Lecture 16 Coulomb Scattering I

is given by $|\langle \psi(t')|$

$$|\langle \psi(t') \,|\, U(t' t_0) \,|\, \psi(t_0)\rangle|^2 .$$

Now to do a calculation we'll do the following:

- do a simple problem that doesn't require $P[\ ]$
- normal order as a matter of course to have a zero-energy vacuum $:\ :$
- $$\hat{P}_{fi} = \lim_{\substack{t_0 \to -\infty \\ t' \to \infty}} \frac{1}{t'-t_0} \,|\, U_{fi}(t' t_0) - \delta_{fi} \,|^2$$

no scattering

$$= \frac{1}{t'-t_0} \,|\, S_{fi} - S^{(0)}_{fi} \,|^2$$

- and then make it more complicated.

So, for first simple case, $e\gamma$ elastic scattering:

$$S^{(1)} - S^{(0)} = -i \int d^4x\, \mathcal{H}_I(x) = i \int d^4x\, \mathcal{L}_I(x) \quad p178$$

where $\mathcal{L}(x) = -e\, \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu(x)$ the physics assumption

We'll do the calculation of a static, Coulomb field only -- a classical field.

The S Matrix element, minus the non-scattering term is called the T-Matrix

$$\mathcal{T} \equiv S - S_{fi}$$

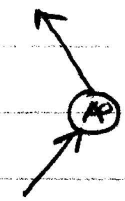
So, $\langle f | S | i \rangle \xrightarrow{\text{scattering}} \langle f | T | i \rangle \equiv \mathcal{T}$

and to first order..

$$S^{(1)}_{fi} \longrightarrow \mathcal{T}^{(1)} = i \langle \text{final} | \int d^4x\, \mathcal{L}(x) \,|\, \text{initial} \rangle$$

We'll choose

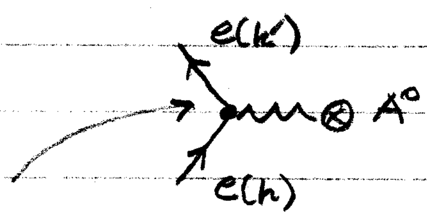
$|\text{final}\rangle = 1$ electron, k'



$|\text{initial}\rangle = 1$ electron, k

So,
$$|\text{initial}\rangle = |e(k)\rangle = a^{\dagger}(k)|0\rangle$$
$$\langle \text{final}| = \langle e(k')| = \langle 0 | a(k')$$

Diagrammatic language – while a shorthand later for real calculational elements – is intuitive, so we'll use it now



vertex

contains the means of: annihilating an electron of k
creating an electron of k'
proportional to e

$$\mathcal{T} = i \int d^4x \, \langle e(k')| : e\bar{\psi}(x)\gamma^\mu \psi(x) : | e(k)\rangle A_\mu(x) \Big|_{\mu=0}$$

remember... classical field

Remember:

$$\psi_j(x) = \sum_{i=1,2} \int dP \left[a^{(i)}(p) u_j^{(i)}(p) e^{-ip\cdot x} + b^{(i)+}(p) v_j^{(i)}(p) e^{ip\cdot x} \right]$$

$$\bar{\psi}_j(x) = \sum_{i=1,2} \int dP \left[\bar{u}_j^{(i)}(p) a^{(i)+}(p) e^{ip\cdot x} + \bar{v}_j^{(i)}(p) b^{(i)}(p) e^{-ip\cdot x} \right]$$

$$\psi = \psi^{(+)} + \psi^{(-)}$$

$\psi^{(-)}$ contains $b^+(p) \rightarrow$ creates positron

$\psi^{(+)}$ contains $a(p) \rightarrow$ annihilates electron

$$\bar{\psi} = \bar{\psi}^{(+)} + \bar{\psi}^{(-)}$$

$\bar{\psi}^{(-)}$ contains $a^+(p) \rightarrow$ creates electron

$\bar{\psi}^{(+)}$ contains $b(p) \rightarrow$ annihilates positron

We can figure out what to do 2 ways:

i) just look -- we want to destroy an electron and create an electron $\Rightarrow$

$$\bar{\psi}^{(-)}\psi^{(+)} \sim a^+(p')a(p)$$

ii) actually do it

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in Fock space:

$$:\bar\psi\psi': = :(\bar\psi^{(-)} + \bar\psi^{(+)})(\psi'^{(+)} + \psi'^{(-)})$$
$$= :\bar\psi^{(-)}\psi'^{(+)} + \bar\psi^{(-)}\psi'^{(-)} + \bar\psi^{(+)}\psi'^{(+)} + \bar\psi^{(+)}\psi'^{(-)}:$$
$$= \bar\psi^{(-)}\psi'^{(+)} + \bar\psi^{(-)}\psi'^{(-)} + \bar\psi^{(+)}\psi'^{(+)} - \psi'^{(-)}\bar\psi^{(+)}$$
$$\sim a^+(p)a(p') + a^+(p)b^+(p') + b(p)a(p') - b^+(p')b(p)$$

which is sandwiched between the |initial> and |final>
states.. (just Fock space action..)

$$\langle e(k')| :\bar\psi\psi': |e(k)\rangle$$
$$= \langle 0| a(k') :\bar\psi\psi': a^+(k)|0\rangle$$
$$= \langle 0| a(k')a^+(p)a(p')a^+(k)|0\rangle \quad ①$$
$$+ \langle 0| a(k')a^+(p)b^+(p')a^+(k)|0\rangle \quad ②$$
$$+ \langle 0| a(k')b(p)a(p')a^+(k)|0\rangle \quad ③$$
$$- \langle 0| a(k')b^+(p')b(p)a^+(k)|0\rangle \quad ④$$

② $\langle 0|a(k')b^+(p')a^+(p)a^+(k)|0\rangle = \langle 0|b^+(p')a(k')a^+(p)a^+(k)|0\rangle$ $= 0$

since: $\{a, a^+\} \neq 0$ & any $\{a, b\} = 0$
$\{b, b^+\} \neq 0$ $\quad \{a,a\} = \{b,b\} = \{a^+,a^+\} = \{b^+,b^+\} = 0$

③ $\langle 0|a(k')a(p')b(p)a^+(k)|0\rangle = \langle 0|a(k')a(p')a^+(k)b(p)|0\rangle$ $= 0$

④ $\langle 0|b^+(p')a(k')b(p)a^+(k)|0\rangle$ $= 0$

① $\langle 0|a(k')a^+(p)a(p')a^+(k)|0\rangle \quad \neq 0$ & survives so,

$$\mathcal{T} = ie \int d^4x \, \langle e(k')| \, \bar{\psi}^{(-)}(x) \gamma^\mu \psi^{(+)}(x) \, |e(k)\rangle A_\mu(x) \Big|_{\mu=0}$$

$$= ie \int d^4x \sum_i \sum_n \int dP \int dP'$$

$$\langle e(k')| \, \bar{u}^{(n)}(p') a^{+(n)}(p') e^{ip'\cdot x} \gamma^\mu a^{(i)}(p) u^{(i)}(p) e^{-ip\cdot x} |e(k)\rangle A_\mu(x) \Big|_{\mu=}$$

keep track of spaces.

$$= ie \int d^4x \sum_i \sum_n \int dP \int dP'$$

$$\langle e(k')| a^{(n)+}(p') a^{(i)}(p) |e(k)\rangle \bar{u}^{(n)}(p') \gamma^\mu u^{(i)}(p) \, e^{-i(p-p')\cdot x} A_\mu(x) \Big|_{\mu=}$$

look at Fock space term..

$$\langle 0| a^{(m)}(k') a^{(n)+}(p') a^{(i)}(p) a^{(l)+}(k) |0\rangle$$

$$= \langle 0| \{ a^{(m)}(k'), a^{(n)+}(p') \} \{ a^{(i)}(p), a^{(l)+}(k) \} |0\rangle$$

(which just adds zero -- $\langle 0| a^{(n)+}(p') a^{(m)}(k')$ - like terms)

$$= (2\pi)^3 2E' \, \delta(\vec{k}' - \vec{p}') (2\pi)^3 2E \, \delta(\vec{p} - \vec{k}) \langle 0|0\rangle \, \delta_{mn} \delta_{il}$$

so

$$\mathcal{T} = ie \int d^4x \sum_i \sum_n \int dP \int dP'$$

$$\times (2\pi)^3 2E' \delta(\vec{k}' - \vec{p}') (2\pi)^3 2E \, \delta(\vec{p} - \vec{k}) \delta_{mn} \delta_{il} \, \bar{u}^{(n)}(p') \gamma^\mu u^{(i)}(p) e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}} A_\mu \Big|_{\mu=}$$

remembering $\quad \int dP = \int \frac{d^3p}{(2\pi)^3 2E} \quad$ etc.

$$\mathcal{J} = ie \int d^4x \int d^3p \int d^3p' \, \delta(\vec{k}' - \vec{p}')\, \delta(\vec{p} - \vec{k})\, \bar{u}^{(m)}(p') \gamma^\mu u^{(\ell)}(p)\, e^{-i(p-p')\cdot x} A_\mu \Big|$$

$$= ie \int d^4x \int d^3p \, \delta(\vec{p} - \vec{k})\, \bar{u}^{(m)}(k') \gamma^\mu u^{(\ell)}(p)\, e^{-i(k_0 - k_0')t + i(\vec{p} - \vec{k}')\cdot\vec{x}} A_\mu \Big|_{\mu=0}$$

$$= ie \int d^4x \, \bar{u}^{(m)}(k') \gamma^\mu u^{(\ell)}(k)\, e^{-i(k_0 - k_0')t + i(\vec{k} - \vec{k}')\cdot\vec{x}} A_\mu(x) \Big|_{\mu=0}$$

$$= ie \int d^4x \, \bar{u}^{(m)}(k') \gamma^\mu u^{(\ell)}(k)\, e^{-i(k-k')\cdot x} A_\mu(x) \Big|_{\mu=0}$$

$$= ie \, \bar{u}^{(m)}(k') \gamma^\mu u^{(\ell)}(k) \underbrace{\int d^4x \, e^{-i(k-k')\cdot x} A_\mu(x)}_{\substack{\text{4d Fourier Transform} \\ \text{of } A_\mu(x) \\ \equiv \\ \tilde{A}(\vec{k} - \vec{k}')}} \Big|_{\mu=0}$$

For pure coulomb interaction – point source

$$A_\mu(x) = \frac{Ze}{4\pi|\vec{x}|} \qquad \mu = 0$$

$$= 0 \qquad \mu = i$$

$$\mathcal{J} = \frac{ie^2 Z}{4\pi} \int d^4x \, e^{-i(k-k')\cdot x} \frac{\bar{u}(k')\gamma^0 u(k)}{|\vec{x}|}$$

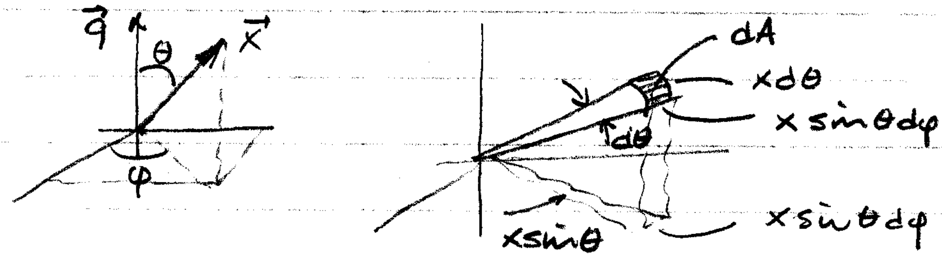
Do the time integration

$$\mathcal{J} = \frac{i Z e^2}{4\pi}\, 2\pi \int d^3x\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\, \bar{u}(k')\frac{\gamma^0}{|\vec{x}|} u(k)\, \delta(E-E')$$

$$= \frac{i Z^2 e^2}{2}\, \delta(E-E')\, \bar{u}(k')\gamma^0 u(k) \int \frac{d^3x}{|\vec{x}|}\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}$$

define $\vec{q} = \vec{k}-\vec{k}'$ then

$$\int \frac{d^3x}{|\vec{x}|}\, e^{i\vec{q}\cdot\vec{x}} = \int \frac{x^2 dx\, d\Omega}{x}\, e^{iqx\cos\theta}$$



$$dA = (x\,d\theta)(x\sin\theta\, d\varphi) = x^2 \sin\theta\, d\theta\, d\varphi$$
$$= x^2 d\Omega$$
$$d\Omega = \sin\theta\, d\theta\, d\varphi = d(\cos\theta)\, d\varphi$$

$$\int \frac{d^3x}{x}\, e^{i\vec{q}\cdot\vec{x}} = \int \frac{dx\, dA}{x}\, e^{i\vec{q}\cdot\vec{x}}$$

$$= \int d\varphi \int_0^\infty x\,dx \int_{-1}^{1} d(\cos\theta)\, e^{iqx\cos\theta}$$

change variables. $\quad y \equiv iqx\eta \qquad \eta = \cos\theta$

$dy = iqx\,d\eta$

$\eta = 1 \quad y = iqx$

$\eta = -1 \quad y = -iqx$

$$= 2\pi \int_0^\infty x\,dx \int d\eta\, e^{iqx\eta}$$

$$= 2\pi \int_0^\infty \frac{x\,dx}{iqx} \int_{-iqx}^{iqx} e^{\eta}\, d\eta$$

$$= -\frac{2\pi i}{q} \int_0^\infty dx\, \frac{iq}{iq} \left[e^{iqx} - e^{-iqx} \right] \qquad z = iqx$$

$$dz = iq\,dx$$

$$= -\frac{2\pi}{q^2} \int_0^\infty dz\, (e^{z} - e^{-z})$$

$$= -\frac{2\pi}{q^2} \underbrace{\left[e^{iqx} + e^{-iqx} \right] \Bigg|_{qx=0}^{qx=\infty}}$$

$$2\cos qx \Bigg|_{qx=0}^{qx=\infty} = 2\left(1 - \frac{(qx)^2}{2!} + \frac{(qx)^4}{4!} + \dots\right)$$

$$\simeq 2$$

$$= -\frac{4\pi}{q^2}$$

so,

$$S = -i\frac{Ze^2}{2} \delta(E-E') \frac{4\pi}{q^2} \bar{u}(k') \gamma^0 u(k)$$

$$= -2i\pi\, Ze^2 \delta(E-E')\, \bar{u}(k') \gamma^0 u(k) \frac{1}{q^2}$$

note we only get energy conservation since the coulomb field approximation is as if the scattering center doesn't recoil... or potential can change any momentum

We must square this,

$$|J|^2 = 4\pi^2 Z^2 e^4 \delta(E-E')\,\delta(E-E') \left| \frac{u(h')\gamma^0 u(h)}{q^2} \right|^2$$

since $$\delta(E-E') = \lim_{T\to\infty} \left(\frac{1}{2\pi}\right) \int_{-T/2}^{T/2} e^{i(E-E')t}\, dt$$

$$\delta(E-E')\,\delta(E-E') = \lim_{T\to\infty} \left(\frac{1}{2\pi}\right) \delta(E-E') \int_{-T/2}^{T/2} e^{i(E-E')t}\, dt$$

this "says" to evaluate this at $E=E'$

$$= \lim_{T\to\infty} \delta(E-E')\frac{T}{2\pi}$$

So,

$$|J|^2 = 2\pi Z^2 e^4\, T\, \delta(E-E') \left| \frac{\bar{u}(h')\gamma^0 u(h)}{q^2} \right|^2$$

want transition <u>rate</u>, so,

$$\frac{|J|^2}{T} = 2\pi Z^2 e^4 \delta(E-E') \left| \frac{\bar{u}(h')\gamma^0 u(h)}{q^2} \right|^2$$

The probability to transition to a final state of energy E to $E+dE$ includes the density of states. (Back to covariant normalization.–)

$$dp = \frac{|\mathcal{J}|^2}{T}\, d\rho$$

$$= 2\pi\, Z^2 e^4\, \delta(E-E') \left| \frac{\bar{u}(k')\gamma^0 u(k)}{q^2} \right|^2 \frac{d^3k'}{(2\pi)^3 2E'}\, V$$

The differential cross section

$$d\sigma = \frac{dp}{\text{flux}} \qquad \text{where} \qquad \text{flux} = \frac{|\vec{v}|\,2E}{V}$$

but V will not show up $= 1$

$$d\sigma = \frac{\pi}{E}\, \frac{1}{|\vec{v}|}\, Z^2 e^4 \delta(E-E') \left| \frac{\bar{u}(k')\gamma^0 u(k)}{q^2} \right|^2 \frac{d^3k'}{(2\pi)^3 2E'}$$

now.. $v = \beta = k/E$

$$d^3k' = |k'|^2 dk' d\Omega'$$

$$k' = \sqrt{E'^2 - m^2}$$

so $$\frac{dk'}{dE'} = \frac{1}{2}\,\frac{2E'}{k'} = E'/k'$$

and $$d^3k' = k'E'dE'd\Omega'$$

so,

$$d\sigma = \left(\frac{\pi}{E}\right)\left(\frac{E}{k}\right) Z^2 e^4 \delta(E-E') \left| \frac{\bar{u}(k')\gamma^0 u(k)}{q^2} \right|^2 \frac{k'dE'd\Omega'}{(2\pi)^3 2}$$

integrate out the energy

$$\frac{d\sigma}{d\Omega'} = \frac{1}{16\pi^2}\left(\frac{k'}{k}\right) \delta(E-E')\, \frac{Z^2 e^4}{q^4} \left| \bar{u}(k')\gamma^0 u(k) \right|^2 dE'$$

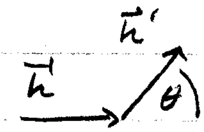
$$\frac{d\sigma}{d\Omega'} = \int \frac{d\sigma}{d\Omega' dE'}\, dE'$$

look at the matrix element - which we'll eventually call T

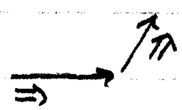
$$T = \bar{u}(k') \frac{\gamma^0}{q^2} u(k)$$

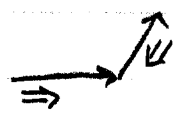
$$= u^+(k') \frac{\gamma^0 \gamma^0 u(k)}{q^2} = \frac{u^+(k') u(k)}{q^2}$$

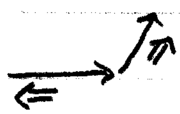
since $u(k) = \sqrt{E+m} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma} \cdot \vec{k}}{E+m} \chi_s \end{pmatrix}$

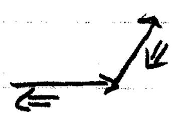


we can consider the following possibilities


$A_{\uparrow\uparrow}$


$A_{\uparrow\downarrow}$


$A_{\downarrow\uparrow}$


$A_{\downarrow\downarrow}$

2 helicity flip and 2 helicity non-flip amplitudes

Define the quantization axis along $\vec{k}$ for the initial state spinors –

$$\chi^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{for electrons.}$$

call $$u_+(k) = \sqrt{E+m} \begin{pmatrix} \chi^+ \\ \frac{\sigma_3 k}{E+m} \chi^+ \end{pmatrix}$$

To find the $\nearrow k'$ state, take $\xrightarrow[|\vec{k}'|]{\Rightarrow}$

and rotate it... homework.

$$u_+(k') = \sqrt{E'+m} \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \\ \frac{k'}{E'+m} \cos\theta/2 \\ \frac{k'}{E+m} \sin\theta/2 \end{pmatrix}$$

shorthand $$\alpha \equiv \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$$

so,
$$u_+(k') = \sqrt{E+m} \begin{pmatrix} \alpha \\ \frac{k'}{E'+m} \alpha \end{pmatrix}$$

so $$A_{\uparrow\uparrow} = \frac{1}{q^2} u_+^\dagger(k') u_+(k)$$

Similarly, by defining $$\beta = \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$$

$$u_-(k') = \sqrt{E'+m}\begin{pmatrix} \beta \\ \frac{-k'}{E'+m}\beta \end{pmatrix}$$

So, this is enough to consider either polarized or unpolarized electron scattering. In the latter, we would $\underline{\text{average}}$ over initial helicities. $\left(\frac{1}{2}\right)$

For elastic scattering $|\vec{k}| = |\vec{k}'|$ so $E = E'$ and

$$A_{\uparrow\uparrow} = (E+m)\left(\cos\theta/2,\ \sin\theta/2,\ \frac{k'}{E'+m}\cos\theta/2,\ \frac{k'}{E'+m}\sin\theta/2\right)\begin{pmatrix} 1 \\ 0 \\ \frac{k}{E+m} \\ 0 \end{pmatrix}$$

$$= (E+m)\left[\cos\theta/2 + \frac{k'k}{(E'+m)(E+m)}\cos\theta/2\right]$$

$$= (E+m)\left[\cos\theta/2 + \frac{k^2}{(E+m)^2}\cos\theta/2\right]$$

$$= (E+m)\cos\theta/2 + \frac{E^2-m^2}{E+m}\cos\theta/2$$

$$= (E+m)\cos\theta/2 + (E-m)\cos\theta/2 = 2E\cos\theta/2$$

The other amplitudes are similarly

$$A_{\uparrow\downarrow} = -2m\sin\theta/2$$

$$A_{\downarrow\uparrow} = 2m\sin\theta/2$$

$$A_{\downarrow\downarrow} = 2E\cos\theta/2$$

Helicity amplitudes are distinct quantum mechanical states, so

$$|T|^2 = \frac{1}{q^4}\left[\,|A|^2 + |A|^2 - |A|^2 + |A|^2\right]$$

$$= \frac{1}{q^4}\left(8E^2\cos^2\theta/_2 + 8m^2\sin^2\theta/_2\right)$$

For an unpolarized cross section, we average over the initial states of which there are 2 — so $1/_2$.

The dE' integration is trivial —

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\,\frac{Z^2e^4}{q^4}\,4\left[E^2\cos^2\theta/_2 + m^2\sin^2\theta/_2\right]$$

$$= \frac{1}{4\pi^2}\,\frac{Z^2e^4}{q^4}\left[1 - \sin^2\theta/_2 + \frac{m^2}{E^2}\sin^2\theta/_2\right]E^2$$

$$= \frac{1}{4\pi^2}\,\frac{Z^2e^4}{q^4}\left[1 - \frac{k^2}{E^2}\sin^2\theta/_2\right]E^2$$

$$\frac{d\sigma}{d\Omega} = \frac{Z^2e^4}{4\pi^2q^4}\,E^2\left[1 - \beta^2\sin^2\theta/_2\right]$$

since, $q^2 = 4k^2\sin^2\theta/_2$, $\alpha = \frac{e^2}{4\pi}$

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4k^4}\left(\frac{E^2}{\sin^4\theta/_2}\right)\underbrace{\left(1 - \beta^2\sin^2\theta/_2\right)}_{\text{a relativistic correction due to spin } 1/_2.}$$

Denote $\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{4z^2\alpha^2 E^2}{q^4}$ where $\alpha = e^2/4\pi$

as the Rutherford cross section.

So, $\left(\frac{d\sigma}{d\Omega}\right)_{\text{Point}} = \left(\frac{d\sigma}{d\Omega}\right)_R (1 - \beta^2 \sin^2\theta/2)$

relativistic correction for spin 1/2

Had we had a totally polarized beam, say 100% RH, we would have had contributions only,

$|A_{\uparrow\uparrow}|^2 \sim 4E^2\cos^2\theta/2$

$|A_{\uparrow\downarrow}|^2 \sim 4m^2\sin^2\theta/2$

and the cross section would have been half as large.

The degree of polarization is defined as the asymmetry,

$P \equiv \frac{N_R - N_L}{N_R + N_L}$ so, for a purely RH beam,

$$P_R = \frac{d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)}{d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)}$$

$$P_R = \frac{4E^2\cos^2\theta/_2 - 4m^2\sin^2\theta/_2}{4E^2\cos^2\theta/_2 + 4m^2\sin^2\theta/_2}$$

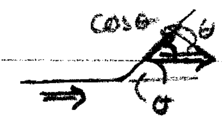
$$= 1 - \frac{2m^2\sin^2\theta/_2}{E^2\cos^2\theta/_2 + m^2\sin^2\theta/_2}$$

In the extreme relativistic limit: $E \gg m$
and

$P_R \rightarrow 1 \quad \Rightarrow$ helicity does not flip -- conserved.

In the extreme nonrelativistic limit $E \approx m$

$$P_R \rightarrow 1 - 2\sin^2\theta/_2 = \cos\theta$$



which is the geometrical overlap of the 2 quantization axes $\longrightarrow$ and $\nearrow_{\theta})$ $\Rightarrow$ spin is not affected by this scattering.

Then, for extreme relativistic scattering $\beta \rightarrow 1$ and,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R \cos^2\theta/_2$$ which is called Mott scattering

$$= \left(\frac{d\sigma}{d\Omega}\right)_M$$ $\rightarrow$ Coulomb scattering from a point source

$$= \frac{Z^2\alpha^2}{4E^2}\,\frac{\cos^2\theta/_2}{\sin^4\theta/_2}$$

Suppose the scattering is from a nucleus, like Al.

$R(\text{nucleus}) \sim r_0 A^{1/3}$ $\qquad r_0 = 1.45 \times 10^{-13}$ cm

$A = 1$ proton

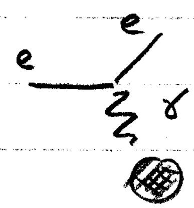
$= 27$ Al

so,

$R(\text{proton}) \sim 1.5 \times 10^{-13}$ cm

$R(\text{Al}) \sim 4 \times 10^{-13}$ cm

To resolve these distances we need a probe of wavelength of order these distances. For



$\vec{q}$ momentum transfer

$$q \sim \frac{hc}{r} \sim 300 \text{ MeV} \quad \text{Al}$$

$$1 \text{ GeV} \quad p.$$

$\Rightarrow$ smaller distances $\Rightarrow$ more momentum transfer $\Rightarrow$ higher energy beams. $\quad q^2 = 4k^2 \sin^2\theta/2$

Right off the bat we see that the electron is really relativistic for any kind of scattering of this sort.

$$\gamma = E/m = \sim \frac{300 \text{ MeV}}{0.5 \text{ MeV}} \sim 600 \Rightarrow \beta = 0.999997$$

How about a Bohr radius? $\quad q \sim \dfrac{hc}{10^{-8}} \sim 2 \text{keV}$