xi_a}{\xi_a}\mathcal{P}_{q\bar{q}}(\xi_a,\tau/\xi_a)$$

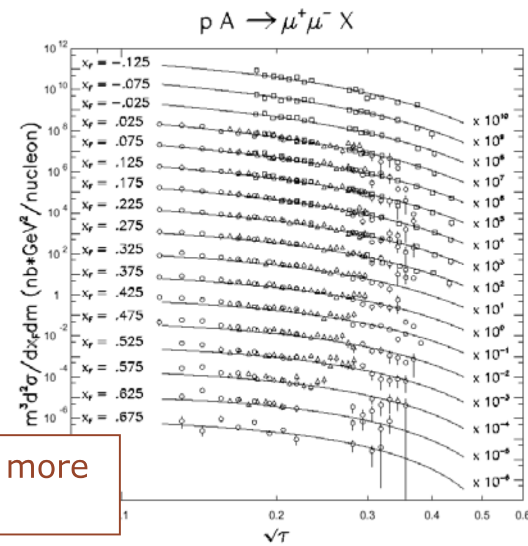
$$Q^4\frac{d\sigma}{dQ^2} = \frac{4\alpha\pi}{9}F(\tau)$$

We have a prediction:
this quantity is independent of Q … only τ

▷ Drell and Yan's explanation of the Lederman/Pope results proved substantially correct

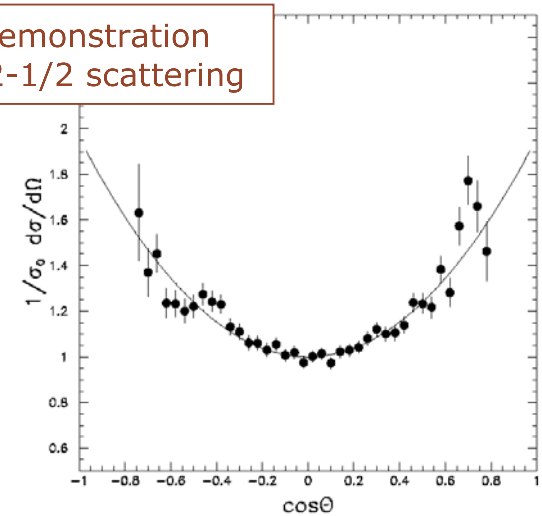


early results demonstrated scaling…



…later results, even more impressively

…including the demonstration of point -like 1/2-1/2 scattering



▷ **well, because of two approximations:**

1. scale invariant parton distribution functions

and

2. presumption of the γ produced with p_T=0

▷ scale-violating pdf's

The Factorization Theorem (Collins, Soper, Sterman) puts the physically appealing quark - parton model on a firm, formal basis:

separate short-distance (calculable) from long distance (not calculable) using a scale, μ_f...hard part is only implicitly dependent on μ_f - in practice, μ_f set = μ.

physical cross section:

calculable, to a particular order in α_S

$$\frac{d\sigma_{AB}}{dQ^2} = \sum_{a,b} f_{a/A}(\xi_a, \mu^2, \mu_f) \otimes \hat{\sigma}_{ab}\left(\xi_a, \xi_b, Q^2, \frac{\mu}{Q}, \frac{\mu_f}{\mu}, \alpha_S(\mu)\right) \otimes f_{b/B}(\xi_b, \mu^2, \mu_f)$$

measurable, universal not calculated, long-distance effects are absorbed into pdf's

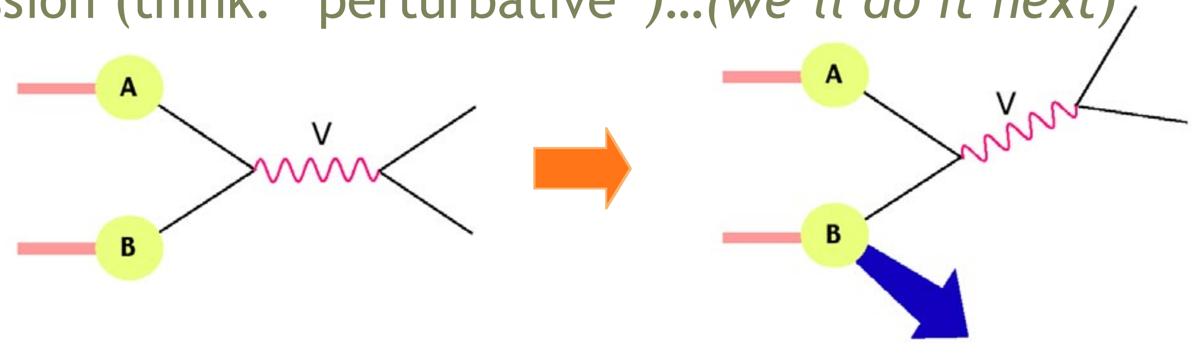
finite effects - the infamous "k-factor"

from now on, we'll presume scale-violating pdf's

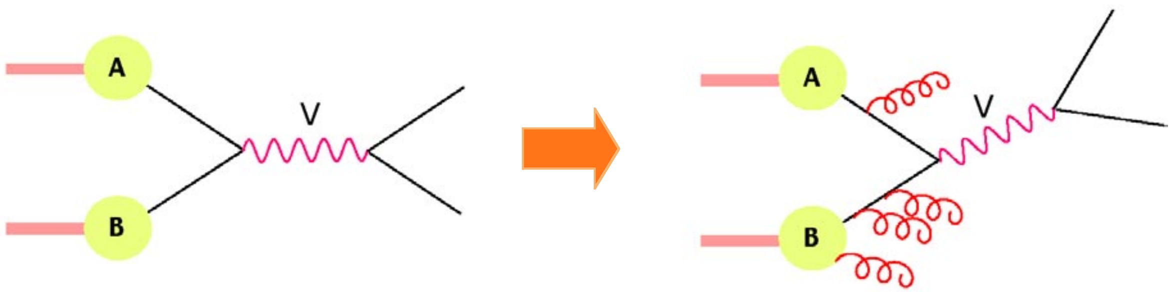
loss of naiveté, 2

▷ **finite p_T … comes in 2 ways:**

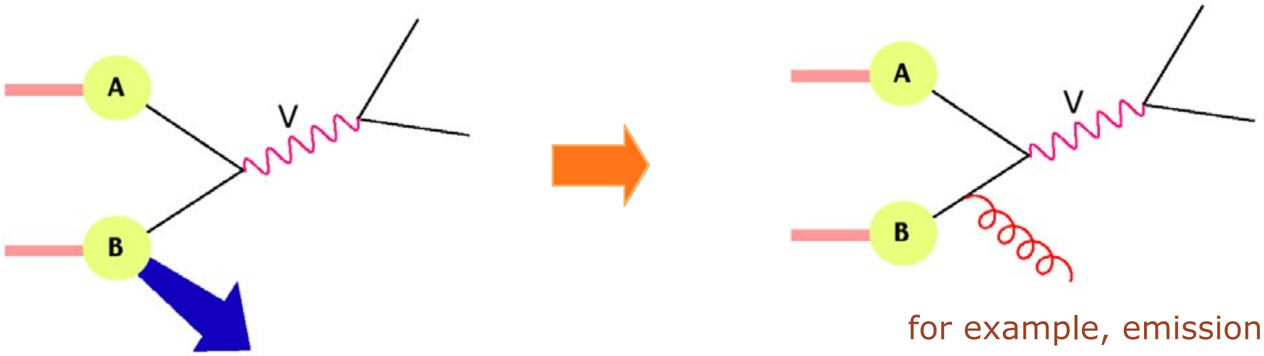
hard emission (think: "perturbative")…*(we'll do it next)*



soft emission (think: "complicated"*!)…(we'll do it later)*



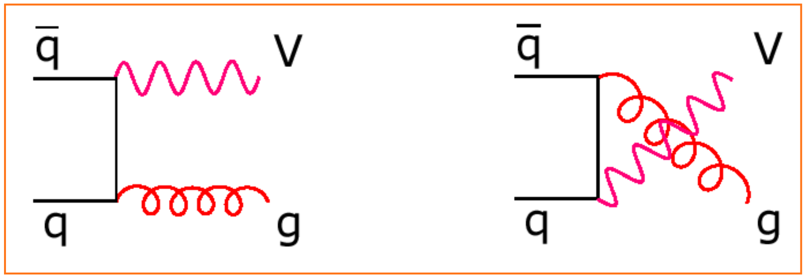
▷ to first order in α_S, the elementary processes are:



for example, emission is 1 gluon

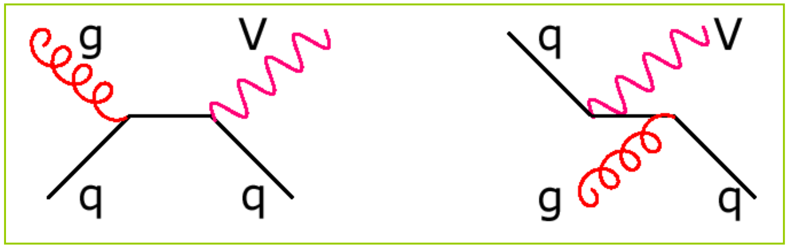
This is a familiar fundamental process: annihilation

…one of a set of order-$\alpha_S\alpha_{EM}$ hard processes which can be treated perturbatively

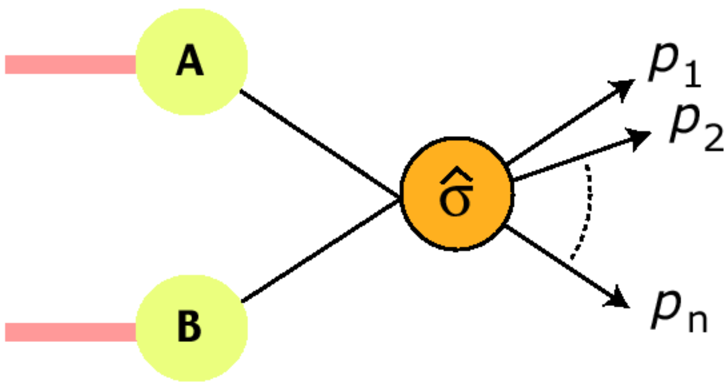


Annihilation graphs, “A”

Compton graphs, “C”



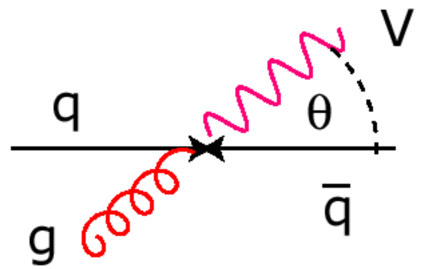
▷ it's harder this time...



in principle, *n* states, for the *i* th particle...

$$
\begin{aligned}
p_i &: \quad (E_i, \vec{p}_{iT}, p_{iL}) \\
p_i^2 &= m_i^2 = (E_i^2 - \vec{p}_{iT}^{\,2} - p_{iL}^2) \\
&= (E_i - p_{iL})(E_i + p_{iL}) - p_{iT}^2
\end{aligned}
$$

presume cm



and consider that the *i*th *is V*

$$
\begin{aligned}
s &= 4P^2 \\
t &= (p - P_A)^2 = M^2 - 2p \cdot P_A = M^2 - 2EP_A + 2pP_A \cos\theta \\
&= M^2 - 2P(E - p_L)
\end{aligned}
$$

kinematics, 2

▷ **from solution for *t*,**

$$t = M^2 - 2P(E - p_L) \quad \Rightarrow \quad E - p_L = \frac{M^2 - t}{2P}$$

$$u = M^2 - 2P(E - p_L) \quad \Rightarrow \quad E + p_L = \frac{M^2 - u}{2P}$$

$$\begin{aligned} p_L &= \left(\frac{M^2 - u}{2P}\right) - \left(\frac{M^2 - t}{2P}\right) \\ &= \left[\left(\frac{M^2 - u}{2P}\right) - \left(\frac{M^2 - t}{2P}\right)\right] P \\ p_L &= [x_1 - x_2]\, P \end{aligned}$$

express the longitudinal p as a fractional difference of P

different from before:

$$p_L = (\xi_a - \xi_b)P$$

▷ **to get...**

$$x_1 \equiv \frac{M^2 - u}{4P^2} = \frac{E + p_L}{\sqrt{s}}$$

$$x_2 \equiv \frac{M^2 - t}{4P^2} = \frac{E - p_L}{\sqrt{s}}$$

solving for M^2:

$$M^2 = (E - p_L)(E + p_L) - p_T^2$$

$$M^2 = s x_1 x_2 - p_T^2$$

so,

$$x_1 x_2 s = M^2 + p_T^2 \equiv M_T^2$$

and therefore,

$$= \xi_a \xi_b s + p_T^2 \text{ so, not the same as } \xi_a \xi_b s \ !$$

also,

$$y = \frac{1}{2} \ln \left(\frac{E + p_L}{E_i - p_L} \right)$$

$$= \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right) \text{ so, not the same as } \ln \left(\frac{\xi_a}{\xi_b} \right) \ !$$

▷ **solving...**

$$x_1 = \frac{M_T}{\sqrt{s}} e^{+y} \qquad x_2 = \frac{M_T}{\sqrt{s}} e^{-y}$$

$$x_1 x_2 = \frac{M_T^2}{s}$$

$$x_T \equiv \frac{p_T}{P} = \frac{2p_T}{\sqrt{s}}$$

$$x_1 x_2 = \tau + \frac{x_T^2}{4} \text{ since, still } \tau = \frac{M^2}{s}$$

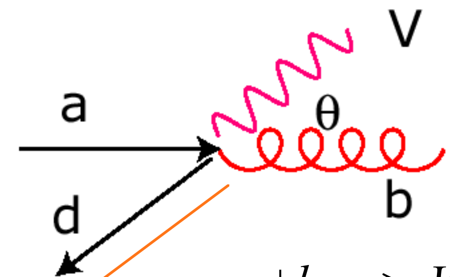
(recalling $\tau = \xi_a \xi_b$ only in the zero p_T case...)

immediate plans…

1. set up the hadronic QCD "Compton" process
2. but, be lazy: calculate the $e\gamma$ Compton cross section
   - *everyone has done this, don't need to really calculate…*
   - *…except we'll do it to a "heavy" photon*
3. convert it to the QCD process by simple substitutions
4. use crossing to infer the Annihilation cross section
5. put it together…and then look at low p_T for trouble

▷ from our earlier phase space outline:

for the simple 2-2:



$a+b \longrightarrow V+d$

$$d\hat{\sigma}(ab \to Vd) = \frac{\hat{\Sigma}_i \Sigma_f |T|^2 d_2\rho(\vec{p}_V, \vec{p}_d)}{4E_a\sqrt{s}}$$

$$d_2\rho = (2\pi)^4 \delta^4 \left(p_a + p_b - p_V - p_d\right) \frac{d^3 p_V}{(2\pi)^3 2E_V} \frac{d^3 p_d}{(2\pi)^3 2E_d}$$

set up to ignore *d*, <u>and</u> embed the process inside hadrons:

a trick

$$d\sigma = \int d\xi_a \int d\xi_b \mathcal{P}_{qg}(\xi_a, \xi_b) \frac{\bar{\Sigma}_i \Sigma_f |T|^2}{32\pi^2 E_a E_b} \delta(p_d^2) \delta^4 \left(\ldots\right) d^4 p_d \frac{d^3 p_V}{2E_V}$$

note!

integrate trivially:

$$= \int d\xi_a \int d\xi_b \mathcal{P}_{qg}(\xi_a, \xi_b) \frac{\bar{\Sigma}_i \Sigma_f |T|^2}{32\pi^2 E_a E_b} \delta(p_d^2) \frac{d^3 p_V}{2E_V} \Big|_{p_d = p_a + p_b - p_V}$$

▷ mess with the delta function:

you can show that $p_d^2 \quad = \quad \hat{s}+\hat{t}+\hat{u}-M^2$

so, $\delta\left(\hat{s}+\hat{t}+\hat{u}-M^2\right) \quad \rightarrow \quad \text{function of } \xi_{a,b}$

you remember
$$\delta\left[f(x)\right] \quad = \quad \left|\frac{\partial f}{\partial x}\right|^{-1}_{x=x_R} \delta(x-x_R)$$

$$f(x_R) \quad = \quad 0$$

so... $$f(\xi_a) \quad = \quad \hat{s}+\hat{t}+\hat{u}-M^2 = (\xi_a\xi_b - \xi_a x_1 - \xi_b x_2 + \tau)s$$

$$\xi_a \quad = \quad \frac{\xi_b\xi_2 - \tau}{\xi_b - x_1} \equiv \xi_a^R$$ is the root...

$$\frac{\partial f}{\partial \xi_a} \quad = \quad \xi_b - x_1$$ is the derivative...

so: $$\delta(p_d^2) \quad \rightarrow \quad \frac{\delta(\xi_a - \xi_a^R)}{s(\xi_b - x_1)}$$

allows us to do the ξ_a integration

▷ so, putting it together:

$$E_V \frac{d\hat{\sigma}}{d^3p_V} = \int d\xi_a \delta(\xi_a - \xi_a^R) \int d\xi_b \mathcal{P}_{qg}(\xi_a, \xi_b) \frac{\hat{\Sigma}_i \Sigma_f |T|^2}{64 E_a E_b \pi^2 s(\xi_b - x_1)}$$

for 2 body kinematics

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{1}{16\pi \hat{s}^2} |T|^2$$

integrating and changing variables...

$$\frac{d\hat{\sigma}}{dQ^2 dy dp_T^2} = \frac{1}{\tau} \int_{\xi_b^{min}}^{1} \frac{d\xi_b}{(\xi_b - x_1)} \mathcal{P}_{qg}(\xi_a, \xi_b) \xi_a^R \xi_b \frac{d\hat{\sigma}}{d\hat{t} dQ^2}$$

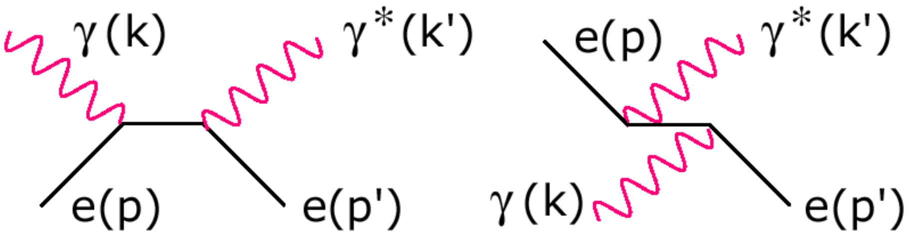
where

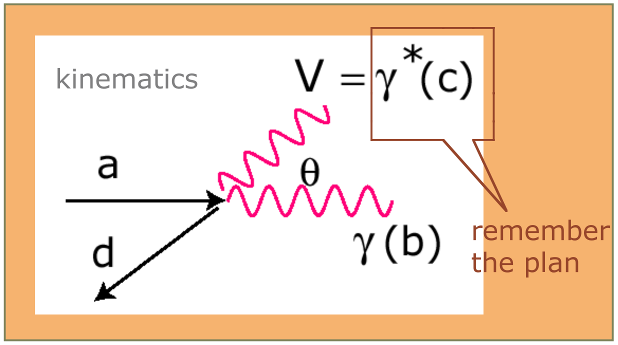
$$\xi_b^{min} = \frac{x_1 - \tau}{1 - x_2}$$

the physics lives here...

"regular" Compton scattering

- **textbook, but for keeping the outgoing photon mass**





$$\begin{aligned}
\hat{s} = (p+k)^2 &= (p'+k')^2 = 2p\cdot k = Q^2 + 2p'\cdot k' \\
\hat{t} = (p-k')^2 &= (k-p')^2 = Q^2 - 2k'\cdot p = -2k\cdot p' \\
\hat{u} = (k-k')^2 &= (p-p')^2 = Q^2 - 2k'\cdot k = -2p\cdot p'
\end{aligned}$$

▷ **actually, an exercise in Halzen and Martin...**

$$\overline{\sum_{\text{initial e}}}\;\overline{\sum_{\text{initial }\gamma}}\;\sum_{\text{final e}}\;\sum_{\text{final }\gamma}|T|^2 = 32\pi^2\alpha\left(-\frac{\hat{t}}{\hat{s}}-\frac{\hat{s}}{\hat{t}}-\frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right)$$

mass only affects the interference term

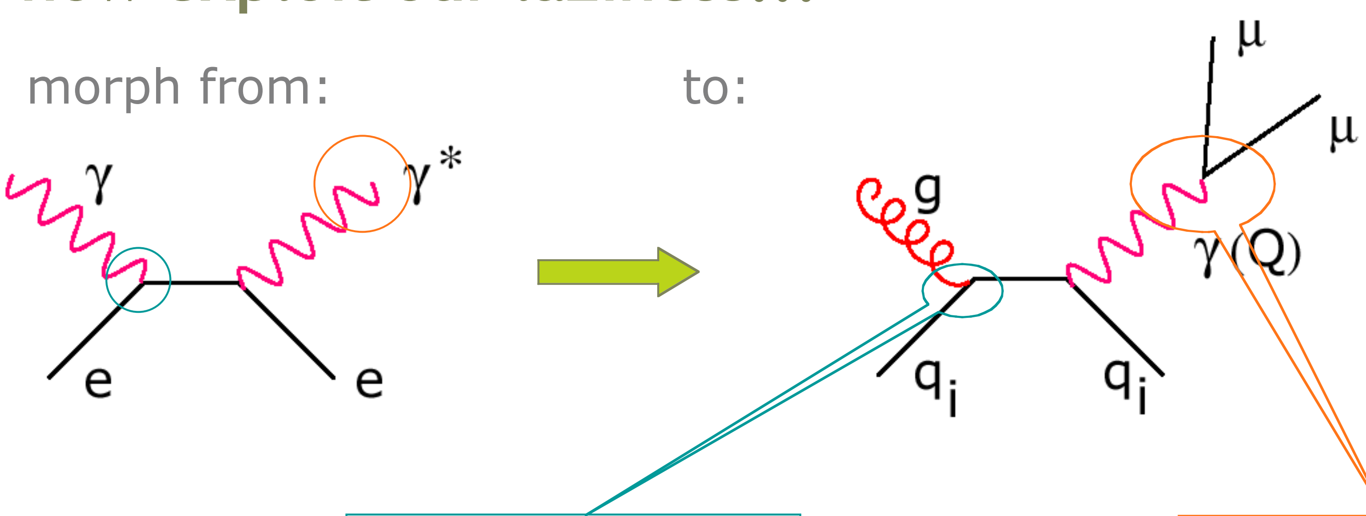
standard 2 body kinematics $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p'}{p}|T|^2$$

Jacobian to go to invariants $$\frac{d\sigma}{d\Omega} = \frac{pp'}{\pi}\frac{d\sigma}{d\hat{t}}$$

so: $$\frac{d\sigma}{d\hat{t}} = \frac{2\pi\alpha^2}{\hat{s}^2}\left(-\frac{\hat{t}}{\hat{s}}-\frac{\hat{s}}{\hat{t}}-\frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right)$$

▷ now exploit our laziness...

morph from: to:



coupling change:

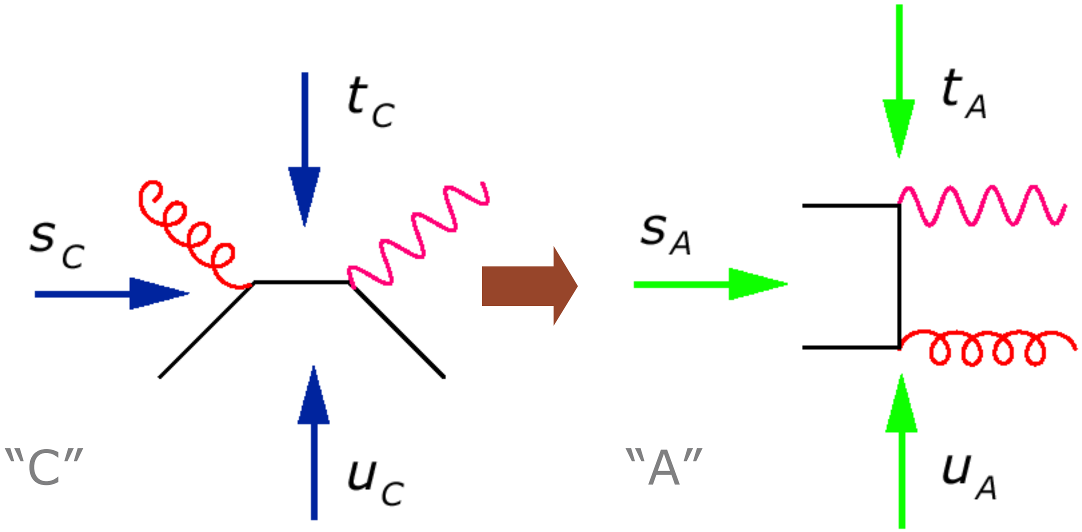
$$\alpha_{EM}^2 \rightarrow \alpha_{EM} e_{q_i}^2 \alpha_S$$
$$\rightarrow \times \frac{1}{6} \text{ color}$$

decay to muons

$$d\sigma(qg \rightarrow \mu\mu q) = \frac{\alpha}{3\pi Q^2} d\sigma(qg \rightarrow \gamma^\star q) dQ^2$$

$$\frac{d\hat{\sigma}_C}{dQ^2 d\hat{t}} = \frac{1}{9} \frac{\alpha^2 e_q^2 \alpha_S}{Q^2 \hat{s}^2} \left(\frac{-\hat{t}^2 - \hat{s}^2 - 2\hat{u}Q^2}{\hat{s}\hat{t}} \right)$$

▷ **use crossing symmetry to go Compton $\rightarrow$ Annihilation:**



$$\hat{u}_C \rightarrow \hat{s}_A$$
$$\hat{s}_C \rightarrow \hat{u}_A$$
$$\hat{t}_C \rightarrow \hat{t}_A$$

plus color changes... $\frac{1}{8} \cdot \frac{1}{3} Tr[T_g, T_g] = \frac{1}{6} \quad \rightarrow \quad \frac{1}{3} \cdot \frac{1}{3} Tr[T_q, T_q] = \frac{4}{9}$

total order-α_s Drell-Yan cross section

▷ putting it together...

$$\frac{d\sigma(AB \to \mu\mu j)}{dQ^2 dy dp_T^2} = \frac{1}{\pi}\int_{\xi_b^{\min}}^{1} \frac{\xi_b}{(\xi_b - x_1)} \xi_a^R \xi_b \mathcal{P}_{q\bar{q}}(\xi_a, \xi_b) \frac{d\hat{\sigma}_A}{dQ^2 d\hat{t}} + \frac{1}{\pi}\int_{\xi_b^{\min}}^{1} \frac{\xi_b}{(\xi_b - x_1)} \xi_a^R \xi_b \mathcal{P}_{qg}(\xi_a, \xi_b) \frac{d\hat{\sigma}_C}{dQ^2 d\hat{t}}$$

where

$$\frac{d\hat{\sigma}_C}{dQ^2 d\hat{t}} = \frac{1}{9}\frac{\alpha^2 e_q^2 \alpha_S}{Q^2 \hat{s}^2}\left(\frac{-\hat{t}^2 - \hat{s}^2 - 2\hat{u}Q^2}{\hat{s}\hat{t}}\right)$$

$$\frac{d\hat{\sigma}_A}{dQ^2 d\hat{t}} = \frac{8}{27}\frac{\alpha^2 e_q^2 \alpha_S}{Q^2 \hat{s}^2}\left(\frac{\hat{t}^2 + \hat{u}^2 + 2\hat{s}Q^2}{\hat{u}\hat{t}}\right)$$

this leads to trouble... the annihilation hard cross section goes like:

$\frac{1}{\hat{s}\hat{t}\hat{u}}$ which, since $\hat{t}\hat{u} = \xi_a \xi_b s p_T^2$ leads to $\frac{1}{p_T^2}$ behavior for $\hat{\sigma}_A$

▷ perhaps not surprisingly, logs float to the surface…

factor out the $1/p_T^2$ behavior and as $x_T \to 0$, the leading terms go like:

$$\int_{\xi_b^{\min}}^{1} \frac{d\xi_b}{\xi_b - x_1}\left[1 + \frac{\tau^2}{(\xi_a^R \xi_b)^2}\right] \quad \to \ln(Q^2/k_T^2)$$

(There'll be others later)

k_T dependence

as $k_T \to 0$

so,in that limit, the form of the order-α_S Drell Yan cross section is:

$$\frac{d\sigma}{d\tau dy dp_T^2} = \left(\frac{d\sigma}{d\tau dy}\right)_B \left[\frac{4\alpha_S}{3\pi}\frac{1}{p_T^2}\ln\left(Q^2/p_T^2\right)\right]$$

which live here…

BUT

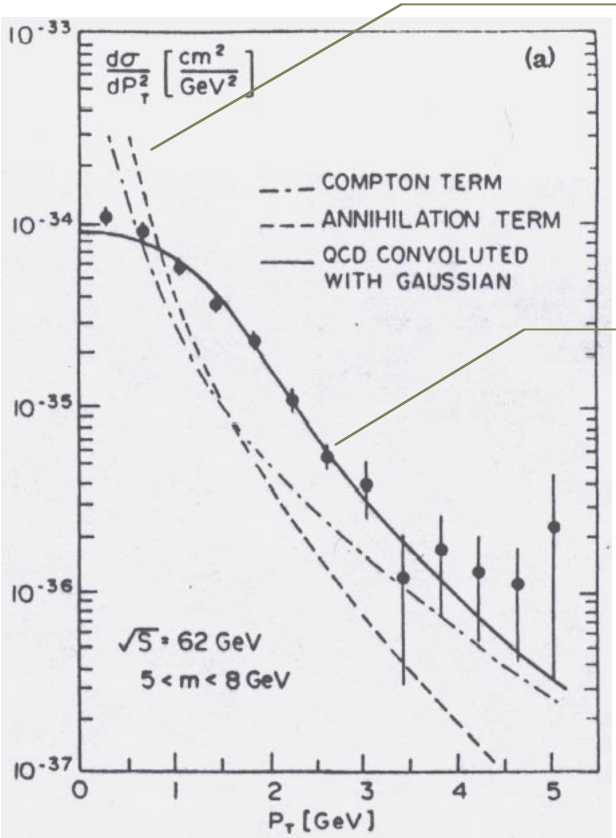
here's a scale (p_T) which cannot be absorbed into the pdf's: the dreaded **"two scale problem"**

where:

$$\left(\frac{d\sigma}{d\tau dy}\right)_B = \sigma_0 \frac{e_q^2}{3}\mathcal{P}_{q\bar{q}}(\xi_a, \xi_b, Q^2)$$

includes leading log pdf's

▷ **what about data?** R209 at the ISR, 1982



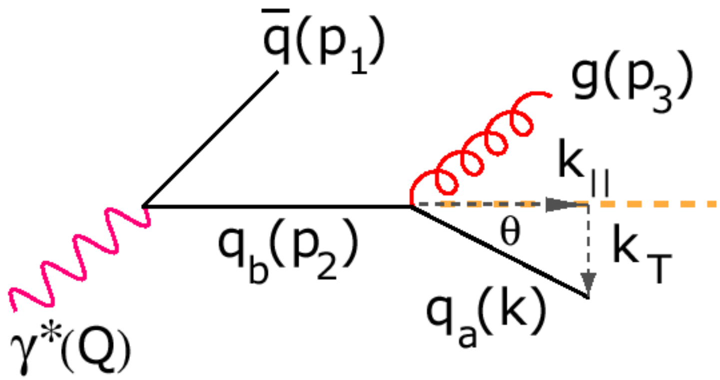
bad behavior at low pT

best fit for a Gaussian-smeared model

hmm...

1. **an interlude, of sorts…**
   - calculate the probability of the emission of a low-pt gluon
   - then calculate the probability of an infinite number of low-pt gluons
   - call them "Sudakov" and identify the approximations as *Leading Pole* and *Leading Double Log*

▷ a side calculations - dealing with soft gluon radiation(s)



define x_1 and x_2 to be the momentum fractions carried by q(p_1) and q(p_2)...then $x_3 = 1 - x_1 - x_2$ is the fraction carried by g.

then: $$\frac{d\Gamma(\gamma^\star \to q\bar{q}g)}{dx_1 dx_2} = 3\alpha e_q^2 Q \frac{32\alpha_S}{3\pi} \frac{x_1^2 + x_2^2}{(1-2x_1)(1-2x_2)}$$

log divergence when $x_{1,2} \to 1/2$

define: $z \equiv E_a/E_b$ (which is small)

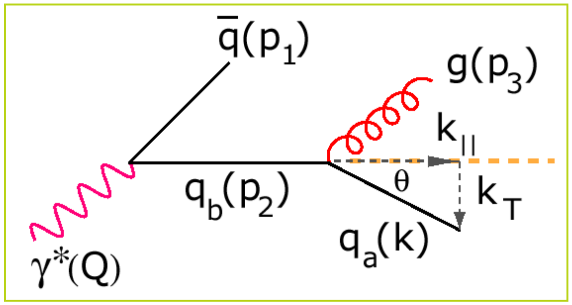
so, $j \equiv (k+p_3)^2/Q^2 = Q^2(1-2x_1)/Q^2 = (1-2x_1)$

(invariant q_a-g mass, which is small)

▷ wave some hands here…

$$\theta \approx \tan\theta = \frac{k_T}{k_{\|}} \cong \frac{k_T}{Q/2}$$

and so, small



change variables, for photon "lifetime":

$$\frac{d\Gamma(\gamma^\star \to q\bar{q}g)}{dj\,dz} = \frac{4\alpha_S \alpha e_q^2 Q^3}{\pi}\left[\frac{j}{1-z} + \frac{1+z^2}{j(1-z)} + \frac{16}{1-z}\right]$$

log divergence, $j \to 0$

when j small (m_{qg} small) and z near 1 (E_g small)

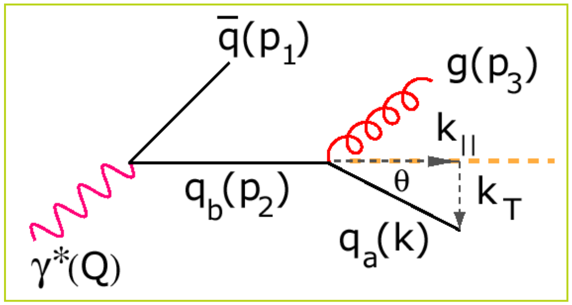
the middle term dominates…

called the "Leading Pole Approximation", (LPA)

$$\frac{1}{\sigma_0}\left(\frac{d\sigma}{dj\,dz}\right)_{\text{LPA}} = \frac{2\alpha_S}{3\pi}\frac{1+z^2}{(1-z)j} = \frac{\alpha_S}{2\pi j}\mathcal{P}_{q\bar{q}}(z)$$

related to the $\mathcal{P}_{qg}$ splitting function

▷ add up a series of emissions



in the limit, the angle is small

$$\theta \cong \tan\theta = \frac{k_T}{k_{||}} = \frac{k_T}{Q/2}$$

define: $\mathcal{S}(\theta) \equiv$ probability of q emitting g with angle $< \theta$

$\mathcal{T}(\theta) \equiv$ probability of q emitting g with angle $> \theta$

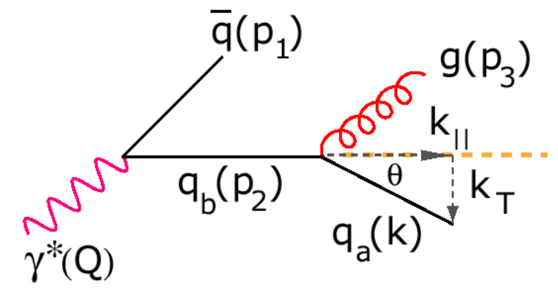
so... $1 = \mathcal{S}(\theta) + \mathcal{T}(\theta)$

as a function of k_T, the probability of > θ

$$\mathcal{T}(k_T^2) = \int_{k_T^2}^{Q^2/4} \frac{1}{\sigma_0}\frac{d\sigma}{dk_T'^2} dk_T'^2$$

so, $$\frac{1}{\sigma_0}\frac{d\sigma}{dk_T^2} = -\frac{d\mathcal{T}}{dk_T^2} = \frac{d\mathcal{S}}{dk_T^2}$$

▷ change variables



$$\mathcal{T}(\theta)_{\rm LPA} = \frac{2\alpha_S}{3\pi}\int\int\frac{1+z^2}{(1-z)j}dzdj$$

from $k_T^2/Q^2 = z(1-z)j \equiv \ell j > \theta^2/4$

$$\mathcal{T}(\theta) = \frac{2\alpha_S}{3\pi}\int_{\theta^2/4}^1\frac{dj}{j}\int_{\theta^2/4j}^1\frac{2d\ell}{\ell}$$

$$\mathcal{T}(\theta)_{\rm LDLA} = \frac{2\alpha_S}{3\pi}\ln^2(\theta^2/4)$$

leading double log approximation, LDLA

likewise, then $\mathcal{S}(\theta)_{\rm LDLA} = 1 - \frac{2\alpha_S}{3\pi}\ln^2(\theta^2/4)$

remember $\frac{1}{\sigma_0}\frac{d\sigma}{dk_T^2} = -\frac{d\mathcal{T}}{dk_T^2} = \frac{d\mathcal{S}}{dk_T^2}$

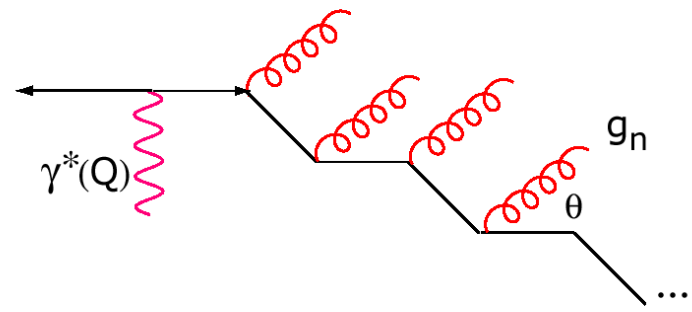
so, $\frac{d\sigma}{dk_T^2} = \frac{4\alpha_S}{3\pi}\frac{1}{k_T^2}\ln(Q^2/k_T^2)$

again, with at least $k^2_T << Q^2$

▷ treat radiations as independent

the probability of *n*, with < θ

$$\mathcal{S}_n(k_T)\,|_{\rm LDLA} \;=\; \frac{1}{n!}\left[-\frac{2\alpha_S}{3\pi}\ln(k_T^2/Q^2)\right]^n$$



adding the probabilities for all possible *n*'s

$$\mathcal{S}_{\rm LDLA}(k_T) \;=\; \sum_{n=0}^{\infty}\mathcal{S}_n(k_T)$$

which is an exponential: $= \exp\left[-\frac{2\alpha_S}{3\pi}\ln(k_T^2/Q^2)\right]$

so

$$\frac{d\mathcal{S}}{dk_T^2} = \frac{1}{\sigma_0}\frac{d\sigma}{dk_T^2}\,|_{\rm LDLA} \;=\; \frac{4\alpha_S}{3\pi}\frac{1}{k_T^2}\ln(Q^2/k_T^2)e^{-\frac{2\alpha_S}{3\pi}\ln(k_T^2/Q^2)}$$

a significant result: as $p_T \to 0$, cross $\sigma \to 0$!

▷ this is where we started

$$\frac{dS}{dk_T^2} = \frac{1}{\sigma_0}\frac{d\sigma}{dk_T^2}\Big|_{\text{LDLA}} = \frac{4\alpha_S}{3\pi}\frac{1}{k_T^2}\ln(Q^2/k_T^2)e^{-\frac{2\alpha_S}{3\pi}\ln(k_T^2/Q^2)}$$

for the k_T of the radiating quark has the same form as our order-α_S cross section for the q…but including the effects of copious radiation of soft glue

we can improve our Drell-Yan cross section the same way:

notice powers of α_S

$$\frac{d\sigma}{d\tau dy dp_T^2} = \left(\frac{d\sigma}{d\tau dy}\right)_B \left[\frac{4\alpha_S}{3\pi}\frac{1}{p_T^2}\ln\left(Q^2/p_T^2\right)e^{-\frac{2\alpha_S}{3\pi}\ln(p_T^2/Q^2)}\right]$$

what we got perturbatively at order-α_S

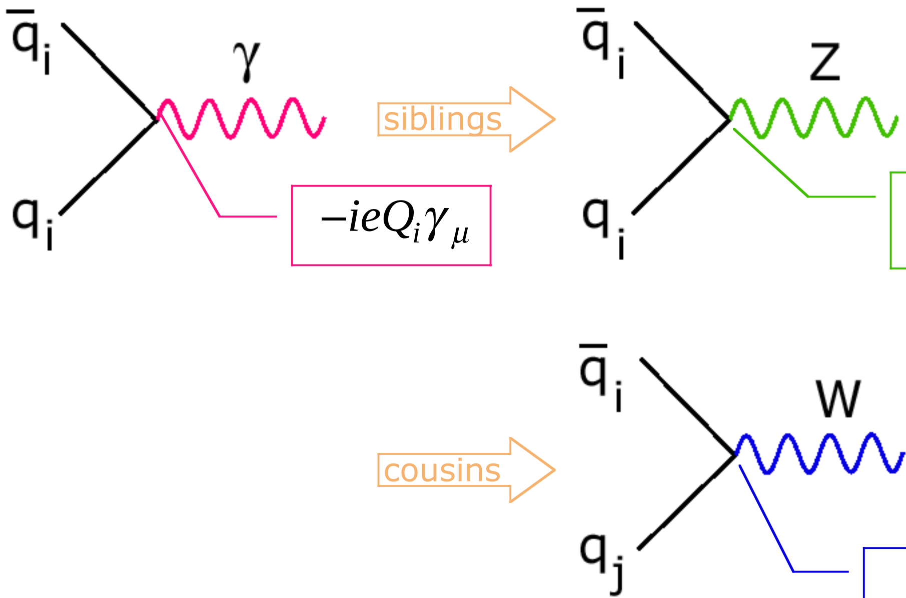
adding infinite-order soft radiation

this “RESUMMATION” (the exponential) tames the bad behavior

1. **move to W/Z production**
   - (*ah*, to) be naïve again in order to adjust to EW parameters
   - get a sense of the rates
   - use our results and calculate for finite p_T

▷ Drell-Yan process is operative here too

the Feynman rules are slightly different



$$-ieQ_i\gamma_\mu$$

$$ie\gamma_\mu\left(v_i - a_i\gamma_\mu\right)$$

$$v_i = \frac{(I_3)_i - 2Q_i\sin^2\theta_W}{2\sin\theta_W\cos\theta_W}$$

$$a_i = \frac{(I_3)_i}{2\sin\theta_W\cos\theta_W}$$

$$ie\gamma_\mu(1-\gamma_5)\frac{1}{2\sqrt{2}\sin^2\theta_W}$$

▷ just an EW lookup:

propagators: $$\frac{d\hat{\sigma}}{dQ^2} \propto \frac{1}{Q^4} \Rightarrow \frac{1}{\left(\hat{s} - M_V{}^2\right)^2 + \left(\Gamma_V M_V\right)^2}$$

couplings:

W bosons

$$e^4 \quad \rightarrow \quad e^4 \left[\frac{2}{(2\sqrt{2}\sin^2\theta_W)^2} \right]^2$$

Z bosons

$$e^2\text{leptons} \quad \rightarrow \quad e^2(v_f^2 + a_f^2)$$
$$e^2 e_q^2\text{quarks} \quad \rightarrow \quad e^2(v_f^2 + a_f^2)$$
$$Q_f^2 \quad \rightarrow \quad 1 + \left[1 - 4|Q_f|\sin^2\theta_W\right]^2$$

plus, the connection between weak and electromagnetic couplings

$$\frac{G_F M_W^2}{\sqrt{2}} = \frac{\pi\alpha}{2\sin^2\theta_W}$$

▷ **write down the, now standard, cross section:**

$$\sigma(AB \to W \to \ell\nu) = \frac{1}{3}\sum_{ij}\int d\xi_a d\xi_b f_{q_i/A}(\xi_a, Q^2) f_{\bar{q}_j B}(\xi_b, Q^2)\hat{\sigma}(q_i\bar{q}_j \to \ell\nu)$$

the hard part:

$$\overline{\sum_{\text{initial}}}\sum_{\text{final}} |T|^2 = 8^4 |V_{q_i q_j}|^2 \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{\hat{u}^2}{(\hat{s} - M_W^2)^2 + (M_W \Gamma_W)^2}$$

$$\frac{d\hat{\sigma}(q_i\bar{q}_j \to \ell\nu)}{d\cos\hat{\theta}} = \frac{|V_{q_i\bar{q}_j}|^2}{8\pi}\frac{G_F^2 M_W^4}{2}\frac{\hat{s}(1+\cos\hat{\theta})^2}{(\hat{s} - M_W^2)^2 + (M_W\Gamma_W)^2}$$

make use of the "narrow width approximation":

$$\frac{1}{(\hat{s} - M_W^2)^2 + (M_W\Gamma_W)^2} \quad \to \quad \frac{\pi}{\Gamma_W M_W}\delta(\hat{s} - M_W^2)$$

define W-specific parton density combination:

$$\mathcal{P}^W_{q_i,\bar{q}_j}(\xi_a, \xi_b, Q^2) \equiv f_{q_i/A}(\xi_a, Q^2) f_{\bar{q}_j B}(\xi_b, Q^2)$$

W - cross section

combine with K-M matrix elements for the isospin-changing process

$\left(\sqrt{\text{cabbibo}^2 + \text{cabibbo}^2}\right)$ angles, actually

~95%

$$\sum_{ij} \mathcal{P}^W_{q_i,\bar{q}_j}(\xi_a, \xi_b, Q^2) =$$

$$\left[f_{u/A}(\xi_a, Q^2) f_{\bar{d}/B}(\xi_b, Q^2) + f_{\bar{d}/A}(\xi_a, Q^2) f_{u/B}(\xi_b, Q^2) \right] |V_{ud}|^2$$

~5%

$$+ \left[f_{u/A}(\xi_a, Q^2) f_{\bar{s}/B}(\xi_b, Q^2) + f_{\bar{s}/A}(\xi_a, Q^2) f_{u/B}(\xi_b, Q^2) \right] |V_{us}|^2$$

so,

$$\mathcal{P}^W(\xi_a, \xi_b, Q^2) \equiv \mathcal{P}^W_{u,d}(\xi_a, \xi_b, Q^2)|V_{ud}|^2 + \mathcal{P}^W_{u,s}(\xi_a, \xi_b, Q^2)|V_{us}|^2$$

forget the Cabibbo-disallowed transition...

$$\sigma_W(AB \to \ell\nu) = \frac{1}{3}|V_{ud}|^2 \pi \sqrt{2} G_F \int d\xi_a d\xi_b \mathcal{P}^W_{u,d}(\xi_a, \xi_b, Q^2) \delta(\hat{s} - M_W^2)\hat{s}$$

some details, delta function gymnastics:

$$\delta(\hat{s}-M_W^2) = \frac{1}{s}\delta(\xi_a\xi_b-\tau) = \frac{\delta(\xi_a-\tau/\xi_b)}{s\xi_b}$$

so,

$$\sigma_{W^+}(AB\to\ell\nu) = \frac{\sqrt{2}G_F\pi|V_{ud}|^2}{3}\tau\underbrace{\int_\tau^1\frac{d\xi_b}{\xi_b}\mathcal{P}^W_{u,d}(\xi_a,\xi_b,Q^2)}_{\tau\left(\frac{d\mathcal{L}}{d\tau}\right)_{u\bar{d}}}$$

defined as the differential Parton Luminosity

Tevatron luminosities

(royal) and ancient; EHLQ

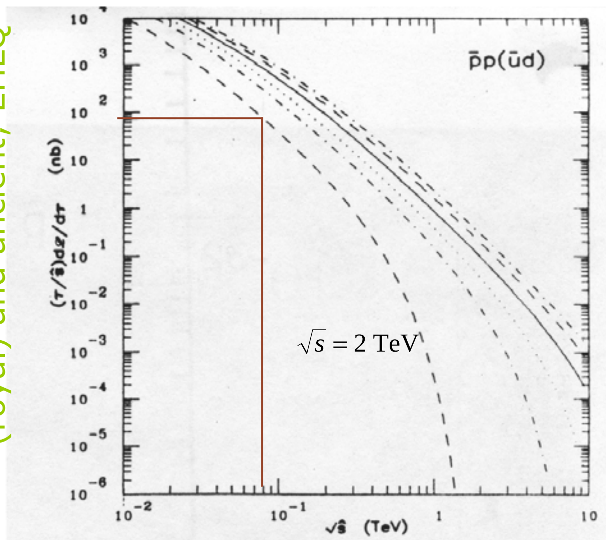


FIG. 55. Quantity $(\tau/\hat{s})d\mathcal{L}/d\tau$ for $u\bar{d}$ or $\bar{u}d$ interactions proton-antiproton collisions.

$$\tau = \frac{Q^2}{s} \cong \frac{80^2}{2000^2} \approx 1.6\times10^{-3}$$

$$\hat{s} \cong 0.08\ \text{TeV}/c^2 \Rightarrow \frac{\tau}{\hat{s}}\frac{d\mathcal{L}}{d\tau} \cong 100\ \text{nb}$$

$$\sigma = (6\ \text{nb})\ \hat{s}\left(\frac{\tau}{\hat{s}}\frac{d\mathcal{L}}{d\tau}\right)$$

$$= (6\ \text{nb})(80\text{GeV})^2\frac{1}{0.39(\text{GeV}^2\text{mb})}\left(\frac{1\,\text{mb}}{10^{-3}\,\text{b}}\right)\left(\frac{10^{-9}\,\text{b}}{\text{nb}}\right)100\ \text{nb}$$

$$= 9.9\ \text{nb}$$

$\sigma(Z)$ is about 1/3 $\sigma(W)$

▷ same idea: just follows the previous pattern…

$$\sigma(AB \to Z \to \ell\ell) = \frac{1}{3}\sum_i \int d\xi_a d\xi_b f_{q_i/A}(\xi_a, Q^2) f_{\bar{q}_i B}(\xi_b, Q^2)\hat{\sigma}(q_i\bar{q}_i \to \ell\ell)$$

$$\sum_{ij} \mathcal{P}^Z_{q_i,\bar{q}_i}(\xi_a,\xi_b,Q^2) = \left[f_{u/A}(\xi_a,Q^2)f_{\bar{u}/B}(\xi_b,Q^2) + f_{\bar{u}/A}(\xi_a,Q^2)f_{u/B}(\xi_b,Q^2)\right]\left(\frac{1}{4} - \frac{2}{3}x_W + \frac{8}{9}x_W^2\right)$$

$$+\left[f_{d/A}(\xi_a,Q^2)f_{\bar{d}/B}(\xi_b,Q^2) + f_{\bar{d}/A}(\xi_a,Q^2)f_{d/B}(\xi_b,Q^2)\right.$$

$$\left.+f_{s/A}(\xi_a,Q^2)f_{\bar{s}/B}(\xi_b,Q^2) + f_{\bar{s}/A}(\xi_a,Q^2)f_{s/B}(\xi_b,Q^2)\right]\left(\frac{1}{4} - \frac{1}{3}x_W + \frac{2}{9}x_W^2\right)$$

where $x_W = \sin^2\theta_W$

$$\mathcal{P}^Z(\xi_a,\xi_b,Q^2) \equiv \mathcal{P}^Z_u(\xi_a,\xi_b,Q^2)\left(\frac{1}{4} - \frac{2}{3}x_W + \frac{8}{9}x_W^2\right) + \mathcal{P}^Z_{d,s}(\xi_a,\xi_b,Q^2)\left(\frac{1}{4} - \frac{1}{3}x_W + \frac{2}{9}x_W^2\right)$$

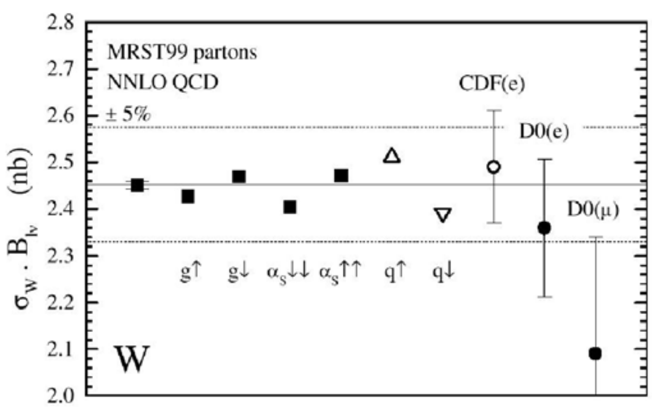
so

$$\sigma_{Z^+}(AB \to \ell\ell) = 2\frac{\sqrt{2}G_F\pi}{3}\tau\int_\tau^1 \frac{d\xi_b}{\xi_b}\mathcal{P}^Z(\xi_a,\xi_b,Q^2)$$

▷ **the calculation is historical…the data and pdf fits are much advanced**

EHLQ gives… $\sigma \cdot \mathrm{BR}(W \to \ell \nu) \approx 2.2 \text{ nb}$

but, data and phenomenology deserve more attention than possible here…



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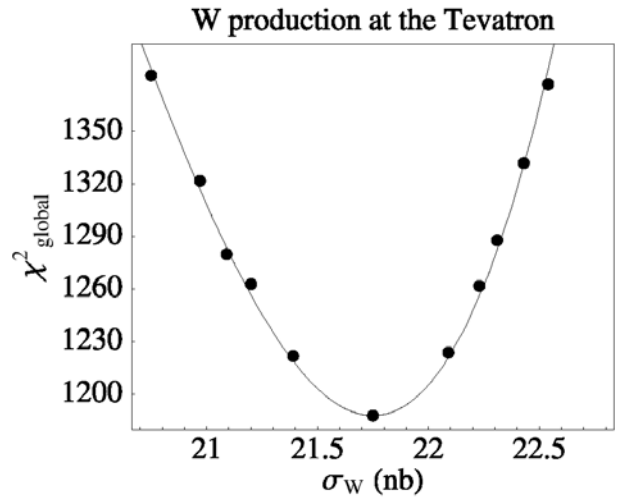

- **let's un-naïve the *W* calculation for finite p_T and then for radiation:**

everything goes through as before...

$$\frac{d\hat{\sigma}_A}{d\hat{t}} = \left(\frac{2\pi\alpha_S}{\hat{s}^2}\right)\left(\frac{G_F M_W^2}{4\pi\sqrt{2}}\right)\frac{8}{9}\left(\frac{\hat{t}^2+\hat{u}^2+2M_W^2\hat{s}}{\hat{t}\hat{u}}\right)$$

$$\frac{d\hat{\sigma}_C}{d\hat{t}} = \left(\frac{2\pi\alpha_S}{\hat{s}^2}\right)\left(\frac{G_F M_W^2}{4\pi\sqrt{2}}\right)\frac{1}{3}\left(\frac{\hat{t}^2+\hat{s}^2+2M_W^2\hat{u}}{-\hat{t}\hat{s}}\right)$$

so that:

$$\frac{d\sigma^{W^\pm}}{dy\,dp_T^2} = \left(\frac{d\sigma^{W^\pm}}{dy}\right)_B\left[\frac{4\alpha_S}{3\pi}\frac{1}{p_T^2}\ln\left(M_W^2/p_T^2\right)e^{-\frac{2\alpha_S}{3\pi}\ln(p_T^2/M_W^2)}\right]$$

so, what's wrong with this?

▷ The Sudakov factor includes the exponentiation of

$$\mathcal{S}(\theta)_{\rm LDLA} = 1 - \frac{2\alpha_S}{3\pi} \ln^2(\theta^2/4)$$

leading double log approximation

I dropped the power of 2 in the exponentiated form

$$\frac{d\sigma}{d\tau dy dp_T^2} = \left(\frac{d\sigma}{d\tau dy}\right)_B \left[\frac{4\alpha_S}{3\pi}\frac{1}{p_T^2}\ln\left(Q^2/p_T^2\right) e^{-\frac{2\alpha_S}{3\pi}\ln^2(p_T^2/Q^2)}\right]$$

please put that in...the web will be correct...

back