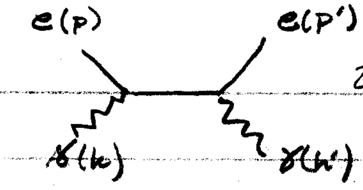
Compton - ~~contd~~

label momenta



so, back to Compton scattering.

$$J^{(2)}(e\gamma \to e\gamma) = \frac{(-ie)^2}{2} \int d^4x_1 \int d^4x_2 \; \langle e\gamma | \gamma^\mu_{ij} \gamma^\nu_{\ell m}$$

$$\Big\{ \bar\psi_i^-(x_1) \psi_m^+(x_2) A_\mu(x_1) A_\nu(x_2) \underbrace{\psi_j(x_1) \bar\psi_\ell(x_2)}$$

$$- \bar\psi_\ell^-(x_2) \psi_j^+(x_1) A_\mu(x_1) A_\nu(x_2) \underbrace{\bar\psi_i(x_1) \psi_m(x_2)} \Big\} \, | e\gamma \rangle$$

$$\psi_m^+(x) = \sum_r \int dP_1 \; a^{(r)}(P_1) u_m^{(r)}(P_1) e^{-iP_1\cdot x}$$

$$\bar\psi_n^-(x) = \sum_s \int dP_2 \; \bar u_n^{(s)}(P_2) a^{(s)+}(P_2) e^{iP_2\cdot x}$$

$$A_\nu^+(x) = \int dK_1 \sum_\lambda \epsilon_{\nu(\lambda)}(k_1) a_{(\lambda)}(k_1) e^{-ik_1\cdot x}$$

$$A_\mu^-(x) = \int dK_2 \sum_{\lambda'} \epsilon_{\mu(\lambda')}(k_2) a^+_{(\lambda')}(k_2) e^{ik_2\cdot x}$$

Put all this stuff, plus the propagators, back. first separate out the Fock space terms for electrons, then do photons.

$$J^{(2)}(e\gamma \to e\gamma) = -\frac{e^2}{2} \int d^4x_1 \int d^4x_2 \Big\{ \langle \gamma | A_\mu(x_1) A_\nu(x_2) | \gamma \rangle \Big[$$

$$\cdot \sum_{r,s} \int dP_1 \int dP_2 \langle 0 | a^{(f)}(p') a^{(s)+}(P_2) a^{(r)}(P_1) a^{(g)+}(p) | 0 \rangle \, \bar u_\ell^{(s)}(P_2) u_m^{(r)}(P_1) e^{iP_2\cdot x_1} e^{-iP_1\cdot x_2}$$

$$\cdot \int \frac{d^4q}{(2\pi)^4} \left(\frac{i}{\not q - m} \right)_{j\ell} e^{-iq\cdot(x_1-x_2)}$$

$$\bar u_\ell^{(s)}(P_2) u_j^{(r)}(P_1)$$

$$+ \sum_{r,s} \int dP_1' \int dP_2 \langle 0 | a^{(f)}(p') a^{(s)+}(P_2) a^{(r)}(P_1) a^{+(g)}(p) | 0 \rangle e^{iP_2\cdot x_2} e^{-iP_1\cdot x_1}$$

$$\cdot \int \frac{d^4q'}{(2\pi)^4} \left(\frac{i}{\not q' - m} \right)_{mi} e^{-iq\cdot(x_2-x_1)} \Big] \Big\} \gamma^\mu_{ij} \gamma^\nu_{\ell m}$$

where $\psi_a(y)\bar\psi_b(x) = -\bar\psi_b(x)\psi_a(y)$

Look at 1st electron Fock space term. Do the standard trick to turn $a(n)a^+(m)$ into $\{a(n), a^+(m)\}$ – twice

$$\langle 0| -- |0\rangle_{\text{electron}_1} = (2\pi)^3 2E'\delta(\vec p' - \vec p_2)(2\pi)^3 2E\,\delta(\vec p - \vec p_1)\,\delta_{fs}\,\delta_{gr} \times \langle\gamma| :AA: |\gamma\rangle\, \underbrace{\psi_j\bar\psi_l}$$

2nd – same, except for matrix indices.

$$\langle 0| — |0\rangle_{\text{electron}_2} = (2\pi)^3 2E'\delta(\vec p - \vec p_2)(2\pi)^3 2E\,\delta(\vec p - \vec p_1)\,\delta_{fs}\,\delta_{gr} \times \langle\gamma| :AA: |\gamma\rangle\, \underbrace{\bar\psi_i\psi_m}$$

Now, the $\int dP_1$ and $\int dP_2$ integrations can be done and the $\sum_r\sum_s$ sum collapsed,

$$J^{(2)}(e\gamma\to e\gamma) = -\frac{e^2}{2}\int d^4x_1\int d^4x_2 \left\{\left[\bar u_i^{(f)}(p')u_m^{(g)}(p)e^{ip'\cdot x_1}e^{-ip\cdot x_2}\right.\right.$$

$$\cdot \int\frac{d^4q}{(2\pi)^4}\left(\frac{i}{\not q - m}\right)_{jl} e^{-iq\cdot(x_1-x_2)}$$

$$\left. + \;\bar u_l^{(f)}(p')u_j^{(g)}(p)e^{ip'\cdot x_2}e^{-ip\cdot x_1}\int\frac{d^4q'}{(2\pi)^4}\left(\frac{i}{\not q' - m}\right)_{mi} e^{-iq'\cdot(x_2-x_1)}\right]$$

$$\left.\cdot\,\langle\gamma|A_\mu^+(x_1)A_\nu(x_2)|\gamma\rangle\right\}\gamma^\mu_{ij}\gamma^\nu_{lm}$$

The photon terms go the same way –
for example, $\langle\gamma| :A_\mu(x_1)A_\nu(x_2): |\gamma\rangle = \langle\gamma| A_\nu^-(x_2)A_\mu^+(x_1) + A_\mu^-(x_1)A_\nu^+(x_2)|\gamma\rangle$
(2 terms)
and

$$A_\nu^+(x_2)|\gamma\rangle = \int dK_1 \sum_{\lambda_1}\epsilon_{\nu(\lambda_1)}(k_1)e^{-ik_1 x_2}a_{(\lambda_1)}(k_1)a^+_{(\lambda)}(k)|0\rangle$$

do the same trick on $a(l)a^+(m) \longrightarrow [a(l), a^+(m)]$

-- twice (which cancels the funny minus signs) --
So, we get

$$A^{+}_{\nu}(x_2)|\gamma\rangle = \int dk_1 \sum_{\lambda_1} \epsilon_{\nu(\lambda_1)}(k_1) e^{-ik_1\cdot x_1} (2\pi)^3 2\omega_1 \delta_{\lambda\lambda_1} \delta(\vec{k}-\vec{k}_1) |0\rangle$$

and we can do the momentum integrals giving

$$\langle\gamma(k'\lambda')| : A_\mu(x_1) A_\nu(x_2) : |\gamma(k\lambda)\rangle =$$

$$\epsilon_{\mu(\lambda')}(k')\,\epsilon_{\nu(\lambda)}(k)\, e^{ik'\cdot x_1} e^{-ik\cdot x_2} + \epsilon_{\nu(\lambda')}(k')\,\epsilon_{\mu(\lambda)}(k)\, e^{ik'\cdot x_2} e^{-ik\cdot x_1}$$

and therefore, 4 terms overall.

$$J^{(2)}(e\gamma\to e\gamma) = -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \int \frac{d^4q}{(2\pi)^4}\, \gamma^\mu_{ij}\gamma^\nu_{lm}$$

$$\cdot \left\{ \bar{u}^{(f)}_i(p')\, u^{(g)}_m(p)\, e^{ip'\cdot x_1} e^{-ip\cdot x_2} \left(\frac{i}{\not{q}-m}\right)_{jl} e^{-iq\cdot(x_1-x_2)} \right.$$

$$\left. + \bar{u}^{(f)}_l(p')\, u^{(g)}_j(p)\, e^{ip'\cdot x_2} e^{-ip\cdot x_1} \left(\frac{i}{\not{q}-m}\right)_{mi} e^{-iq\cdot(x_2-x_1)} \right\}$$

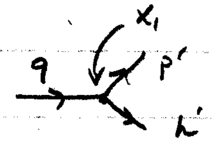
$$\cdot \left\{ \epsilon_{\mu(\lambda')}(k')\,\epsilon_{\nu(\lambda)}(k)\, e^{ik'\cdot x_1} e^{-ik\cdot x_2} + \epsilon_{\nu(\lambda')}(k')\,\epsilon_{\mu(\lambda)}(k)\, e^{ik'\cdot x_2} e^{-ik\cdot x_1} \right\}$$

All of this is tied together through the exponentials —

$$J^{(2)} = (e_1 + e_2)(\gamma_1 + \gamma_2) = e_1\gamma_1 + e_2\gamma_1 + e_2\gamma_1 + e_2\gamma_2$$
$$= 1 + 2 + 3 + 4$$

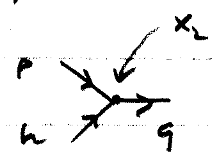
notice $\quad \int d^4x_1 \, e^{ip'\cdot x_1} e^{-iq\cdot x_1} e^{ik'\cdot x_1}$ in ①

$$= (2\pi)^4 \delta(p' - q + k')$$
$$= (2\pi)^4 \delta(q - p' - k')$$



4 momentum Kirchhoff's law. —

and $\quad \int d^4x_2 \, e^{-ip\cdot x_2} e^{iq x_2} e^{-ik x_2} = (2\pi)^4 \delta(q - p - k)$



then, $\int d^4q$ integral can be trivially done.

$$\int d^4q \, (2\pi)^4 (2\pi)^4 \delta(q - p' - k') \, \underbrace{\delta(q - p - k)}_{\longrightarrow \; \Rightarrow \; q = p + k}$$

$$= (2\pi)^8 \delta(p + k - p' - k')$$

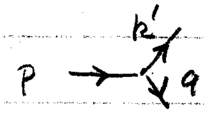
and

$$① = -\frac{e^2}{2} \Big\{ (2\pi)^4 \delta(p + k - p' - k') \, \underline{\bar{u}^{(r)}_i}(p') \, \underline{\gamma^\mu_{ij}} \, \epsilon_{\mu(\lambda')}(k') \left(\frac{i}{\underline{\not{k} + \not{p} - m}} \right)_{jl}$$
$$\cdot \, \underline{\gamma^\nu_{lm}} \, \epsilon_{\nu(\lambda)}(k) \, \underline{u^{(s)}_m}(p) \Big\}$$

this has the matrix structure in Dirac space,

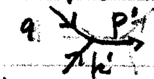
$$(- - -)\left(\begin{bmatrix} \; \end{bmatrix}\right)\left(\begin{bmatrix} \; \end{bmatrix}\right)\left(\begin{bmatrix} \; \end{bmatrix}\right)\left(\Big|\right) = \text{a number}$$

Look at ② = $e_2 \gamma_1$



The spacetime integrals give $(2\pi)^4 \delta(q-p+k') \Rightarrow q = p-k'$

$(2\pi)^4 \delta(-q-k+p') \Rightarrow \delta(q+k-p')$



$q = p'-k$

$$② = -\frac{e^2}{2}\left\{ (2\pi)^4 \delta(p-k'+k-p')\, \bar{u}_l^{(r)}(p')\, \gamma^\nu_{lm} \left(\frac{i}{\not{p}-\not{k}'-m}\right)_{mi} \gamma^\mu_{ij}\, u_j^{(s)}(p) \right\}$$

$$\cdot \left[\epsilon_{\mu(\lambda')}(k')\, \epsilon_{\nu(\lambda)}(k)\right]$$

Keep this up and we would find ① = ④ & ② = ③

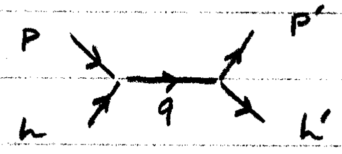
So, $\quad J^{(2)} = 2\left[① + ② \right]$

$$= -e^2 (2\pi)^4 \delta(p+k-p'-k')\, \bar{u}^{(r)}(p') \Big[\not{\epsilon}_{(\lambda')}(k') \frac{i}{\not{p}+\not{k}-m} \not{\epsilon}_{(\lambda)}(k)$$

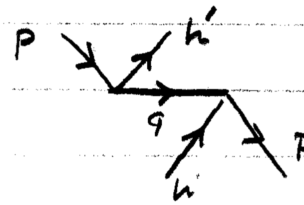
$$+ \not{\epsilon}_{(\lambda)}(k) \frac{i}{\not{p}-\not{k}'-m} \not{\epsilon}_{(\lambda')}(k') \Big] u^{(s)}(p)$$

Look at what the "momentum flow" results imply at the 2 vertices.—

From ①



From ②

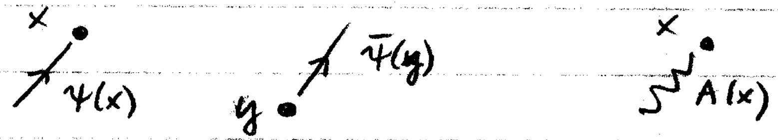


$q = p - k' \Rightarrow p = q + k'$

$q = p' - k \Rightarrow p' = q + k$

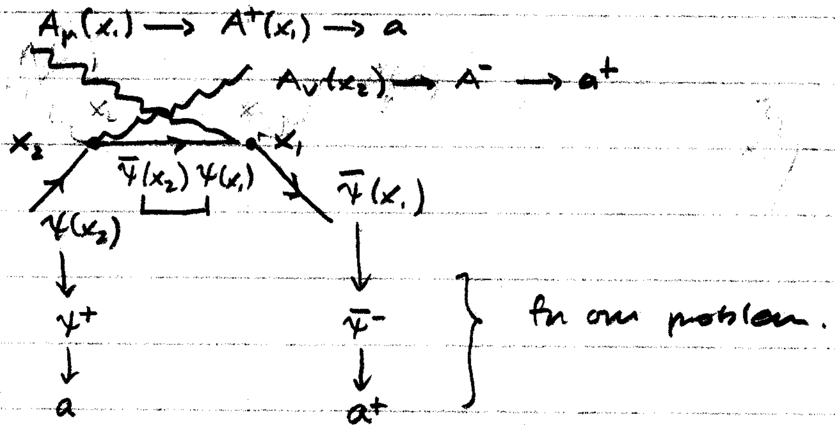
Earlier, I attached a graphical meaning to the Wick expansion terms. Let's recap that according to what we've calculated.

$$\underbrace{\bar{\psi}(x)\psi(y)} = \langle 0 | T[\bar{\psi}(x)\psi(y)] | 0 \rangle$$

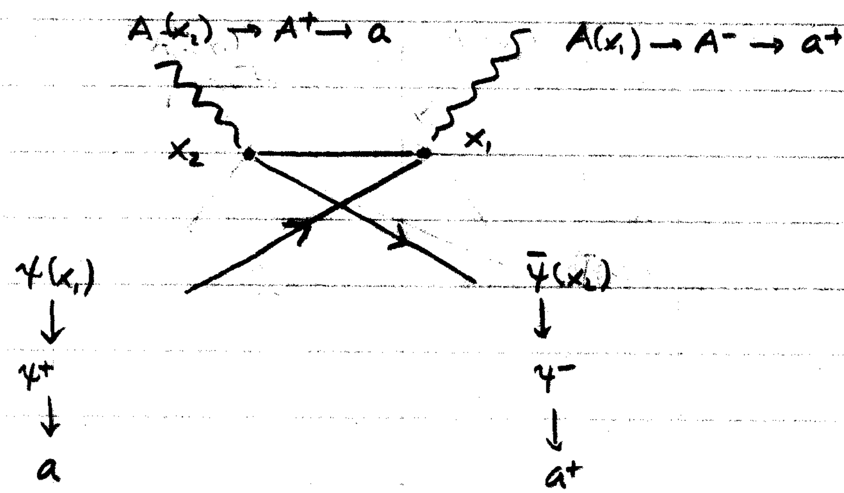


So, the first (① or ③) graph would be

generally:

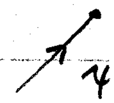


and ② or ④

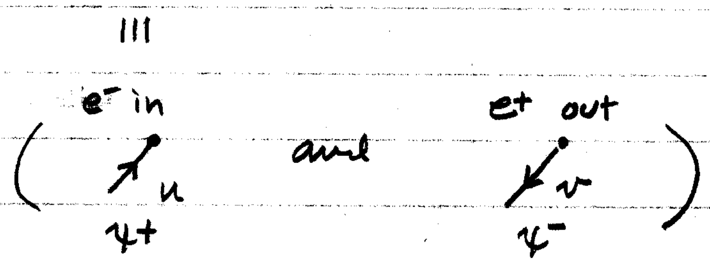


Remember the Feynman interpretation

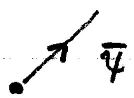
$\uparrow$ time



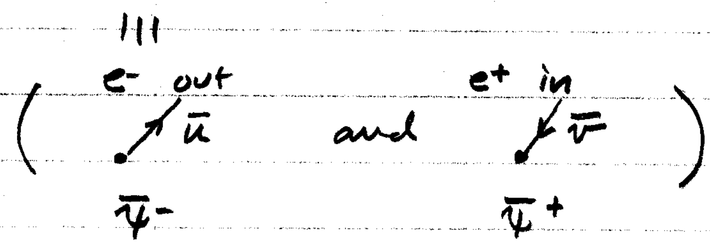
ψ^+ both annihilates electrons (a)
ψ^- and creates positrons (b^+)



likewise



annihilates positrons (b)
& creates electrons (a^+)



So, our process can be relabeled with the frequency components indicated.

The really useful Feynman rules eliminate <u>all</u> of the previous calculation and go directly to momentum space.

Standard Model on ~~one~~ ~~two~~ three pages

primordial group conserved: $SU(2) \otimes SU(3) \otimes U(1)$.

$\underset{\text{weak isospin}}{SU(2)} \otimes \underset{\text{color}}{SU(3)} \otimes \underset{\text{weak hypercharge}}{U(1)}$

$T_{universe} > 10^{15}$ K
$\tau_{universe} < 10^{-12}$ s

constituents are:

g^μ SU(3) octet, massless
$\vec{B}^\mu$ SU(2) triplet, massless 2dof
a^μ U(1) singlet, massless 2dof

appearance in problem $\longrightarrow$ $\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$, $\Phi^+ = \begin{pmatrix}\phi^-\\ \phi^0\end{pmatrix}$, massless

$-\frac{\lambda}{4}(\varphi^4 - 2a\varphi^2 + a^2)$

$-\frac{\lambda}{4}\varphi^4 + \frac{\lambda a}{2}\varphi^2 - \frac{\lambda a^2}{4}$

wrong sign for mass if $\lambda a > 0$

as many massless quarks & leptons as needed + built-in V-A weak interactions

@ $T \sim 10^{15}$ K, a 2nd order phase transition...

$B^{\mu 1} + \phi^+ \longrightarrow W^+$, massive
$B^{\mu 2} + \phi^- \longrightarrow W^-$, massive
} propagators of weak interactions 3dof

$B^{\mu 3} + \phi^0 \longrightarrow W^0$ 3dof

$g \longrightarrow g$ no change, SU(3) unbroken.

$\phi^0 \longrightarrow h$, massive Higgs Boson

& also: $\begin{pmatrix}A^\mu\\ Z^\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}a^{\mu 1}\\ W^{\mu 0}\end{pmatrix}$

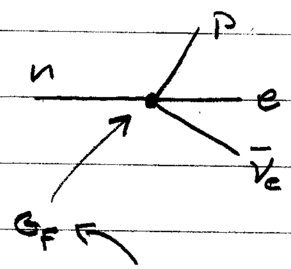
mass eigenstates $\longrightarrow$ QED interactions + new weak-electromagnetic interactions

mixing angle, θ_W, is a <u>measurable</u> ("Weinberg angle")

$\theta_W \sim 28°$

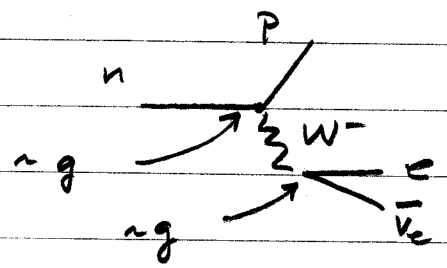
Historically, the weak interactions were characterized by Fermi's original idea:

old way:

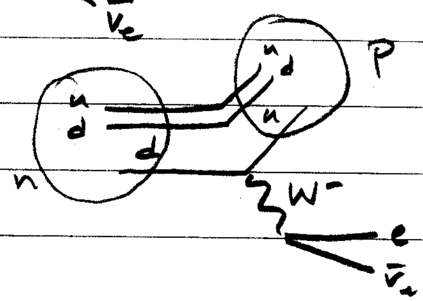


strength characterized by Fermi constant
$G_F = 1.03 \times 10^{-5}\, m_p^{-2}$

modern way: exploit the virtual "exchange" of W --



even more modern



unification links various constants..

$g^2 = \frac{8 M_W^2 G_F}{\sqrt{2}}$ easy to see for heavy W propagator

$g \sin\theta_W = e$ ← explicit weak & electromagneti

The leptons and quarks align themselves into $SU(2)$ doublets and singlets ... $T_f = 1/2$

$$T_f^3 = \begin{array}{c} +1/2 \\ -1/2 \end{array} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad u_R, d_R, c_R, s_R, t_R, b_R$$

$$\begin{array}{c} +1/2 \\ -1/2 \end{array} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

weak interactions

electromagnetic interactions (except ν's)