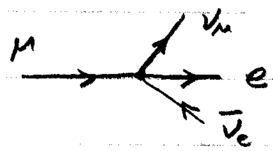
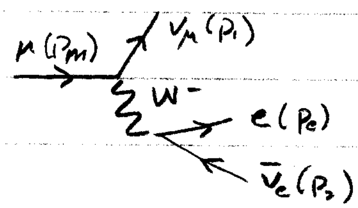
Muon Decay

One of the simplest weak interactions is muon decay, originally thought to be another 4-fermion interaction,



but in the 1950's, thought to be the result of



$k^\mu = p_m^\mu - p_1^\mu$

The utility of this is of 2 sorts. First, it is precisely the semileptonic decay sequence of quarks lighter than about M_W and second, various new physics scenarios predict the presence of additional gauge groups - which in turn require various additional W and Z bosons. Very precise μ decay measurements are sensitive to such a mixing with the "regular" W - especially if a RH weak interaction is involved.

The T matrix for the exchange graph is,

$$T = (-i)\left(\frac{ig}{2\sqrt{2}}\right)\left[\bar{u}_{\nu_\mu}\gamma_\mu(1-\gamma_5)u_m\right]\left[\frac{g^{\mu\nu} - k^\mu k^\nu/M_W^2}{k^2 - M_W^2}\right]\left(\frac{ig}{2\sqrt{2}}\right)$$

$$\times\left[\bar{u}_e\gamma_\nu(1-\gamma_5)v_{\bar{\nu}_e}\right]$$

Certainly, $|k^2| \ll M_W^2$ so,

$$\frac{g^{\mu\nu} - k^\mu k^\nu / M_W^2}{k^2 - M_W^2} \longrightarrow -\frac{g^{\mu\nu}}{M_W^2}$$

Using the definition $\dfrac{g^2}{8 M_W^2} = \dfrac{G_F}{\sqrt{2}}$ we

would have the graph for the 1st diagram.

$$T = -i \frac{G_F}{\sqrt{2}} \bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma_5) u_\mu \, \bar{u}_e \gamma^\mu (1-\gamma_5) v_{\bar{\nu}_e}$$

The rate will be

$$d\Gamma = (2\pi)^4 \delta(p_e + p_1 + p_2 - p_\mu) \frac{1}{2E_\mu} \sum |T|^2 d^3p_e \, d^3p_1 \, d^3p_2$$

By the now familiar techniques,

$$|T|^2 = \frac{1}{2} G_F^2 \sum_\mu \sum_e \sum_{\nu_\mu} \sum_{\bar{\nu}_e} u_2 \bar{u}_2 \gamma_\mu (1-\gamma_5) u_\mu \bar{u}_\mu \gamma_\nu (1-\gamma_5)$$

$$\times \, u_e \bar{u}_e \gamma^\mu (1-\gamma_5) v_1 \bar{v}_1 \gamma^\nu (1-\gamma_5)$$

We will <u>not</u> average over initial μ helicities and <u>will</u> consider outgoing electron helicities...

So, for example we will project out both the + energy and helicity electrons--

$$\sum_{i=1}^{2} u_e^{(i)} \bar{u}_e^{(i)} = \frac{1}{2}(\not{p}_e + m_e)(1 + \gamma_5 \not{s}_e) \qquad \text{ditto for muon}$$

where, remember, in the μ rest frame $s_{m_\mu} = (0, \hat{s}_m)$

The traces are actually quite involved algebraically, and we would get,

$$|T|^2 = \frac{1}{2} G_F^2 \, (p_e - m_e s_e) \cdot p_2 \, (p_m - m_m s_m) \cdot p_1 \qquad \text{4vectors}$$

The neutrinos will both be undetected, so integrate them away — with a trick.

Write,

$$d\Gamma = \frac{2G_F^2}{(2\pi)^5} \frac{d^3p_e}{E_e E_m} \, (p_e - m_e s_e)^\mu (p_m - m_m s_m)^\nu I_{\mu\nu}$$

where

$$I_{\mu\nu} \equiv \iint \delta^4(p_1 + p_2 - q) \, p_{2\mu} \, p_{1\nu} \, \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2}$$

Define $\quad q^\mu \equiv p_m^\mu - p_e^\mu = p_1^\mu + p_2^\mu$

The only vectors and tensors that $I_{\mu\nu}$ can depend on are q^μ and $g^{\mu\nu}$, so generally parameterize it as,

$$I_{\mu\nu} = A q^2 g_{\mu\nu} + B q_\mu q_\nu$$

Multiply by $g^{\mu\nu}$ and $q^\mu q^\nu$

$$g^{\mu\nu} I_{\mu\nu} = 4Aq^2 + Bq^2 = \iint \frac{\delta(\ )}{E_1 E_2} p_1 \cdot p_2 \, d^3p_1 \, d^3p_2$$

and

$$q^\mu q^\nu I_{\mu\nu} = Aq^4 + Bq^4 = \iint \frac{\delta(\ )}{E_1 E_2} (p_1 \cdot p_2)^2 \, d^3p_1 \, d^3p_2$$

We can evaluate these invariants in any frame,

choose the one in which $\vec{p}_1 = -\vec{p}_2 \Rightarrow q^\mu = (q^0, \vec{0})$.

There, $\vec{p}_1 + \vec{p}_2 - \vec{q} = 0$ and so $\int \delta^3(0)\, d^3p_2 = 1$

so,

$$g^{\mu\nu} I_{\mu\nu} = 8\pi \int E_1^2 \, dE_1 \, \delta(2E_1 - q^0) = \pi q^2$$

$$q^\mu q^\nu I_{\mu\nu} = 16\pi \int E_1^4 \, dE_1 \, \delta(2E_1 - q^0) = \tfrac{1}{2} \pi q^2$$

Solving for A & B ..

$$A = \pi/6 \quad \text{and} \quad B = \pi/3$$

$$p_1 \cdot p_2 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2 = E_1 E_2 + E_1 E_2 = 2 E_1 E_2$$

$$d^3 p_1 = p_1^2 dp_1 \, d\Omega = 4\pi \, E_1^2 dE_1$$

$$E_1 = E_2 \qquad \int \delta(2E_1 - q^0) 4\pi \, 2 \cdot E_1 E_2 \frac{\cancel{E_1^2} dE_1}{\cancel{E_1^2}} = \int 8\pi \, E_1^2 dE_1 \, \delta(2E_1 - q^2)$$

$$\delta(2E_1 - q^2) \qquad \frac{1}{2} \delta(E_1 - q^2/2)$$

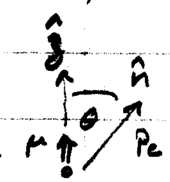
$$4\pi \, E_1^2 \Big|_{E_1 = q^{0 2}/4}$$

$$4\pi \frac{q^2}{4} = \pi q^2$$

So, $$I_{\mu\nu} = \frac{1}{6}\pi \left(q^2 g_{\mu\nu} + 2 q_\mu q_\nu \right)$$

and

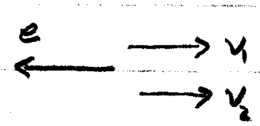
$$d\Gamma = \frac{\pi}{3} \frac{G_F^2}{(2\pi)^5} \frac{d^3 p_e}{E_e E_m} \left(p_e - m_e s_e \right)^\mu \left(p_m - m_m s_m \right)^\nu \left(q^2 g_{\mu\nu} + 2 q_\mu q_\nu \right)$$



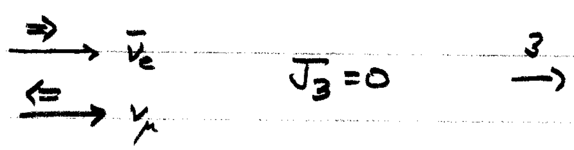
Choose μ rest frame and use $m_e \ll m_\mu$ w/

$$d\Gamma = \frac{\pi G_F^2}{96 m_m} d\Omega_e \, p_e \, dE_e \left(1 - \hat{n}\cdot\hat{s}_e \right) E_e m_m^2$$
$$\times \left[(3m_\mu - 4E_e) + (m_m - 4E_e)\, \hat{n}\cdot\hat{s}_m \right]$$

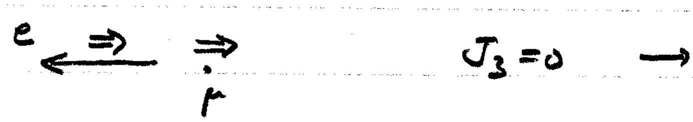
The maximum electron energy would be when



Because of helicity assignments for neutrinos and antineutrinos



then, at this max E_e configuration, $\hat{s}_e$ and $\hat{s}_m$ must be related:



In max E_e state, $E_\mu = E_e^M + 2E_\nu = m_\mu$

and $\vec{P}_e^M = 2\vec{p}_\nu = 2E_\nu \hat{z}$

so $E_e^M + |\vec{P}_e^M| = m_\mu$

$E_e^M = m_\mu/2$

since $m_e \ll m_\mu$

Define $x = E_e/E_e^M$ so $E_e = \frac{x m_\mu}{2}$

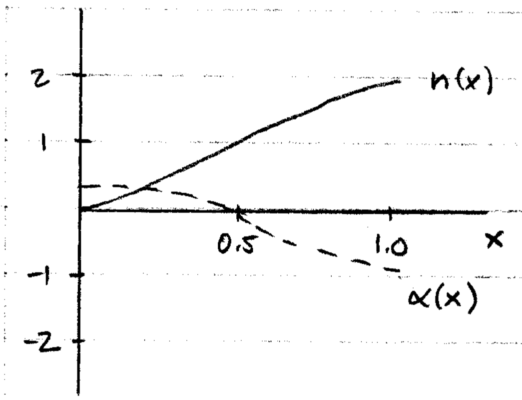
and with $\hat{s}_m = \hat{z}$

$$d\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \underbrace{\left[2x^2(3-2x)\right]}_{n(x)} \underbrace{\left[1+\alpha\cos\theta\right]}_{\text{asymmetry wrt } \hat{s}_m} \underbrace{\left[\frac{1-\hat{n}\cdot\hat{s}_e}{2}\right]}_{h(e),\text{ helicity of } e} dx \frac{\sin\theta\, d\theta\, d\varphi}{4\pi}$$

where $\alpha = \frac{1-2x}{3-2x}$ ($= -1$ for $x=1$ when $E_e = E_e^M$)

Further, $h(e) = -1$, independent of x.

These normalized quantities have the following energy dependence:



Integrating, the total rate is,

$\tau = \Gamma^{-1}$.

$\Gamma = \dfrac{G^2 m_\mu^5}{192\pi^3}$ and an early measured value was 2.2×10^{-6} s which gave $G_\mu \simeq G_\beta$.

A traditional way to analyze μ decay (and τ and subsequently top) is to parameterize

$$T = \frac{G_F}{\sqrt{2}} \sum_i \bar{u}_e \Theta_i u_m \bar{u}_2 \Theta_i (C_i + C_i' \gamma_5) v_1 \quad \text{w/ } g_i^2 = |C_i|^2 + |C_i'|^2$$

which is reordered into the charge-retention form.--

Then, various couplings are formed:

collectively called the Michel Parameters

$\rho = (3g_A^2 + 3g_V^2 + 6g_T^2)/D$ ← often (incorrectly) called "The Michel Parameter" or (correctly) "the ρ param

$\eta = (g_S^2 - g_P^2 + 2g_A^2 - 2g_V^2)/D$

$\xi = (6g_S g_P \cos\phi_{SP} - 8g_A g_V \cos\phi_{AV} + 14 g_T^2 \cos\phi_{TT})/D$

$\delta = (-6g_A g_V \cos\phi_{AV} + 6g_T^2 \cos\phi_{TT})/D\xi$

$h = (2g_S g_P \cos\phi_{SP} - 8g_A g_V \cos\phi_{AV} - 6g_T^2 \cos\phi_{TT})/D$

$D = g_S^2 + g_P^2 + 4g_V^2 + 6g_T^2 + 4g_A^2$

$\cos\phi_{ij} = \mathrm{Re}(C_i^* C_j' + C_i' C_j^*)$

In general, the rate becomes.

$$d\Gamma = \frac{D}{16} \frac{G^2 m_\mu^5}{192\pi^3} \frac{\sin\theta\, d\theta\, d\phi}{4\pi}$$

$$\times \left\{ \left[12(1-x) + \frac{4}{3}\rho(8x-6) \right] \right.$$

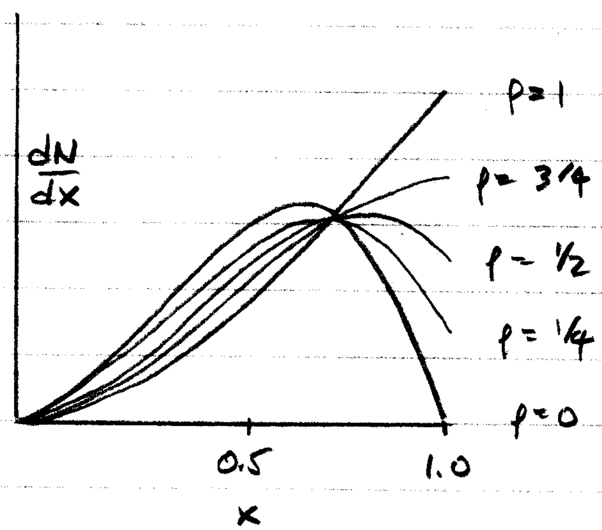
$$\left. \times\ \xi \cos\theta \left[4(1-x) + \frac{4}{3}\delta(8x-6) \right] \right\} x^2 dx$$

ρ & δ characterize the high end of the β spectrum.

ξ characterizes the asymmetry

η drops out in a massless electron limit

For V & A	V-A
$\rho = 3/4$	$\rho = 3/4$
$\delta = 3/4$	$\delta = 3/4$
$\xi = \frac{2 g_A g_V \cos\phi_{AV}}{g_A^2 + g_V^2}$	$\xi = -1$
	$\eta = 0$



Best $\rho = 0.7503 \pm 0.0026$

Peoples, unpub.

$\eta = 0$

$\rho = 0.7518 \pm 0.0026$

w/ $\eta = -0.12 \pm 0.21$

in 2 parameter fit

My favorite plot shows the evolution of ρ -- 1957/58 was the V-A prediction. Also shown is a modern measurement.

$$\rho = 0.7518 \pm 0.0026 \tag{3.68}$$

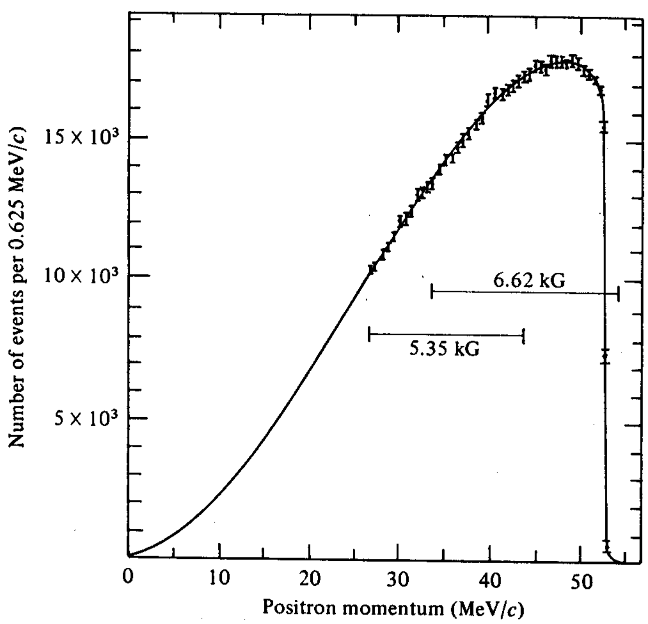
The quantity η, which should be zero according to the V–A law, was measured by stopping π^+ and μ^+ in a bubble chamber in a strong magnetic field. Over 400,000 exposures were taken, including 2,070,000 μ^+ decays. Momentum measurements calibrated with internal conversion e^- tracks of known momentum from a radioactive source inside the bubble chamber yielded (Derenzo 69):

$$\eta = -0.12 \pm 0.21 \tag{3.69}$$

3.4.4 *Asymmetry in the decay of polarized muons*

The parameter ξ should take the value -1 according to the V–A law, and experimental measurement of ξ is important because observation of departure of ξ from -1 would signify the existence of right-handed (V + A) currents, as expected in left–right symmetric models (see Section 2.12). The experimental problem is very difficult because one actually observes the product $P\xi$, where P is the polariza-

Figure 3.7. Results of experiments to determine ρ. Experimental points are plotted, together with a theoretical curve for $\rho = \frac{3}{4}$ corrected for radiative effects and ionization loss. (From Bardon et al. 65. Reprinted with permission.)



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tion of the muons. This quanti
because of miscellaneous muc
which can easily be account
grated asymmetry $a = \xi/3$ w
stopping in photoemulsion in
the muon spin. The result (Gu

$$\xi = -0.972 \pm 0.013$$

was found, in accord with the
cient to constrain left–right sy
tion 2.12). In a measuremen
stopped in the magnetic field
which is used for both moment
eters δ and P are determined
to experimental results. The b
berger 68) and others is

$$\delta = 0.7551 \pm 0.0085$$

3.4.5 *Summary*

Table 3.2 presents the
them with predictions of the V

Table 3.2. *Properties*

Parameter
Muon mass m_μ
m_μ/m_e
ν_μ mass m_{ν_μ}
Maximum electron ener in muon decay assum $m_{\nu e} = m_{\nu\mu} = 0$
Muon mean life
Michel parameters
ρ
η
ξ
δ
$h(e^-)$

* At 90% confidence lin

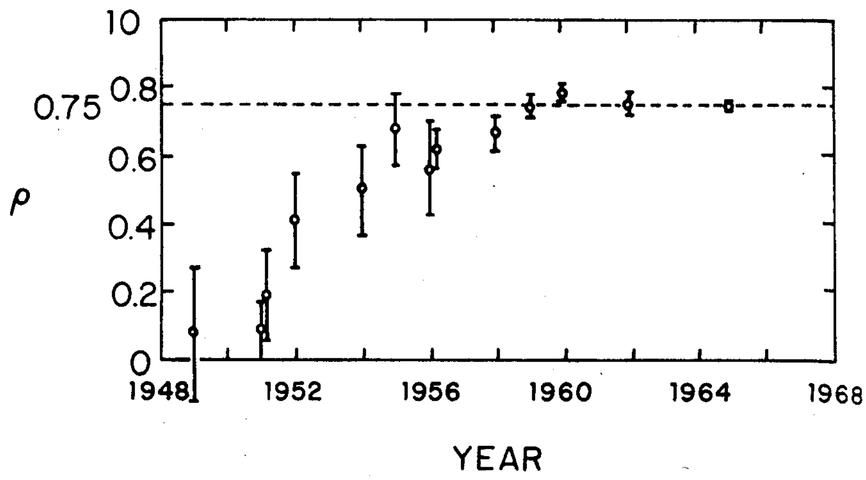

Fig. 21.2. Experimental determination of the Michel parameter ρ versus time.

It is instructive to plot the experimental value of ρ against the year when the measurement was made. As shown in Figure 21.2, historically it began with $\rho \cong 0$ and then slowly drifted upwards; only after the theoretical prediction in 1957 did it gradually become $\rho = \frac{3}{4}$. Yet, it is remarkable that at no time did the "new" experimental value lie outside the error bars of the preceding one.

(ii) When $x = 1$, the distribution (21.8) becomes

$$\frac{d^2 N_e}{dx\, d\cos\theta} = 1 \mp \cos\theta \quad .$$

As we shall see, this expression can be derived without any actual computation. In the rest system of the muon, the momentum of e, $\vec{p}_e$, has its maximum magnitude at $x = 1$. The neutrino and antineutrino momenta $\vec{p}_\nu$ and $\vec{p}_{\bar{\nu}}$ must therefore be parallel to each other, but antiparallel to $\vec{p}_e$. Because ν is lefthanded and $\bar{\nu}$ righthanded, when these two particles are moving in the same

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direction, they transform toget[...]
momentum

$$\vec{p}_\nu + \vec{p}_{\bar{\nu}} = -\vec{p}_e$$

under a Lorentz transformation. [...] as in (13.101), the angular dis[...] tion of $\cos\theta$.

In the μ^- decay, the fi[...] 21.3, we see that the angular [...] to be forbidden, but $\theta = \pi$ is [...] of e^- has to be $1 - \cos\theta$; li[...]

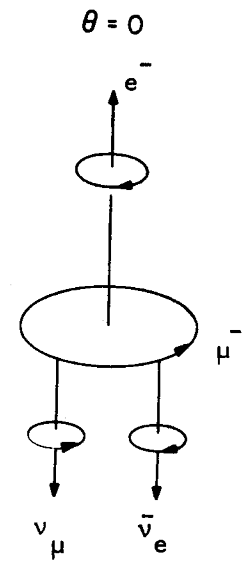

Fig. 21.3. The curled arrows i[...] in μ^- decay.

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which do not require production of ν_2, can still set a limit on the mixing angle ζ.

Hadronic weak processes set limits on right-handed currents independently of ν_R masses. In a class of models, called "manifestly" L-R symmetric, the left-handed and right-handed Kobayashi-Maskawa quark mixing angles are assumed to be identical, and CP invariance is assumed to hold. In these models the K_L-K_S mass difference requires $m(W_2)>1.6$ TeV (Ref. 6), and current-algebra analysis of $\Delta S=1$ decays yields $\zeta \leq 0.004$, $m(W_2)>300$ GeV for $\zeta=0$ (Ref. 7). If left-handed and right-handed mixing angles are not identical hadronic processes are consistent with $m(W_2)\geq 300$ GeV (Ref. 8). Another strong limit $\zeta<0.005$ has been obtained in a model-dependent analysis of semileptonic weak processes, again assuming manifest L-R symmetry.[9]

Figure 1 exhibits contours corresponding to 90% confidence limits[10] on ϵ and ζ from experiments in β decay, μ decay, and $\nu N, \bar{\nu} N$ scattering. The allowed regions contain the origin $\epsilon=\zeta=0$, which is the $V-A$ limit. Manifest L-R symmetry has been assumed. The contours from μ- and β-decay experiments have been plotted with the assumption that the right-handed neutrinos are sufficiently light not to affect the kinematics. The bold ellipse in Fig. 1 is the combined result from the analysis of the muon-decay spectrum end point opposite the μ^+ spin, presented in this paper, and from our μSR analysis.[11,12] The other muon-decay contours are derived from the measurement[13] of the polarization parameter ξP_μ (dotted curve) and the Michel parameter[14] ρ (solid curve). Nuclear β-decay contours are derived from the Gamow-Teller β polarization[15] (dot-dashed curve); the comparison of Gamow-Teller and Fermi β polarizations[16] (long-dashed curves); and the ^{19}Ne asymmetry $A(0)$ and ft ratio,[17] with the assumption of conserved vector current (short-dashed curves). The limits from the y distributions[18] in $\nu N, \bar{\nu} N$ scattering (double lines) are valid irrespective of ν_R mass.

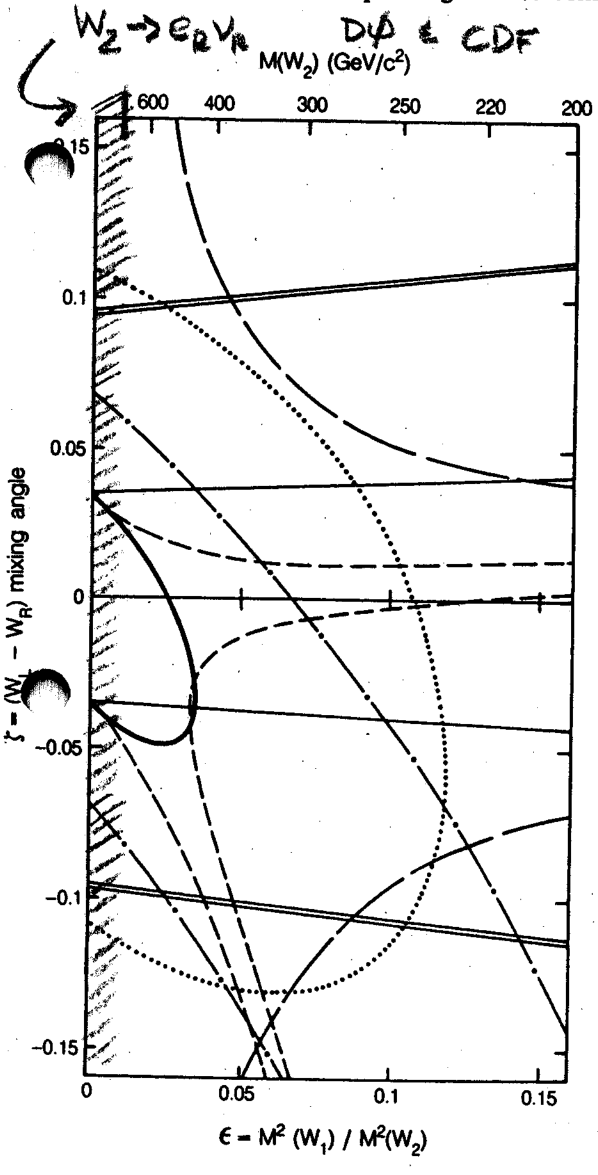

FIG. 1. Experimental 90% confidence limits on the mass-quared ratio ϵ and mixing angle ζ for the gauge bosons W_1 and W_2. The allowed regions are those which include $\epsilon=\zeta=0$. The old ellipse is the combined result from the analysis presented in is paper and from our μSR analysis (Refs. 11 and 12). The urces of the other limits are described in the text.

Section II of this paper discusses the properties of the muon-decay spectrum and their application to the data analysis. The beamline and experimental apparatus are discussed in Sec. III. Event reconstruction and selection are considered in Sec. IV. Data analysis and data fitting results are presented in Sec. V, and systematic errors are discussed in Sec. VI. The conclusions from the experimental result are drawn in Sec. VII.

II. MUON-DECAY SPECTRUM

The muon differential decay rate for an interaction mediated by a heavy vector boson W differs from the decay rate computed with the corresponding four-fermion contact interaction Hamiltonian by terms[19] of order $(m_\mu/M_W)^2$. These terms are $\approx 10^{-6}$ for $M_W \approx 80$ GeV/c^2 and are negligible at the present level of experimental precision. Consequently we will use the expression for the muon-decay spectrum computed for a four-fermion contact interaction. We will also assume that neutrinos are sufficiently light not to affect the kinematics. We will return to the question of massive neutrinos in Sec. VII G.

Without radiative corrections, the muon differential decay rate,[20] integrated over e^+ spin directions, is given by

$$\frac{d^2\Gamma}{x^2 dx\, d(\cos\theta)} \approx \left[(3-2x)+(\tfrac{4}{3}\rho-1)(4x-3) + 12\frac{m_e}{m_\mu}\frac{x-1}{x}\eta\right] - [(2x-1)+(\tfrac{4}{3}\delta-1)(4x-3)]\xi P_\mu \cos\theta \,. \tag{2.1}$$

Here x is the reduced energy $E_e/E_{\max}$, where $E_{\max}=(m_e{}^2+m_\mu{}^2)/(2m_\mu)=52.83$ MeV is the maximum energy and m_e and m_μ are the particle masses. The effects of finite positron mass are neglected in the above formula but not in the analysis. The angle between the positron momentum and the muon polarization vector

This is not just a silly effort at getting a number precisely. If the gauge group of the electroweak interactions is larger than $SU(2) \times U(1)$ .. more gauge bosons would be required. In particular, a RH W' is interesting as it would rationalize the lack of respect for parity.

So, W_L and W_R would presumably mix and one defines

$$W_1 = \cos\zeta\, W_L + \sin\zeta\, W_R \quad = \text{observed } W \text{ boson}$$

$$W_2 = -\sin\zeta\, W_L + \cos\zeta\, W_R \quad = \text{new, unobserved}$$

physical masses

μ decay provides a means of testing this, which because of extreme precision, reaches to very high mass exclusion and mixing angle limits. Shown on the next page are limits from the most precise measurement at TRIUMF in Vancouver.