finally, above $\Upsilon(b\bar{b})$ (and below Z^0 tail)

$$R = 3.3 + 3\left(-\tfrac{1}{3}\right)^2 \simeq 3.7$$

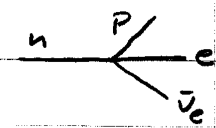
WEAK INTERACTIONS

A sordid history of confusion, stimulated first by the energy crisis in β decay and then confusion about the 'real' form of the matrix element. Shortly after Fermi modeled β decay on QED (which implied particular selection rules ($\Delta J = 0$)) by writing in 1934

$$\mathcal{L}_{int} = \frac{G_F}{\sqrt{2}} \bar{\psi}_p \gamma^\mu \psi_n \bar{\psi}_e \gamma_\mu \psi_{\nu_e}$$

→ a real field theory w/ creation of particles

and $G_F = 1.03 \times 10^{-5}\, m_p^{-2}$.



Positron decay was found in 1934 and accomodated but then decays with a different selection rule ($\Delta J = 0, \pm 1$) were found and carefully described by Gamow & Teller in 1936. This led to the need to characterize the interaction more generally:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \sum_{\substack{\text{nucleons}\\ \text{in nucleus}}} \sum_{j=S,V,T,A,P} \int d^3x \; C_j \bar{\psi}_p(x) O_j \psi_n(x) \bar{\psi}_e(x) O_j \psi_\nu(x)$$

and the next 30 years were spent trying to pin down the Lorentz structure of the weak interaction.

"Weakness" of various interactions was characterized by long lifetimes primarily, related presumably to a small coupling constant.

By the mid 1950's, there were a dizzying array of decays which had common characteristics.

$n \to p e \bar{\nu}$

$p \to n e^+ \nu$ (inside nucleus)

$\pi \to \mu \nu$, $e\nu$ } discovered in cosmic rays -- a real tale

$\mu \to e \nu \nu$

$K \to \pi \pi$

$\Lambda \to p \pi$

but K decay appeared to come in 2 flavors.

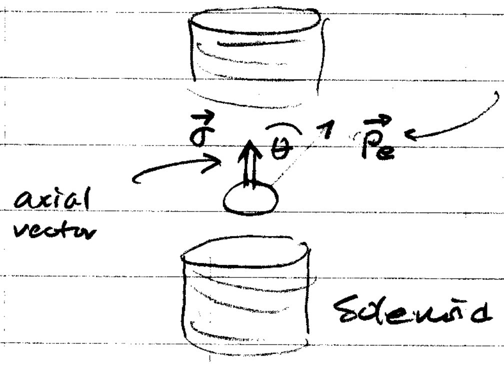
$\tau \to \pi^+ \pi^+ \pi^-$ } τ & θ: same mass, same charge, same spin

$\theta \to \pi^+ \pi^0$

final states have different parities -1 & $+1$.

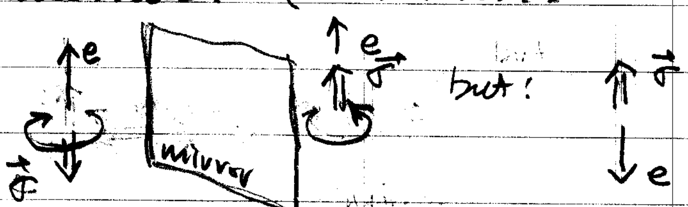
This led Lee and Yang to examine all proofs of parity conservation and found that there were none in weak interactions... they made some suggestions in 1956.

Within the year C.S. Wu did an experiment following their suggestion to look at the correlation of the β from a polarized Co^{60} source



By flipping direction of current, she changed sign of $\vec{\sigma}$ and measured (a mirror..

$$I(\theta) = 1 + \alpha\left(\frac{\vec{\sigma}\cdot\vec{p}_e}{E}\right)$$

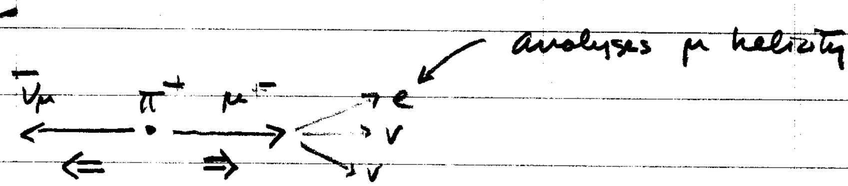


Preferentially, β followed $-\vec{\sigma}$ but when current is flipped, β were opposite: the mirror universe is not the same as the original.

In the same issue of PRL, Lederman and Garwin showed that the β in

$$\pi^- \to \mu^- \bar{\nu}_\mu \quad \text{(at rest)}$$
$$\hookrightarrow e^+ \bar{\nu}_e \nu_\mu$$

peaks at maximum value expected for neutrinos of different identities



This preferential helicity for $\bar{\nu}$ and opposite for ν was also evidence for parity violation:

neutrinos, LH $\Rightarrow$ lepton matrix element involves.

$$(1-\gamma_5)\psi_\nu$$

This was generally parameterized

$$O_j(C_j - C_j'\gamma_5)\psi_\nu$$

measurables.

Now, the parity-violating matrix element for β decay looks like
(most general parity-violating matrix element)

$$\mathcal{M} = G_F \sum_{\text{Z nucleons}} \sum_j \int d^3x\, C_j \left[\bar{\psi}_p O_j \psi_n \bar{\psi}_e O_j \left(1 - \frac{C_j'}{C_j}\gamma_5\right)\psi_\nu \right]$$

$\Rightarrow$ lots of possible couplings. T invariance $\Rightarrow$ C_j & C_j' are real.
(Data showed early that $C_j' = C_j \rightarrow$ maximal P.V.)
Feynman and Gell Mann took an amazing step in 1958 — they hypothesized that the coupling for all fermions would be the same as that for neutrinos for weak interactions. — namely

$$\sum_j C_j \left[\overline{a\psi_p}\, O_j\, a\psi_n\, \overline{a\psi_e}\, O_j\, a\psi_\nu \right]$$

where $a = (1-\gamma_5)$

picks out the LH piece

Since, $\overline{a\Psi} = \bar{\Psi}\bar{a}$ then

$$\sum_j C_j \left[\bar{\Psi}_p \underline{\bar{a}\, O_j a}\, \Psi_n \bar{\Psi}_e \underline{\bar{a}\, O_j a}\, \Psi_\nu \right]$$

note, $\bar{\gamma}_5 = -\gamma_5$ so $\overline{(1+\gamma_5)} = 1-\gamma_5$

$\overline{(1-\gamma_5)} = 1+\gamma_5$

and we have terms like

$$(1+\gamma_5)\, O_j\, (1-\gamma_5).$$

Not all O_j will survive this Feynman-Gell-Mann suggestion (called the "Universal V-A" interaction)...

$$\begin{aligned}
O_j = S &\Rightarrow (1+\gamma_5)(1-\gamma_5) = 0 \\
P &\Rightarrow (1+\gamma_5)\gamma_5(1-\gamma_5) = (\gamma_5+1)(1-\gamma_5) = 0 \\
T &\Rightarrow (1+\gamma_5)\gamma_\mu\gamma_\nu(1-\gamma_5) = 0 \\
V &\Rightarrow (1+\gamma_5)\gamma_\mu(1-\gamma_5) = \gamma_\mu(1-\gamma_5)(1-\gamma_5) = 2\gamma_\mu(1-\gamma_5) \\
A &\Rightarrow (1+\gamma_5)(\gamma_\mu\gamma_5)(1-\gamma_5) = \gamma_\mu(1-\gamma_5)\gamma_5(1-\gamma_5) \\
&= \gamma_\mu \,(\gamma_5 - 1)(1-\gamma_5) \\
&= \gamma_\mu(\gamma_5\gamma_5\gamma_5 - \gamma_5\gamma_5)(1-\gamma_5) \\
&= \gamma_\mu\gamma_5(1-\gamma_5)(1-\gamma_5) \\
&= 2\gamma_\mu\gamma_5(1-\gamma_5)
\end{aligned}$$

So, this suggestion forced the interaction form to be

$$\frac{G_\beta}{\sqrt{2}}\left[C_V \bar{\psi}_p \gamma_\mu \psi_n \ \bar{\psi}_e \gamma^\mu(1-\gamma_5)\psi_\nu + C_A \bar{\psi}_p \gamma_\mu \gamma_5 \psi_n \ \bar{\psi}_e \gamma^\mu(1-\gamma_5)\psi_\nu\right]$$

$$= \frac{G_\beta}{\sqrt{2}}\left[\underbrace{\bar{\psi}_p \gamma_\mu (C_V - C_A\gamma_5)\psi_n}_{} \ \underbrace{\bar{\psi}_e \gamma_\mu (1-\gamma_5)\psi_\nu}_{}\right]$$

leaves open measurables for F and G-T decays.

insures LH e and RH ν explicitly -- from Co^{60} and π decay.

The Feynman and Gell Mann universality statement required all weak interactions to have this form — for purely leptonic interaction, like μ decay, the pure V-A

$$\mathcal{L}_\mu = \frac{G_\mu}{\sqrt{2}}\left[\bar{\psi}_\nu \gamma_\mu (1-\gamma_5)\psi_\mu \ \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_\nu\right]$$

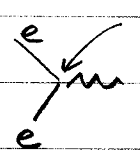
w/ $G_\mu = G_\beta$.

Measurements in nuclear β decays find $\frac{C_A}{C_V} = 1.250$

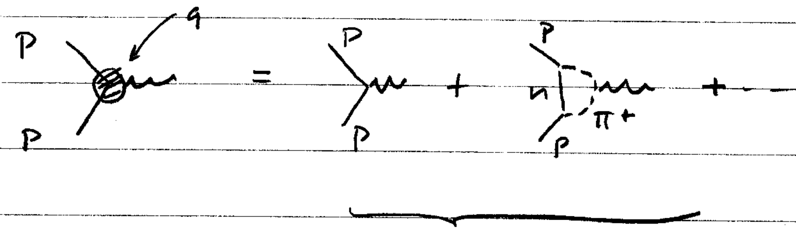
That it is not 1.0 was interpreted as a renormalization effect.

This was bold at the time as the data had favored T for a well regarded Argonne experiment on He^6 GT. They said that the experiment had to be wrong — and it was.

They were pushed to this notion of universality by the observation of something rather remarkable — and we now know, subtle.

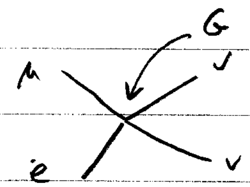
In QED 

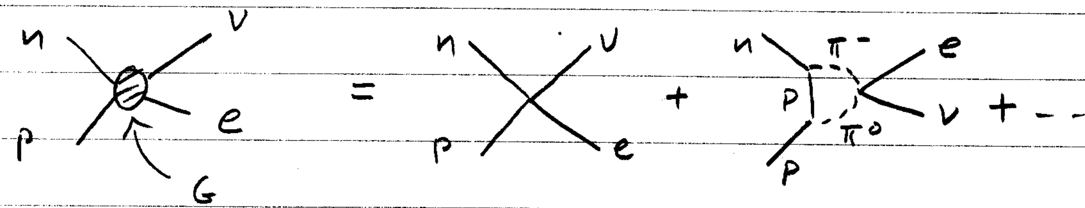
has the same

strength as 

all of this high order stuff doesn't change the equality of q in QED.

The same thing happens with weak interactions.





in 1958:
this suggested weak & electromagnetic int were related!

Pre- Weinberg Salam days led to a description of weak interactions in terms of a current- current interaction -- just like QED. -

$$\mathcal{L} = \frac{1}{2} \frac{G_F}{\sqrt{2}} \left[j^W_\mu \, j^{\mu W+} + j^{\mu W+} \, j_\mu \right]$$

drop the "W" and remember there is always the h.c. term and we have,

$$\mathcal{L}^W_{int} = \frac{G_F}{\sqrt{2}} \, j_\mu j^\mu \qquad \text{* next}$$

These currents are really rather complex. Simply,

$$j_\mu = \underset{\text{leptonic}}{l_\mu} + \underset{\text{hadronic}}{h_\mu}$$

$$l_\mu = \sum_{i=e,\mu,\tau} \bar{\Psi}_i \gamma_\mu (1-\gamma_5) \Psi_{\nu_i}$$

$$h_\mu = \sum_{\substack{i=d,s,b \\ j=u,c,t}} \bar{\Psi}_i \gamma_\mu (1-\gamma_5) \Psi_j$$

isospin lowering (h^+_μ would be isospin-raising, accounting for β decay)

Some of the features of various weak interaction processes are

Feynman and Gell-Mann also speculated in not only a current - current interaction, but also an IVB exchange.

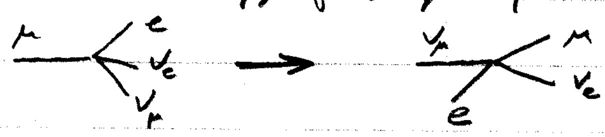
$j^\mu W_\mu$ like $J^\mu A_\mu$ for QED

(they didn't call it "W"). —

PROCESS	~ COUPLING	$\lvert\Delta Q\rvert$	$\lvert\Delta S\rvert$	$\lvert\Delta I\rvert$
$\mu \to e \nu_e \nu_\mu$	G_μ			
$\tau \to \ell \nu_\tau \nu_\ell$	G_μ			
$\pi \to \ell \nu_\ell$	$0.98\, G_\mu$	1	0	1
$K \to \ell \nu_\ell$	$0.22\, G_\mu$	1	0	$1/2$
$n \to p e \bar{\nu}_e$	$0.98\, G_\mu$	1	0	1
$K \to \pi^0 \ell \nu_\ell$	$0.22\, G_\mu$	1	1	$1/2$
$\Lambda \to p \ell \nu_\ell$	$0.22\, G_\mu$	1	1	$1/2$
$\Lambda \to p \pi^-$	$0.22\, G_\mu$	0	1	$1/2, 3/2 \ldots$
$K \to \pi\pi$	$0.22\, G_\mu$	0	1	$1/2, 3/2 \ldots$

& hermitian conjugate reactions.

& reordering (phase space permitting) of outgoing and incoming legs. eg



At least 3 couplings (how were weak interactions unified among themselves, let alone w/ QED?)

$$G_\mu \;;\; \overbrace{0.98\, G_\mu \;;\; 0.22\, G_\mu}^{\text{now understood to be a mixing phenomenon among quarks..}}$$

many selection rules

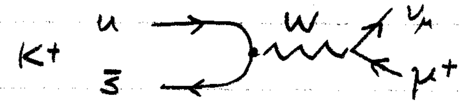
$\Delta Q = 0, \pm 1$

$\Delta S = 0, \pm 1$

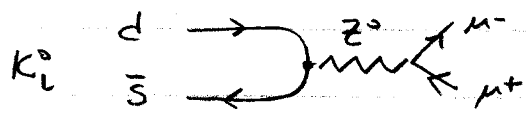
$\Delta I = 1/2, 1, 3/2, \ldots$

Because $K_L^0 \to \mu^+\mu^-$ happens at a rate of 10^{-9} when $K^+ \to \mu^+\nu_\mu$ is fully allowed was puzzling...

The latter occurs via



and the former should happen via



Strangeness changing neutral currents

... something suppresses it. The key was to predict the presence of the charmed quark by Glashow, Iliopoulos, and Maiani (GIM mechanism) and propose that quarks mixed via

$$d_c = d\cos\theta_c + s\sin\theta_c$$
$$s_c = -d\sin\theta_c + s\cos\theta_c$$

So,
$$h_\mu = \bar{d}_c\gamma_\mu(1-\gamma_5)u + \bar{s}_c\gamma_\mu(1-\gamma_5)c$$

Then, the neutral component of the hadronic weak current (suppressing spacetime quantities)

$$\bar{u}u + \bar{c}c + \bar{d}d + \bar{s}s = \bar{u}u + \bar{c}c + \bar{d}_c d + \bar{s}_c s$$

$\Rightarrow$ no cross terms $\bar{d}s$ or $\bar{s}d$

θ_c is called the Cabibbo angle, $\sim 13^\circ$ such that $\cos^2\theta_c \mathrel{\hat{=}} 0.95$ and $\sin^2\theta_c \mathrel{\hat{=}} 0.22$.

The arithmetic of hadronic selection rules are accounted for by,

$$Q = \frac{B+S}{2} + I_3$$

For all of these, $\Delta B = 0$ so

$$\Delta Q = \tfrac{1}{2}\Delta S + \Delta I_3 \qquad \text{works.}$$

So, h_μ can be divided into

$\Delta S = 0$	semileptonic
$\lvert\Delta S\rvert = 1$	semileptonic
$\lvert\Delta S\rvert = 1$	non-leptonic

In turn:

- $\Delta S = 0$ involves hadronic matrix elements of the form

$$\langle B' | j | B \rangle \quad \text{or} \quad \langle M' | j | M \rangle$$

SO $\Delta Q = \Delta I_3$ ... eg.

$$\langle p | j | n \rangle$$

I_3: $\tfrac{1}{2}$ $\quad -\tfrac{1}{2}$ $\quad\Rightarrow \Delta I_3 = +1 \Rightarrow \Delta I = 1$

$$\langle 0 | j | \pi \rangle \quad \Rightarrow \Delta I_3 = \pm 1 \Rightarrow \Delta I = 1$$

I_3: 0 $\quad 1$

• $|\Delta S| = 1$ includes semileptonic matrix elements like

$$\langle B' | j | B \rangle \quad \text{or} \quad \langle M' | j | M \rangle$$

$$\Delta Q = \tfrac{1}{2}\Delta S + \Delta I_3 \quad \text{w/ 2 possibilities}$$

- $\Delta S = +\Delta Q \Rightarrow |\Delta I_3| = 1/2 \Rightarrow \Delta I = 1/2, 3/2 \ldots$
- $\Delta S = -\Delta Q \Rightarrow |\Delta I_3| = 3/2 \Rightarrow \Delta I = 3/2, 5/2 \ldots$

eg $\langle \pi^0 | j | K \rangle$

I_3:	0	$\pm 1/2$	$\Rightarrow \Delta I_3 = \pm 1/2 \Rightarrow \Delta I = 1/2, 3/2 \ldots$
S:	0	1	$\Delta S = -1$
Q:	0	± 1	$\Delta Q = \pm 1$

• $|\Delta S| = 1$ w/ nonleptonic, like:

$$\langle B'M | j | B \rangle$$

obviously $\Delta Q = 0$

so $\Delta I_3 = -\tfrac{1}{2}\Delta S$ so $|\Delta I_3| = 1/2$ always

$\Rightarrow \Delta I = 1/2, 3/2$

a rule,
as like $K^+ \to \pi^+ \pi^0$
is suppressed .. not
entirely understood..

So, this mess is described by.

$$h_\mu = h_\mu^0 + h_\mu^1$$

$|\Delta S|$: 0 1

<u>and</u>

$$h_\mu^0 = v_\mu^0 + a_\mu^0$$
$$h_\mu^1 = v_\mu^1 + a_\mu^1$$

vector + axial vector

All weak interactions can be described (pre-Weinberg Salam) by,

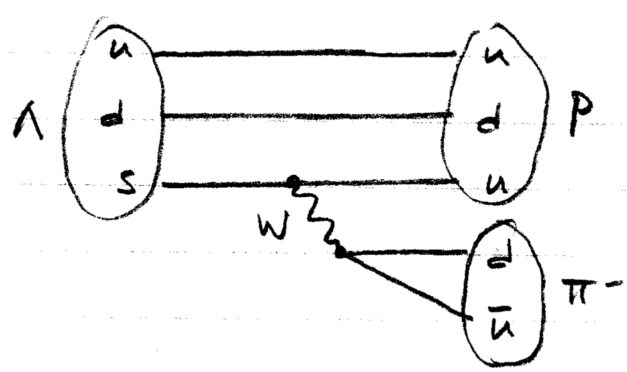
$$\mathcal{L}^W = \frac{G_\mu}{\sqrt{2}} j_\mu j^{\mu +}$$

$$= \frac{G_\mu}{\sqrt{2}} \left(l_\mu + h_\mu^0 + h_\mu^1 \right)\left(l^{\mu +} + h^{\mu 0 +} + h^{\mu 1 +} \right)$$

-- pair them up and all reactions are accounted for.

The quark model w/ SU(3) as the underlying symmetry describes it all. Planar diagrams like

$\Lambda^0 \rightarrow p\pi^-$



etc.