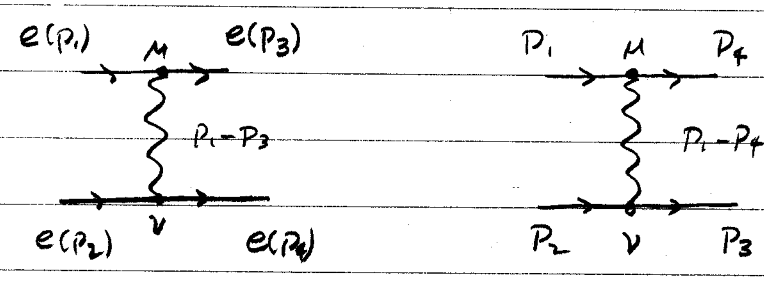
Moller Scattering $\quad e^- e^- \rightarrow e^- e^- \quad$ (1932)



(a) (b)

$$T^{(a)} = (-i)\,\bar{u}(p_3)(-ie\gamma_\mu)u(p_1)\,\frac{-ig^{\mu\nu}}{(p_1-p_3)^2}\,\bar{u}(p_4)(-ie\gamma_\nu)u(p_2)$$

$$T^{(b)} = (-1)(-i)\,\bar{u}(p_4)(-ie\gamma_\mu)u(p_1)\,\frac{-ig^{\mu\nu}}{(p_1-p_4)^2}\,\bar{u}(p_3)(-ie\gamma_\nu)u(p_2)$$

$$T_{fi} = T_{fi}^{(a)} + T_{fi}^{(b)}$$

$$= e^2\Big[\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\gamma^\mu u(p_2)\,\frac{1}{(p_1-p_3)^2}$$

$$- \bar{u}(p_4)\gamma_\mu u(p_1)\bar{u}(p_3)\gamma^\mu u(p_2)\,\frac{1}{(p_1-p_4)^2}\Big]$$

rule

Look at just the first term --

$$\bar{u}(p_3)\gamma_\mu u(p_1)\,\bar{u}(p_4)\gamma^\mu u(p_2) = a\cdot b = a_\mu b^\mu$$

square and spin sum and average -- just that term

$$\bar{\Sigma}\Sigma\bar{\Sigma}\Sigma\,(a\cdot b)(a\cdot b)^\dagger = \Sigma\, a^\mu b_\mu\,(a^\nu b_\nu)^\dagger$$

$$= \bar{\Sigma}\Sigma\bar{\Sigma}\Sigma\; a^\mu a_\nu^{\dagger}\, b_\mu b_\nu^\dagger$$

$$= \bar{\Sigma}\Sigma\, a^\mu a_\nu^\dagger\;\bar{\Sigma}\Sigma\, b^\mu b_\nu^\dagger$$

$$= \sum\sum \bar{u}(p_3)\gamma_\mu u(p_1)\left(\bar{u}(p_3)\gamma_\nu u(p_4)\right)^\dagger \bar{\sum}\sum\left(\bar{u}(p_4)\gamma^\mu u(p_2)\right)\left(\bar{u}(p_4)\gamma^\nu u(p_1)\right)^\dagger$$

$$= \sum\sum \bar{u}(p_3)\Gamma_\mu u(p_1)\left(\bar{u}(p_3)\Gamma_\nu u(p_1)\right)^\dagger \bar{\sum}\sum (\mu)(\nu)$$

$$= \mathrm{Tr}\left[(\not{p}_3+m)\Gamma_\mu(\not{p}_4+m)\bar{\Gamma}_\nu\right]\mathrm{Tr}\left[(\not{p}_4+m)\Gamma^\mu(\not{p}_2+m)\bar{\Gamma}^\nu\right]$$

with in massless limit

$$\bar{\sum}\sum|T|^2 = \frac{e^4}{4}\Bigg\{\frac{1}{(p_1-p_3)^4}\mathrm{Tr}\left[\not{p}_3\gamma_\mu\not{p}_1\gamma_\nu\right]\mathrm{Tr}\left[\not{p}_4\gamma^\mu\not{p}_2\gamma^\nu\right]$$

$$+\frac{1}{(p_1-p_4)^4}\mathrm{Tr}\left[\not{p}_4\gamma_\mu\not{p}\gamma_\nu\right]\mathrm{Tr}\left[\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu\right]$$

$$-\frac{1}{(p_1-p_3)^2(p_1-p_4)^2}\mathrm{Tr}\left[\not{p}_3\gamma_\mu\not{p}_1\gamma_\nu\not{p}_4\gamma^\mu\not{p}_2\gamma^\nu\right]$$

$$-\frac{1}{(p_1-p_3)^2(p_1-p_4)^2}\mathrm{Tr}\left[\not{p}_4\gamma_\nu\not{p}_1\gamma_\mu\not{p}_3\gamma^\nu\not{p}_2\gamma^\mu\right]\Bigg\}$$

1st term: $= 16\left(p_{3\mu}p_{1\nu}+p_{3\nu}p_{1\mu}-g_{\mu\nu}p_1\cdot p_3\right)\left(p_4^\mu p_2^\nu+p_4^\nu p_2^\mu-g^{\mu\nu}p_2\cdot p_4\right)$

$$= 16\Big(p_3\cdot p_4\, p_1\cdot p_2 + p_3\cdot p_2\, p_1\cdot p_4 - p_1\cdot p_3\, p_2\cdot p_4$$

$$+ p_3\cdot p_2\, p_1\cdot p_4 + p_3\cdot p_4\, p_1\cdot p_2 - p_1\cdot p_3\, p_2\cdot p_4$$

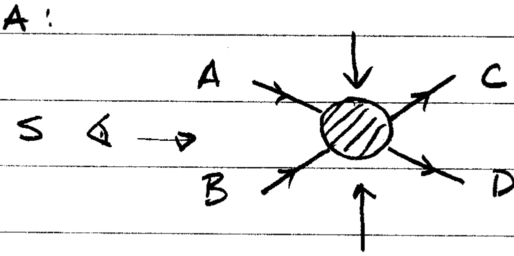
$$- p_1\cdot p_3\, p_2\cdot p_4 - p_1\cdot p_3\, p_2\cdot p_4 + \underbrace{g^{\mu\nu}g_{\mu\nu}}_{4}\, p_1\cdot p_3\, p_2\cdot p_4\Big)$$

$$= 32\left[p_1\cdot p_2\, p_3\cdot p_4 + p_1\cdot p_4\, p_2\cdot p_3\right]$$

more jargon -- for general scattering some invariants are typically defined called "Mandelstam Variables"

For a general scattering process $A+B \to C+D$

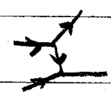
focus on A:



$s \equiv (p_A+p_B)^2$ →

"s channel"

$t \equiv (p_A-p_C)^2$



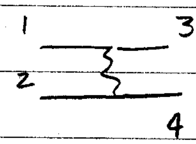
"t channel" 4-momentum transfer

$u \equiv (p_A-p_D)^2$

there is a fundamental algebraic constraint

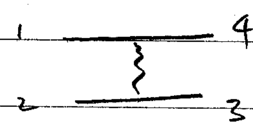
$$s+t+u = m_A^2+m_B^2+m_C^2+m_D^2$$

For our problem



~~define this to be t channel~~

then the 2nd diagram



is the u channel.

so, $s \equiv (p_1+p_2)^2 = (p_3+p_4)^2 = 2p_1\cdot p_2 = 2p_3\cdot p_4 \quad (m_i=0)$

$t \equiv (p_1-p_3)^2 = (p_2-p_4)^2 = -2p_1\cdot p_3 = -2p_2\cdot p_4$

$u \equiv (p_1-p_4)^2 = (p_2-p_3)^2 = -2p_1\cdot p_4 = -2p_2\cdot p_3$

So, with this definition, the 1st term can be written,

$$8\left[s^2+u^2\right]$$

and the denominator for that term is t^2.

The other traces work out to be

2nd $\quad 32\left[(p_1\cdot p_2)^2+(p_1\cdot p_3)^2\right] = 8(s^2+t^2)$

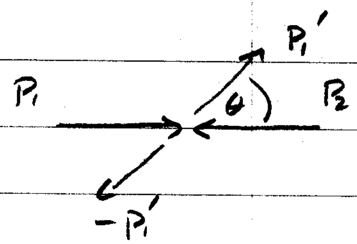
3rd $\quad -32\, p_1\cdot p_2\, p_3\cdot p_4 = -8s^2$

4th $\quad -32(p_1\cdot p_2)^2 = -8s^2$

So,

$$\sum\sum|T|^2 = \overset{2}{\cancel{\frac{e^4}{4}\cdot 8}}\left\{\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}+\frac{2s^2}{tu}\right\}$$

Now we can pick a frame – center of momentum....



$P_1 = (E, \vec{p})$
$P_2 = (E, -\vec{p})$
$P_3 = (E, \vec{p}\,')$
$P_4 = (E, -\vec{p}\,')$

so $\vec{p}\cdot\vec{p} = |p||p'|\cos\theta$

and for massless electrons $|\vec{p}| = |\vec{p}\,'| = E$

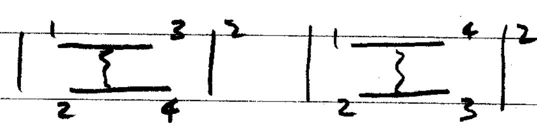
then
$$P_1\cdot P_2 = \frac{s}{2} = E^2 + p^2 = 2E^2$$

$$P_1\cdot P_3 = \frac{t}{2} = E^2 - \vec{p}\cdot\vec{p}\,' \cong E^2(1-\cos\theta) = 2E\sin^2\theta/2$$

$$P_1\cdot P_4 = \frac{u}{2} = E^2 + \vec{p}\cdot\vec{p}\,' = E^2(1+\cos\theta) = 2E\cos^2\theta/2$$

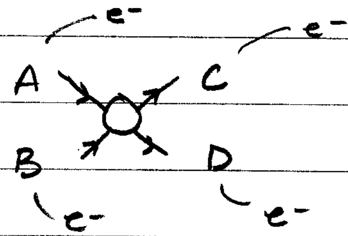
and
$$\bar{\sum}\sum|T|^2 = 2e^4\left\{\frac{1+\cos^4\theta/2}{\sin^4\theta/2} + \frac{1+\sin^4\theta/2}{\cos^4\theta/2} + \frac{2}{\sin^2\theta/2\,\cos^2\theta/2}\right\}$$

t channel　　u channel　　interference



We can use "crossing symmetry" to predict another process.

Møller –



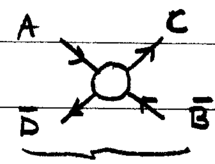
$AB \to CD$

can predict Bhabha, $e^- e^+ \to e^- e^+$, or $A\bar{D} \to C\bar{B}$ by looking at the Mandelstam invariants

Møller –

$$s = (A+B)^2$$
$$t = (A-C)^2$$
$$u = (A-D)^2$$

Bhabha – $B \to \bar{D}$ $\quad D \to \bar{B}$



reverse momenta

$$s' = (A - \bar{D})^2 = u$$
$$t' = (A - C)^2 = t$$
$$u' = (A - (-\bar{B})) = s$$

so can get matrix element for Bhabha by exchanging $s_M \to u_B$ and $u_M \to s_B$

$$|\mathcal{M}|^2_B = \frac{u^2 + s^2}{t^2} + \frac{u^2 + t^2}{s^2} + \frac{2u^2}{ts}$$