Precise Representation The Enlightenment:

Newton

“What is a Philosophe? “One who, trampling on prejudice, tradition, universal content, authority–in a word, all that enslaves most minds–**dares to think for himself**, to go back and search for the clearest general principles, to admit nothing except on the testimony of his experience and his reason.

Denis Diderot

The Enlightenment

time of defiant expectations for Progress

Confidence that all could be known.

materialism, determinism, atheism, freedom

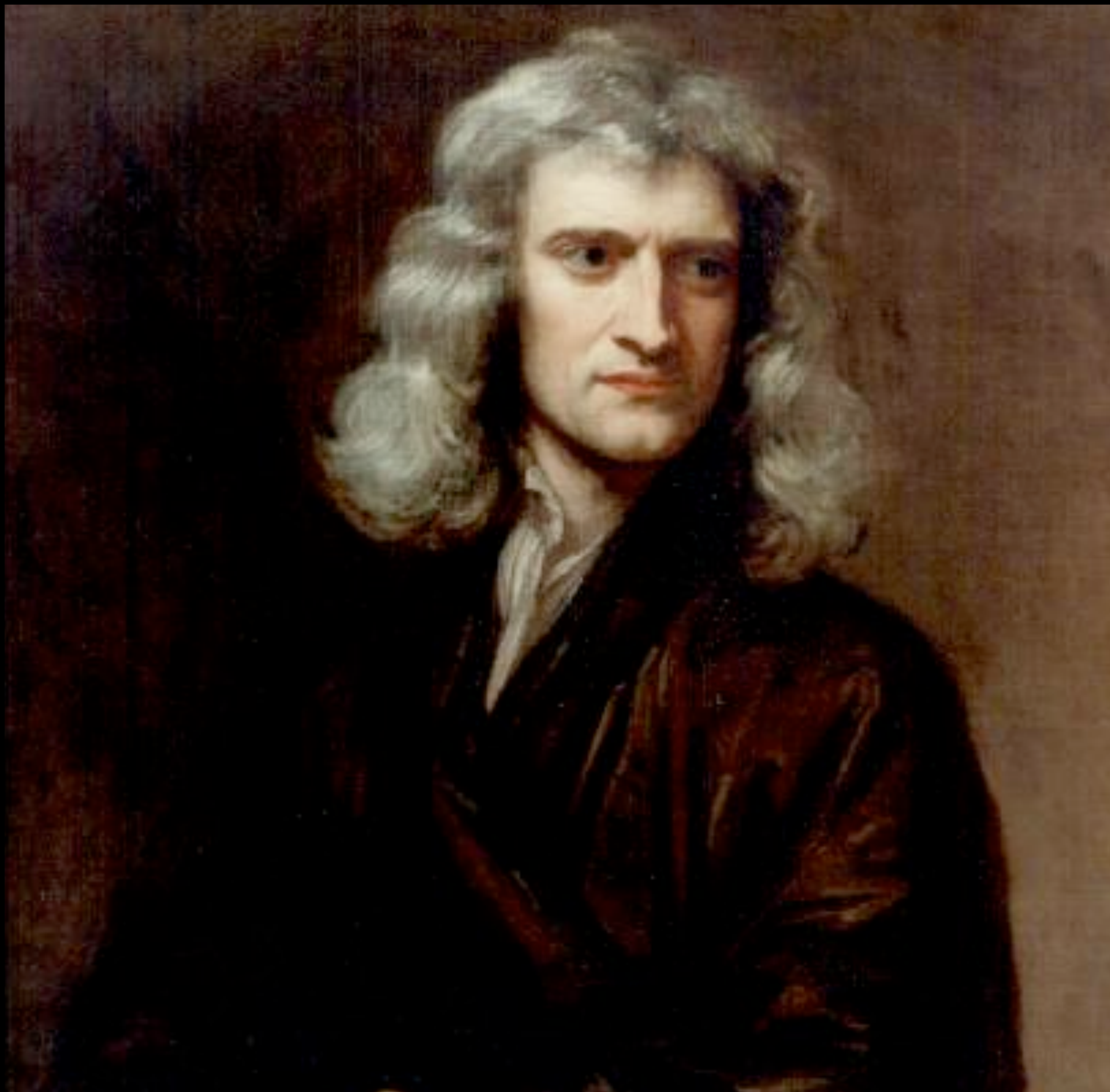
Locke, Swift, Pope, Johnson, Voltaire, Rousseau, d'Alembert, Grimm, Montesquieu, Franklin, Madison, Jefferson, Paine, Hume, Kant, Gibbon, Boswell...

“...man's emergence from his self-incurred immaturity. Immaturity is the inability to use one's own understanding without the guidance of another. This immaturity is self-incurred if its cause is not lack of understanding, but lack of resolution and courage to use it without the guidance of another. The motto of enlightenment is therefore: ... Have courage to use your own understanding!

Kant, to question: what is the Enlightenment?

due, in no small part

to the success of science.



one of a handful of unique system-builders

Newton

Maxwell

Einstein

Feynman

Isaac Newton, 1642-1727

physicist
mathematician
alchemist
politician& administrator
religious historian and zealot

born at a very early age

simply born during civil war

not ideal childhood

socially stunted

Cambridge to study Law

discovered Descartes, Hobbes, Boyle

“

Threatening my father and mother Smith to burn them and the house over them.

Newton assesses his sins in a diary entry at 19:

plague hit London, 1665

the universities emptied

he went home for ~2 years

and changed the world.

In the beginning of the year 1665 I found the Method of approximating series & the Rule for reducing any dignity of any Binomial into such a series. The same year in may I found the method of Tangents…, & in November had the direct method of fluxions & the next year in January had the Theory of Colors & in May following I had entrance into the inverse method of fluxions. And the same year I began to think of gravity extending to the orb of the Moon & (having found out how to estimate the force with which [a] globe revolving with in a sphere presses the surface of the sphere) from Kepler's rule…I deducted that the forces which keep the Planets in their Orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve: & thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the earth, & found them answer pretty nearly. All this was in the two plague years of 1665 & 1666. For in those days, I was in the prime of my age for invention & minded Mathematicks & Philosophy more than at any time since.

Newton on his vacation:

on his own, he had mastered:

Euclid, Descartes, Wallis, van Shooten, de Witt, van Heuraet, Barrow

by 1665, all of known mathematics

by 1666, had in hand the Three Laws of Mechanics, Gravitation, and essentially optics

with an incorrect idea of circular motion

1667: returned to Cambridge to finish next degree

mathematics was the thing

discovered by Isaac Barrow,

the first Lucasian Professor of Mathematics

1669: published first work on infinite series

Barrow steps down in favor of Newton

one lecture per week for life

1671: fluxions circulated privately,

1672: introduction to Royal Society

- **reflecting telescope**
- **theory of colors: particulate theory of light**
- enemy for life: Robert Hooke
- `N. did not take criticism well..um...at all.`
- **elected Fellow**

1675: Hypothesis of Light submitted to RS

- **Hooke claims ideas stolen**
- 1676, Newton writes “stood on shoulders of giants” letter
- **Optics book burns in fire**
- does not publish until after Hooke’s death

good with his hands

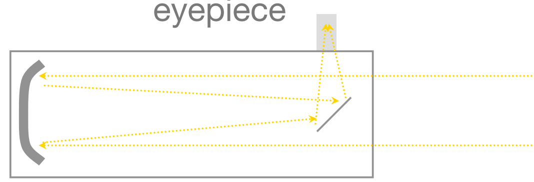
we tend to think of Newton as a theoretical physicist

- **very proficient as an experimenter**
- indeed, since he was a child, he was good at fabricating mechanical devices
- this evolved into optical and chemical experiments...and some ill-fated experimentation on himself

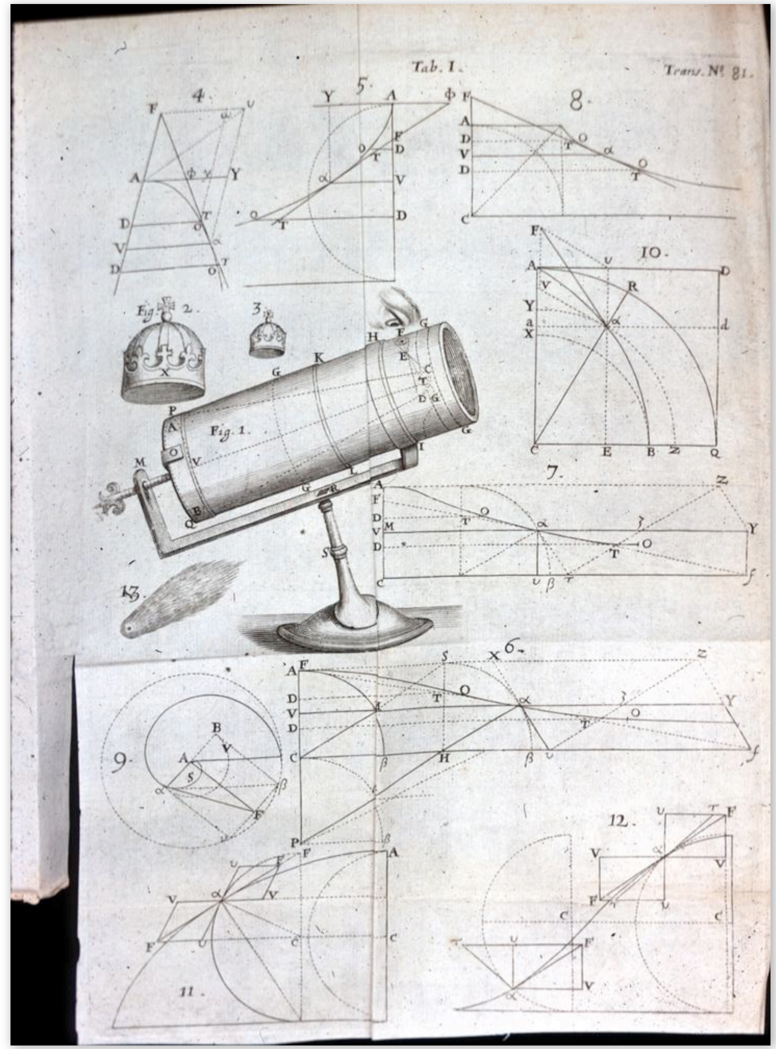
example:

telescope from his induction meeting of the RS in 1672

- The only other telescopes were refracting telescopes - involving multiple lenses.
- Newton had a theory of light that suggested to him that lenses would lead to an irreducible error as the different colors would focus to different spots due to refraction
- His telescope uses only mirrors - so, no refraction issues.
- His 6” long device had the same magnification of a 6’ refractor



This theory led to Hooke’s attack, as he held a different theory.


Tab. I.
Trans. Nº 81.
Fig. 1

1675: Dispensation to N. regarding Anglican Orders

1679: Mother dies

- **Hooke elected President of Royal Society**
- **N. begins manic study of alchemy and religion**
- **he and Hooke correspond briefly about gravitational attraction**
- Hooke had been thinking about an inverse square force since 1666
- Newton thought: central force (in) and centrifugal force (out): this was wrong.
- Hooke was right....could not, or would not, demonstrate it.
- Hooke insisted on a direct, physical force contact (who?)

the delegation

1684: Halley, Hooke, and Wren

- **band together to solve the planetary force problem**
- **if the force is $1/r^2$, what is the orbit's shape?**
- they tire of Hooke's empty boasting
- **Deputized Halley to ask Newton.**
- **Immediately:**
- "an ellipse...because I have calculated it."
- `but can't find it.`

later that year:

Halley receives a 9 page manuscript proving:

if orbit is an ellipse, force must be proportional to $1/r^2$

if force is $1/r^2$, then orbit must be a conic

`ellipse, circle, parabola, or hyperbola`

personal coach

Halley prodded and pushed and cajoled and paid £ !

- all N.'s considerable energy into physics
- 2 years of totally manic, dawn-to-dusk work:

Philosophiae naturalis principia mathematica

aka: "The Principia"

laws of motion, motion of bodies with and without resistance, circular motion, planetary motion, universal gravity, fluid mechanics, tides, precession of earth's axis

in Three Books:

Book I.	his theories, laid down axiomatically "Motion of Bodies"
Book II.	fluids and waves
Book III.	applications, "System of the World,"

in 1726 edition: attack on Descartes' vortices, and his philosophy of science

Principia

uses no explicit calculus

only geometry and series

nearly impenetrable

so difficult in order to protect his priority?

Hooke:

accuses Newton of plagiarism

fight until Hooke’s death in 1703

whereupon, Newton systematically erases any mention of Hooke in *Principia*

REGULÆ PHILOSOPHANDI.
~~HYPOTHESES.~~

~~Hypoth.~~ [Reg.] I. *Causas rerum naturalium non plures admitti debere, quàm quæ & vera sint & earum Phænomenis explicandis sufficiant.*

Natura enim simplex est & rerum causis superfluis non luxuriat.

~~Hypoth.~~ [Reg.] II. *Ideoque effectuum naturalium ejusdem generis eadem sunt causæ.*

Uti respirationis in Homine & in Bestia; descensûs lapidum in *Europa* & in *America*; Lucis in Igne culinari & in Sole; reflexionis lucis in Terra & in Planetis.

Hypoth. III. *Corpus omne in alterius cujuscunque generis corpus transformari posse, & qualitatum gradus omnes intermedios successivè induere.*

Hypoth. IV. *Centrum Systematis Mundani quiescere.*

Hoc ab omnibus concessum est, dum aliqui Terram alii Solem ~~in centro quiescere contendant.~~ PHÆNOMENA.

~~Hypoth.~~ [Phæn.] ~~V~~ I. *Planetas circumjoviales, radiis ad centrum Jovis ductis, areas describere temporibus proportionales, eorumque tempora periodica esse in ratione sesqui~~altera~~ [plicata] distantiarum ab ipsius centro.*

Constat ex observationibus Astronomicis. Orbes horum Planetarum non differunt sensibiliter à circulis Jovi concentricis, & motus eorum in his circulis uniformes deprehenduntur. Tempora verò periodica esse in ratione sesqui~~altera~~ [plicata] semidiametrorum orbium consentiunt Astronomi: & ~~*Flamstedius*, qui omnia Micrometro & per Eclipses Satellitum accuratius definivit, literis ad me datis, quinetiam numeris suis mecum communicatis, significavit rationem illam sesquialteram tam accuratè obtinere, quàm sit possibile sensu deprehendere. Id quod~~ [et] ex Tabula sequente manifestum est.

the rest of the story:

1687: Newton stands up to King James

Principia published

1688: Glorious Revolution

1693: another nervous breakdown

1696: parliament, Warden, then Master of the Mint

moves to London, toast of the town.

President of Royal Society

priority over calculus?

another decades-long fight

with Leibnitz in 1693

Newton didn't publish, was first

Leibnitz' notation, continental prestige spread calculus further, faster

Newton continued the argument

after Leibnitz' death in 1716

Newton

said everything there was to say about mechanics until 1905

that’s good

totally dominated the British scientific scene

that’s not good

progress came after Newton in France and Switzerland

(after they finally rid themselves of vortices)

1727…at the age of 85, buried in Westminster Abbey

bit of a scandal: alchemy

Fighting drove Newton into privacy

decades and more words than on science studying:

Alchemy (Boyle had made respectable)

obsessed with the search for the Philosopher's Stone and the transmutation to gold from lesser materials

A huge library and thousands of words written

known periods where the furnaces in his lab were not out for 5-6 weeks continuously

breakdowns due to mercury poisoning?

The fire that completely destroyed his first work on Optics, "… everyone thought that he would have run mad, he was so much troubled… that he was not himself for a month…" - from alchemical experiments? His furnace fires burned furiously for months at a time

…he abandoned the optics work

and theology

Researched the entirety of religious history

Learned Hebrew and translated the Bible

and its competing literature

Conclusion: Christianity & Trinity, were a 4C corruption

Further, believed that his scientific work was re-creating ancient wisdom known from before Moses: prisca sapientia…connected with his alchemy

spent years trying to recreate the plans for the Temple of Solomon

His religious eccentricities were heretical and could have caused him considerable difficulty, if not prison.

Suppressed by British science after his death…but he had published in Holland

the man

Newton was known as a highly moral individual

- **The "whitest soul" - perhaps to match his hair that turned white when he was in his 30's**
- Totally honest and straightforward, by all accounts
- Sensitive to criticism to an almost unhealthy degree
- **Unattached romantically with any woman**
- **Said to have only smiled once**

Stories of his ability to concentrate abound:

- **He could have people over for dinner and forget they were there**
- **He was known to have stood with reins in hand, but not notice that his horse was gone**
- He would concentrate to the degree of forgetting to eat, forgetting to sleep…for days on end

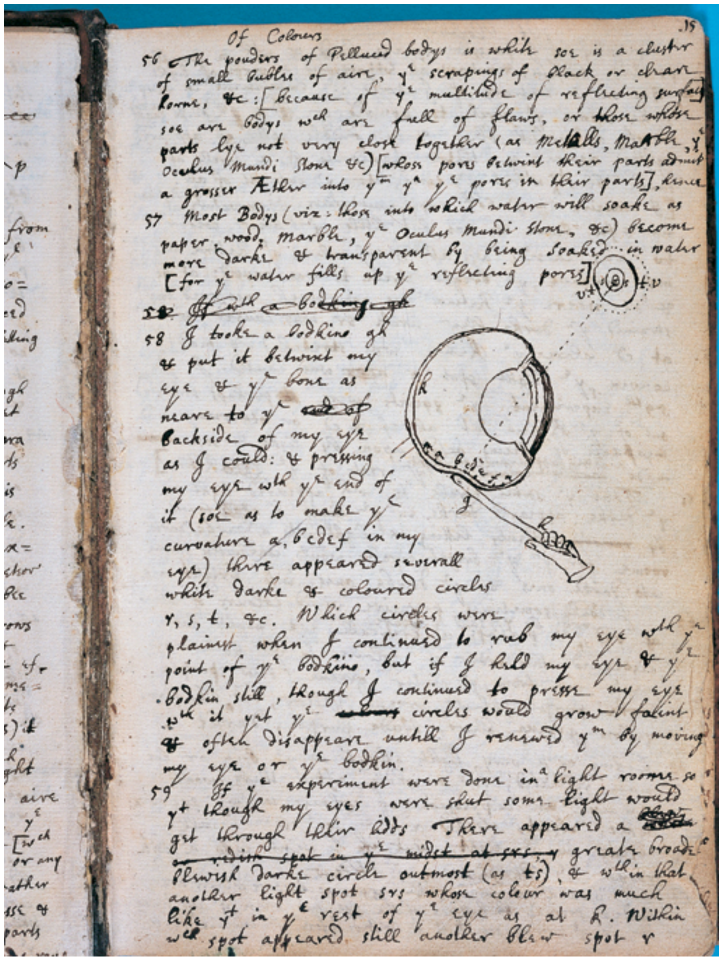
He did experiments that involved

- **staring at the sun so long**

    it took days for him to recover his sight

- **pushed “bodkins” behind his eyeball to study the optical effects of changing his eyeball shape**

- **tasted many of the chemical concoctions that he made with his alchemy**



Of Colours 15

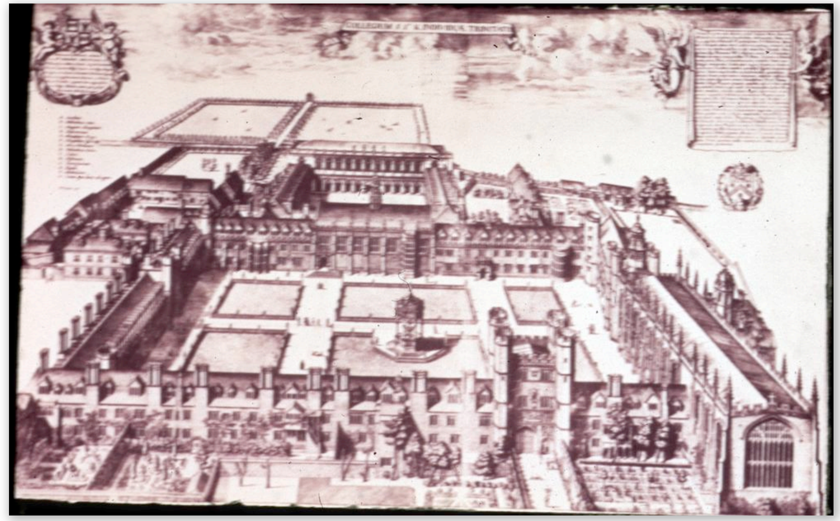
56 The powders of Pellucid bodys is white soe is a cluster of small bubles of aire, ye scrapings of black or charcorne, &c: [Because of ye multitude of reflecting surfaces] soe are bodys wch are full of flaws, or those whose parts lye not very close together (as metalls, marble, ye Oculus Mundi stone &c) [whose pores betwixt their parts admit a grosser Æther into ym ye ye pores in their parts], hence

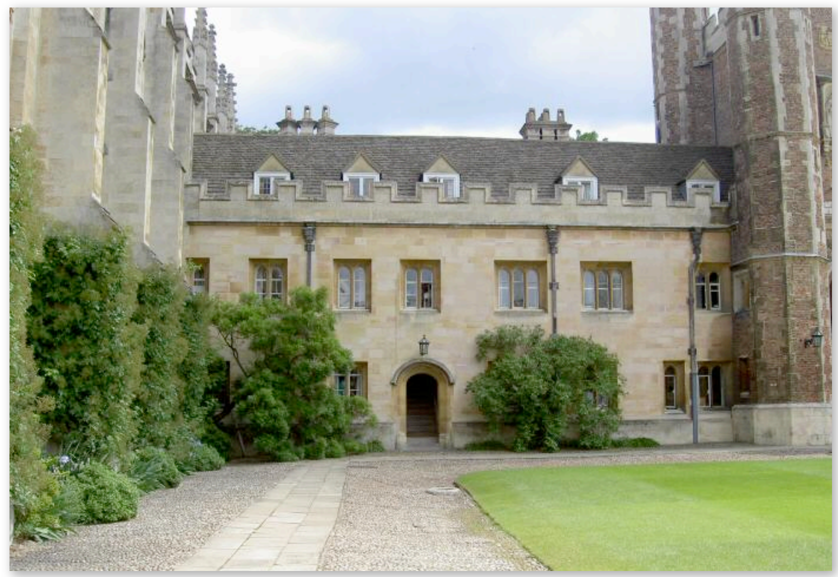
57 Most Bodys (viz: those into which water will soake as paper, wood, marble, ye Oculus Mundi stone, &c) become more darke & transparent by being soaked in water [for ye water fills up ye reflecting pores]

~~58 If wth a bodkin gh~~

58 I tooke a bodkine gh & put it betwixt my eye & ye bone as neare to ye ~~end of~~ backside of my eye as I could: & pressing my eye wth ye end of it (soe as to make ye curvature a, bcdef in my eye) there appeared severall white darke & coloured circles r, s, t, &c. Which circles were plainest when I continued to rub my eye wth ye point of ye bodkine, but if I held my eye & ye bodkin still, though I continued to presse my eye wth it yet ye ~~circles~~ circles would grow faint & often disappeare untill I renewed ym by moving my eye or ye bodkin.

59 If ye experiment were done in a light roome so yt though my eyes were shut some light would get through their lidds There appeared a ~~reddish spot in ye midst at sxs~~ greate broade blewish darke circle outmost (as ts), & wthin that another light spot srs whose colour was much like yt in ye rest of ye eye as at k. Within wch spot appeared still another blew spot r





complicated

His achievements came at personal cost?

But, he knew fame and took pleasure in his later years in London from his fame

lonely, however.

Nicholas Wickins describing his father, John’s first encounter with Newton

“My father’s first Chamber-fellow being very disagreeable to him, he retired one day into the walks, where he found Mr Newton solitary and dejected; Upon entering into discourse they found their cause of retirement the same and thereupon agreed to shake of their present disorderly Companions and Chum together, which they did as soon as conveniently they could do so and so continued for as long as my Father stayed at College.

They shared the suite of home-lab at Trinity for 28 years

"I do not know what I may appear to the world; but to myself I seem to have been only like a boy, playing on the sea-shore, and diverting myself, in now and then finding a smoother pebble, or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Newton, in old age:

what we'll talk about:

infinite series and calculus

important definitions and three laws

circular motion

universal gravitation

examples of 2nd and 3rd laws

a bit of his philosophy of science

optics

Newton's Mathematics

much modern-looking progress by his time

the idea of a function was developing

The free-fall argument:

Galileo used words:

"The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time intervals employed in traveling those distances."

By 1714, Leibniz would say about this: "x is a function of t^2"

By 1734, Euler would write about this: "$x(t) \propto t^2$"

functions were known

geometrically and/or in tabular form

sin(x), log(x), etc...

4 problems being attacked

1. Given a formula for the distance a body covers as a function of time, find the velocity and acceleration at any instant.

Or, the inverse: Given a formula for the velocity (acceleration), find the distance (velocity, distance).

2. Find the tangent to and area under any curve

optics motivated, calculating of angle of reflection/refraction from glass

3. Finding the maximum or minimum of a function

motivated by the angle required for a maximum trajectory of a projectile

Begun by Kepler with his suspicions about wine casks

4. Finding the length of curves

Isaac Barrow came close

to the question of tangents

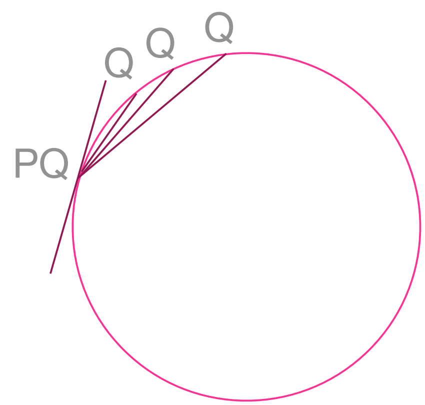
Pascal and Fermat came close

to the issue of areas

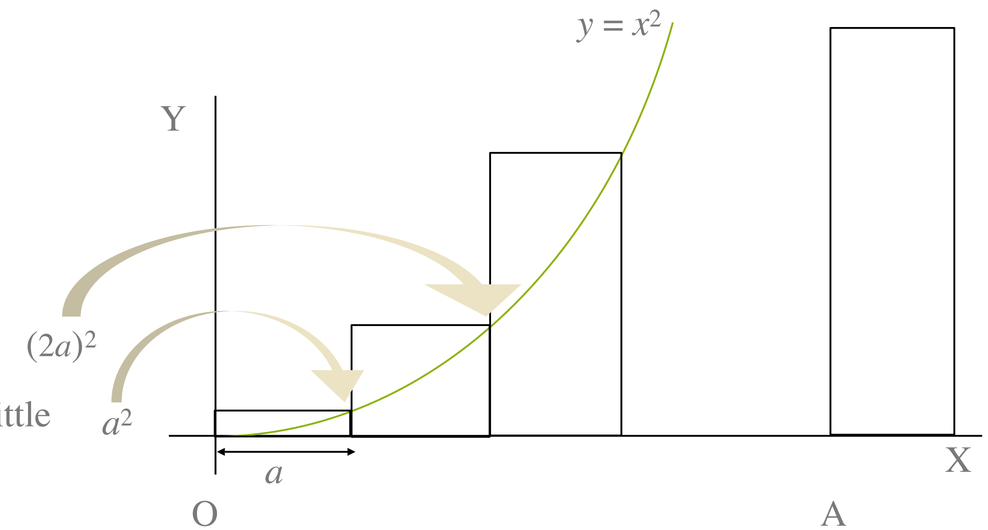
Tangents

Barrow found that a tangent is the limit of a chord of a circle as Q and P approach one another

it's the slope of that limiting chord



Areas



the area can be estimated by adding up the areas of little rectangles:

$$area = a^2 + a(2a)^2 + a(3a)^2 + \ldots$$
$$= a^3(1 + 2^2 + 3^2 + \ldots + n^2)$$

This, in turn, is a known arithmetic series:

$$= a^3\left(\frac{2n^3 + 3n^2 + n}{6}\right)$$
$$= OA^3\left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right)$$

Then, the argument was that this becomes

$OA^3/3$ when $n \to \infty$

abstraction

had returned to mathematics during this period

continuity...length of a curve related to straight lines...

infinity and infinitesimals

Given the bad rap that infinity had carried since Aristotle, it's a remarkable leap to infinite series and continuity

- The concept of **infinity** was not put on a rigorous footing until the 19th Century by Georg Cantor
  - *This is not the first mathematical concept used to advantage in physics without true validity, nor the last.*

Binomial Expansions

known in a rudimentary form by the Arabs in the 13C

One can insert the coefficients in the expansion of any arbitrary integer power of $(a+b)^n$

$$
\begin{aligned}
(a+b)^1 &= a^1 + \begin{pmatrix} 1 \\ 1 \end{pmatrix} a^0 b = a + b \\
(a+b)^2 &= a^2 + \begin{pmatrix} 2 \\ 1 \end{pmatrix} a^1 b + \begin{pmatrix} 2 \\ 2 \end{pmatrix} a^0 b^2 = a^2 + 2ab + b^2 = a + b \\
(a+b)^3 &= a^3 + \begin{pmatrix} 3 \\ 1 \end{pmatrix} a^2 b + \begin{pmatrix} 3 \\ 2 \end{pmatrix} ab^2 + \begin{pmatrix} 3 \\ 3 \end{pmatrix} a^0 b^3 = a^3 + 3a^2 b + 3ab^2 + b^3
\end{aligned}
$$

which can be designated as $\begin{pmatrix} n \\ k \end{pmatrix}$:

$n \backslash k$	0	1	2	3
0	1			
1	1	1		
2	1	2	1	
3	1	3	3	1

For an integer power, this can be proved using mathematical induction - it has a **finite number of terms**

1
1 1
1 2 1
1 3 3 1

what Newton did was interesting

c1664: Newton extended the idea to an algorithm

suggesting that it held for negative as well as fractional powers

not proved rigorously until the 19C

now called the Binomial Theorem

with the especially useful representatives,

The Taylor and Maclaurin Series

Binomial Theorem

Newton's Binomial Theorem says, (in modern language):

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 \ldots \qquad (15)$$

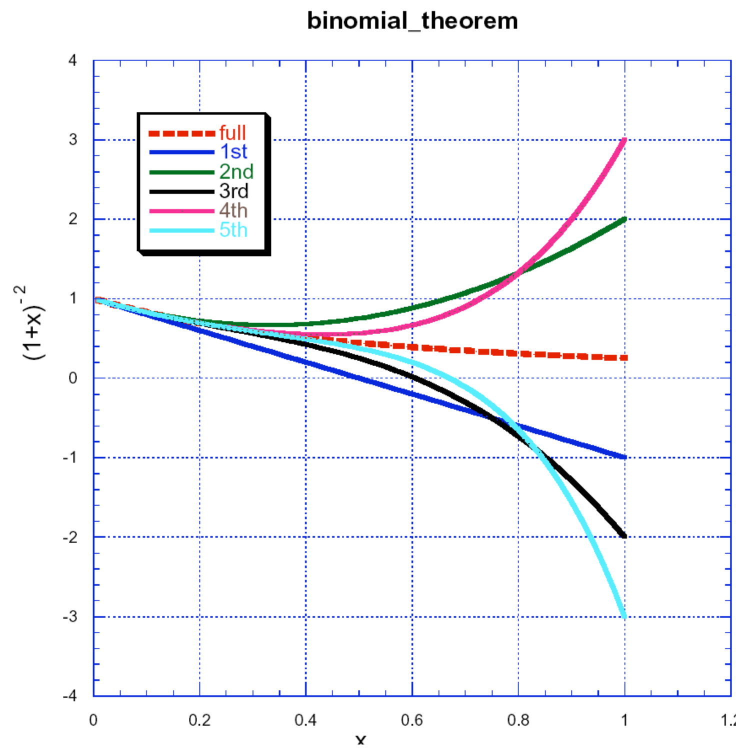
This series is a closed form for integer powers and an infinite series for negative or fractional powers. Notice that each term increases as a power of, here, b. If b is very small compared to a, then each term gets progressively smaller than the one before.

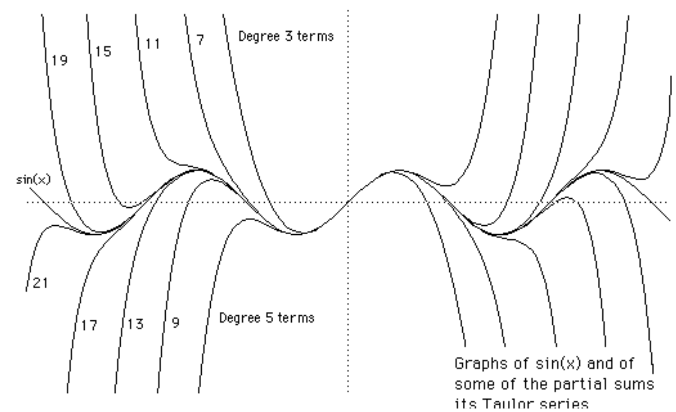
For example, suppose that $a = 1$, $b = x$, and $n = -2$. Then, 15 says:

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots \qquad (16)$$

The top figure shows this function in the red and the various terms added together, slowly approaching the full function. Clearly, for practical use, one needs for x to not be very large. The figure at the bottom shows the sine function plus many of the terms required to make it up. Here, the expansion is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots \qquad (17)$$





Often, for small angles, it is useful to be able to approximate $\sin\theta \sim \theta$.

1669: a small pamphlet

On Analysis by Equations Unlimited in the Number of Their Terms

in it, the invention of differentiation

he makes use of his Binomial Theorem

Et ſublatis ($\frac{4}{9}x^3$ & zz) æqualibus, reliquiſque per o diviſis, reſtat $\frac{4}{9}$ in $3x^2 + 3xo + o^2 = 2zv + ov^2$. Si jam ſupponamus $B\beta$ in infinitum diminui & evaneſcere, ſive o eſſe nihil, erunt v & y æquales, & termini per o multiplicati evaneſcent, quare reſtabit $\frac{4}{9} \times 3xx = 2zv$, ſive $\frac{2}{3}xx\ (= zy) = \frac{2}{3}x^{\frac{3}{2}}y$, ſive $x^{\frac{1}{2}}\ (= \frac{x^2}{x^{\frac{3}{2}}}) = y$. Quare e contra ſi $x^{\frac{1}{2}} = y$, erit $\frac{2}{3}x^{\frac{3}{2}} = z$.

Demonſtratio.

Vel generaliter, ſi $\frac{n}{m+n} \times ax^{\frac{m+n}{n}} = z$; ſive, ponendo $\frac{na}{m+n} = c$, & $m+n = p$, ſi $cx^{\frac{p}{n}} = z$, ſive $c^n x^p = z^n$: tum $x + o$ pro x, & $z + ov$ (ſive, quod perinde eſt, $z + oy$) pro z, ſubſtitutis, prodit c^n in $x^p + pox^{p-1}$, &c. $= z^n + noyz^{n-1}$, &c. reliquis nempe terminis, qui tandem evaneſcerent, omiſſis. Jam ſublatis $c^n x^p$ & z^n æqualibus, reliquiſque per o diviſis, reſtat $c^n p x^{p-1} = nyz^{n-1}\ (= \frac{nyz^n}{z} = \frac{nyc^n x^p}{cx^{\frac{p}{n}}}$ ſive, dividendo per $c^n x^p$, erit $px^{-1} = \frac{ny}{cx^{\frac{p}{n}}}$ ſive $pcx^{\frac{p-n}{n}} = ny$; vel reſtituendo $\frac{na}{m+n}$ pro c, & $m+n$ pro p, hoc eſt, m pro $p-n$, & na pro pc, fiet $ax^{\frac{m}{n}} = y$. Quare e contra, ſi $ax^{\frac{m}{n}} = y$, erit $\frac{n}{m+n}ax^{\frac{m+n}{n}} = z$. Q. E. D.

by expanding functions in an infinite series

to invent the derivative

“he notes that the logic he uses is “...more...shortly explained than accurately demonstrated...”

Newton

1671, *The Method of Fluxions and Infinite Series*

He starts to think of variables ("fluents") *flowing* in time

not necessary, but a useful picture

Think about points on a curve tracing out lines in time
and curves tracing out surfaces in time

hard not think that he's not got motion on his mind

Fluxions

He imagines a tiny increment of time, “delta t” or Δt and invents a notation for the time-variation of x and that of y:

*These, he calls “**fluxions**”…*

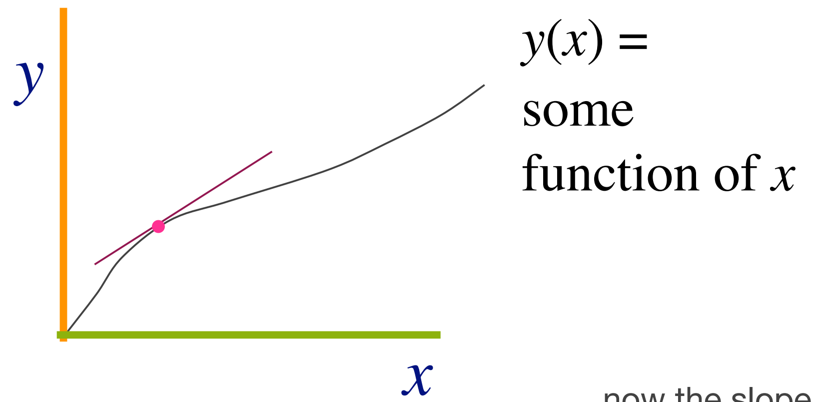
$$\dot{x} = \frac{\Delta x}{\Delta t}; \;\; \dot{y} = \frac{\Delta y}{\Delta t}$$

So, here both x and y are changing in time - you could think of a trajectory

motion followed from functions

arbitrary functions...

finding tangents...



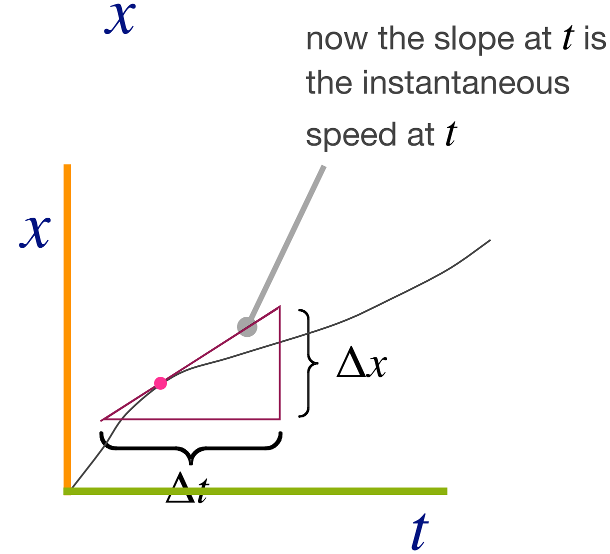
motion:

go some distance, Δx

take some time to do it, Δt

your speed is $\Delta x/\Delta t$

can plot that too

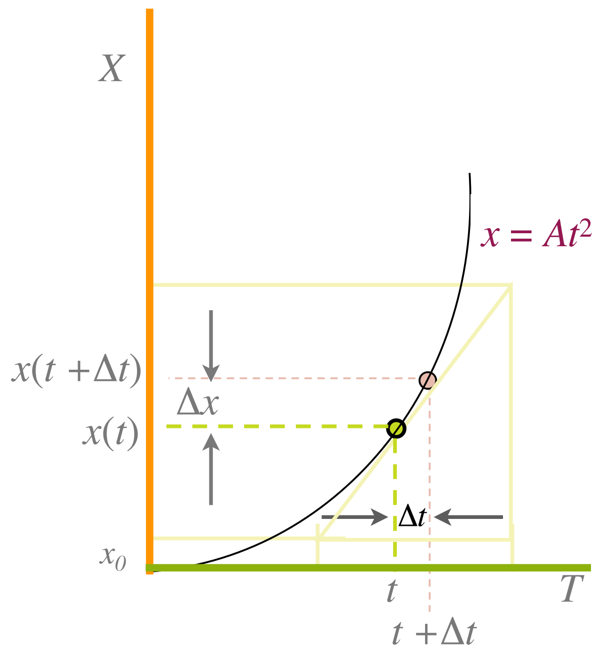


Imagine only one space dimension, using Newton's thought process…

- suppose we have $x = At^2$, (like free-fall?)
- how would you find the instantaneous velocity at a particular value of t?
- since $v = \Delta x / \Delta t$, geometrically, you'd try to draw the tangent at that value of t and measure its slope.

The position at an arbitrary time is $x(t)$ is:

$$x(t) = At^2$$



and at the tiny, little time later,

$$x(t+\Delta t) = A(t+\Delta t)^2$$

The increment during Δt is

$$\begin{aligned}\Delta x &= x(t+\Delta t) - x(t) \text{ substituting} \\ &= A(t+\Delta t)^2 - At^2\end{aligned}$$

Using the Binomial Theorem on the second term,

$$\begin{aligned}A(t+\Delta t)^2 &= At^2 + 2At^1\Delta t + \frac{2(2-1)}{2!}t^0\Delta t^2 \\ &\quad + \frac{2(2-1)(2-2)}{3!}t^{-1}\Delta t^3 \ldots \\ &= A(t^2 + \Delta t^2 + 2t\Delta t) - At^2\end{aligned}$$

cancelling

$$\Delta x = A\Delta t^2 + 2At\Delta t$$

As in Barrow's sense, form the slope:

$$\begin{aligned}\frac{\Delta x}{\Delta t} &= \frac{A\Delta t^2 + 2At\Delta t}{\Delta t} \text{ ...and cancel} \\ &= A\Delta t + 2At\end{aligned}$$

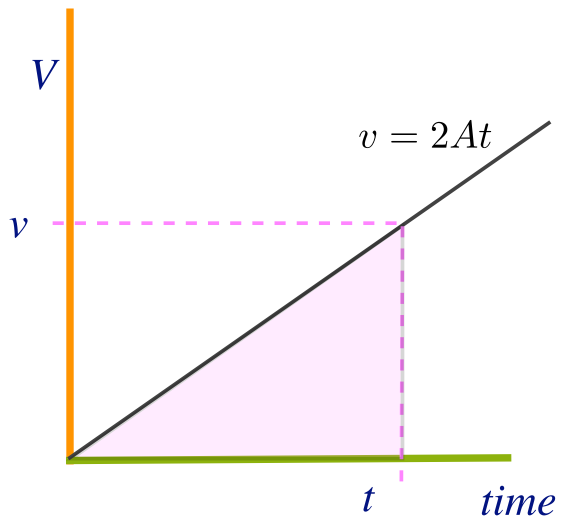
Newton reasoned: Δt is smaller than any number...so he suggested that it can be set to zero.

In modern terminology, this is called the Limit, which was not a rigorously acceptable procedure for more than a century. He did it anyway.

$$v(t) = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right)$$

$$v(t) = 2At$$

This is the **derivative** of x with respect to t, the **instantaneous velocity**.



go backwards?

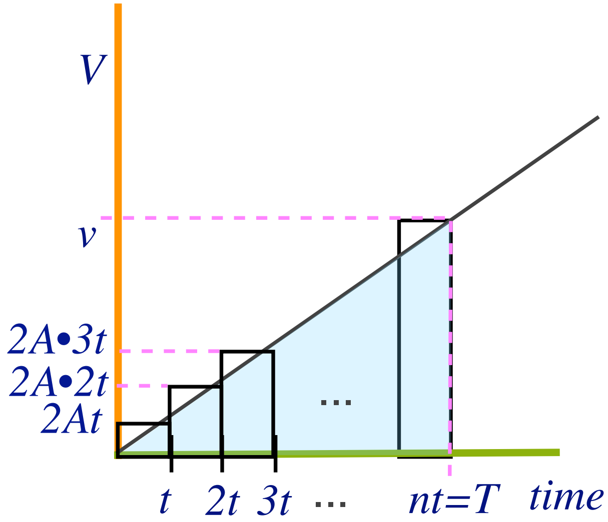
What is the area under a graph of speed that varies like: $v = 2At$?

The area = 1/2 base • height

$$\begin{aligned} \text{area} &= 1/2t \cdot v \\ &= 1/2t \cdot 2At \end{aligned}$$

$$\text{area} = At^2$$ which is the distance! $$x = At^2$$

The area under a curve is called the **integral** of the function that defines that curve, the **distance**



go backwards?

What is the area under a graph of speed that varies like: $v = 2At$?

The area = lots of boxes. The width of each box is t; the height of each box is $2A$ • total t

Add each little rectangle: $\text{area} = 2At \cdot t + 2A2t \cdot t + 2A3t \cdot t + \ldots$

$$\begin{aligned}
\text{area} &= 2At^2(1 + 2 + 3 + \ldots n) \\
&= 2At^2 \frac{(n^2 + n)}{2} \\
&= A(nt)^2(1 + \frac{1}{n}) \rightarrow \lim_{n \rightarrow \infty} = A(nt)^2 \\
area &= AT^2
\end{aligned}$$

which is the distance!

The area under a curve is called the **integral** of the function that defines that curve, the **distance**

EXCERPTA

Ex Epiſtolis D. *NEWTONI* Ad Methodum FLUXIONUM, ET SERIERUM INFINITARUM Spectantibus.

Fragmentum **Epiſtolæ ad D.* Oldenburgium 13 Junii 1676 *miſſæ.*

FRactiones in Infinitas Series reducuntur per diviſionem; & quantitates radicales per extractionem radicum, perinde inſtituendo operationes iſtas in ſpeciebus ac inſtitui ſolent in decimalibus numeris. Hæc ſunt fundamenta harum reductionum; ſed extractiones radicum, multum abbreviantur per hoc *Theorema.*

$$\overline{P+PQ}^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ + \frac{m-2n}{3n}CQ + \frac{m-3n}{4n}DQ + \&c.$$

Ubi $P+PQ$ ſignificat quantitatem cujus Radix, vel etiam dimenſio quævis, vel radix dimenſionis, inveſtiganda eſt. P, primum terminum quantitatis ejus; Q, reliquos terminos diviſos per primum. Et $\frac{m}{n}$, numeralem indicem dimenſionis ipſius $P+PQ$: Sive dimenſio illa integra ſit; ſive (ut ita loquar) fracta; ſive affirmativa, ſive negativa. Nam, ſicut Analyſtæ, pro aa, aaa, &c. ſcribere ſolent a^2, a^3, &c. ſic ego, pro $\sqrt{a}$, $\sqrt{a^3}$, $\sqrt{c.a^5}$, &c. ſcribo $a^{\frac{1}{2}}$, $a^{\frac{3}{2}}$, $a^{\frac{5}{3}}$; & pro $\frac{1}{a}$, $\frac{1}{aa}$, $\frac{1}{a^3}$, ſcribo a^{-1}, a^{-2}, a^{-3}.

Et

* Extat Epiſtola in Tom. 3. Operum Walliſii.

$\frac{\frac{1}{2}n-1}{2}$, $\frac{\frac{3}{2}n-3}{4}$, $\frac{\frac{5}{2}n-5}{6}$, $\frac{\frac{7}{2}n-7}{8}$, $\frac{\frac{9}{2}n-9}{10}$, &c. Ubi n ſignificat numerum dimenſionum ipſius c in denominatore iſtius ſupremi termini. E. g. ut terminorum infra $\frac{1}{22r^4c^6}$, numerales coefficientes inveniantur, pono $n=6$, ducoque $\frac{1}{22}$ (numeralem coefficientem ipſius $\frac{1}{22r^4c^6}$) in $\frac{\frac{1}{2}n-1}{2}$, hoc eſt, in 1; & prodit $\frac{1}{22}$, numeralis coefficiens termini proxime inferioris: dein duco hunc $\frac{1}{22}$ in $\frac{\frac{3}{2}n-3}{4}$, ſive in $\frac{n-3}{4}$, hoc eſt, in $\frac{3}{4}$; & prodit $\frac{3}{88}$ numeralis coefficiens tertii termini in iſta columna. Atque iſta $\frac{3}{88}\times\frac{\frac{5}{2}n-5}{6}$ facit $\frac{5}{176}$ numeralem coefficientem quarti termini; & $\frac{5}{176}\times\frac{\frac{7}{2}n-7}{8}$ facit $\frac{35}{1408}$ numeralem coefficientem infimi termini. Idem in aliis ad infinitum uſque columnis præſtari poteſt: Adeoque valor ipſius DG per hanc Regulam pro lubitu produci.

Adhæc, ſi BF dicatur x, ſitque r latus rectum Ellipſeos, & $e = \frac{r}{AB}$; Erit Arcus Ellipticus

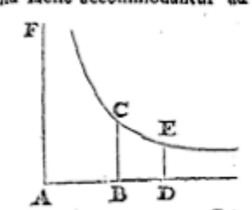
$$BG = \sqrt{rx} \text{ in } \overline{1 + \frac{2 - \frac{1}{2}e}{3r}x + \frac{-2 + 3e - \frac{5}{8}e^2}{5r^2}x^2 + \frac{4 - 9e + \frac{33}{4}e^2 - \frac{7}{16}e^3}{7r^3}x^3 + \frac{-10 + 30e - \frac{133}{4}e^2 + \frac{25}{2}e^3 - \frac{45}{128}e^4}{9r^4}x^4, + \&c.}$$

Quare, ſi ambitus totius Ellipſeos deſideretur; Biſeca CB in F, & quære Arcum DG, per prius Theorema, & Arcum BG per poſterius.

6. Si, vice verſâ, *ex dato arcu Elliptico* DG, *quæratur Sinus ejus* CF; tum dicto $CD = r$, $\frac{CB^2}{CD} = c$, & arcu illo $DG = z$; Erit

$$CF = z - \frac{1}{6c^2}z^3 - \frac{1}{10rc^3}z^5 - \frac{1}{14r^2c^4}z^7 - \&c.$$
$$\qquad\qquad + \frac{13}{120c^4}z^5 + \frac{71}{420rc^5}z^7$$
$$\qquad\qquad\qquad - \frac{493}{5040c^6}z^7$$

Quæ autem de Ellipſi dicta ſunt, omnia facile accommodantur ad Hyperbolam; mutatis tantum ſignis ipſorum c & e ubi ſunt imparium dimenſionum.



7. Præterea, ſi ſit CE Hyperbola, cujus Aſymptoti AD, AF rectum angulum FAD conſtituant; & ad AD erigantur utcunque perpendicula BC, DE occurrentia Hyperbolæ in C & E: & AB dicatur a, BC b, & area BCED z;

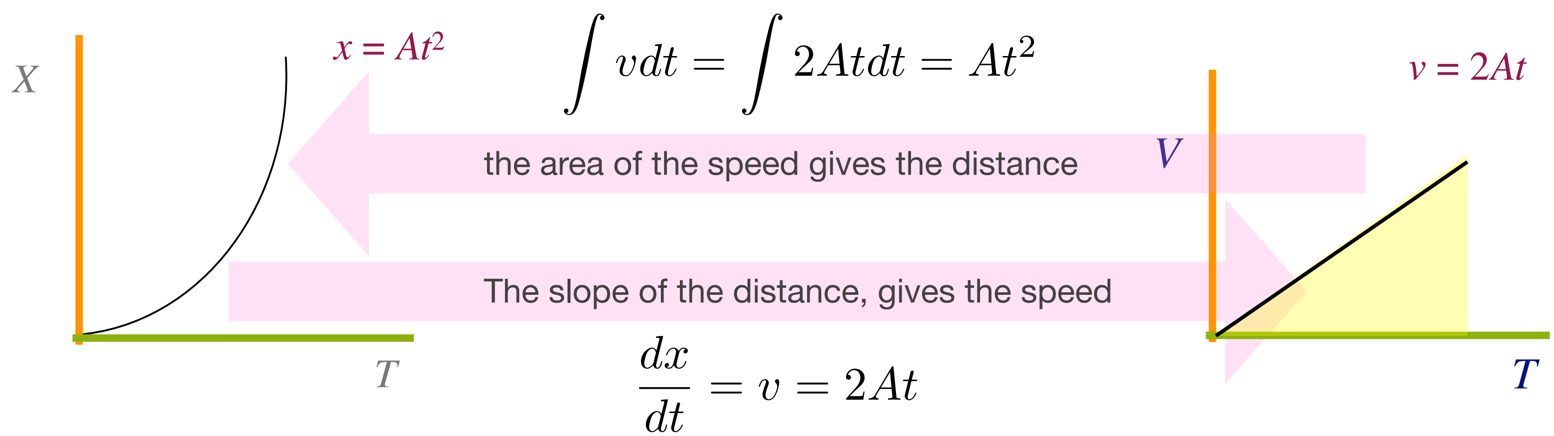
Erit

what he's saying is

The slope - the derivative - of x = f(t)

is the inverse of

the area of the result of that derivative



the classic priority dispute:

over discovery/invention(?) of Calculus

between Newton and

Gottfried Wilhelm Leibnitz (1646-1716)

German diplomat, philosopher, mathematician had first-hand contact with Newton, Huygens and both scientific societies in Paris and London

Had asked for and received explanation of the Binomial Theorem from N in the 1670's

Between 1675 and 1685, he (probably) independently invented the differential and integral calculus

and promptly published in 1684…writing well, and inventing a notation that people could adopt easily: dx, dy, ∫ f(x) dx were his inventions

it's clear

from letters and drafts that (luckily)

Newton provided to others

that he was thinking about the issues in Calculus as early as 1664

His neurotic inhibitions towards publication got in his way

He published his first mathematical works only in 1704

actually, they got along

...but their friends got in the way

and their relation became very sour

The Royal Society commissioned a study

It came down firmly on the side of Newton

who was president of the RS at the time

and wrote most of the report himself.