$\boxed{B'}$

$$B' = \lim_{\not p \to m} \frac{\Sigma(p) - A}{\not p - m}$$

$$= \frac{e^2}{8\pi^2}\int_0^1 dx\,(x-1)\ln\left\{\frac{(1-x)\Lambda^2}{m^2x^2}\right\}$$

$$- \lim_{\not p \to m}\frac{e^2}{8\pi^2}\frac{1}{\not p - m}\int_0^1 dx\,[2m+(x-1)\not p]\ln\left\{1-\overbrace{\frac{x(1-x)(p^2-m^2)}{m^2x^2+(1-x)\lambda^2}}^{\text{small}}\right\}$$

$$\equiv B_1 + B_2 \qquad *$$

$$B_1 = \frac{e^2}{8\pi^2}\int_0^1 dx\,(x-1)\ln(\Lambda^2/m^2) + \frac{e^2}{8\pi^2}\underbrace{\int_0^1 (x-1)\ln\left(\frac{1-x}{x^2}\right)}_{5/4 \text{ (next page)}}$$

$$= \frac{e^2}{8\pi^2}\ln(\Lambda^2/m^2)\underbrace{\int_0^1 dx\,(x-1)}_{-1/2} + \frac{e^2}{8\pi^2}(-5/4)$$

$$= \frac{\alpha}{2\pi}\left[-\frac{1}{2}\ln(\Lambda^2/m^2) - 5/4\right] \qquad *$$

$$B_2 = -\lim_{\not p \to m}\frac{1}{\not p - m}\frac{e^2}{8\pi^2}\int_0^1 dx\,[2m+(x-1)\not p]\,\underbrace{\frac{x(1-x)(p^2-m^2)}{m^2x^2+(1-x)\lambda^2}}_{\text{expansion of } \ln(1+\_)}$$

$$= -\lim_{\not p \to m}\frac{(p^2-m^2)}{\not p - m}\frac{e^2}{8\pi^2}\int_0^1 dx\,[2m+(x-1)\not p]\,\frac{x(1-x)}{m^2x^2+(1-x)\lambda^2}$$

$$\frac{p^2-m^2}{\not p - m} = \frac{(\not p - m)(\not p + m)}{\cancel{\not p - m}} = \not p + m \quad \text{take limit} \to 2m$$

$$B_2 = -\frac{2me^2}{8\pi^2}\int_0^1 dx\,[2m+(x-1)m]\,\frac{x(1-x)}{m^2x+(1-x)\lambda^2}$$

...

$$= -\frac{2m^2e^2}{8\pi^2}\underbrace{\int_0^1 dx\,\frac{x-x^3}{m^2x+(1-x)\lambda^2}}_{\frac{1}{2m^2}\left[\ln(m^2/\lambda^2)-1\right]}$$

$$-\int_0^1 dx\,(1-x)\ln\left(\frac{1-x}{x^2}\right)$$

$$= \int dx\,\ln\left(\frac{1-x}{x^2}\right) - \int dx\, x\ln\left(\frac{1-x}{x^2}\right)$$

$$= \int dx\,\ln(1-x) - \int dx\,\ln x^2 - \int dx\, x\ln(1-x) + \int dx\, x\ln x$$

$$A \quad - \quad B \quad - \quad C \quad + \quad D$$

$$= -1 + 2 + 3/4 + -1/2 = 1 - \tfrac{1}{2} + 3/4 = 1/2 + 3/4 = \frac{2+3}{4}$$

$$= 5/4$$

so,
$$B' = -\frac{\alpha}{2\pi}\left[\ln(\Lambda/m) + \ln(m^2/\lambda^2) + 9/4\right] \quad *$$
$$= B(\Lambda) + B(\lambda)$$

and we have so far,

$$\Sigma(p) = A(\Lambda) + (\not p - m)\left[B(\Lambda) + B(\lambda)\right] + (\not p - m)^2 \Sigma_R$$

If we then directly solve for A

$$(\not p - m)^2 \Sigma_R = \frac{e^2}{8\pi^2}\int_0^1 dx\,\left[2m + (x-1)\not p\right]\frac{x(1-x)(p^2-m^2)}{m^2x^2 + (1-x)\lambda^2}$$

call D.

so
$$\Sigma_R = \frac{(p^2-m^2)}{(\not p - m)^2}\,\frac{\alpha}{2\pi}\left\{\int_0^1 dx\,\frac{\left[2mx(1-x) + \not p(x-1)x(x-1)\right]}{D}\right\}$$

$$\frac{(p^2-m^2)}{(\not p - m)^2} = \frac{\cancel{(\not p - m)}(\not p + m)}{(\not p - m)^{\cancel{2}}}$$

$$\{\ \} = (2m - \not p)\int \frac{x\,dx}{D} - (2m - 2\not p)\int\frac{x^2dx}{D} - \not p\int\frac{x^3dx}{D}$$

all are integrals like
$$\int\frac{(x, x^2, x^3)\,dx}{ax^2 + bx + c}$$
which can be explicitly done

and we get

$$\{\} = \frac{1}{m^2}\left\{ \tfrac{1}{2}\ln(m^2/\lambda^2)\left[2m - \not{p}\right] - 2m + 3\not{p} \right\}$$

Useful is

$$(\not{p}-m)\Sigma_R = \frac{\cancel{(\not{p}-m)}}{\cancel{(\not{p}-m)}}(\not{p}+m)\frac{\alpha}{2\pi}\{\ \}$$

more algebra

$$(\not{p}-m)\Sigma_R = (\not{p}+m)\frac{\alpha}{2\pi m}\left\{\tfrac{1}{2}\ln(m^2/\lambda^2) + 1\right\} \qquad A'$$

Let's combine B & $(\not{p}-m)\Sigma_R$

$$(\not{p}-m)\left[B(\Lambda) + B(\lambda) + \Sigma_R \right]$$

$$= -(\not{p}-m)\frac{\alpha}{2\pi}\left[\ln\Lambda/m + 9/4 + \ln(m^2/\lambda^2) \right.$$

$$\left. - \frac{(\not{p}+m)}{m}\tfrac{1}{2}\ln(m^2/\lambda^2) - \frac{(\not{p}+m)}{m} \right]$$

we're working $\not{p} \to m$ inside [ ]

$$[\ ] = \ln\Lambda/m + 9/4 + \ln(m^2/\lambda^2) - \frac{2m}{m}\tfrac{1}{2}\ln(m^2/\lambda^2) - 2$$
$$= \ln\Lambda/m + 9/4 - 2$$

ie we can combine $B(\lambda)$ and part of Σ_R to form $\Sigma_f(\lambda, p)$, which is infrared finite as $\not{p} \to m$.

IR finite as $\not p \to m$

so,

$$\Sigma(p) = A(\Lambda) + (\not p - m) B(\Lambda) + (\not p - m)\Sigma_f(\lambda, p)$$

we started with $\quad \bar u(p')\, i\Sigma\, u(p)$

$$= \bar u(p')\left[A(\Lambda) + (\not p - m)B(\Lambda) + (\not p - m)\Sigma_f \right] u(p)$$

$\uparrow$ dimensions of mass.

remember.

$i S_p' =$ — +  +  + ...

$$= \frac{i}{\not p - m_0 - \Sigma(p)}$$

to order $\frac{\alpha}{\xi}$ only

$$= \frac{i}{\not p - m_0 - A - (\not p - m_0)B - (\not p - m_0)\Sigma_f}$$

look at pole: — denominator $= \not p - m_0 - A(\Lambda) - (\not p - m_0)B(\Lambda) - (\not p - m_0)\Sigma_f(\lambda, p)$

write $\quad B \equiv 1 - \xi = \mathcal{O}(\alpha)$

$$\begin{aligned} \text{den} &= \not p - m_0 - (\not p - m_0)\left[1 - \xi - \Sigma_f\right] - A(\Lambda) \\ &= \cancel{(\not p - m_0)} - \cancel{(\not p - m_0)} + (\not p - m_0)\xi + (\not p - m_0)\Sigma_f - A(\Lambda) \\ &= \xi\left[(\not p - m_0) - \frac{A}{\xi} - \frac{\Sigma_f}{\xi}(\not p - m_0)\right] \end{aligned}$$

$B \sim \theta(\alpha)$

$A \sim \theta(\alpha)$

$BA \sim \theta(\alpha^2) \longrightarrow 0$ we're doing $\theta(\alpha)$

$A(1-\xi) \sim \underset{\theta(\alpha^2)}{AB} \to 0 = A - A\xi$

$$\Rightarrow \frac{A}{\xi} \sim A - \theta(\alpha^2)$$

likewise $\frac{\Sigma_f}{\xi} \sim \Sigma_f + \theta(\alpha^2)$

so

$$\text{denominator} = \xi\left[(\not{p} - m_0) - A - \Sigma_f(\not{p} - m_0)\right] + \theta(\alpha^2)$$

since $A \cdot \Sigma_f = \theta(\alpha^2) \sim 0$ .. add.

$$= \xi\left[(\not{p} - m_0 - A) - \Sigma_f(\not{p} - m_0 - A)\right] + \theta(\alpha^2)$$

$$= \xi(\not{p} - m_0 - A)\left[1 - \Sigma_f\right]$$

So, $$i S_F'(p) = \frac{1/\xi \; i}{[\not{p} - m_0 - A(\Lambda)][1 - \Sigma_f]}$$

DEFINE $m_0 + A(\Lambda) \equiv m + \delta m = m_{physical}$

called mass renormalization $\longrightarrow$ never see a bare mass.. only dressed propagators

$$iS_F' = \frac{1/\xi \; i}{(\not p - m_{phy})(1-\Sigma_f)}$$

$\Sigma_R \notin B_2$ — note

remember $\Sigma_f \sim (p^2 - m_0^2)(\alpha) f(\lambda, p)$

$$= (p^2 - (m_{phy} + \delta m)^2)(\alpha) \; f(\lambda, p)$$

$$= [p^2 - m_p^2 + 2m\,\delta m + (\delta m)^2](\alpha) f(\lambda, p)$$

$$\quad\quad\quad \mathcal{O}(\alpha) \quad \mathcal{O}(\alpha^2) \quad \mathcal{O}(\alpha)$$

$$\simeq (p^2 - m_p^2)(\alpha) f(\lambda, p)$$

so, when $p^2 \to m_p^2$, then Σ_f vanishes. and when near the physical pole,

$$iS_F' = \frac{1/\xi \; i}{(\not p - m_p)}$$ .. for real electrons. ie, external legs

Conventional notation — $1/\xi \equiv Z_2$

so,

$$B = \left(1 - \frac{1}{Z_2}\right)$$

$$iS_F' = \frac{Z_2 \, i}{(\not p - m_p)} = Z_2 S_F$$

cutoff dependent

Now. If we start with a Lagrangian with bare quantities

free electron — "external" legs

$$\mathcal{L}_0 = \bar{\psi}_0 (i\not\partial - m_0)\psi_0$$
$$= \bar{\psi}_0 (i\not\partial - m_p)\psi_0 + \underbrace{\bar{\psi}_0\psi_0\,\delta m}$$

"counter term"

correct to treat this as a separate self-interaction w/ Feynman diagram —

and get a Lagrangian with physical quantities

must use counter terms

$i\delta m$

so m_0

Note further, in configuration space,

$$iS_F'(x-x') = \langle 0|T(\bar{\psi}_0(x)\psi_0(x'))|0\rangle$$
$$= \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\, iS_F'(p)$$

cut-off dependent., $A(\Lambda)$

DEFINE renormalized propagator

$$iS_F(p) \equiv \frac{iS_F'(p)}{Z_2} = \frac{i}{(\not p - m_p)(1-\Sigma_f)}$$

cut-off independent ξ

finite as $\Lambda \to \infty$

$\xi = \xi(\Lambda) \Rightarrow Z_2(\Lambda)$

$$i S_F(x-x') = Z_2^{-1}\, S_F'(x-x')$$
$$= Z_2^{-1} \langle 0 | T(\bar\psi_0 \psi_0) | 0 \rangle$$

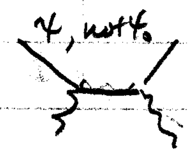
define $\psi \equiv \sqrt{1/Z_2}\, \psi_0$ wavefunction renormalization.

so, $i S_F = \langle 0 | T(\bar\psi(x)\psi(x')) | 0 \rangle$ no bare parameters
no cut-off parameters

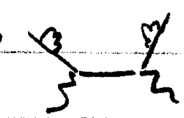
The Lagrangian in terms of physical wavefunctions...

$$\mathcal{L}_0 = \bar\psi_0 (i\gamma\!\!/ - m_0)\psi_0$$
$$= \bar\psi_0 (i\gamma\!\!/ - m_p)\psi_0 + \delta m\, \bar\psi_0\psi_0 + \bar\psi(i\gamma\!\!/ - m_p)\psi - \bar\psi(i\gamma\!\!/ - m_p)\psi$$
$$= Z_2 \bar\psi (i\gamma\!\!/ - m_p)\psi + \delta m\, Z_2 \bar\psi\psi + \bar\psi(i\gamma\!\!/ - m_p)\psi - \bar\psi(i\gamma\!\!/ - m_p)\psi$$
$$= \bar\psi(i\gamma\!\!/ - m_p)\psi + \underbrace{(Z_2 - 1)\bar\psi(i\gamma\!\!/ - m_p)\psi + \delta m\, Z_2 \bar\psi\psi}_{\text{counter terms}}$$

So now, don't need to worry for external legs in diagrams.

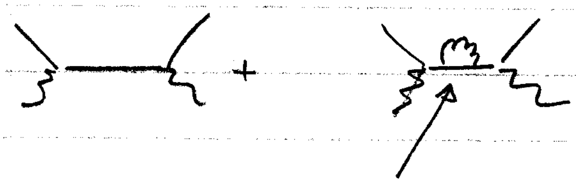


and don't need to be concerned with



what about internal lines?

for example



renormal finite

always have $(-ie_0\gamma_\mu)\, iS_F'\, (-ie_0\gamma_\nu)$

$$\downarrow$$

$$Z_2\, i S_F(p)$$

since there are 2 electron lines for each factor of e_0

we can perform a charge renormalization $e = Z_2 e_0$ ...
The origin of this charge renormalization is the process

.