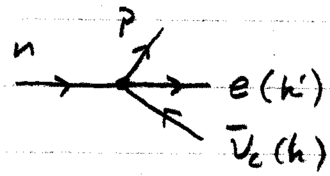
β decay.

To do the standard (old fashioned) calculation of neutron β decay, we start with

$$\mathcal{T} = \int d^4x \, \langle p e \nu_e | \mathcal{L} | n \rangle$$

I'll proceed a bit differently, because I want to display the exponentials explicitly.--

The graph is an effective interaction,




"4-fermion interaction"

$k' = (\varepsilon, \vec{k}')$
$k = (\omega, \vec{k})$

The hadronic vertex really contains all sorts of ugly virtual physics — non-perturbative — and must be handled with parameterizations which respect the symmetries, but are form factors.

Note, a complete job involves real nuclear wavefunctions and complicated Coulombic effects on the electron escaping — not here! (Actually, Alex Barnes is an expert...)

Using the maximal parity violation assumption for leptons and a general matrix element for hadron -

$$\mathcal{T} = \frac{i G_F}{\sqrt{2}} \int d^4x \; \bar{\Psi}_p(x)\, \Gamma^\mu \Psi_n(x)\, \bar{\Psi}_e(x)\, \gamma_\mu (1-\gamma_5) \Psi_\nu$$

burying uncertainties in $\Gamma^\mu$. For a general analysis, this is quite complicated - for just V and A, it's still complicated

separately, write $\qquad h_\mu^0 = v_\mu^0 + a_\mu^0$

$$\langle p | v_\mu^0 | n \rangle = \langle p | \Gamma_\mu^V | n \rangle$$

where generally,

"weak magnetism"

"induced pseudoscalar"

$$\Gamma_\mu^V = \gamma_\mu f_1(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{M_p + M_n} f_2(q^2) + f_3(q^2)\, q_\mu$$

This is the most general Rank 1 tensor, using the available tensors. The $f$'s are scalar structure functions.

Also, $\qquad \langle p | a_\mu^0 | n \rangle = \langle p | \Gamma_\mu^A | n \rangle$

$$\Gamma_\mu^A = \gamma_\mu \gamma_5\, g_1(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{M_p + M_n} \gamma_5\, g_2(q^2) + g_3(q^2)\, q_\mu \gamma_5$$

"2nd class current"

There are reasons why these 6 terms reduce to just a few. Since momentum transfer is so low (for example $M_n - M_p \simeq 1.3$ MeV), we can ignore the terms proportional to $q^\mu = p_n^\mu - p_p^\mu$ ..

So, write $\Gamma^\mu = f_1(q^2)\gamma^\mu + g_1(q^2)\gamma^\mu\gamma_5$.

Let's write a nuclear wavefunction as.

$$\psi_p(x) = \sqrt{E_p + M_p} \begin{pmatrix} \chi_p(x) \\ \frac{\vec{\sigma}\cdot\vec{p}_p}{E_p + M_p}\chi_p(x) \end{pmatrix}$$

and use plane waves

$$\chi_p(x) = \chi_p^\prime(x) e^{ip_p\cdot x}$$

nuclear wavefunctions.. presumably products of nucleon wavefunctions.

-- same for neutron --

So,

$$\mathcal{J} = \frac{iG_F}{\sqrt{2}} \int d^4x \sqrt{E_p + M_p}\sqrt{E_n + M_n}$$

$$\chi_p^\dagger \left(1, \frac{\vec{\sigma}_p\cdot\vec{p}_p}{E_p + M_p}\right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \left[f_1(q^2)\gamma^\mu + g_1(q^2)\gamma^\mu\gamma^5\right] \begin{pmatrix} 1 \\ \frac{\vec{\sigma}_n\cdot\vec{p}_n}{E_n + M_n} \end{pmatrix} \chi_n$$

$$\bar{\nu}_e \gamma_\mu (1-\gamma_5) v_\nu \, e^{-i(k+k')\cdot x} \, e^{i(p_n - p_p)\cdot x}$$

now, a series of approximations and conventions...

Since energies of the leptons are small $\sim m_e c$, the deBroglie wavelengths are $\sim \hbar/m_e c \sim 10^{-11}$ cm which is very much larger than a typical nucleus.

so

$$e^{i(\vec{k}'+\vec{k})\cdot\vec{x}} \simeq 1 \qquad \text{"\underline{Allowed} \underline{decays}"}$$

The nucleons are very <u>non-relativistic</u> (fermi motion is few MeV's and kinetic energy of recoil in decay is few keV's or less)..

Then,

$$\bar{\psi}_p \gamma^\mu \psi_n \rightarrow \psi_p^+ \psi_n = \bar{\psi}_p \gamma^0 \psi_n$$

$$\bar{\psi}_p \gamma^\mu \gamma_5 \psi_n \rightarrow \psi_p^+ \vec{\sigma} \psi_n = \bar{\psi}_p \gamma^0 \gamma^i \psi_n$$

$$q^2 \simeq 0$$

Defining, conventionally, $\Delta \equiv E_n - E_p = M_n - E_p$,

$$\mathcal{T} = \frac{iG_F}{\sqrt{2}} 2\sqrt{M_n M_p} \int d^4x \left[ \chi_p^+ f_1(0) \chi_n \bar{u}_e \gamma_0 (1-\gamma_5) v_\nu \right.$$

$$\left. + \chi_p^+ g_1(0) \vec{\sigma} \chi_n \bar{u}_e \vec{\gamma} (1-\gamma_5) v_\nu \right] e^{i(\omega+\varepsilon-\Delta)t} e^{i(\vec{p}_n-\vec{p}_p)\cdot\vec{x}}$$

Do the $dx^0$ integration

$$\mathcal{T} = \frac{iG_F}{\sqrt{2}} (2\pi) \delta(\omega+\varepsilon-\Delta) 2\sqrt{M_n M_p} \int d^3x \left[ \quad \right] e^{i(\vec{p}_n-\vec{p}_p)\cdot\vec{x}}$$

Standard notation is to define

$$C_V \equiv f_1(0)$$
$$C_A \equiv g_1(0)$$

and the <u>Impulse Approximation</u> presumes quasi-independence among nucleons. This ignorance is sometimes parameterized

classical notation

$$\langle 1 \rangle \equiv \sum_{i=1}^{A} \int \chi_i^+(x)\, \tau_i^+ \chi_i(x)\, d^3x \qquad \text{"Fermi M.E."}$$

$$\langle \vec{\sigma} \rangle \equiv \sum_{i=1}^{A} \int \chi_i^+(x)\, \vec{\sigma}\, \tau_i^+ \chi_i(x)\, d^3x \qquad \text{"G.T. M.E."}$$

and the $\tau^+$'s are the isospin raising operators to transform general nuclear transitions

$$(A, Z) \longrightarrow (A, Z+1) + e^- + \bar{\nu}$$

$$\Delta I_3 = 1$$

So,

$$\mathcal{T} = \frac{iG_F}{\sqrt{2}} (2\pi)^4 \delta(\omega - \varepsilon - \Delta)\, \delta^3(\vec{p}_n - \vec{p}_p)\, 2\sqrt{M_p}\sqrt{M_n}$$

$$\times \left[ C_V \langle 1 \rangle\, \bar{u}_e \gamma_0 (1-\gamma_5) v_\nu + C_A \langle \vec{\sigma} \rangle \cdot \bar{u}_e \vec{\gamma} (1-\gamma_5) v_\nu \right]$$

The decay rate is

$$d\Gamma = \frac{\bar{\sum}\sum|T|^2}{2E_n} \frac{d^3\vec{k}}{(2\pi)^3 2\omega} \frac{d^3\vec{k}'}{(2\pi)^3 2\varepsilon} \frac{d^3\vec{p}_p'}{(2\pi)^3 2E_p}$$

$\langle 1\rangle\langle\vec{\sigma}\rangle$ cross terms vanish when nuclear spins are averaged, so

$$d\Gamma = G^2 \frac{4\cancel{M_p}\cancel{M_n}}{2}\; 2\pi\,\delta(\omega+\varepsilon-\Delta)\,\frac{1}{2\cancel{M_n}}\sum_{\varepsilon,\nu}$$
$$\times\left\{ C_V^2 |\langle 1\rangle|^2\, |\bar{u}_e\gamma_0(1-\gamma_5)v_\nu|^2 + C_A^2 |\langle\vec{\sigma}\rangle|^2 |\bar{u}_e\vec{\gamma}(1-\gamma_5)v_\nu|^2 \right\}$$
$$\times \frac{d^3k}{(2\pi)^3 2\omega}\frac{d^3k'}{(2\pi)^3 2\varepsilon}\frac{1}{2\cancel{M_p}}$$

Let's calculate for a pure Fermi transition $\Rightarrow$ $C_A = 0$.
In particular, $O^{14} \to N^{14} e^+ \nu_e$
so, change lepton current to $\bar{u}_\nu(k)\gamma^0(1-\gamma_5)v_e(k')$. We need to keep track of the isospin structure,

$$\langle 1\rangle = \langle N^A_{Z-1} | \sum_j \tau_j^+ | N^A_Z\rangle$$

$$= \langle I, I_3-1 |\, I^- \,| I, I_3\rangle \quad \left(\times\ F(Z, E_p)\ \text{coulomb correction}\right)$$

w/

$$I^\pm |I\, I_3\rangle = \sqrt{(I\pm I_3+1)(I\mp I_3)}\;|I, I_3\pm 1\rangle$$

$O^{14}$, $N^{14*}$, $O^{14}$ form an isotriplet, so

$$\left.\begin{array}{l} I_3 \text{ of } O^{14} = 1 \\ I_3 \text{ of } N^{14} = 0 \end{array}\right\}\quad I^-|O^{14}\rangle = \sqrt{2}\,|N^{14}\rangle$$

So,

$$d\Gamma = \frac{G^2}{2}(2\pi)\,\delta(\varepsilon+\omega-\Delta)\sum_{e\nu}\left|\bar{u}_\nu\gamma_0(1-\gamma_5)v_e\right|^2\cdot \underset{\uparrow}{2}\cdot dK\,dK'\,|C_V|^2$$

Simple .. not really practical

neglecting lepton masses.

$$\Sigma|\quad|^2 = \sum_e\sum_\nu\left[\bar{u}\gamma_0(1-\gamma_5)v\bar{v}(1+\gamma_5)\gamma_0 u\right]$$

$$= \mathrm{Tr}\left[\not{k}\gamma_0(1-\gamma_5)\not{k}'(1+\gamma_5)\gamma_0\right]$$

$$= \mathrm{Tr}\left[\not{k}\gamma_0\not{k}'(1+\gamma_5)(1+\gamma_5)\gamma_0\right]$$

$$= 2\,\mathrm{Tr}\left[\not{k}\gamma_0\not{k}'(1+\gamma_5)\gamma_0\right]$$

$$= 2\cdot 4\left[k^0k^{0\prime}+k^{0\prime}k^0-(k\cdot k')g^{00}\right]$$

$$= 8\left[2\omega\varepsilon-(\omega\varepsilon-\omega\varepsilon\cos\theta_{e\nu})\right]$$

$$= 8\omega\varepsilon\,(1+\cos\theta_{e\nu}) \qquad (= 8\omega\varepsilon(1-\cos\theta) \text{ for } \underline{\text{scalar}})$$

by late 1950's,
by measuring nuclear recoil, the $\nu$ direction was inferred and $\theta_{e\nu}$ could be measured.

Doing the same thing for pure G.T. gives

$$\Sigma|\text{lepton}|^2 = 8\omega\varepsilon\left(1-\tfrac{1}{3}\cos\theta_{e\nu}\right) \qquad \left(= 8\omega\varepsilon(1+\tfrac{1}{3}\cos\theta) \text{ for } \underline{\text{tensor}}\right)$$

In general, for mixed decays,

define $\xi \equiv |C_V|^2|\langle 1\rangle|^2 + |C_A|^2|\langle\sigma\rangle|^2$

$$\xi a \equiv |C_V|^2|\langle 1\rangle|^2 - \tfrac{1}{3}|C_A|^2|\langle\sigma\rangle|^2$$

$$d^3k' = k'^2 dk' \, d\Omega'$$

we can write $\quad d^3k = \omega^2 d\omega \, d\Omega$

generally. –

$$d\Gamma_F = \frac{G^2}{\cancel{2}(2\pi)^5} \delta(\varepsilon+\omega-\Delta) \cancel{8\omega\varepsilon} \left(1 + \frac{\vec{p}_e \cdot \vec{p}_\nu}{\omega\varepsilon}\right) |C_V|^2 |\langle 1 \rangle|^2 \frac{\varepsilon^2 d\varepsilon \, d\Omega'}{\cancel{2\varepsilon}} \frac{\omega^2 d\omega \, d\Omega}{\cancel{2\omega}}$$

$$d\Gamma_{GT} = \frac{G^2}{2(2\pi)^5} \delta(\varepsilon+\omega-\Delta) \, 8\omega\varepsilon \left(1 - \frac{1}{3} \frac{\vec{p}_e \cdot \vec{p}_\nu}{\omega\varepsilon}\right) |C_A|^2 |\langle \sigma \rangle|^2 \frac{\varepsilon^2 d\varepsilon \, d\Omega'}{2\varepsilon} \frac{\omega^2 d\omega \, d\Omega}{2\omega}$$

putting them together, with the above definitions...
Integrating over neutrino energies gives

$$\varepsilon + \omega - \Delta = 0$$

$$\omega = \Delta - \varepsilon$$

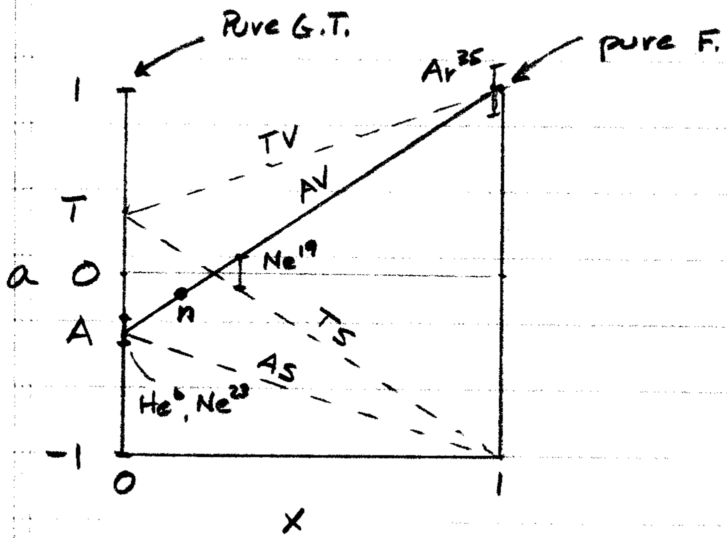
so, generally,

$$\frac{d\Gamma}{d\varepsilon \, d\Omega \, d\Omega'} = \frac{G^2}{(2\pi)^5} \varepsilon^2 (\Delta - \varepsilon)^2 \, \xi \left(1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{\varepsilon(\Delta-\varepsilon)}\right)$$

$$x \equiv \frac{C_V \langle 1 \rangle}{C_V\langle 1 \rangle + C_A \langle \sigma \rangle} \qquad \text{"Fermi fraction"}$$

and

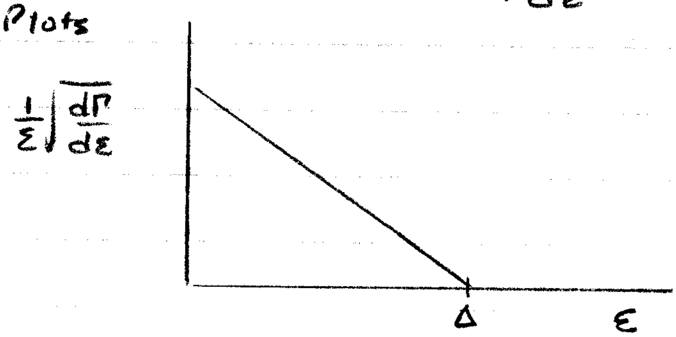
$$\xi a \equiv |C_V|^2 |\langle 1 \rangle|^2 - \tfrac{1}{3} |C_A|^2 |\langle \sigma \rangle|^2$$




by choosing the coordinate system, we can first integrate over all neutrino angles — $2\pi$ — then over the electron angle, and get,

$$\frac{d\Gamma}{d\varepsilon} = \frac{G_F^2}{2\pi^3} \xi \varepsilon^2 (\Delta - \varepsilon)^2$$

What's traditionally plotted is $\frac{1}{\varepsilon}\sqrt{\frac{d\Gamma}{d\varepsilon}}$ vs. $\varepsilon$ called Kurie Plots



Actually, the coulomb effects are very important and it's proper to write,

$$\Gamma = \frac{G^2}{2\pi^3} \xi \underbrace{\int_{m_e}^{\Delta} F(Z,\Delta)\, \varepsilon^2 (\Delta-\varepsilon)^2 \, d\varepsilon}_{\text{called } f_0(Z,\Delta), \text{ the "Fermi Integral"}}$$

$$\Gamma = \frac{G^2}{2\pi^3} \xi f_0$$

calculated and tabulated for many species. The lifetime is $\tau = \Gamma^{-1}$, but usually the "comparative half-life", $f_{0,corr}\, t_{1/2}$ or "$ft$",

$$ft = \frac{2\pi^3 \ln 2}{G_F^2 \xi} = 3080.6 \pm 2.1 \text{ s}$$

.. actually, early indication for $Z^0$ between 70 - 150 GeV was predicted from radiative corrections.

Suppose the neutrino has a mass.. ,

the $\delta(\omega + \varepsilon - \Delta)$ changes – when $\varepsilon$ is maximum, $\omega \to m_\nu$ and $\varepsilon = \Delta - m_\nu$

So, $\quad d^3k = 4\pi k^2 dk$

$$k\,dk = \omega\, d\omega$$

$$d^3k = 4\pi k \omega\, d\omega$$

and neutrino energy integration is

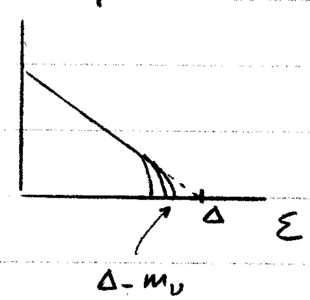
$$k\omega\, \delta(\omega + \varepsilon - \Delta)\, d\omega$$
$$\sqrt{\omega^2 - m_\nu^2}\; \omega\, d\omega\, \delta(\omega + \varepsilon - \Delta)$$

or – replace $\omega$ by $\Delta - \varepsilon$: $\sqrt{(\Delta-\varepsilon)^2 - m_\nu^2}\,(\Delta - \varepsilon)$

and $\beta$ spectrum is

$$\frac{d\Gamma}{d\varepsilon} \propto \varepsilon^2 \left[ (\Delta - \varepsilon)^2 - m_\nu^2 \right]^{1/2} (\Delta - \varepsilon)$$

and the Kurie plots look like




There are a couple of experiments with positive results, but there are also directly disagreeing experiments.

α, proportional to $(E_0 - E)^2$, which was required to account for an apparent nonlinearity in the spectrum (possibly caused by electrons scattered in the spectrometer, or by variations with energy in the spectrometer acceptance). To account for the multiplicity of the spectrum as a result of the large number of levels in the final state of the valine molecule (a total of 51 levels were used (Kaplan et al. 83); see also the discussion below), the required folding was included in the analysis. From a fit to the parameters $m_\nu$, $E_0$, and α, Boris et al. (85, 87) have derived a neutrino mass, based on the valine final state, of

$$m_\nu = (30.3\ ^{+2}_{-8})\ \text{eV}, \tag{2.5}$$

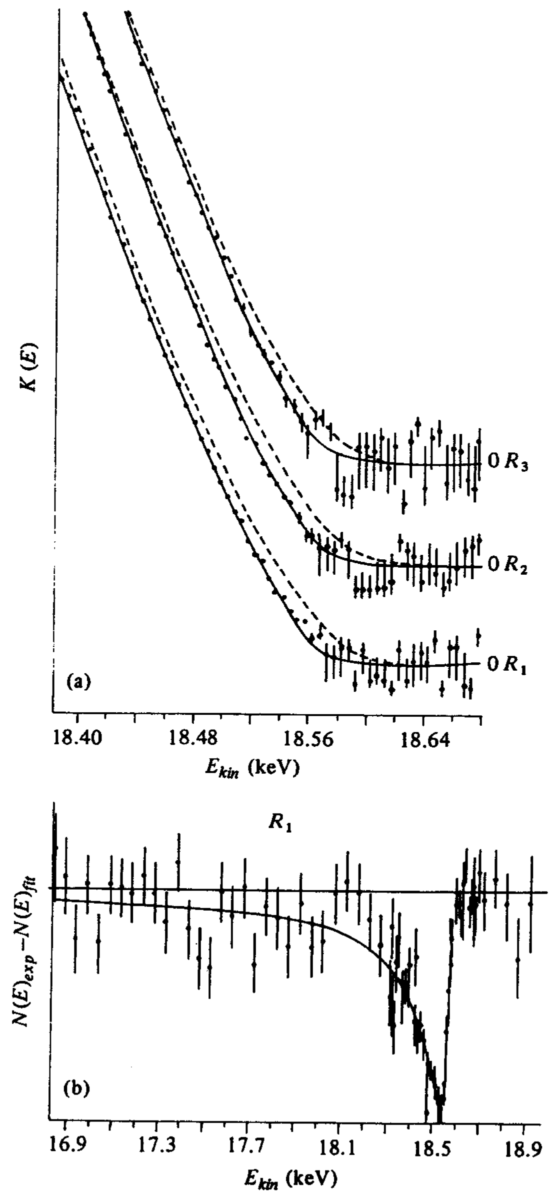
as well as a value for the total decay energy of $E_0 = (18{,}600.3 \pm 0.4)$ eV. For the case (probably unjustified) in which the final state configurations are allowed to vary over a wide range, a conservative range of mass values, $17 < m_\nu < 40$ eV, is obtained. The observed linearized spectrum near the endpoint, as well as the difference between the experimental spectrum and the calculated spectrum for $m_\nu = 0$ are shown in Figure 2.5.

The ITEP measurement, while not contradicting Bergkvist's results, is the only experiment to date that has provided positive evidence for a finite neutrino mass. Over the course of this series of experiments, the values reported for the mass have changed only little and appear to have been unaffected by several drastic changes in procedures such as improvement of the resolution or inclusion of the Lorentzian width (Simpson 83). While this seems at first reassuring, it is, on closer inspection, rather difficult to understand. As mentioned above, the resulting spectrum depends strongly on the structure of the low energy tail in the resolution function, as well as on the procedure for determining the extrapolated endpoint. In an analysis, endpoint and mass are strongly correlated, and a small change in the extrapolated endpoint energy, such as that resulting from a distortion of the spectrum (the α term), may change the mass in a significant way.

*The Zurich experiment*. A toroidal spectrometer similar to that used by the ITEP group was built by Kündig and his collaborators in Zurich, Switzerland (Fritschi et al. 86, 91; Kündig et al. 86). This spectrometer is depicted schematically in Figure 2.6. The tritium source was prepared by implanting a monolayer of tritiated hydrocarbon onto a thin $SiO_2$ layer which, in turn, was deposited on a carbon backing. The source area was 157 cm$^2$ and the source strength amounted to about 50 millicuries. The electrons emerging from the source were decelerated by about 15 keV with the help of an electric potential. The resulting truncated spectrum was accepted by the momentum analyzing spectrometer set at a constant mag-



**Figure 2.5.** (a) Observed tritium beta spectrum (Kurie plot) near the endpoint for three runs ($R_1$,$R_2$,$R_3$) from the ITEP experiment of Boris et al. (85). The solid curves are overall fits based on the valine final state. The dashed curves are calculated spectra for $m_\nu = 0$. (b) Difference between "best fit" to the experimental data and calculated spectrum for $m_\nu = 0$. The best fit was achieved for a set of $E_0$, α and $m_\nu = 34.8$ eV. The $m_\nu = 0$ horizontal line is based on the same parameters $E_0$ and α.



is now fixed: $P = -(p + q)$.] So the number of final states is

$$d^2N = \frac{16\pi^2}{h^6} p^2 q^2 \, dp \, dq.$$

Also, for given values of $p$ and $E$ the neutrino momentum is fixed,

$$q = (E_0 - E)/c,$$

within the range $dq = dE_0/c$. Hence

$$\frac{dN}{dE_0} = \frac{16\pi^2}{h^6 c^3} p^2 (E_0 - E)^2 \, dp. \qquad (4.2$$

If $|M|^2$ is regarded as a constant, this last expression gives the electron spectrur

$$N(p) \, dp \propto p^2 (E_0 - E)^2 \, dp,$$

and thus if we plot $[N(p)/p^2]^{1/2}$ against $E$, a straight line cutting the $x$-axis a $E = E_0$ should result. This is called a *Kurie plot*. For many $\beta$-transitions, th Kurie plot is linear, as shown for the decay $H^3 \rightarrow He^3 + e^- + \bar{\nu}$ (mixed). I Fig. 4.3, $F(z, p)$ is a correction factor ($\sim 1$ in this case) to account for the Couloml effects on the electron wave function. For a nonzero neutrino mass, the plo bends over near the endpoint, and cuts the axis at $E' = E_0 - m_\nu c^2$. Thi particular decay, with small $E_0$, has been used to set an upper limit, $m_\nu < 60$ eV

The total decay rate is obtained by integrating (4.2) over the electro spectrum. As a very crude approximation in some decays we can consider th

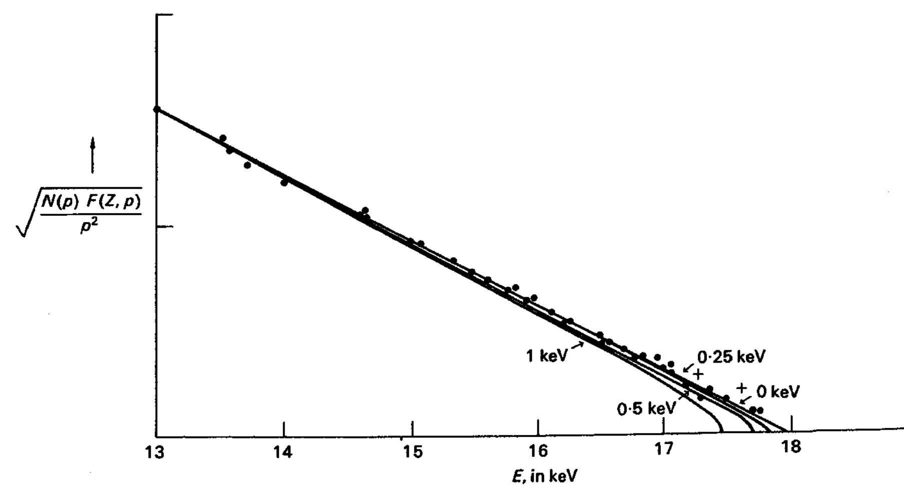



**Fig. 4.3** Kurie plot for tritium $\beta$-decay. Deviations near the end point for various neutrino mass values are indicated. (After Langer and Moffat, 1952.)

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