NOW FOR SOMETHING *COMPLETELY* DIFFERENT...

THE BASES ON WHICH THE ARCANE MECHANICS OF THE "STANDARD MODEL" ARE BUILT ARE NOT ALWAYS TAUGHT / WRITTEN ... *but they are fun.*

- THE DEVELOPMENT OF ESPECIALLY THE ELECTROWEAK MODEL IS FULL OF INTERESTING HISTORY, FALSE STARTS, INTRIGUE, MYSTERY, & SOME PRETTY <u>NON</u>-HEP.
- I PROPOSE TO SCHEMATICALLY... WITH A MINIMUM OF MATHEMATICS...

TALK AROUND THE EDGES OF

THE STANDARD MODEL OF ELECTRO-WEAK PHYSICS

R. BROCK
MICHIGAN STATE

GAUGE - THEORIES

- an eccentric introduction

A RECREATION IN THE HISTORICAL AND CROSS-CULTURAL ROOTS OF MODERN GAUGE THEORIES OF THE ELECTROMAGNETIC, WEAK, AND STRONG INTERACTIONS

introduction
uses of symmetry & invariance in physics
gauge principle
weak interactions
critical phenomena
BROKEN symmetry
Higgs, et. al. mechanism
PUTTING IT TOGETHER → WEINBERG & SALAM MODEL

I WANT TO TELL YOU a story...

introduction: field theory primer

A CATALOG WILL SUFFICE...

FREE LAGRANGIANS — EQUATIONS OF MOTION

scalar fields:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \qquad\qquad \partial_\mu\partial^\mu\phi + m^2\phi^2 = 0$$

spin 1/2 fields:

$$\mathcal{L} = \bar{\Psi}(x)\left[i\gamma^\mu\partial_\mu - m\right]\Psi(x) = 0 \qquad\qquad (i\gamma^\mu\partial_\mu - m)\Psi(x) = 0$$

spin 1, massless fields:

$$\mathcal{L} = -1/4\, F_{\mu\nu}F^{\mu\nu} \qquad\qquad \partial_\mu F^{\mu\nu} = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

spin 1, massive fields:

$$\mathcal{L} = -\frac{1}{4} f^{\mu\nu}f_{\mu\nu} + \frac{1}{2}M^2 B^\mu B_\mu \qquad\qquad \partial_\mu f^{\mu\nu} + m^2 B^\nu = 0$$

$$f^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

INTERACTIONS -

$$\mathcal{L}_{\text{electromagnetic - spin } 1/2} = e_f \bar{f}(x)\gamma^\mu f(x) A_\mu(x)$$

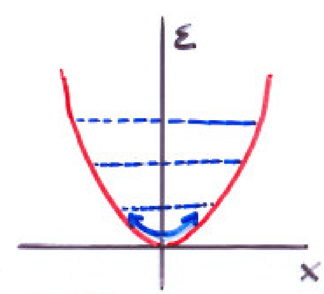
$$\mathcal{L}_{\text{YUKAWA}} = g\,\phi(x)\bar{\Psi}(x)\Psi(x)$$

PARTICLE SPECTRA -

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k}\left[a(k)e^{-ikx} + a^\dagger(k)e^{ikx}\right]$$

creation operator

annihilation operator



... just like
quantum oscillator from 1st year quantum mechanics

quick lesson on symmetry in quantum mechanics 1/4:

OUR FAITH HAS COME FULL CIRCLE...

- We are amused at the image of Kepler, among many others, trying to bend the observed universe into an *a priori* notion of how it ought to be — for Kepler, it *ought* to have something to do with the Platonic Solids. For others, a "perfect geometry". circles... then ellipses...
- We are no different now! One of my messages...
- WHAT DID EINSTEIN DO IN SPECIAL RELATIVITY?
  → HE DIDN'T INVENT THE TRANSFORMATIONS NECESSARY
  Lorentz did that earlier
  → HE DIDN'T ESTABLISH THE MATHEMATICAL RIGOR
  Poincaré did that earlier
- WHAT HE DID WAS *DERIVE* THOSE RESULTS BY ARGUING FROM AN A PRIORI PREDJUDICE REGARDING A PREFERANCE FOR SYMMETRY

THAT WAY OF THINKING CAUGHT ON... SPACETIME SYMMETRIES TOOK ON A FUNDAMENTAL IMPORTANCE IN PHYSICS...

only to be confused & frustrated by:

1. The discovery of non-spacetime symmetries (eg, isospin.. the "INTERNAL" SYMMETRIES)
2. ★ The discovery that Nature is actually RARELY symmetric! .. approximate symmetries!

quick lesson on symmetry in quantum mechanics I:

QUANTUM MECHANICS:

- GROUP OPERATIONS REPRESENTED BY UNITARY OPERATORS, $\mathcal{U}$, IN A LINEAR VECTOR SPACE OF STATE VECTORS, $|\alpha\rangle$

generated by G

vectors transform: $|\alpha\rangle \longrightarrow |\alpha'\rangle = \mathcal{U}|\alpha\rangle$

operators transform: $\Theta \longrightarrow \Theta' = \mathcal{U}\Theta\mathcal{U}^{-1}$

- IF SYSTEM IS SYMMETRIC wrt GROUP, $[\mathcal{H}, G] = 0$

  - An important theorem came out of the incredible mathematics group of F. Klein in Göttingen – written down by Emmy Noether:

SYMMETRY $\Longleftrightarrow$ CONSERVATION LAW

- OF PARTICULAR INTEREST ARE SYMMETRY GROUPS WITH REPRESENTATIONS LIKE $\mathcal{U}(\varepsilon) = e^{-i\sum_j \varepsilon^j Q^j}$

INFINITESIMAL PARAMETERS

"GENERATORS" OF THE GROUP & OPERATORS HAVING QUANTUM #'S AS EIGENVALUES

- CONNECTION THROUGH "CHARGE" & A CONSERVED "CURRENT" –

$$Q \equiv \int d^3x \, j^0(x)$$

where $\partial_\mu j^\mu(x) = 0$ signifies a conservation law quick lesson on symmetry in quantum mechanics?:

QUANTUM FIELD THEORY:

- $\phi(x)$ IS AN OPERATOR $\quad \phi \to \phi' = U \phi U^{-1}$

$$= (1 - i\sum_j \varepsilon^j Q^j)\phi(1 + i\sum_j \varepsilon^j Q^j)$$
$$\vdots$$
$$= \phi + i\sum_j \varepsilon^j [Q^j, \phi(x)]$$

SO $[Q^j, \phi(x)] = \phi(x) \Rightarrow$

(note: often $U\phi U^{-1} = \exp(i\sum_j \varepsilon^j q^j)\,\phi(x)$ ... a phase)

$\uparrow$ eigenvalues of Q^j

- SUPPOSE $[H, Q] = 0 \quad \Rightarrow \quad \partial_0 Q = 0$

LET $H|\vec{p}_n\rangle = E_n|\vec{p}_n\rangle$

THEN
$$\left.\begin{array}{l} QH|\vec{p}_n\rangle = E_n Q|\vec{p}_n\rangle \\ \quad \parallel \\ HQ|\vec{p}_n\rangle = E_n Q|\vec{p}_n\rangle \end{array}\right\}$$
$|\vec{p}_n\rangle$ & $Q|\vec{p}_n\rangle$ ARE BOTH EIGENSTATES OF H WITH SAME E_n – degenerate

$\rightarrow$ MAY REPRESENT ORTHOGONAL STATES WITH DISTINCT QUANTUM NUMBERS...

- THERE IS A SPECIAL EIGENSTATE OF H ... THE VACUUM.

$H|0\rangle = 0$ IS ALWAYS TRUE FOR VACUUM STATE

USUALLY, IT IS ASSUMED THAT, FOR $U = e^{iQ\alpha}$

$U|0\rangle = |0\rangle$ FOR ALL SYMMETRIES

$\Rightarrow Q|0\rangle = 0$

IF $Q|0\rangle \neq 0$, THEN THERE MUST BE DEGENERATE VACUA IF ALSO $[H, Q] = 0$. stay tuned!

HISTORICALLY...

- SOON AFTER GENERAL RELATIVITY WAS WRITTEN BY EINSTEIN, H. WEYL PROPOSED A MODIFICATION...

HE ADDED INVARIANCE WITH RESPECT TO

a. $g'_{\mu\nu} = \lambda(x) g_{\mu\nu}$ — same $\lambda(x)$ phase

b. $A'_\mu = A_\mu - \frac{\partial \lambda(x)}{\partial x^\mu}$

b. is the regular ambiguity required of electromagnetic potentials.

a. is weird. $\Rightarrow$ $ds^2 = g_{\mu\nu} dx^\mu dx^\nu \longrightarrow \lambda ds^2$: LENGTHS ARE RE-"GAUGED"

- suggests an invariance even though space & time can change <u>over</u> all space and time.
- the mediator which holds the spacetime structure together would be the electromagnetic field.

→ ALL CALLED A "GAUGE TRANSFORMATION"

"Your ideas show a wonderful cohesion. Apart from the agreement with reality, it is at any rate a grandiose achievement of mind." A. Einstein to H. Weyl 1919.

THE THEORY -- AN EARLY ATTEMPT TO UNIFY GRAVITATION WITH ELECTROMAGNETISM -- DIDN'T WORK.

... but, the name stuck.

IN 1927 London revived the idea... but the symmetry isn't the scale of spacetime, rather the phase of the wave function.

GLOBAL, U(1) SYMMETRIES:

$$U(\theta) = e^{i\theta Q}$$

"GLOBAL" ⇒ SAME PHASE, INDEPENDENT OF SPACETIME $\theta \neq \theta(x)$

"U(1)" ⇒ 1 PARAMETER LIE GROUP HAVING Q AS GENERATOR

$$\psi(x) \to \psi'(x) = U\psi(x)U^{-1} = e^{i\theta q}\psi(x)$$

SIMPLE EXERCISE 1: For the Dirac free field, show that a local U(1) transformation leads to an invariance, and hence conserved quantum numbers, q.
ie. show $\delta\mathcal{L} = \mathcal{L}(\psi) - \mathcal{L}(\psi') = 0$.

GLOBAL SYMMETRIES NOT VERY RESTRICTIVE & NOT REALLY CONSISTENT WITH RELATIVITY & LOCAL FIELD THEORY...

LOCAL, U(1) SYMMETRIES:

$$U(\theta) = e^{i\theta(x)Q}$$

"LOCAL" ⇒ POTENTIALLY DIFFERENT PHASE AT ALL SPACETIME POINTS $\theta = \theta(x)$

$$\psi(x) \to \psi'(x) = e^{i\theta(x)q}\psi(x)$$

NOT SO SIMPLE...

$$\mathcal{L}(\psi) \to \mathcal{L}(\psi') = e^{-i\theta(x)q}\,\bar{\psi}(x)\left[i\underline{\gamma^\mu\partial_\mu} - m\right]e^{i\theta(x)q}\psi(x)$$

$$= \bar{\psi}(x)\left[i\gamma^\mu\partial_\mu - m\right]\psi(x) - \underline{q\,\partial_\mu\theta(x)\,\bar{\psi}(x)\gamma^\mu\psi(x)} \neq \mathcal{L}(\psi)$$

SIMPLE EXERCISE 2: For $\psi = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$ and $U = e^{i\theta Q}$ ... what is the physical symmetry for Global U(1)?

Derivative term causes trouble ... define a new divergence operator to cancel the unwanted term!

$$D_\mu \equiv \partial_\mu + X_\mu$$ as-yet unnamed vector operator

goal is to get the gradient term to transform simply...

$$(D_\mu \psi) \longrightarrow (D_\mu \psi)' = e^{iq\theta(x)}(D_\mu \psi)$$

- START OUT WITH $\mathcal{L} = \bar{\psi}(x)\left[i\gamma^\mu D_\mu - m\right]\psi(x)$
  $= \bar{\psi}(x)\left[i\gamma^\mu \partial_\mu + i\gamma^\mu X_\mu - m\right]\psi(x)$

transform $\psi \to \psi'$

$$\mathcal{L}(\psi) \to \mathcal{L}(\psi') = \bar{\psi}'(x)\left\{i\gamma^\mu\left[\partial_\mu + \underline{X_\mu - iq\partial_\mu\theta(x)}\right] - m\right\}\psi'(x)$$

STILL NOT RIGHT!

must simultaneously transform $X_\mu \longrightarrow X_\mu' = X_\mu - iq\partial_\mu\theta(x)$

aha! Denote $X_\mu \equiv iqA_\mu(x)$ so the gradient looks like

$$D_\mu \equiv \partial_\mu + iqA_\mu$$

& TOTAL TRANSFORMATION NECESSARY TO LEAVE $\mathcal{L}$ ALONE IS:

$$\psi(x) \longrightarrow \psi'(x) = e^{iQ\theta(x)}\psi(x)$$

$$A_\mu(x) \longrightarrow A_\mu'(x) = A_\mu(x) - \partial_\mu\theta(x)$$

GAUGE INVARIANCE OF 2nd KIND

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi}_{\text{free } \psi} - \underbrace{qA_\mu\bar{\psi}\gamma^\mu\psi}_{\text{"interaction"}} \left(\underbrace{-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{added free } A_\mu}\right)$$

$\mathcal{L}$ IS GAUGE INVARIANT

TURNING THE UTILITY OF SYMMETRY

upside-down...

IF INVARIANCE WITH RESPECT TO LOCAL, U(1) SYMMETRY

IS, *a priori*, OF PARAMOUNT IMPORTANCE...

one is forced to invent the photon.

DEMAND OF A SYMMETRY... GET NEW FIELDS <u>AND</u> DYNAMICS !!

OTHER SYMMETRIES $\longrightarrow$ NEW SPIN 1,2... FIELDS ?

THE INTRIGUING RESEARCH PROJECT IN 1954 OF
YANG & MILLS.. AND INDEPENDENTLY BY SHAW

a) LOCAL SU(2) SYMMETRY $\longrightarrow$ ISOTRIPLET OF SPIN 1 FIELDS

b) GRAVITON?

DEMANDING $U = e^{i \sum_a \vec{\theta}(x) \cdot \vec{\tau}/2} \longrightarrow \vec{b}_\mu(x) \left\langle \begin{array}{l} 2 \text{ charged} \\ 1 \text{ neutral} \end{array} \right.$

isovector &
Lorentz vector

Yang Mills 1:

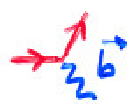
AGAIN: $\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi$

now $\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$ as bases for SU(2) operators

DEFINE A NEW COVARIANT DERIVATIVE...

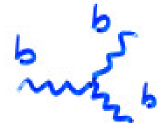
$$\mathcal{D}_\mu \equiv \partial_\mu + ig\,\vec{b}_\mu \cdot \vec{\tau}/_2$$ & substitute & lots of algebra-

$$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi - \frac{g}{2}\bar\psi\gamma^\mu\vec{\tau}\psi\cdot\vec{b}_\mu \underbrace{- \tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}^{\mu\nu}}_{\text{complicated}}$$



$$-\tfrac{1}{4}\vec{f}^{\mu\nu}\cdot\vec{f}_{\mu\nu} = -\tfrac{1}{2}(\partial_\nu\vec{b}_\mu - \partial_\mu\vec{b}_\nu)\cdot\partial^\nu\vec{b}^\mu$$

$$+ g\,\vec{b}_\nu\times\vec{b}_\mu\cdot\partial^\nu\vec{b}^\mu$$



$$-\tfrac{1}{4}g^2\left[(\vec{b}_\nu\cdot\vec{b}^\nu)^2 - (\vec{b}_\nu\cdot\vec{b}_\mu)(\vec{b}^\mu\cdot\vec{b}^\nu)\right]$$

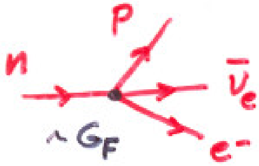
– get self-couplings for $\vec{b}$'s.

$\vec{b}_\mu$ FIELD IS STILL MASSLESS

ONE MIGHT HAVE HOPED THAT THE $\vec{b}_\mu$ WOULD HAVE FOUND WORK AS $\vec{W}_\mu$ ... but masslessness is a fatal flaw.

Weak interactions, circa 1960 :

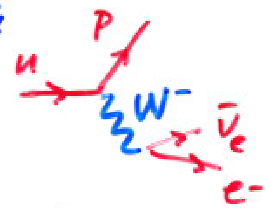
SINCE PAULI & FERMI IN 1930's...



$G_F \sim 10^{-5}/M_p^2$

20 YEARS OF CONTRADICTORY EXPERIMENTAL RESULTS, SURPRISES, BEAUTIFUL THEORY (1958 Feynman & Gell-Mann)... A RAG-TAG BUNDLE OF DECAYS WERE FINALLY ALL RECOGNIZED TO BE "WEAK" & PARITY-VIOLATING

... HURISTICALLY DESCRIBED BY:

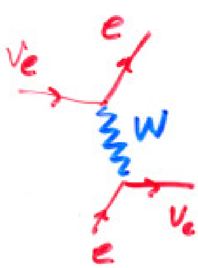


$W^\pm$: charged

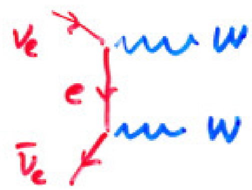
isospin raising/lowering

massive

THERE WERE WELL-KNOWN PROBLEMS



violates Unitarity

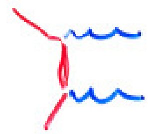


σ unbounded

2W Production

Contains an important hint...

hint 1:

THE PROBLEM WITH  LIES WITH THE

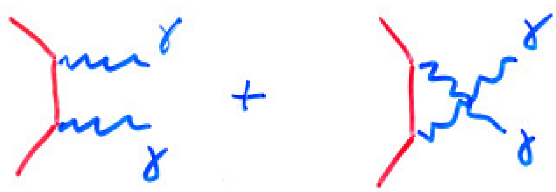
LONGITUDINAL DEGREE OF FREEDOM

- MASSLESS SPIN 1 FIELDS HAVE 2 dof --- polarizations, L, R (Gauge invariance)

- MASSIVE SPIN 1 FIELDS HAVE 3 dof... USUALLY TAKEN AS L, R, & LONGITUDINAL

$\varepsilon^{\mu}(\lambda=0) \sim \frac{k^{\mu}}{M}$ at high energy

HINT IN ELECTROMAGNETISM... 2γ PRODUCTION



BOTH GRAPHS REQUIRED BECAUSE REQUIRE GAUGE INVARIANCE...

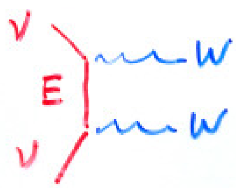
PRETEND THAT γ HAD A MASS... & THEREFORE A LONGITUDINAL dof.

THIS BADLY-BEHAVED POLARIZATION TERM CANCELS BETWEEN THE GRAPHS...

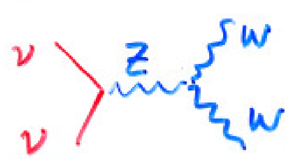
IN HINDSIGHT, CANCELATION CAN BE ARRANGED FOR W.I.

either, require a new, heavy electron

$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}$



or, require a new, heavy spin 1 field



stay tuned.

hint 2:

ENCOURAGEMENT (!)...

ELECTROMAGNETISM EXHIBITS A MAGICAL BEHAVIOR...

(clean lepton)

(messy hadron)

SAME COUPLING (in limit)

SO DO WEAK INTERACTIONS...

(clean leptons)

(messy hadrons)

SAME COUPLING ! (vector weak charge)

↳ conservation of something

↳ a gauge symmetry?

spin 1 propagator...

CVC...

COULD THE WELL-BEHAVED ELECTROMAGNETIC INTERACTION BE RELATED TO THE ILL-BEHAVED, BADLY-BRED WEAK ?

Schwinger, Salam, Ward, Glashow, Weinberg... — using Yang-Mills ideas...!

$\begin{pmatrix} W^+ \\ \gamma \\ W^- \end{pmatrix}$? $\quad \begin{pmatrix} W^+ \\ Z^\circ \\ W^- \end{pmatrix}$ & γ

BUT... YANG-MILLS FIELDS MUST BE MASSLESS... * sigh *

... AN INTERLUDE ...

MEANWHILE – CONDENSED MATTER PHYSICS WAS HAVING
GREAT CONCEPTUAL & EXPERIMENTAL SUCCESS
WITH 2^{nd} ORDER PHASE TRANSITIONS
... cooperative phenomena in many-body physics

~ MINI-AGENDA ~

- LIGHT-SPEED REVIEWS OF
  - the thermodynamics of phase transitions
  - Mean Field Theory & the Ginsburg-Landau phenomenology
- FERROMAGNETISM AS AN EXAMPLE OF A "BROKEN SYMMETRY"

  ... AN INTERLUDE WITHIN AN INTERLUDE ...
- GOLDSTONE THEOREM
- DILUTE BOSE GAS AS AN EXAMPLE OF THE GOLDSTONE THEOREM
- GOLDSTONE – not!
  - superconductivity

BACK TO PARTICLE PHYSICS WITH THE SOLUTION — 1967 thermodynamics of phase transitions I:

WHAT IS A PHASE ?

FORMALLY... A REGION OF ANALYTICITY OF THE FREE ENERGY...

$f = -k_B T \ln Z$ from statistical mechanics

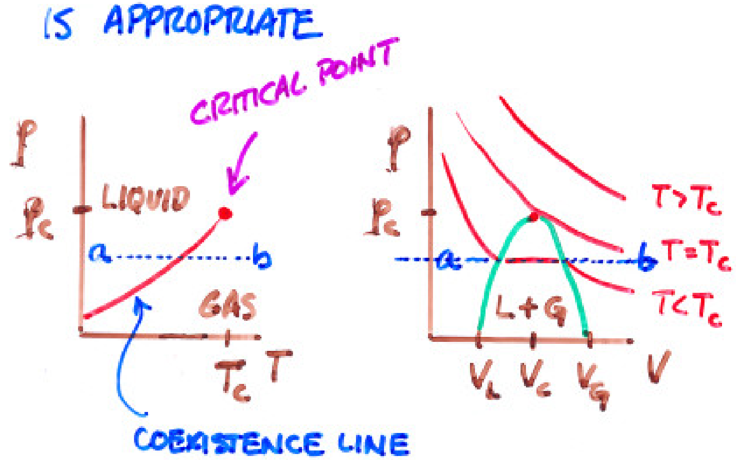
$Z \to \mathrm{Tr}\, e^{-H/k_B T}$

thermodynamics comes from derivatives of f

$f:$ $F = U - TS$ (Helmholtz)
$G = F + pV$ (Gibbs)
from thermodynamics

$$S = \left(-\frac{\partial G}{\partial T}\right)_{P,N} = \left(-\frac{\partial F}{\partial T}\right)_{V,N}$$

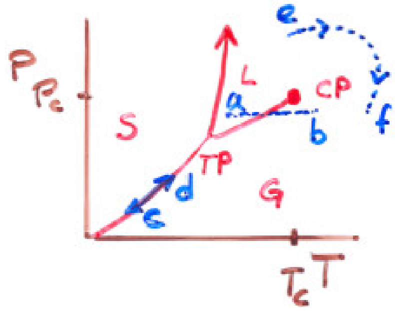
- A PARTICULAR PHASE MIGHT BE REALIZED WITH MINIMUM G...
- MORE THAN 1 PHASE MIGHT BE POSSIBLE (WITH SAME H), SUGGESTING THAT ANALYSIS OF f FOR NON-ANALYTIC BEHAVIOR IS APPROPRIATE



HEAT AT CONSTANT P & DENSITY, $a \to b$

$\Rightarrow dG_L = dG_G$ ACROSS COEXISTENCE LINE

thermodynamics of phase transitions 2:



IMAGINE HEATING, WHILE MAINTAING EQUILIBRIUM BETWEEN S & G, c→d

$dG_S = dG_G$ where $dG_i = V_i dP - S_i dT$

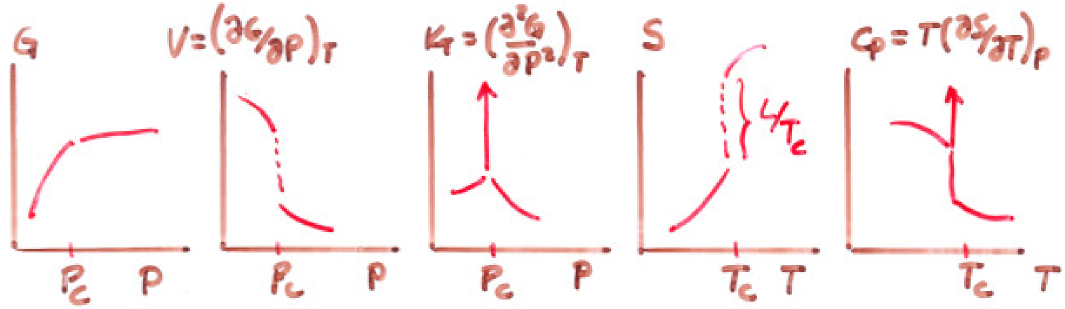
$\Downarrow$

$$\frac{dP}{dT} = \frac{S_S - S_G}{V_S - V_G} = \frac{\Delta S}{\Delta V} = \frac{L}{T\Delta V}$$

entropy change ⇒ heat absorbed in "crossing the line"

(Clausius-Clapeyron)

→ latent heat



FIRST DERIVATIVE OF G IS DISCONTINUOS ⇒ "1st ORDER P.T."

TAKES PLACE ACROSS COEXISTENCE CURVE

CRUCIAL CONCEPT IS THE SYMMETRY OF THE PHASES...

- A SYSTEM EITHER HAS A SYMMETRY... OR IT DOESN'T
- IF THERE IS A SYMMETRY CHANGE → P.T. HAS TAKEN PLACE

HIGH DEGREE OF SYMMETRY ⇒ LACK OF ORDER

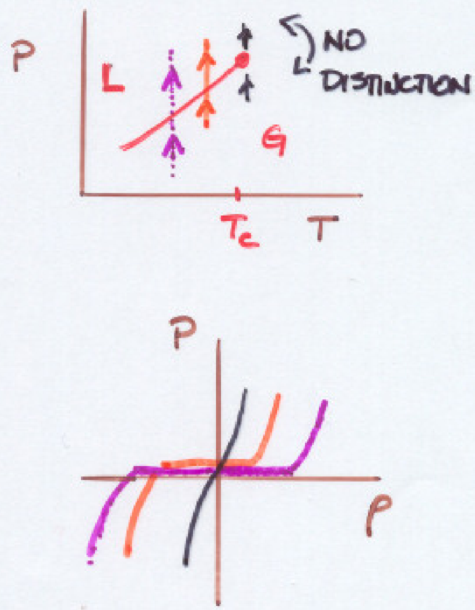
$\Downarrow$ $\Downarrow$

MORE SYMMETRY OPERATIONS ⇒ HIGH ENTROPY

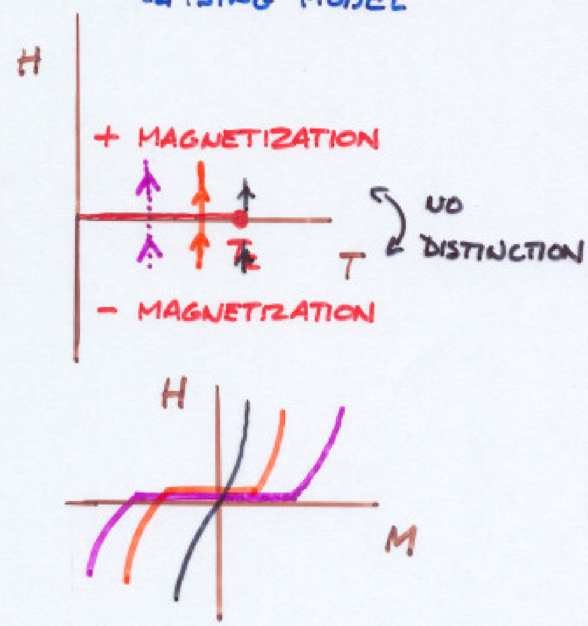
@ HIGH TEMPERATURE

NOTE: e → f DOESN'T INVOLVE A SYMMETRY CHANGE.

LIQUID-GAS



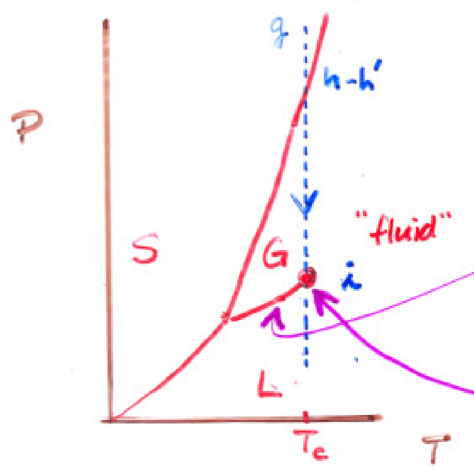
2d ISING MODEL



WHILE VERY DIFFERENT, THERE IS CLEARLY SOMETHING THE SAME ABOUT DENSITY IN A FLUID & MAGNETIZATION IN A FERROMAGNET.

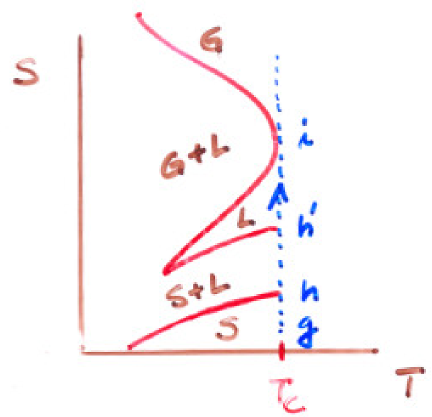
... a universality.

thermodynamics of phase transitions $2\frac{1}{2}$:



PHASE BOUNDARY IS A LINE SEPARATING TWO DISTINCT PHASES OF A SYSTEM... what distinguishes them here? DENSITY

@ T_c, THAT DISTINCTION VANISHES



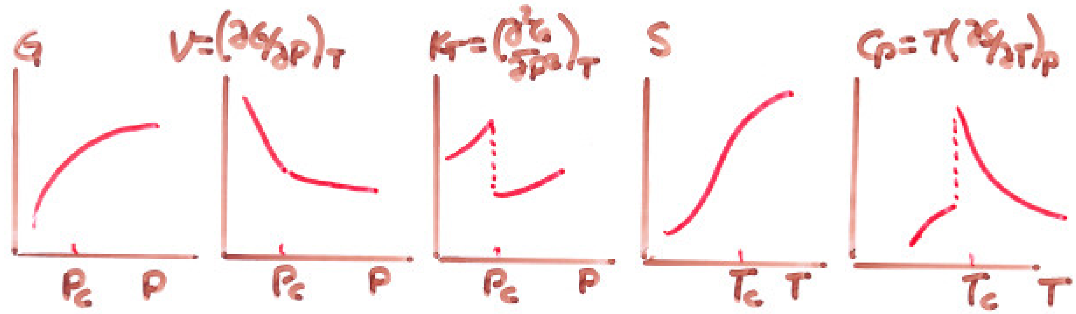
G & L ARE EQUALLY ORDERED AT T_c

PASSING THROUGH COEXISTENCE CURVE → LATENT HEAT

TRANSITIONS WHICH DON'T DISPLAY A SUDDEN STATE CHANGE & HAVE A CONTINUOUS ENTROPY CHANGE --- CALLED "ORDER-DISORDER" TRANSITIONS ... DERIVATIVES ARE DISCONTINUOS .. "LAMBDA" TR.

thermodynamics of phase transitions 3:

THERE ARE P.T. WHICH ARE CONTINUOUS AT 1^{st} ORDER



↑

2^{nd} DERIVATIVE ⇒ "2^{nd} ORDER P.T."

- FOLLOWING ON SYMMETRY FOCUS.. LANDAU & GINSBURG INVENTED A DEGREE OF FREEDOM TO MEASURE THE ORDER IN A SYSTEM : THE ORDER PARAMETER , $\eta(T)$.

  ... and in so doing, universalized the study of phase transitions

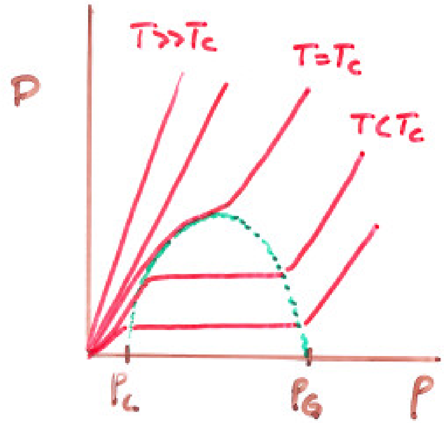
- IF $\eta = 0$ THEN SYSTEM IS IN ORDERED PHASE

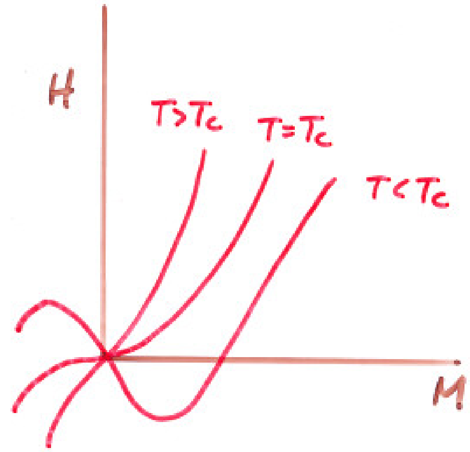
  $|\eta| \neq 0$ THEN SYSTEM IS IN DISORDERED PHASE

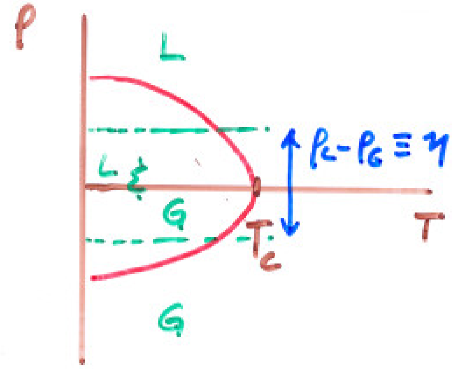
- IF $\eta(T) \rightarrow 0$ CONTINUOUSLY THEN P.T. IS 2^{nd} ORDER

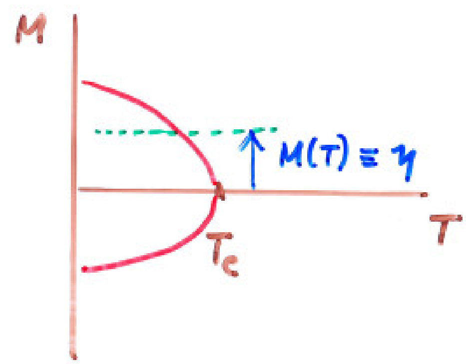
SYSTEM	η	EXAMPLE	T_c (K)
liquid-gas	$\rho_L - \rho_G$	H_2O	647
ferromagnet	M	Fe	1044
superfluid	$\psi_{\text{ground state}}$	^4He	2
superconductivity	$\psi_{\text{cooper pairs}}$	Pb	7
ferroelectrics	P	tryglycerine sulfate	323
binary alloys	concentration	Cu-Zn	739

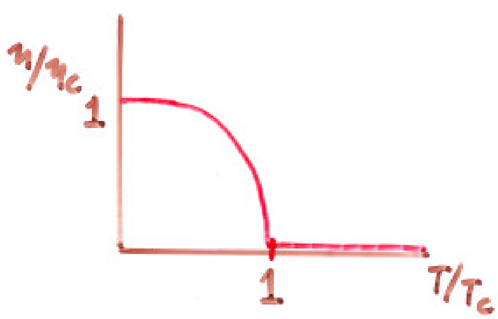
thermodynamics of phase transitions 4:











thermodynamics of phase transitions 5:

NEAR T_c, LANDAU POSTULATED THAT WE CAN WRITE A FUNCTION, L (Landau free energy) ... RELATED TO G. $\propto V$

$$L(P,T,\eta) = L_0 + \beta(P,T)\eta^2 + \delta(P,T)\eta^4 \ldots$$

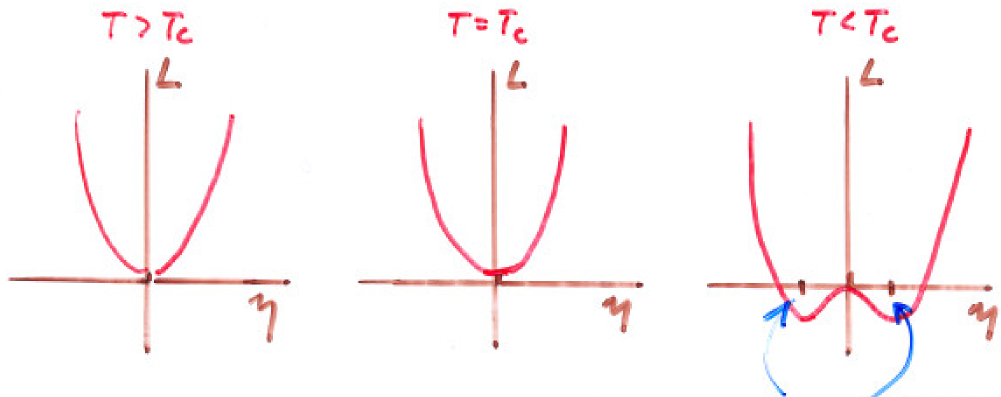
$\beta > 0 \Rightarrow T \geq T_c$ } FLIP, ABOVE & BELOW
$\beta < 0 \Rightarrow T < T_c$ } PHASE TRANSITION

$\delta > 0$ ALWAYS

- MINIMIZATION OF L GUARANTEES A STABLE GROUND STATE

... PRESUME GOOD BEHAVIOR NEAR T_c: $\beta(P,T) = b(P)(T-T_c) + \ldots$

$$L = L_0 + b(T-T_c)\eta^2 + \delta\eta^4$$



SIMPLE EXERCISE 3: SHOW

$$\left\{ \begin{aligned} \eta &= \pm\sqrt{\frac{b}{2\delta}|T-T_c|} \\ L &= -\frac{b^2}{2\delta}(T-T_c)^2 \end{aligned} \right.$$

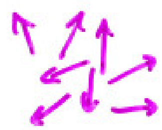
TWO CHARACTERISTICS FOR $T < T_c$:

1. GROUND STATE ENERGY LOWERED
2. MULTIPLE GROUND STATE CONFIGURATIONS POSSIBLE

SYMMETRY IN PHASE TRANSITIONS

... FERROMAGNET

$T > T_c$

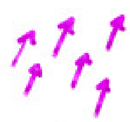


$\langle M \rangle = 0$

- ALL DIRECTIONS EQUALLY PROBABLE... GROUND STATE IS INVARIANT wrt $SO(3)$, U_3
- $[H, G] = 0$

  $\Rightarrow$ HAMILTONIAN INVARIANT wrt $SO(3)$

$T < T_c$



$\langle M \rangle \neq 0$

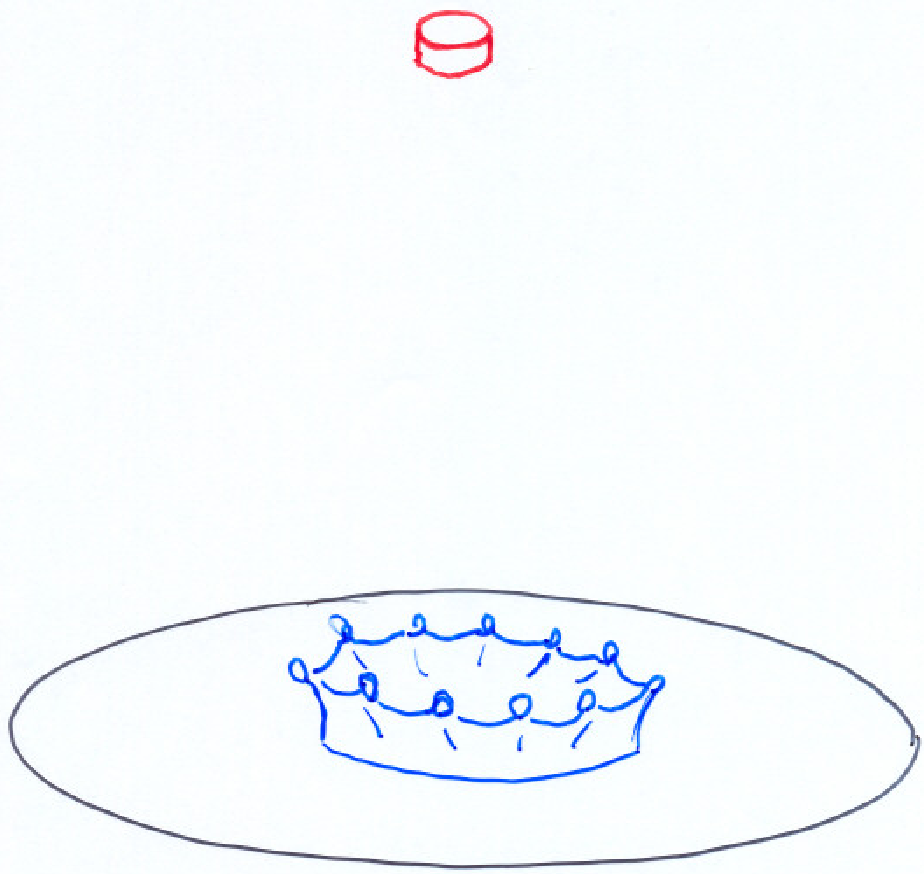
- A SINGLE, RANDOM DIRECTION IS SINGLED OUT
- SYMMETRY OF GROUND STATE IS LOWERED $SO(3) \rightarrow SO(2)$
- $[H, G] = 0$ still

- SPECIAL STATE OF AFFAIRS.. common to 2^{nd} order p.t. ... SYMMETRY OF GROUND STATE IS LOWERED FROM THAT OF THE HAMILTONIAN

- SYMMETRY IS SAID TO BE "SPONTANEOUSLY BROKEN". (lousy phrase.. better is "HIDDEN SYMMETRY")

SYSTEMS WITH SYMMETRIES WHICH ARE NOT BROKEN ARE RARE! classically & quantum mechanically

WORTHINGTON – turn of the century



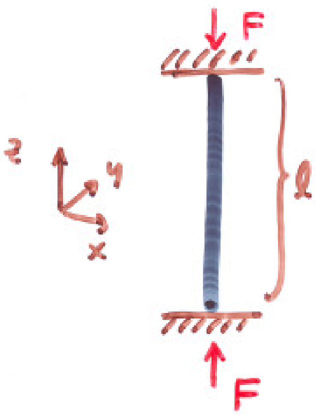
Where does the symmetry "go"?

It's still there... inside of the
ensemble of all potential splashes

Spontaneously broken symmetries !:

HOW DOES SYMMETRY GET LOST? WHERE DOES IT GO?

CLASSIC... CLASSICAL... EXAMPLE (solved by Euler):



$$\left.\begin{aligned} EI\frac{d^4x}{dz^4} + F\frac{d^2x}{dz^2} &= 0 \\ EI\frac{d^4y}{dz^4} + F\frac{d^2y}{dz^2} &= 0 \end{aligned}\right\} \; x=y=0 \text{ is a solution}$$

BUT, WHEN $F > \frac{4\pi^2 EI}{l^2} \equiv F_c$

$x \text{ (or } y) = C \sin kz \qquad k = \sqrt{|F|/_{EI}}$

SYMMETRY IS LOST... HIDDEN (same equation of motion)

→ ROD COULD HAVE PICKED AN INFINITE NUMBER OF DIRECTIONS TO BULDGE... IN ACCORDANCE WITH ORIGINAL SYMMETRY

...just like ferromagnet

~ INTERLUDE ~ AROUND 1960 HEP THEORIST'S WERE STRUGGLING WITH A NUMBER OF BROKEN SYMMETRIES: SU(3), SU(2), PARITY...

★ Weinberg got a whiff of CMP's success & began trying to apply some of these ideas → idea that symmetry isn't gone, but hidden was appealing to him... and wrong.

WRONG BECAUSE ...

GOLDSTONE THEOREM (G.T.): A SYSTEM WHICH HAS A SPONTANEOUSLY BROKEN CONTINUOUS SYMMETRY <u>MUST</u> HAVE MASSLESS, BOSE-EXCITATIONS.

(This spoiled Weinberg's hopes, as there are no massless spin zero particles...)

G.T. WORKS FINE FOR CMP...

eg ferromagnetism

↑ ↑ ↑ ↑ ↑ ↑ ↑ GROUND STATE

↑ ↑ ↑ ↓ ↑ ↑ ↑ 1 EXCITED STATE

but that's not what magnets do (<u>large</u> magnets..!)

energetics favor: ↑ ↗ ↗ → ↘ ↓ ↓ ↙ ↙ ← ← ↖ ↑ ↑

get a long-wavelength MACROSCOPIC, QUANTIZABLE excitation with energy

$$\epsilon = \hbar^2 S \sum_{\vec{a}} (1 - \cos \vec{q}\cdot\vec{a}) \qquad (\text{"dispersion"})$$

AS $\vec{q} \to 0$, $\epsilon \to 0$ $\Rightarrow$ "MASSLESS"

... as if the ground state is full of SPIN WAVE excitations...

IF YOU LIVED INSIDE AT $T < T_c$, HOW WOULD YOU RECOGNIZE THAT THE SYMMETRY OF THE HAMILTONIAN IS SO(3)!?

... that's our situation.

Goldstone Theorem 2:

PROOF:

- SUPPOSE WE HAVE A CONSERVED CURRENT, $\partial_\mu j^\mu(x) = 0$

  FOR SOME SYSTEM CHARACTERIZED BY FIELDS $\phi(x)$

$$\partial_\mu [j^\mu(x), \phi(x')] = 0$$

$$\partial_0 [j^0(x), \phi(x')] - \vec{\nabla}\cdot[\vec{j}(x), \phi(x')] = 0$$

$$\partial_0 \int d^3x\, [j^0(x), \phi(x')] - \int d\vec{S}\cdot[\vec{j}(x), \phi(x')] = 0$$

(using Divergence theorem)

if $\int d\vec{S}\cdot[\vec{j}(x), \phi(x')] = 0$ over the surface

field operator

then $\partial_0 [Q(t), \phi(x')] = 0 \Rightarrow [Q, \phi(x')] = \text{constant}, C$

- TAKE EXPECTATION VALUE OF THIS QUANTITY IN VACUUM...

$$\langle 0|[Q, \phi(x')]|0\rangle = \langle 0|C|0\rangle$$

without identifying this quantity, yet.

- USE COMPLETENESS TO INSERT THE SPECTRUM OF A COMPLETE SET OF INTERMEDIATE STATES OF THE ϕ's, $|n\rangle$...

$$\sum_n \left[\langle 0|Q|n\rangle\langle n|\varphi(x')|0\rangle - \langle 0|\varphi(x')|n\rangle\langle n|Q|0\rangle\right] = \langle 0|c|0\rangle$$

- WRITE Q IN TERMS OF $j(x)$ & SHIFT SPACETIME ARGUMENT USING

$$j^0(x) = e^{-iPx} j^0(0)\, e^{iPx} \quad \text{with} \quad e^{iPx}|n\rangle = e^{ik_n x}|n\rangle, \quad e^{iPx}|0\rangle = |0\rangle$$

$$\int d^3x \left\{ \sum_n \langle 0|j^0(0)|n\rangle\langle n|\phi(x')|0\rangle e^{ik_n x} - \langle 0|\varphi(x')|n\rangle\langle n|j^0(0)|0\rangle e^{ik_n x} \right\} =$$

- INTEGRATE

  exponentials contain only $\vec{x}$ dependence $\longrightarrow \delta(\vec{k}_n)$

Goldstone theorem 3:

only time dependence

$$\sum_n (2\pi)^3 \delta(\vec{k}_n)\Big[\langle 0|j^0(0)|n\rangle\langle n|\varphi(x')|0\rangle e^{-iE_n t} - \langle 0|\varphi(x')|n\rangle\langle n|j^0(0)|0\rangle e^{iE_n t}\Big] = \langle 0|c|0\rangle \equiv RHS$$

GO BACK: $\langle 0|[Q,\varphi(x')]|0\rangle = \langle 0|Q\varphi'(x')|0\rangle - \langle 0|\varphi(x')Q|0\rangle \equiv LHS$

LEAVING 2 CONSEQUENCES, DEPENDING ON VACUUM & GROUP.

a) $\mathcal{U}(Q)|0\rangle = |0\rangle \Rightarrow Q|0\rangle = 0$ "Weyl Symmetry"

OR

b.) $\mathcal{U}(Q)|0\rangle \neq |0\rangle \Rightarrow Q|0\rangle \neq 0$ "Goldstone Symmetry"

a.) IS THE USUAL SITUATION... the vacuum "carries the trivial, one dimensional representation of all symmetry groups." Roman

b.) HAPPENS ALL THE TIME IN CMP... $|0\rangle \equiv$ GROUND STATE ☆ NAMBU

IF b) IS THE SITUATION...

$LHS = \langle 0|[Q,\varphi(x')]|0\rangle \neq 0$

$RHS =$ independent of time $\Rightarrow$ in $e^{\pm iE_n t}$ terms $E_n \to 0$

HERE WE GO AGAIN: AS $\underline{\vec{k}_n \to 0}$, $\underline{E_n \to 0}$ $\Rightarrow$ MASSLESS $|n\rangle$'s.

NOTE: IF $\phi(x)$ IS NOT A SINGLET UNDER THE GROUP

$[Q,\phi(x')] = \phi'(x')$ ... some ϕ' must exist

THEN OUR $\langle 0|c|0\rangle \longrightarrow \underbrace{\langle 0|\phi'(x')|0\rangle}$ VEV OF FIELD ITSELF

vacuum expectation value

vacuum •- - - - -• vacuum $\phi'(x)$

ϕ CONNECTS VACUUM TO ITSELF... VACUUM IS FULL OF ϕ's.

OBSERVATION THAT VEV OF FIELD $\neq 0$ IS A TRIGGER FOR GOLDSTONE THEOREM.

DILUTE BOSE GAS: STATISTICAL MECHANICS

$n_i > 0 \Rightarrow$
$\varepsilon_i - \mu \geq 0 \Rightarrow$
$\mu \leq 0$

RECALL: # OCCUPATION NUMBER FOR BOSONS $n_i = \dfrac{g_i}{e^{(\varepsilon_i-\mu)/kT}-1}$ (BE)

TOTAL OCCUPATION: $N = 2\pi V(2m)^{3/2}\int_0^\infty \dfrac{\varepsilon^{1/2}\,d\varepsilon}{e^{(\varepsilon-\mu)/kT}-1}$

continuum limit $\longrightarrow$

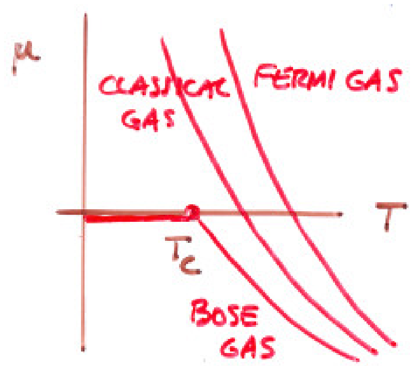
$\dfrac{1}{e^{-x}-1} = \sum_{j=1}^{\infty} e^{jx}$

$g(\varepsilon)\,d\varepsilon = 2\pi V(2m)^{3/2}\varepsilon^{1/2}\,d\varepsilon$
non-relativistic

$N = V(2m\pi kT)^{3/2}\left[\sum_{j=1}^{\infty}\dfrac{1}{j^{3/2}}e^{j\mu/kT}\right]$

as $T\to\infty$, $N \to e^{-\mu/kT} \longrightarrow$ M.B. $\quad \mu \to -\infty$, $T \to \infty$
hot $\to$ classical

@ $\mu = 0$, CALL $T \equiv T_c$: $T_c = \dfrac{1}{[\zeta(3/2)]^{2/3}}\left(\dfrac{N}{V}\right)^{2/3}\dfrac{1}{2m\pi k}$



BELOW T_c? SEPARATE GROUND STATE FROM EXCITED STATES...

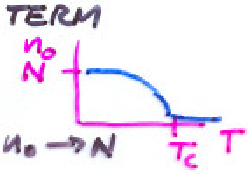
$N = n_0 + N_\varepsilon$

Ground state
$\varepsilon = 0$ $\quad n_0 = \dfrac{g_0}{e^{-\mu/kT}-1}$ $\qquad \mu = 0^-$ below T_c $\qquad$ (to keep n +)

Excited states
$\varepsilon \neq 0$ $\quad n_\varepsilon = \dfrac{g_i}{e^{(\varepsilon_i-\mu)/kT}-1}$ $\qquad \mu = 0$ at $T = T_c$

$N = n_0 + V(2\pi mkT)^{3/2}\sum \dfrac{1}{j^{3/2}}e^{j\mu/kT}$ $\qquad$ FOR $T < T_c$ $\mu = 0$ IN SECOND TERM

$N = n_0 + V(2\pi mkT)^{3/2}\zeta(3/2)$



$N = n_0 + N\left(T/T_c\right)^{3/2} \Rightarrow \boxed{n_0 = N\left[1-\left(\dfrac{T}{T_c}\right)^{3/2}\right]}$ $\quad$ AS $T\to 0$ $n_0 \to N$

CONDENSATE IN G.S.

DILUTE BOSE GAS: QUANTUM MECHANICS

- CONDENSATION INTO GROUND STATE IS A PROBLEM FOR A FIELD THEORY

  RECALL: USE WICK'S THEOREM TO ORDER a's AND $a^\dagger$'s IN VEV's ... NEED $a|0\rangle = 0$ & $\langle 0|a^\dagger = 0$ TO BUILD A PERTURBATION THEORY...

  need an empty vacuum – Bose-Einstein Condensate is a full vacuum!

$|0\rangle_N \equiv$ VACUUM STATE WITH N PARTICLES

as a Fock state...

$|0\rangle_N = |N, 0, 0 \ldots 0\rangle$

ALL OCCUPY THE $\varepsilon = 0$ STATE AT $T = 0$

- A WAY OUT INVENTED BY BOGOLIUBOV...

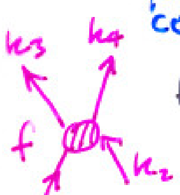
$$H = \int d^3x \; \psi^\dagger(x)\left[-\frac{\hbar^2}{2m}\nabla^2\right]\psi(x) \qquad \text{K.E. term}$$

$$+ \int d^3x \int d^3x' \; \psi^\dagger(x)\psi^\dagger(x')\, v(x,x')\, \psi(x)\psi(x') \qquad \text{P.E. term} \quad \psi^4$$

$$+ \mu \int d^3x \; \psi^\dagger(x)\psi(x) \qquad \text{C.P. term} \quad \psi^2$$

$\mu = 0$ IN CONDENSATE:

★ [Reminiscent of Landau free energy with $\psi_{G.S.}$ as order parameter] ★



$$H = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + \sum_{k_1}\sum_{k_2}\sum_{k_3}\sum_{k_4} \underbrace{\delta_{k_1+k_2,\, k_3+k_4}}_{\text{conserves momentum}} f(\vec{k}_2 - \vec{k}_4)\, a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4}$$

SINCE ${}_N\langle 0|a^\dagger a|0\rangle_N \neq 0$ & $a|0\rangle_N = N^{\frac{1}{2}}|0\rangle_{N-1} \simeq N^{\frac{1}{2}}|0\rangle_N$ large N

WE HAVE ${}_N\langle 0|\psi|0\rangle_N = {}_N\langle 0|\psi^\dagger|0\rangle_N \neq 0$ FOR FIELDS OF SYSTEM... SHOULD EXPECT TO SEE GOLDSTONE KICKING IN ?

Bose gas 3!

- NEED A BROKEN SYMMETRY... $u = e^{i\lambda N}$ ← $a^\dagger a$, NUMBER OPERATOR IS TRIVIALLY SATISFIED BY HAMILTONIAN BUT $u|0\rangle_N \neq |0\rangle_N$

  SINCE $N|0\rangle_N \neq 0$

YUP.. WE GOT GOLDSTONE.

BUT WE ALSO HAVE THE FIELD THEORY PROBLEM...

- BOGOLIUBOV NOTED $a^\dagger$ AND a ARE ALMOST c-numbers...

SIMPLE EXERCISE 4: SHOW THAT $[a^\dagger, a] \simeq 0$ IN CONDENSATE GROUND STATE AND THAT $\underbrace{a^\dagger \simeq a \simeq \sqrt{n_0}}_{\sim \text{ C-NUMBERS}}$.

BOGOLIUBOV TRANSFORMATION;

two parts -

1. SHIFT AWAY FROM GROUND STATE

$$\psi(x) = e^{i\alpha}\sqrt{n_0} + \chi(x)$$

original field op's... eg ^{4}He atoms

G.S. stuff: number

EXCITED STATE STUFF

$$\chi(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k} \neq 0} a_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}$$

IN CONDENSATE: ${}_N\langle 0|\chi(x)|0\rangle_N = 0$

SUBSTITUTE THIS INTO HAMILTONIAN...

$$H = N^2 + \sum_{\vec{k} \neq 0} \omega_k a_k^\dagger a_k + N \sum_{\vec{k} \neq 0} (a_k a_{-k} + a_k^\dagger a_{-k}^\dagger)$$

where $\omega_k = \frac{\hbar^2 k^2}{2m} + 2N f(\vec{k})$

MESSY EXERCISE 1: SHOW THIS.

NOTE: PRICE IS NON-DIAGONAL INTERACTION --

Bose gas 4:

(2.) DIAGONALIZE WITH A CANONICAL TRANSFORMATION

$$\left.\begin{array}{l}\alpha_{\vec{k}} \equiv u_k a_k^{\dagger} + v_k a_{-k}^{\dagger} \\ \alpha_{-\vec{k}} \equiv u_{-k} a_{-k} + v_{-k} a_k^{\dagger}\end{array}\right\}$$ α's have same commutation relations as a's.

α'S CREATE & ANNIHILATE A NEW PARTICLE SPECTRUM ... A "QUASI-PARTICLE" SPECTRUM & $\alpha_k |0\rangle_N = 0 \quad \forall_{k\neq 0}$

$$H = N^2 - \underbrace{\frac{1}{2}\sum_{k\neq 0} (\omega_k - \varepsilon_k)}_{\text{G.S. energy level lowered}} + \underbrace{\frac{1}{2}\sum_{k\neq 0} \varepsilon_k \, \alpha_k^{\dagger}\alpha_k}_{\text{energy spectrum of quasi particles near ground state}}$$

where $\varepsilon_k = \sqrt{\omega_k^2 - 4N^2 f^2(k)}$

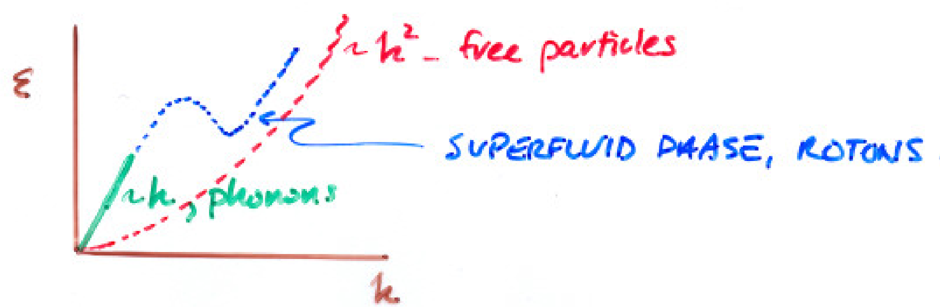
$$\varepsilon_k = \sqrt{\frac{\hbar^4 k^4}{4m^2} + \frac{4\hbar^2 k^2 f(k)}{2m}}$$ DISPERSION

AS $k \to$ LARGE, $\varepsilon_k \sim k^2$ such as free particles...

AS $k \to$ SMALL, $\varepsilon_k \sim k\sqrt{\frac{4\hbar^2 f(0)}{2m}} \propto k$ LIKE PHONONS...

$\left.\begin{array}{l}\to 0 \\ \text{as } k \to 0\end{array}\right\}$ massless

* THE MASSLESS QUASI EXCITATIONS ARE PHONONS ... GOLDSTONE BOSONS OF BOSE GAS



REMEMBER THE BOGOLIUBOV SOLUTION...

recap:

MANY PHENOMENA INVOLVE BROKEN SYMMETRIES:
full symmetry is "really" there... natural manifestation hides that fact

$\Rightarrow$

GROUND STATE FULL: $\langle 0|\phi|0\rangle \neq 0$

BROKEN CONTINUOUS SYMMETRY
$\Rightarrow$ MASSLESS GOLDSTONE BOSONS

$\downarrow$

SHIFT FIELD OPERATORS INTO

c-number vacuum term + quasi-particle operator term

$\downarrow$

INSERT INTO MODEL & TRANSFORM INTO <u>QUASIPARTICLE SPECTRUM</u>

GINSBURG-LANDAU PHENOMENOLOGY:
identify order parameter & mechanically induce phase transition

PUT THESE IDEAS TOGETHER WITH THE ASSUMPTION THAT THE MANY-BODY GROUND STATE IS ANALOGOUS TO THE ELEM. PARTICLE VACUUM & $\psi \equiv \phi$

TOY THEORY: ... mix these ideas up!

- INCORPORATE ALL OF ABOVE NOTIONS INTO A RELATIVISTIC QUANTUM FIELD THEORY... Goldstone 1960
  "Field Theories with 'Superconductor' Solutions"

- FOLLOWING LANDAU FORM... OR EQUIVALENTLY, THE BOSE GAS:

$$\mathcal{L} = \underbrace{\tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi}_{\text{KE term}} - \underbrace{a\tfrac{|\mu^2|}{2}\varphi^2}_{\substack{\text{mass term}\\ \text{(like }\mu\text{ in}\\ \text{Bose gas)}}} - \underbrace{\tfrac{1}{4}\lambda\varphi^4}_{\text{self interaction}}$$

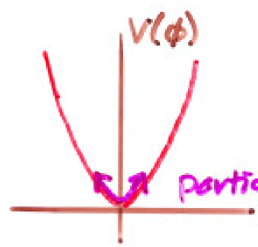
Euler-Lagrange equations of motion: $\partial_\mu\partial^\mu\phi + a\mu^2\phi = \lambda\phi^3$ ✓

ϕ has mass $\sqrt{a}\,\mu$ ...

- INVESTIGATE SYMMETRY —
  REFLECTION SYMMETRY $\phi \to -\phi$ LEAVES $\mathcal{L}$ ALONE

- IDENTIFY LANDAU FREE ENERGY WITH PE TERM OF $\mathcal{L}$

$$V(\phi) = \frac{a\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$



- MINIMIZE TO FIND GROUND STATE:
  minimum @ $V(\varphi) = 0$

$\Rightarrow \langle 0|\phi|0\rangle = 0$

- BUT, a la LANDAU, ALLOW A 2nd ORDER "PHASE TRANSFORMATION"...
  $a \to -|a|$

$$V(\phi) = -\frac{a\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

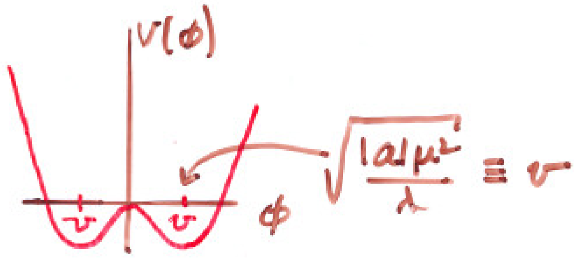
... mass interpretation for $\sqrt{a}\,\mu$ is destroyed... now a complicated interaction for massless ϕ particles

$$\partial_\mu\partial^\mu\varphi - a\mu^2\varphi = \lambda\varphi^3$$

wrong for mass term in K.G. equation

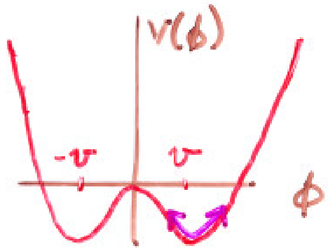
relativistic Goldstone 2:

- MINIMIZE



"VACUUM" OCCURS AT FINITE $\phi \Rightarrow$ "$\langle 0|\varphi|0\rangle \neq 0$"
$= \pm v$

- FULLY REALIZE THE THEORY AS <u>BROKEN</u>, BY CHOOSING ONE OF THE VACUA AS <u>THE</u> VACUUM FROM WHICH THE PARTICLE SPECTRUM IS BUILT..



$\Rightarrow$ A BOGOLIUBOV-LIKE SHIFT $\phi(x) = \langle 0|\varphi|0\rangle + \chi(x)$
$= v + \chi(x)$

& SUBSTITUTE BACK..

$\mathcal{L}(\chi) = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - |a|\,\mu^2\chi^2 +$ quartic & cubic self interactions

Hold the phone... this is the Lagrangian for a χ field of mass $\sqrt{2a}\,\mu$ $\rightarrow$ WHERE HAS GOLDSTONE GONE?

WHAT'S WRONG WITH THE GOLDSTONE THEOREM?
nothing

MESSY EXERCISE 2: SHOW $\mathcal{L}(\chi)$.

HERE, THE SYMMETRY WAS A BROKEN DISCRETE SYMMETRY..

GOLDSTONE THEOREM INVOLVED BROKEN CONTINUOUS SYMMETRIES.

notch it up one step...

NEW TOY:

- FOR A CONTINUOUS SYMMETRY.. NEED MORE THAN 1-COMPONENT OBJECT: φ_1 & φ_2 or φ & $\varphi^\dagger = \dfrac{\varphi_1 \pm i\varphi_2}{\sqrt{2}}$

$$\mathcal{L}(\varphi) = \tfrac{1}{2}\partial_\mu \varphi^\dagger \partial^\mu \varphi - \tfrac{1}{2} a \varphi^\dagger \varphi - \tfrac{1}{4}\lambda(\varphi^\dagger\varphi)^2$$

SYMMETRY: $\varphi \rightarrow \varphi' = e^{i\theta}\varphi$ LEAVES $\mathcal{L}$ ALONE...

or $\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} \rightarrow \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}' = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$

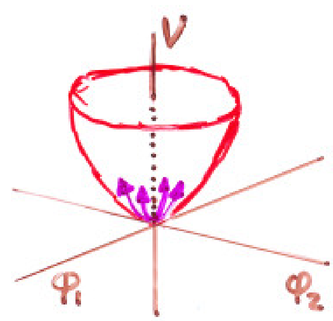
... Global $U(1)$ or $SO(2)$, which are isomorphic.

2 COMPONENT "ISODOUBLET-LIKE" MORE INSTRUCTIVE:

$$\mathcal{L}(\varphi) = \tfrac{1}{2}\partial_\mu \varphi_1 \partial^\mu \varphi_1 + \tfrac{1}{2}\partial_\mu \varphi_2 \partial^\mu \varphi_2 - \frac{a\mu^2}{2}(\varphi_1^2+\varphi_2^2) - \frac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2$$

$$V(\varphi_1,\varphi_2) = \frac{a\mu^2}{2}(\varphi_1^2+\varphi_2^2) + \frac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2$$

MINIMIZATION LEADS TO:

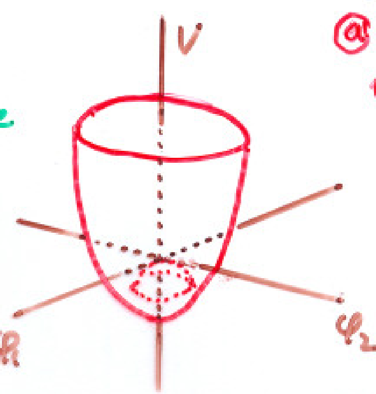


- NOW, $a \rightarrow -|a|$

MINIMIZATION LEADS TO: $\varphi_1^2 + \varphi_2^2 = \dfrac{|a\mu^2|}{\lambda}$ ... a loci which is a circle @ RADIUS $v = \sqrt{\dfrac{a\mu^2}{\lambda}}$

number of vacua is now infinite

→ CHOICE OF ONE INVOLVES A SLICE IN $\varphi_1 - \varphi_2$ & BREAKS THE $SO(2)$ SYMMETRY



BROKEN TOY:

LOCUS: $\langle 0|\varphi|0\rangle = v e^{i\alpha} = v\cos\alpha + i v \sin\alpha$

CHOOSE TO BREAK SYMMETRY BY $\alpha = 0$

$$\left.\begin{array}{l}\langle 0|\varphi_1|0\rangle = v \\ \langle 0|\varphi_2|0\rangle = 0\end{array}\right\} \text{ A } \varphi_2 = 0 \text{ SLICE}$$

$$\langle 0|\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}|0\rangle = \begin{pmatrix} v \\ 0\end{pmatrix}$$

SHIFT FIELDS USING COMPLEX REPRESENTATION...

$$\varphi = \underbrace{v + \sigma(x)}_{\varphi_1} + \underbrace{i\eta(x)}_{i\varphi_2}$$

TO QUASI PARTICLE SET, $\sigma(x)$ & $\eta(x)$.

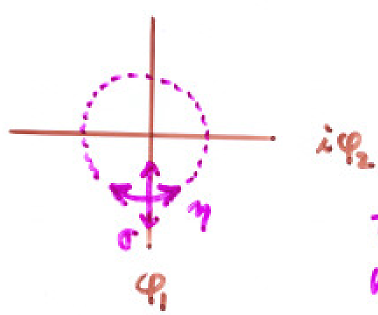
SUBSTITUTE -

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \eta\, \partial^\mu \eta + \tfrac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma - a|\mu^2|\sigma^2 + \underbrace{\text{cubic}}_{\text{no } \eta^2 \text{ term}} \text{ \& quartic interactions}$$

φ_2 LOST ITS MASS... η IS MASSLESS (THE GOLDSTONE BOSON)

σ IS MASSIVE, $m_\sigma = \sqrt{2a}\,\mu$

LOOK DOWN ON $V=0$ PLANE...



THE η OSCILLATES WITHIN THE WELL (MASSLESS).. CONNECTING OTHERS OF THE DEGENERATE VACUUMS

Higgs mechanism I:

GOLDSTONE THEOREM IRON-CLAD

- PROVEN BY WEINBERG, SALAM, & GOLDSTONE
  $\Rightarrow$ USE BY HEP TO ACCOUNT FOR APPROXIMATE SYMMETRIES WAS DEAD
- EXCEPT.. UNNOTICED BY MANY (EXCLUDING ANDERSON, BY THE WAY...)

  THERE IS A LOOPHOLE...

- WE DID GLOBAL $U(1)$ SYMMETRY... WHAT ABOUT LOCAL $U(1)$ SYMMETRY?

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi^+\partial^\mu\varphi - \frac{a\mu^2}{2}\varphi^+\varphi - \frac{\lambda}{4}(\varphi^+\varphi)^2$$

WE KNOW HOW TO MAKE THIS LOCALLY GAUGE INVARIANT...

$\partial^\mu \rightarrow \partial^\mu + iga^\mu$ SUBSTITUTION + TRANSFORMATIONS:

$\varphi \rightarrow \varphi' = e^{ig\theta(x)}\varphi(x)$ & $a^\mu \rightarrow a^{\mu\prime} = a^\mu - \partial_\mu\theta(x)$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{\frac{1}{2}(D_\mu\varphi)^+D^\mu\varphi}_{} - \frac{a\mu^2}{2}\varphi^+\varphi - \frac{\lambda}{4}(\varphi^+\varphi)^2$$

encryption of a-φ interaction

$\frac{1}{2}\partial_\mu\varphi^+\partial^\mu\varphi + \frac{1}{2}g^2a^\mu a_\mu\varphi^+\varphi$

FORCE $a \rightarrow -|a|$ AND SHIFT FIELDS...

$\langle 0|\varphi_1|0\rangle = v \equiv \frac{a\mu^2}{\lambda}$ $\qquad \langle 0|\varphi_2|0\rangle = 0$

$\varphi = v + \sigma + i\eta$ AGAIN

Higgs mechanism 2:

- SUBSTITUTE

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$-\frac{1}{2}2gv\,\partial_\mu\eta\, a^\mu + g^2 v\sigma a^2 + \frac{1}{2}g^2v^2a^2 - a\mu^2\sigma + \text{cubic \& quart. interactions}$$

LOOK AT ▬▬ TERMS...

$$\frac{1}{2}\left(\partial_\mu\eta\partial^\mu\eta - 2gv\,\partial_\mu\eta\, a^\mu + g^2v^2a^2\right)$$

$$= \frac{1}{2}\left(gva_\mu - \partial_\mu\eta\right)^2 = \frac{1}{2}g^2v^2\left(a_\mu - \frac{1}{gv}\partial_\mu\eta\right)^2$$

(RE)DEFINE $\quad \alpha_\mu \equiv a_\mu - \frac{1}{gv}\partial_\mu\eta \quad$ (looks like gauge transformation -- doesn't affect F's)

so, $\quad \Phi_{\mu\nu} \equiv \partial_\mu\alpha_\nu - \partial_\nu\alpha_\mu \quad$ -- or Φ's...

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{2}g^2v^2\alpha^2 - a\mu^2\sigma^2 + \text{int terms}$$

LOTS OF MAGIC HERE

- η HAS DISAPPEARED! THERE ARE NO MASSLESS BOSONS!
- σ HAS A MASS! $\quad m_\sigma = \sqrt{2a\mu^2}$
- a_μ HAS DISAPPEARED... AND BEEN REPLACED BY α_μ WHICH HAS GAINED A MASS!! $\quad m_\alpha = \frac{gv}{\sqrt{2}}$
- THE GRADIENT OF THE GOLDSTONE BOSON COMBINED WITH THE MASSLESS a_μ ... IN MOM. SPACE $\underbrace{k_\mu\eta}$

behavior of longitudinal dof for SPIN 1.

a_μ ATE THE η FIELD...

ACTUALLY.. IT WAS "GAUGED AWAY".

THIS WAS DISCOVERED BY...

Anderson, Nambu, Englert, Brout, Gilbert, Guralnik, Higgs, Hagen, & Kibble

SO IT IS NATURALLY CALLED THE HIGGS MECHANISM.

σ IS THE HIGGS BOSON... A NECESSARY RELIC OF THIS APPROACH

- START OUT WITH:
  - 2 COMPONENT, DEGENERATE BOSON PAIR
  - MASSLESS SPIN 1 BOSON which insures local U(1) symmetry
- EMPLOY THE LANDAU MECHANISM...
- END UP WITH:
  - 1 MASSIVE SPIN 0 BOSON
  - 1 MASSIVE SPIN 1 BOSON

MAGIC? nope... that's superconductivity.

superconductivity 1:

- THE CURRENT DENSITY IN SUPERCONDUCTIVITY IS

$$\vec{j} = \frac{-ie\hbar}{2m}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - \frac{2e^2}{mc}|\psi|^2\vec{A}$$

ψ – WAVEFUNCTION FOR COOPER PAIRS .. THE ORDER PARAMETER

$\vec{A}$ – ELECTROMAGNETIC FIELD, INTRODUCED BY DEMANDING LOCAL GAUGE INVARIANCE IN H.

$$\psi \to \psi' = \sqrt{\frac{n_s}{2}}\; e^{-2ie\phi(x)/\hbar c}$$

SO

$$\vec{j} = \frac{n_s e^2}{mc}\left(\vec{\nabla}\phi - \vec{A}\right)$$

$$\vec{\nabla}\cdot\vec{j} = 0 = \vec{\nabla}\cdot\vec{\nabla}\varphi - \underset{\substack{\parallel \\ 0 \text{ in Coulomb gauge}}}{\vec{\nabla}\cdot\vec{A}}$$

$\Rightarrow \vec{\nabla}\varphi$ CONSTANT... a number

LONDON EQUATION $\quad \vec{j} = -\frac{n_s e^2}{mc}\vec{A}$

MANIPULATE: $\quad \vec{\nabla}\times\vec{j} = -\frac{e^2 n_s}{mc}\vec{B}$

WITH Ampere's law $\quad \vec{\nabla}\times\vec{B} = \vec{j}$

$$\nabla^2\vec{B} = -\frac{e^2 n_s}{mc}\vec{B} \qquad *$$

WITH SOLUTION $\quad \vec{B} = \vec{B}_0\, e^{-x/\lambda} \qquad \lambda = \frac{mc}{e^2 n_s}$

A SHORT PENETRATION INTO SUPERCONDUCTOR OF MAGNETIC FIELD .. THE MEISNER EFFECT

ANOTHER INTERPRETATION...

Superconductivity 2:

* LOOKS LIKE $$\nabla^2 \vec{B} + \frac{1}{\lambda} \vec{B} = 0$$

WHICH LOOKS LIKE THE KLEIN GORDON EQUATION FOR A PHOTON OF MASS $(1/\lambda)$...

EXPULSION OF $\vec{B}$ FROM GROUND STATE (actually arranged by collective "super currents" of Cooper pair electrons) MAKES IT APPEAR TO BE "HEAVY".

- THE COOPER PAIRS ARE ELECTRONS PAIRED WITH SPINS $\downarrow\uparrow$ --- $J=0$ $\Rightarrow$ LOOK LIKE BOSONS $\rightarrow$ THEY ARE THE HIGGS FIELDS
  - composite
  - macroscopic $\rightarrow$ yet quantum mechanical
  - screen out em fields $\rightarrow$ making γ massive

ANDERSON KNEW THIS... but nobody asked!

standard model !:

ALL TOOLS IN PLACE...

WEAK INTERACTIONS
NEED SPIN 1 $W^{\pm}$
PROPAGATOR

↓ YANG-MILLS, 1954

DEMAND LOCAL SU(2) GAUGE INVARIANCE $\xrightarrow{\text{RESULT}}$ GET MASSLESS SPIN 1 TRIPLET

↓ GINSBURG/LANDAU, 1950

PLUS... ISODOUBLET
SCALAR FIELDS
LIKE BOSE-GAS

↓ HIGGS & friends, 1964

SPONTANEOUSLY BREAK GAUGE SYMMETRY $\xrightarrow{\text{RESULT}}$ GET MASSIVE SPIN 1 TRIPLET & MASSIVE SPIN 0 SINGLET
NO PHOTON

↓ WEINBERG, 1967

SPONTANEOUSLY BREAK PRODUCT GAUGE SYMMETRY.. $SU(2) \otimes U(1)$ $\xrightarrow{\text{RESULT}}$ GET MASSIVE SPIN 1 TRIPLET & MASSLESS SPIN 1 SINGLET & MASSIVE SPIN 0 SINGLET

standard model 2:

BUILD A MODEL: OF LEPTONS.

WEINBERG PRL 19, 1264, 1967.

— JUST DETAILS FROM THIS POINT.

- $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\phi + \mathcal{L}_{\phi \ell}$

- $\mathcal{L}_0 = \bar{L}\, i\gamma^\mu \left(\partial_\mu + \frac{i}{2} g' a_\mu - ig \frac{\vec{\tau}\cdot\vec{b}_\mu}{2} \right) L$

$$+ \bar{R}\, i\gamma^\mu (\partial_\mu + ig' a_\mu) R$$

$$- \frac{1}{4} f^i_{\mu\nu} f^{\mu\nu\, i} - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu}$$

WHERE $L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$ $\quad R \equiv \ell_R$ & $\ell_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\ell$ $\quad \nu_L =$ ditto

$$f^i_{\mu\nu} \equiv \partial_\mu b^i_\nu - \partial_\nu b^i_\mu + g\, \varepsilon^{ijk} b^j_\mu b^k_\nu$$

$$\Phi_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$$

- $\mathcal{L}_\phi = (\Delta_\mu \phi^\dagger)(\Delta^\mu \phi) - V(\phi)$

WHERE $\Delta_\mu \phi = \left(\partial_\mu - ig \frac{\vec{\tau}\cdot\vec{b}_\mu}{2} - i\frac{g' a_\mu}{2} \right) \phi$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- $\mathcal{L}_{\phi \ell} = -G_\ell (\bar{R} \phi^\dagger L + \bar{L} \phi R)$

A FANCY WAY TO IMPLEMENT SYMMETRY BREAKING IS TO TAKE ADVANTAGE OF A DIFFERENT GAUGE.. U-gauge

$$\langle 0|\varphi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad v = \sqrt{\mu^2/\lambda}$$

$$\phi(x) = \exp\left(-i\, \vec{\xi}(x)\cdot\vec{\tau}/2v\right) \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$ like polar coordinates...

standard model 3:

$$\varphi \to \varphi' = \underbrace{\exp\left(-i \frac{\vec{\xi}\cdot\vec{\tau}}{2v}\right)}_{u(\vec{\xi})} \varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} = \frac{v+\eta(x)}{\sqrt{2}} \, \chi$$

where $\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

OTHER OBJECTS TRANSFORM...

$$L' = u L$$

$$\frac{\vec{\tau}\cdot\vec{b}'_\mu}{2} = u(\xi) \frac{\vec{\tau}\cdot\vec{b}_\mu}{2} u^{-1}(\xi) - \frac{i}{g}\left[\partial_\mu u(\xi)\right] u^{-1}(\xi)$$

$$a'_\mu = a_\mu$$

$$R' = R$$

$$f' = f \qquad \Phi' = \Phi$$

- ALL OF THE ACTION IS IN THE $\mathcal{L}_\phi$ TERM...

$$\mathcal{L}_\phi \to \mathcal{L}'_\phi = \left(\partial_\mu - ig\frac{\vec{\tau}\cdot\vec{b}'_\mu}{2} \underbrace{- ig'\frac{a_\mu}{2}\right)\left(\text{ditto}^\mu\right)}_{\text{leads to terms quadratic in spin 1 fields}} \left(\frac{v+\eta(x)}{\sqrt{2}}\right)\chi$$

just that piece:

$$= \frac{v^2}{8}\left\{ g^2\left[(b'^1_\mu)^2 + (b'^2_\mu)^2\right] + \underbrace{(g' a_\mu - g b'^3_\mu)^2}_{\text{2 "neutral" fields mixed up...}} \right\}$$

DEFINE $\quad W^\pm_\mu \equiv \sqrt{\frac{1}{2}}\left(b'^1_\mu \mp b'^2_\mu\right)$

$$\left.\begin{aligned} Z_\mu &\equiv \frac{-g' a_\mu + g b'^3_\mu}{\sqrt{g^2+g'^2}} \\ A_\mu &\equiv \frac{g a_\mu + g' b'^3_\mu}{\sqrt{g^2+g'^2}} \end{aligned}\right\} \text{DIAGONALIZATION TO FORCE } A_\mu \text{ TO BE MASSLESS...}$$

standard model 4:

this becomes = $M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \; (+ \; 0 \cdot A_\mu A^\mu)$

where $M_W^2 = (\tfrac{1}{2} g v)^2$

$$M_Z^2 = \frac{v^2}{4}\sqrt{g^2 + g'^2}$$

for convenience $\tan\theta_W \equiv g'/g$

θ_W = "WEINBERG ANGLE"

= "WEAK ANGLE"

$$M_Z = \frac{1}{2}\frac{g v}{\cos\theta_W} = \frac{M_W}{\cos\theta_W}$$

AFTER MUCH ALGEBRA, THE REST OF $\mathcal{L}_\phi$ IS..

$$\mathcal{L}_\phi = \underline{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2 \eta^2} - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4$$
$$+ M_W^2 W_\mu^- W^{\mu +} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$
$$+ \frac{g^2}{8}(\eta^2 + 2v\eta)\left[\frac{1}{\cos^2\theta_W} Z_\mu Z^\mu + 2 W_\mu^+ W^{\mu -}\right]$$

— INDICATES HIGGS FREE PORTION ⇒ $m_{HIGGS} = \sqrt{2}\,\mu$

- THE REGULAR WEAK INTERACTIONS LIVE IN THE $\mathcal{L}_0$

$$\mathcal{L}_0 = \ldots \bar{R}' i\gamma^\mu \partial_\mu R' + \bar{L}' i\gamma^\mu \partial_\mu L + \underbrace{g\,\vec{b}_\mu' \cdot \bar{L}\frac{\vec{\tau}}{2}\gamma^\mu L}_{\text{CONTAINS REGULAR W.I.}} + \underbrace{\frac{g'}{2} a_\mu \left[2\bar{R}\gamma^\mu R + \bar{L}\gamma^\mu L\right]}_{\text{CONTAINS REGULAR EM}}$$

standard model 5:

AFTER WRITING IN TERMS OF W's, Z's, and A ... THIS BREAKS DOWN INTO

$$\mathcal{L}_c = \frac{g}{\sqrt{2}}\left(j_\mu^+ W^{+\mu} + \text{H.C.} \right)$$

where $j_\mu^+ = \bar{\nu}_L \gamma_\mu l = \frac{1}{2} \bar{\nu}_l \gamma_\mu (1-\gamma_5) l$

IDENTIFY $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \Rightarrow v \doteq 260 \text{ GeV}$

$$\mathcal{L}_N = g\, j_\mu^3\, b^{3\mu} + \frac{g'}{2} a_\mu j_Y^\mu$$

$\vdots$

$$= g \sin\theta_W A_\mu J^\mu + \frac{g Z^\mu}{\cos\theta_W}\left[j_\mu^3 - \sin^2\theta_W J_\mu \right]$$

IDENTIFY $e = g\sin\theta_W$

WHERE $j_{\mu Y} \equiv 2\bar{R}\gamma_\mu R + \bar{L}\gamma_\mu L$ & $j_{\mu 3} = \bar{L}\frac{\tau_3}{2}\gamma_\mu L$

$J_\mu \equiv j_\mu^3 + \frac{1}{2} j_{\mu Y}$

- FROM $\mathcal{L}_{\phi l}$ COME ...

$$\mathcal{L}_{\phi l} = -G_l\left[\frac{v}{\sqrt{2}}\left(\bar{l}'_R l'_L + \bar{l}'_L l_R \right) + \frac{\eta}{2}\left(\bar{l}_R l_L + \bar{l}' l_R \right) \right]$$

... the charged lepton mass comes from the primordial Yukawa coupling, G_l, & VEV. SPONTANEOUSLY GENERATED AS WELL.

IMMEDIATE CONCLUSIONS:

- BEAT THE SPIN 1 MASS PROBLEM .. Higgs Mech.
- GAIN NEW WEAK INTERACTION MEDIATED BY Z_μ, A NEW SPIN 1 FIELD.
- GAIN PREDICTION FOR M_Z, IN TERMS OF MIXING PARAMETER, θ_W & M_W.
- GAIN A NEW WAY OF LOOKING AT THE WORLD...! DEMAND OF SYMMETRY → DYNAMICS

IMMEDIATE IMPACT IN 1967:

ZERO

- WEINBERG'S PHYSICAL REVIEW LETTER WAS 3 PAGES LONG
  PRL 19, 1264, 1967. still as readable today as then...

NUMBER OF CITATIONS THROUGH 1969:

one (Salam)

3 SIGNIFICANT OCCASSIONS IN 1970's:

1. 1971 't Hooft shows model to be renormalizable
2. 1973 weak neutral currents discovered at CERN
3. 1973 Politzer & Gross & Wilczek "discover" asymptotic freedom... and learn to leave scalars out of SU(3) gauge invariant theory → QCD.

SO... WHERE ARE WE ?

WEINBERG'S WONDERFUL ANALOGY...

SUPPOSE YOU LIVED INSIDE A FERROMAGNET BELOW T_c ... WHAT EXPERIMENTS COULD YOU PERFORM WHICH WOULD TELL YOU THAT THE SYMMETRY OF THE HAMILTONIAN OF YOUR SO(2)-APPEARING UNIVERSE IS SO(3) ?

IF THE ANALOGIES THAT I'VE TALKED ABOUT ARE REASONABLE ... then, that's our situation!

HOWEVER, WE HAVE A CLUE ... θ_W & PREDICTIONS

→ THE ELECTROWEAK PROGRAM WORLDWIDE.