PHYSICS 584 "Quantum Electrodynamics" Lecture 1

or "Relativistic Quantum Field Theory." 0

1) Class times. — block out time.

2) Email to me --

chipclass @ pa.msu.edu

everyone send me a handshake and I'll reply via distribution list

Do this by Wednesday noon.

3) Web site

www.pa.msu.edu/courses/current/phy854/

visit regularly, esp before class.

announcements
problem assignments (pdf's)
calendar

4) who are you?

5) What we'll do

- Lagrangian techniques.
- 2nd Quantization; spin 0, spin ½, spin 1 fields.
- S Matrix techniques. ⇒ scattering Theory.
- Development of Feynman Rules.
  Lotsa problems.
- Renormalization of Coulomb field
  so-called Pauli-Villars techniques.
  & dimensional regularization
- Maybe... Spontaneous Symmetry Breaking
  Weak Interactions.

ϕa

6) grades: all problem sets, no exams.

7) books: none required.

Best? Sakurai <u>Advanced Quantum Mechanics</u>.

gotta find it on-line, used.

Great: Peskin <u>An Introduction to Quantum Field Theory</u>.

Sterman <u>An Introduction to Quantum Field Theory</u> hmm..

Weinberg <u>The Quantum Theory of Fields I, II</u>

Greiner, Reinhardt <u>Field Quantization</u>

<u>Quantum Electrodynamics</u>

→ I'll get em on reserve.

First, a review of my conventions, normalizations, etc.

METRIC, et al.

Minkowski tensor $$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & 0 \\ & 0 & -1 & \\ & & & -1 \end{pmatrix}$$

having "signature" $(1,-1,-1,-1)$

conventionally refer to spacetime coordinates as <u>contravariant</u> 4-vectors, tensors of the 1st rank. ⇒ index up.

$$x^\mu = (x^0, x^1, x^2, x^3)$$
$$= (ct, x, y, z)$$
$$= [x^0, \vec{x}] \text{ or } (x^0, \vec{x})$$

} of course, "blackboard units" for sure... $c = \hbar = 1$

Context will govern... ( ) or [ ]

So, <u>contract</u> with $g_{\mu\nu}$ to change contravariant tensors into <u>covariant</u> tensors.

$$T_\mu = g_{\mu\nu} x^\nu = \sum_{\nu=0}^{3} g_{\mu\nu} x^\nu = (ct, -x, -y, -z) \neq x^\mu$$

$$T_0 = ct = \sum_{\nu=0}^{3} g_{0\nu} x^\nu = g_{00} x^0 = ct$$
$$T_1 = \sum g_{1\nu} x^\nu$$
$$= g_{11} x^1 = (-1) x^1 = -x$$

etc.

So, $$x_\mu = (t, -x, -y, -z)$$
$$= \cancel{(x_0, -x_1}, $$
$$= (x_0, x_1, x_2, x_3)$$
$$= [x_0, -\vec{x}] = [t, -\vec{x}]$$

or,

$$g_{\mu\nu} x^\nu = x_\mu \rightarrow \text{using Einstein summation convention.}$$

The identity of $g_{\mu\nu}$ is $g^{\mu\nu}$ $\qquad g_{\mu\nu} = g^{\mu\nu}$

and $\qquad g_{\mu\nu}\, g^{\mu\nu} = 1 \quad \left(= \sum_{\nu=0}^{3} \sum_{\mu=0}^{3} \cdots \right)$

and we can conventionally write,

$$\underbrace{g^{\mu\nu} g_{\nu\alpha}}_{\substack{\text{lower the} \\ \text{index } \nu \to \alpha}} = g^{\mu}{}_{\alpha} \equiv \delta^{\mu}{}_{\alpha}$$

4d Kronecker delta "function"
1's on diagonal.

put it together element by element...

$$g^{\mu\nu} g_{\nu\alpha} = \sum_{\nu=0}^{3} g^{\mu\nu} g_{\nu\alpha}$$

$$g^{\mu}{}_{\alpha} = g^{\mu 0} g_{0\alpha} + g^{\mu 1} g_{1\alpha} + g^{\mu 2} g_{2\alpha} + g^{\mu 3} g_{3\alpha}$$

element by element.

$$g^{0}{}_{\alpha} = g^{00} g_{0\alpha} + \cancel{g^{01}}^{\to 0} g_{1\alpha} + 0 + 0$$

$$= (1)\, g_{0\alpha}$$

$$= 1 \quad \alpha = 0, \qquad = 0 \quad \alpha \neq 0$$

$$g^{1}{}_{\alpha} = \cancel{g^{10}}^{\to 0} g_{1\alpha} + g^{11} g_{1\alpha} + \cancel{g^{12}}^{\to 0} g_{1\alpha} + 0$$

$$= (-1)(g_{1\alpha}) = (-1)(-1), \quad \alpha = 1 \qquad 0, \ \alpha \neq 1$$

$$= 1$$

etc.

Generally, then, for tensors of any rank,

$$A_{\mu\nu} = g_{\mu\alpha}\, g_{\nu\beta}\, A^{\alpha\beta} \qquad \text{etc.}$$

4d dot product, or inner product...

$$A^\mu B_\mu = A^0 B_0 + A^1 B_1 + A^2 B_2 + A^3 B_3$$
$$= A^\mu B^\alpha g_{\alpha\mu}$$
$$= A^0 B^\alpha g_{\alpha 0} + A^1 B^\alpha g_{\alpha 1} + \cdots$$
$$= A^0 B^0 + A^1 B^1 (-1) + \cdots$$
$$= A^0 B^0 - A^1 B^1 - A^2 B^2 - A^3 B^3$$
$$= A^0 B^0 - \vec{A} \cdot \vec{B}$$
$$\equiv \underbrace{A \cdot B}_{\text{context will be obvious.}} \qquad \text{a \underline{scalar}}$$

scalar: any quantity which can be "measured with a scale" -- doesn't depend on the coordinate system.

scalar field: a function of coordinates which itself doesn't change under coordinate transformation

$\boxed{\text{Coordinate Transformations:}}$ Consider 2 neighboring points

B $\cdot\ x^\mu + dx^\mu$

A $\cdot\ x^\mu$

and a different coordinate system defined in terms of the "old" coordinates

$$x'^\mu = f^\mu(x^\mu)$$

then

$$dx^{\mu'} = \frac{\partial f^\mu(x^\mu)}{\partial x^\nu} dx^\nu$$

$$dx^{\mu'} = \frac{\partial x'^\mu}{\partial x^\nu} dx^\nu \equiv \Lambda^\mu{}_\nu\, dx^\nu$$

a defining relation.

* Any quantity which transforms like a differential coordinate is a <u>4-vector</u>, a <u>contravariant vector</u>.

Now, consider a scalar function of coordinates $\phi(x^\mu)$ and form the gradient...

$$A_\nu \equiv \frac{\partial \phi(x^\mu)}{\partial x^\nu}$$

down up

by rules of differentiation...

$$\frac{\partial \phi(x^\mu)}{\partial x^{\nu'}} = \frac{\partial \phi(x^\mu)}{\partial x^\sigma} \frac{\partial x^\sigma}{\partial x^{\nu'}}$$

A_σ

$$= \frac{\partial x^\sigma}{\partial x^{\nu'}} A_\sigma = A'_\nu$$

this is not $\Lambda^\sigma{}_\nu$... look at "primes"

* So, This is the transformation equation of the <u>gradient</u>, which is a <u>contravariant vector</u>

Contract covariant and contravariant vectors... & transform:

$$A_\mu B^\mu \longrightarrow A'_\mu B^{\mu'} = A_\nu \frac{\partial x^\nu}{\partial x^{\mu'}} B^\alpha \frac{\partial x'^\mu}{\partial x^\alpha}$$

move terms around sharp w/ indices exposed...

$$= A_\nu B^\alpha \frac{\partial x^\nu}{\partial x^{\mu'}} \frac{\partial x'^\mu}{\partial x^\alpha}$$

$$= A_\nu B^\alpha \frac{\partial x^\nu}{\partial x^\alpha}$$

$$\frac{\partial x^\nu}{\partial x^\alpha} = \delta^\nu{}_\alpha \quad \text{so,} \quad A'_\mu B^{\mu'} = A_\nu B^\alpha \delta^\nu{}_\alpha = A_\nu B^\nu$$

same form $\Rightarrow$ scalar

$\Lambda^\mu{}_\nu$ has 16 components — constraints are

$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ the interval must be invariant

$$ds'^2 = ds^2$$

$$g_{\mu\nu}dx'^\mu dx'^\nu = g_{\alpha\beta}\,dx^\alpha dx^\beta$$

$$g_{\mu\nu}\Lambda^\mu{}_\rho\, dx^\rho \Lambda^\nu{}_\xi\, dx^\xi =$$

$$g_{\mu\nu}\Lambda^\mu{}_\rho \Lambda^\nu{}_\xi\, dx^\rho dx^\xi = g_{\alpha\beta}\,dx^\alpha dx^\beta$$

just summed dummy indices

$$= g_{\rho\xi}\,dx^\rho dx^\xi$$

so,

$$\Lambda^\mu{}_\rho\, g_{\mu\nu} \Lambda^\nu{}_\xi = g_{\rho\xi} = \Lambda^T{}_\rho{}^\mu\, g_{\mu\nu}\Lambda^\nu{}_\xi \quad \left\{ \Lambda_{\nu\rho}\Lambda^\nu{}_\xi = g_{\rho\xi} \right.$$

"pseudo orthogonality"

10 constraint equations: 10 conditions on 16 components
$\Rightarrow$ 6 independent parameters

6 $\begin{cases} 3 & \text{components of relative velocity} \\ 3 & \text{components of angles relating orientation of } x \text{ and } x' \text{ axes} \end{cases}$

For zero relative orientation $\Rightarrow$ PURE LORENTZ TRANSFORMATIONS

eq along x or 1 axis.

$$\Lambda^\mu_{(1)\nu} = \begin{pmatrix} \gamma & -\beta_1\gamma & 0 & 0 \\ -\beta_1\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial x^{\mu'}} \underbrace{dx^{\mu'}}$$

inverse transformation

$$= \underbrace{\frac{\partial x^{\alpha}}{\partial x^{\mu'}} \frac{\partial x'^{\mu}}{\partial x^{\nu}}}_{\delta^{\alpha}_{\nu}} dx^{\nu} \quad \checkmark$$

define inverse by

$$dx^{\alpha} = \Lambda^{-1\alpha}{}_{\mu}\, \underbrace{dx^{\mu'}}$$

$$= \underbrace{\Lambda^{-1\alpha}{}_{\mu} \Lambda^{\mu}{}_{\beta}}_{\text{must be } \delta^{\alpha}{}_{\beta}} dx^{\beta} \quad \checkmark$$

the tensorial character of objects is defined by their Lorentz Transformation properties.

rank

0 trans -- no change $\Rightarrow$ scalar

1 $A^{\mu'} = \Lambda^{\mu}{}_{\nu} A^{\nu}$ $\Rightarrow$ vector, tensor of rank 1

2 $A'^{\alpha\beta} = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} A^{\mu\nu}$ $\Rightarrow$ tensor, rank 2

2 $A'^{\mu}{}_{\nu} = \frac{\partial x'^{\mu}}{\partial x^{\rho}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} A^{\rho}{}_{\sigma} = \Lambda^{\mu}{}_{\rho} A^{\rho}{}_{\sigma} (\Lambda^{-1})^{\sigma}{}_{\nu}$ mixed, rank 2

etc.

consider.

$$A'^{\mu} = \Lambda^{\mu}{}_{\nu} A^{\nu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} A^{\nu}$$

$$\frac{\partial A'^{\mu}}{\partial x'^{\rho}} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} \frac{\partial A^{\nu}}{\partial x'^{\rho}} + \frac{\partial^2 x'^{\mu}}{\partial x^{\nu} \partial x'^{\rho}} A^{\nu}$$

$$= \frac{\partial x'^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\alpha}}{\partial x'^{\rho}} \frac{\partial A^{\nu}}{\partial x^{\alpha}} + \frac{\partial^2 x'^{\mu}}{\partial x^{\nu} \partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\rho}} A^{\nu}$$

if not zero, then $\frac{\partial A^{\mu}}{\partial x'^{\rho}}$ is not a tensor.

we'll deal with transformations linear in coordinates.

related to affine connection in non-Eucl.

Useful tensors

① Metric $\qquad g_{\mu\nu} = g^{\mu\nu}$

useful -- $\quad T^{\sigma}{}_{\mu\rho} = g_{\mu\nu} T^{\nu\sigma}{}_{\rho}$ $\quad$ etc.

note
$$g_{\alpha\mu} \underbrace{g_{\beta\nu} g^{\mu\nu}}_{\delta^{\mu}_{\beta}} = g_{\alpha\mu} g^{\mu}{}_{\beta}$$
$$\parallel$$
$$g_{\alpha\mu} \delta^{\mu}_{\beta} = g_{\alpha\beta} \qquad \text{so } g^{\mu}{}_{\beta} = \delta^{\mu}{}_{\beta} \checkmark$$

then
$$g_{\alpha\mu} g_{\beta}{}^{\mu} g^{\alpha\eta} = g_{\alpha\beta} g^{\alpha\eta}$$

from right w/ $g^{\alpha\eta}$

$$\parallel$$
$$g_{\alpha\mu} \delta^{\mu}{}_{\beta} g^{\alpha\eta} =$$
$$g_{\mu}{}^{\eta} \delta^{\mu}{}_{\beta} = \delta_{\mu}{}^{\eta} \delta^{\mu}{}_{\beta} = \delta^{\eta}{}_{\beta} = g_{\alpha\beta} g^{\alpha\eta}$$

consistent with the assertion that $g_{\alpha\beta}$, $g^{\alpha\beta}$ are reciprocals.

② the gradient — not a vector.

$$\frac{\partial}{\partial x^{\mu}} \longrightarrow \equiv \partial_{\mu} = [\partial/\partial t, +\vec{\nabla}]$$
$$\partial^{\mu} = \frac{\partial}{\partial x_{\mu}} = [\partial/\partial t, -\vec{\nabla}]$$

so. $\quad \dfrac{\partial A_{\mu}}{\partial x^{\mu}} = \partial^{\mu} A_{\mu} = \dfrac{\partial A_0}{\partial t} + (-\vec{\nabla}).(-\vec{A})$ $\qquad$ 4-divergence

$$= \frac{\partial A_0}{\partial t} + \vec{\nabla}.\vec{A}$$

③ Invariant D'Alembertian operator

$$\frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial x_{\mu}} \equiv \partial_{\mu} \partial^{\mu} \equiv \Box = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$$

④ Antisymmetric tensors (convention here)

$$\epsilon_{123} = -\epsilon^{123} = -1$$

$$\epsilon_{0123} = -\epsilon^{0123} = -1 \qquad (\text{"Levi Cevita tensor"})$$

note

$$\epsilon_{\mu\nu\alpha\beta} = -\epsilon_{\nu\mu\alpha\beta} \qquad \text{etc.}$$

and $\epsilon_{\mu\nu\alpha\beta}\, g^{\alpha\beta} = 0$ or any symmetric tensor in indices $\epsilon_{\mu\nu\alpha\beta}\, S^{\alpha\beta} = 0$

$S^{\alpha\beta} = S^{\beta\alpha}$.

Some useful contractions:

$$\epsilon^{\mu\nu\sigma\rho}\,\epsilon_{\mu\nu\gamma\delta} = -2\,\delta^{\sigma}_{\gamma}\,\delta^{\rho}_{\delta} + 2\,\delta^{\rho}_{\gamma}\,\delta^{\sigma}_{\delta}$$

$$\epsilon^{\mu\nu\sigma\rho}\,\epsilon_{\mu\nu\sigma\delta} = -3!\,\delta^{\rho}_{\delta}$$

$$\epsilon^{\mu\nu\sigma\rho}\,\epsilon_{\mu\nu\sigma\rho} = -4!$$