

now for
something

completely different.

"gauge theories"

a story

introduction

uses of symmetry

gauge principle

weak interactions

critical phenomena

Broken symmetry

Higgs, et al. mechanism

*all together:
the Weinberg-
Salam Model*

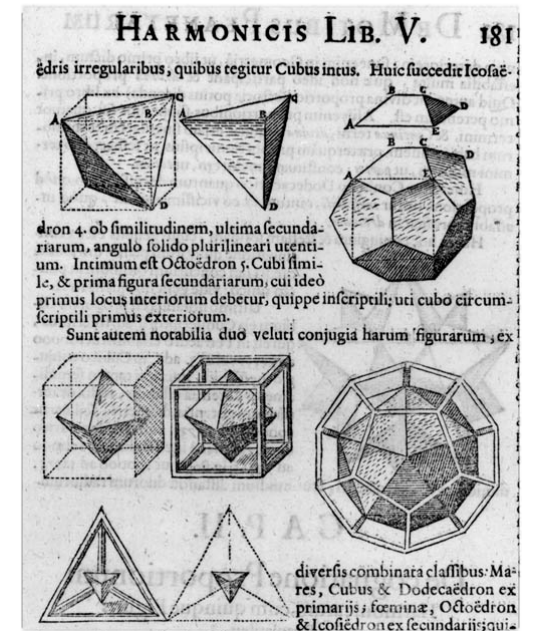
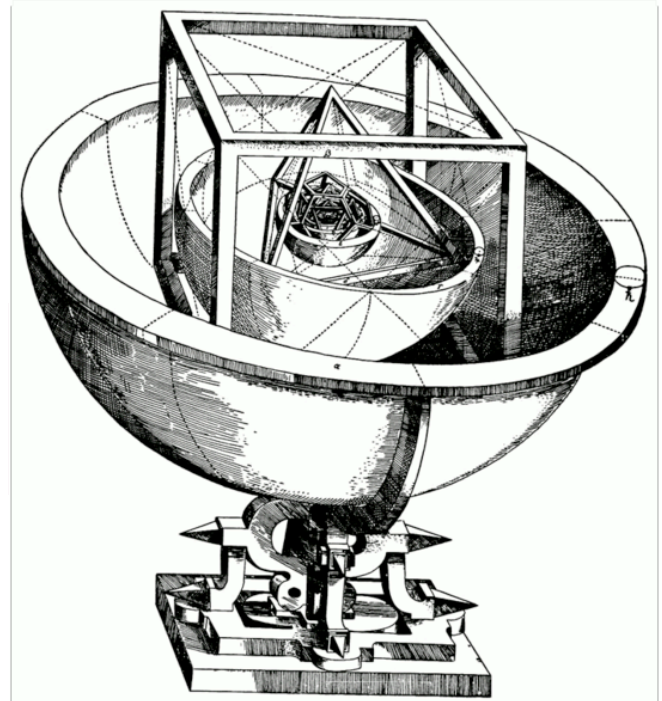
we laugh at Kepler now

he believed in a symmetry
built his world around it

for others:

perfect symmetry ruled-circles

are we different?



no

what about Einstein?

He didn't invent the transformations

done before him.

He didn't establish the mathematical rigor

done before him.

He derived the results

arguing for an a-priori prejudice about symmetry

field theory

primer

the players

Spin 0 Bosons:

ϕ

Spin 1 Vector bosons:

A_μ, B_μ, W_μ

Spin 1/2 Fermions:

ψ

use Lagrangians

Lagrange's Equations

→ *quantum equations of motion*

$$\partial_\mu \rightarrow \partial/\partial x^\mu$$

a catalog will suffice

FREE LAGRANGIANS

scalar fields:

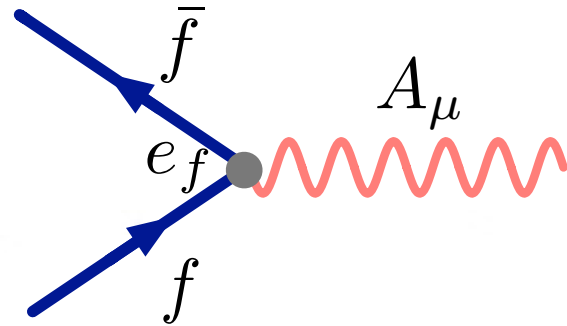
$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$$

EQUATIONS OF MOTION

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

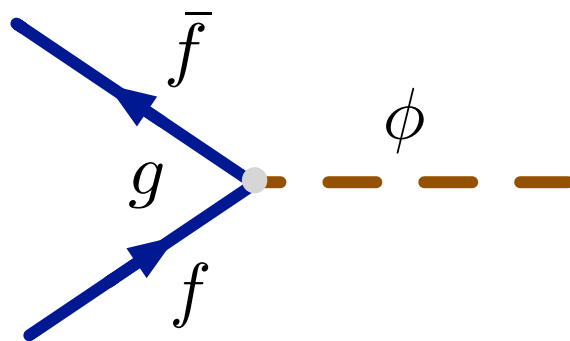
interactions

INTERACTIONS -



$$\mathcal{L}_{\text{electromagnetic-spin } \frac{1}{2}} = e_f \bar{f}(x) \gamma^\mu f(x) A_\mu(x)$$

$$\mathcal{L}_{\text{YUKAWA}} = g \phi(x) \bar{\psi}(x) \psi(x)$$



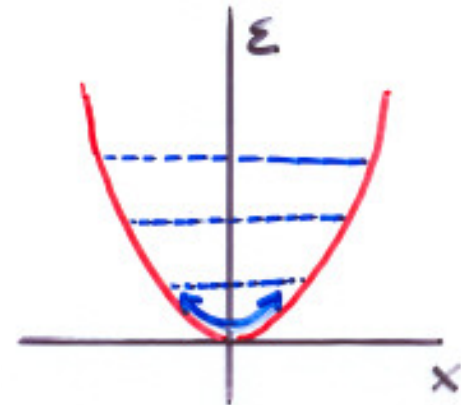
particle creation

PARTICLE SPECTRA -

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a(k)e^{-ikx} + a^\dagger(k)e^{ikx}]$$

creation operator
annihilation operator

just like the quantum oscillator from 1st year quantum mechanics



symmetry in quantum mechanics

Group operations represented by operators, U ,

generated by G

in a linear vector space of vectors $|\alpha\rangle$

vectors transform: $|\alpha\rangle \rightarrow |\alpha'\rangle = U|\alpha\rangle$

operators transform: $\Theta \rightarrow \Theta' = U\Theta U^{-1}$

If a system is symmetric wrt U , $[\mathcal{H}, G] = 0$

Noether's Theorem

If a system has a symmetry

there is an associated conservation law

space translation \rightarrow momentum conservation, \mathbf{p}

time translation \rightarrow energy conservation, E

Also, for “internal symmetries”

phase transformation \rightarrow charge conservation, Q

- OF PARTICULAR INTEREST ARE SYMMETRY GROUPS WITH

REPRESENTATIONS LIKE $U(\epsilon) = e^{-i \sum_j \epsilon^j Q^j}$

(INFINITESIMAL
PARAMETERS


“GENERATORS” OF THE
GROUP \hat{Q} OPERATORS
HAVING QUANTUM #’S
AS EIGENVALUES

charges and conserved currents

- CONNECTION THROUGH "CHARGE" & A CONSERVED "CURRENT" -

$$Q \equiv \int d^3x j^0(x)$$

where $\partial_\mu j^\mu(x) = 0$ signifies a conservation law



Q plays a dual role: both a "charge" and the generator of the transformation

quantum field theory: 1 slide

- $\phi(x)$ IS AN OPERATOR $\phi \rightarrow \phi' = U\phi U^{-1}$
 $= (1 - i \sum_j \epsilon^j Q^j) \phi (1 + i \sum_j \epsilon^j Q^j)$
 \vdots
 $= \phi + i \sum_j \epsilon^j [Q^j, \phi(x)]$

so $[Q^j, \phi(x)] = \phi(x) \Rightarrow$

(note: often $U\phi U^{-1} = \exp(i \sum_j \epsilon^j Q^j) \phi(x) \dots$ a phase)
 \uparrow eigenvalues of Q^j

- SUPPOSE $[H, Q] = 0 \Rightarrow \partial_0 Q = 0$

LET $H|\vec{p}_n\rangle = E_n|\vec{p}_n\rangle$

THEN $Q H|\vec{p}_n\rangle = E_n Q|\vec{p}_n\rangle$

" $H Q|\vec{p}_n\rangle = E_n Q|\vec{p}_n\rangle$

} $|\vec{p}_n\rangle \& Q|\vec{p}_n\rangle$ ARE
BOTH EIGENSTATES OF H
WITH SAME E_n - degenerate
 \rightarrow MAY REPRESENT ORTHOGONAL
STATES WITH DISTINCT
QUANTUM NUMBERS...

I lied...2 slides

- THERE IS A SPECIAL EIGENSTATE OF H ... THE VACUUM.

$H|0\rangle = 0$ IS ALWAYS TRUE FOR VACUUM STATE

USUALLY, IT IS ASSUMED THAT, FOR $U = e^{iQ\alpha}$

$U|0\rangle = |0\rangle$ FOR ALL SYMMETRIES

$$\Rightarrow Q|0\rangle = 0$$

IF $Q|0\rangle \neq 0$, THEN THERE MUST BE DEGENERATE VACUA

IF ALSO $[H, Q] = 0$. stay tuned!

a little bit of history...repeating



Soon after general relativity

H. Weyl proposed:

HE ADDED INVARIANCE WITH RESPECT TO

$$\begin{aligned} \text{a. } g'_{\mu\nu} &= \lambda(x) g_{\mu\nu} \\ \text{b. } A'_\mu &= A_\mu - \frac{\partial \lambda(x)}{\partial x^\mu} \end{aligned}$$

same $\lambda(x)$ phase



note: b. is E&M...a. is strange.

*space and time can change...all over space and time
he called it a "gauge"*

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \rightarrow \lambda ds^2 : \text{LENGTHS ARE RE-"GAUGED"}$$

The thing that holds spacetime together? The Photon

Einstein dug it...sorta

"Your ideas show a wonderful cohesion.

Apart from agreement with reality, it is at any rate a grandiose achievement of mind."

This early attempt to unify E&M with gravity failed.

1927

London revived the idea

Not a scale of spacetime

*A **phase** in quantum fields*

first kind: GLOBAL U(1) symmetry

$$U(\theta) = e^{i\theta Q}$$

$$\begin{aligned}\psi(x) \rightarrow \psi'(x) &= U \psi(x) U^{-1} \\ &= e^{i\theta Q} \psi(x)\end{aligned}$$

"GLOBAL" \Rightarrow SAME PHASE, INDEPENDENT OF SPACETIME $\theta \neq \theta(x)$

"U(1)" \Rightarrow 1 PARAMETER LIE GROUP HAVING Q AS GENERATOR

other kind: LOCAL U(1) symmetry

$$U(\theta) = e^{i\theta(x)Q}$$

"LOCAL" \Rightarrow POTENTIALLY DIFFERENT PHASE
AT ALL SPACETIME POINTS $\theta = \theta(x)$

$$\psi(x) \rightarrow \psi'(x) = e^{i\theta(x)q} \psi(x)$$

NOT SO SIMPLE...

the derivative is trouble

define a new divergence

to cancel the unwanted term

$$D_\mu \equiv \partial_\mu + X_\mu \quad \text{as-yet unnamed vector operator}$$

Goal: get the gradient to transform invariantly

$$(D_\mu \Psi) \rightarrow (D_\mu \Psi)' = e^{iq\theta(x)} (D_\mu \Psi)$$

• START OUT WITH

$$\begin{aligned} \mathcal{L} &= \bar{\Psi}(x) [i\gamma^\mu D_\mu - m] \Psi(x) \\ &= \bar{\Psi}(x) [i\gamma^\mu \partial_\mu + i\gamma^\mu X_\mu - m] \Psi(x) \end{aligned}$$

transform $\Psi \rightarrow \Psi'$

$$\mathcal{L}(\Psi) \rightarrow \mathcal{L}(\Psi') = \bar{\Psi}'(x) \left\{ i\gamma^\mu \left[\partial_\mu + X_\mu - iq\partial_\mu\theta(x) \right] - m \right\} \Psi'(x)$$

STILL NOT RIGHT!

one more ingredient

must simultaneously transform $X_\mu \rightarrow X'_\mu = X_\mu - iq\partial_\mu\theta(x)$

aha! Denote $X_\mu \equiv iqA_\mu(x)$ so the gradient looks like

$$D_\mu \equiv \partial_\mu + iqA_\mu$$

∴ TOTAL TRANSFORMATION NECESSARY TO LEAVE \mathcal{L} ALONE IS:

freaking amazing

Turns the utility of $U(1)$ symmetry

upside down

If invariance with respect to a local $U(1)$ symmetry is, a priori, of paramount importance:

**ONE IS FORCED
TO INVENT THE PHOTON**

demanding a symmetry

forces the inclusion of a spin-1 field

specifies the interaction with spin-1/2 fields

$U(1)$ is good

How about $SU(2)$?

The project of Yang and Mills in 1954

A local $SU(2)$ symmetry

leads to an isotriplet of spin-1 fields

DEMANDING $U = e^{i \sum_a \vec{\theta}^a(x) \cdot \vec{\tau}^a / 2}$

$\rightarrow \vec{b}_\mu(x) \begin{cases} 2 \text{ charged} \\ 1 \text{ neutral} \end{cases}$

isovector &
Lorentz vector

Yang - Mills Theory

AGAIN: $\mathcal{L} = \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi$

now $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ as bases for $SU(2)$ operators

close, but not so much

one might have hoped that the b might have turned out to be the W^\pm Boson

*But the weak interaction is short-ranged
and so the W would be heavy*

Masslessness of b_μ was a fatal flaw.

weak interactions

circa 1960...a primer

Since Pauli and Fermi in 1930s

There had been

*20 years of contradictory experimental results
a beautiful theory—1958 Feynman and Gell-Mann*



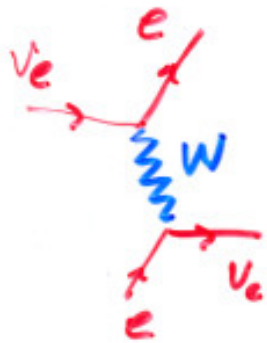
$$G_F \sim 10^{-5} / M_P^2$$

... HISTORICALLY DESCRIBED BY:

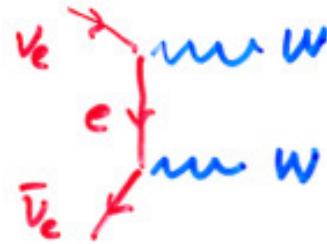
W^\pm : charged
isospin raising/lowering
massive



there were problems:



violates Unitarity



σ unbounded

2W Production

hint



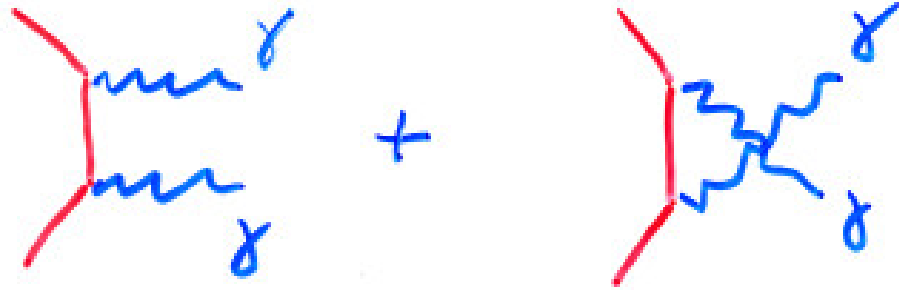
infinity is associated with the longitudinal degree of freedom

Massless spin 1: 2 dof...e.g. L,R polarizations

MassIVE spin 1: 3 dof...e.g. L,R polarizations
+ longitudinal polarization

$$\epsilon^\mu(\lambda = 0) \sim \frac{k^\mu}{M}$$

in E&M...2 photon production:



both graphs required
because of gauge
invariance

If you pretend that the photon
had a mass...

the bad behavior term cancels
between the graphs

spoiler:

in hindsight: this cancellation can be arranged for weak interactions:

either, require a new, heavy electron

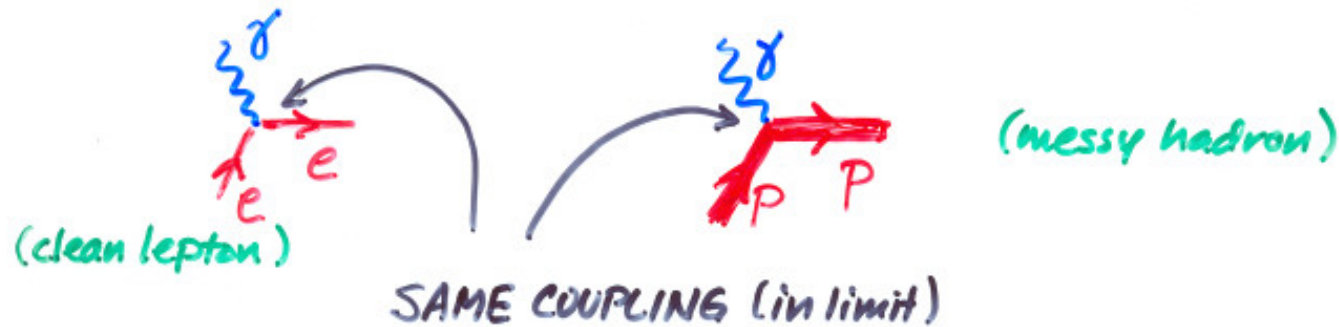


or, require a new, heavy spin 1 field

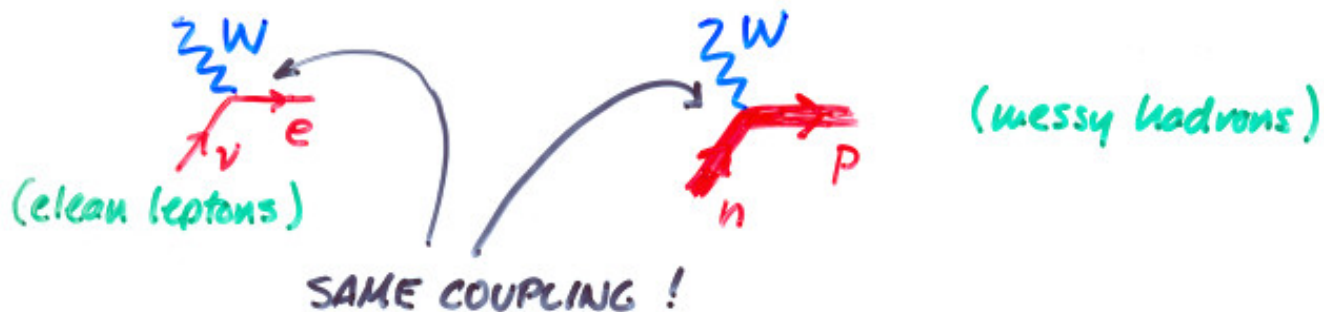


E&M is magic

same coupling of photon to electron & proton:



ditto, weak interactions:



could it be?

that the regal

electromagnetic interaction

might be related

to the rag-tag, ill-behaved, badly-bred

weak interaction?

many tried:

Schwinger, Salam, Ward, Glashow, Weinberg...

all used Yang-Mills theory

Salam: "dream" of Weak and Electromagnetic interaction unification...

$$\begin{pmatrix} W^+ \\ \gamma \\ W^- \end{pmatrix} ?$$

$$\begin{pmatrix} W^+ \\ Z^0 \\ W^- \end{pmatrix} \& \gamma$$

masslessness of W always
blocked progress

critical phenomena

circa 1960...a primer



MENU

UN APERITIF

thermodynamics of phase transformations

UNE ENTRÉE

Mean Field theory and Ginsburg-Landau phenomenology

LE PLAT PRINCIPAL

Ferromagnetism as an example of a broken symmetry

LE FROMAGE

Goldstone Theorem

LE DESSERT

Dilute Bose Gas as an example of the G.T.

UN DIGESTIF

Superconductivity as an example of the loophole

what's a phase?

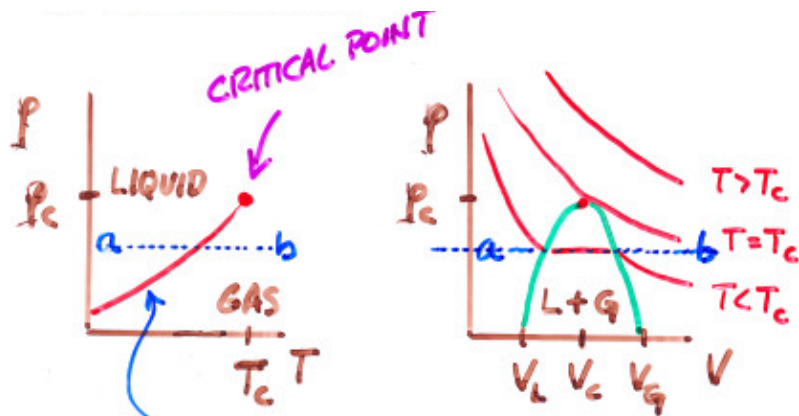
a region of analyticity of the free energy

$$f = -k_B T \ln Z \quad \text{from statistical mechanics} \quad \left. \begin{array}{l} \text{thermodynamics} \\ \text{comes from} \\ \text{derivatives of} \\ f \end{array} \right\}$$

$$\hookrightarrow \text{Tr} e^{-H/k_B T}$$

$$f: \quad \left. \begin{array}{l} F = U - TS \quad (\text{Helmholtz}) \\ G = F + pV \quad (\text{Gibbs}) \end{array} \right\} \text{from thermodynamics}$$

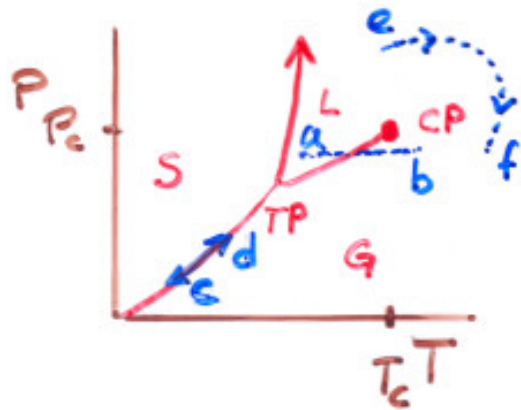
$$S = \left(-\frac{\partial G}{\partial T} \right)_{P,N} = \left(-\frac{\partial F}{\partial T} \right)_{V,N}$$



HEAT AT CONSTANT P &
DENSITY, $a \rightarrow b$

$$\Rightarrow dG_L = dG_G \quad \text{ACROSS COEXISTENCE LINE}$$

latent heat



IMAGINE HEATING, WHILE MAINTAINING
EQUILIBRIUM BETWEEN S & G, c → d

$$dG_S = dG_G$$

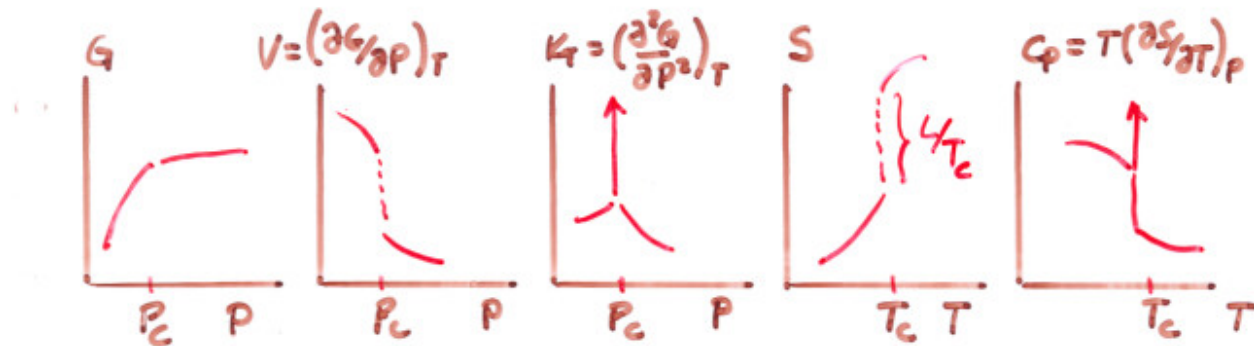
$$dG_i = V_i dP - S_i dT$$

$$\frac{dP}{dT} = \frac{S_S - S_G}{V_S - V_G} = \frac{\Delta S}{\Delta V}$$

$$= \frac{L}{T \Delta T}$$

latent heat

action in the derivatives



FIRST DERIVATIVE
OF G IS DISCONTINUOUS \Rightarrow "1st ORDER P.T."
TAKES PLACE ACROSS
COEXISTENCE CURVE

Crucial: the concept of the symmetry of the phases

symmetry

due to Pierre Curie, actually:

*If there is a symmetry change,
a Phase Transition has occurred.*

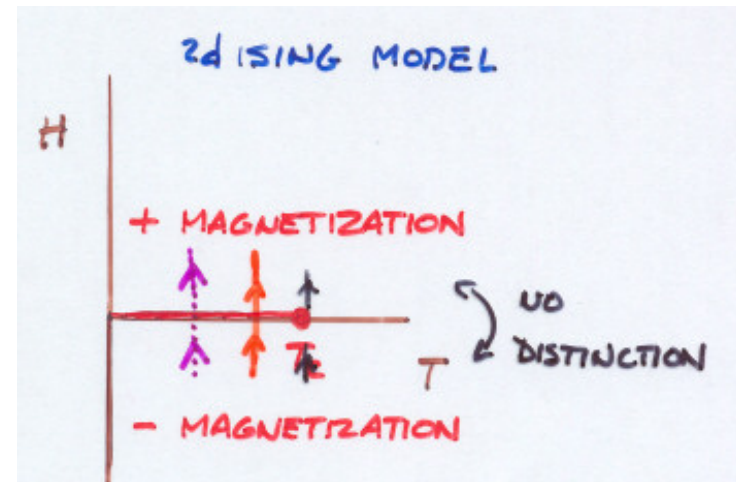
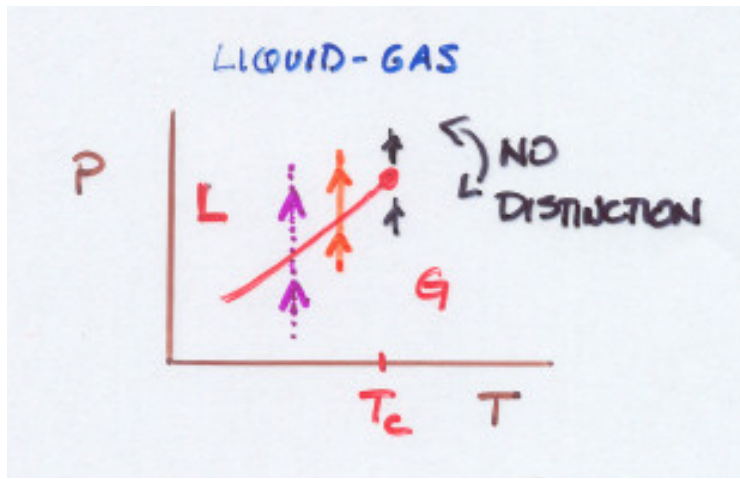
high degree of symmetry \Rightarrow lack of order



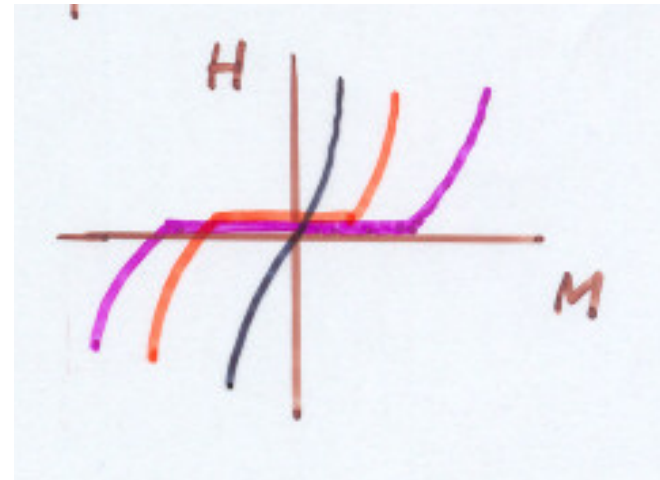
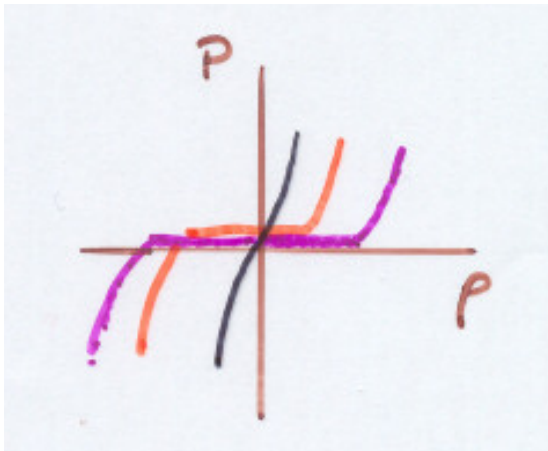
more symmetry operations \Rightarrow high entropy

related to higher
temperatures

the more things are the same

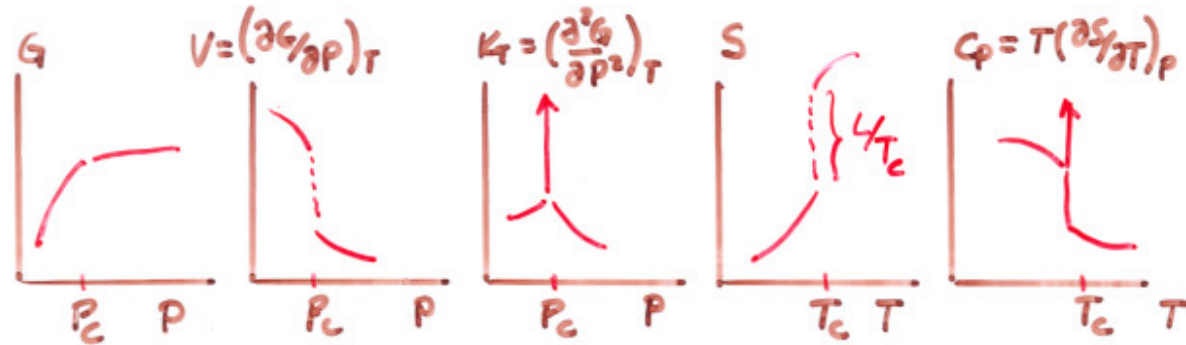


but...plot differently:

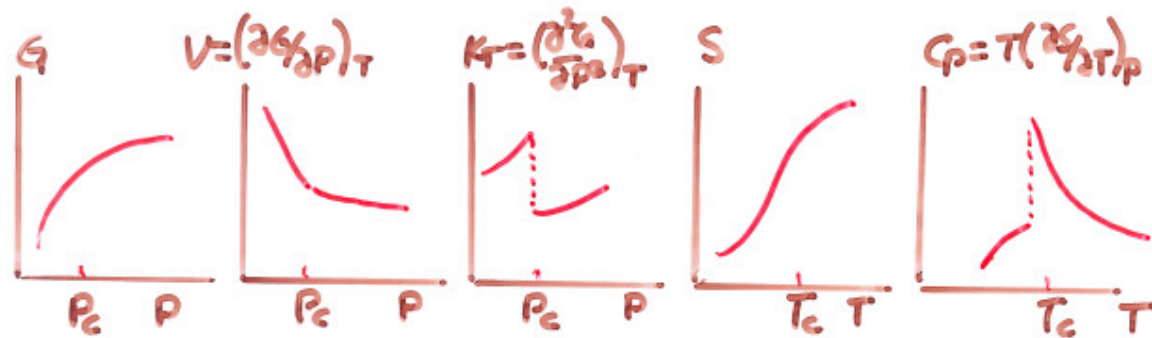


2nd order

1st



2nd



\uparrow
 2nd DERIVATIVE \Rightarrow "2nd ORDER P.T."

& no latent heat

order parameter

Landau and Ginsberg invented a parameter

to measure the order in a system

$\eta(T)$ *the order parameter*

universalizing the study of phase transitions

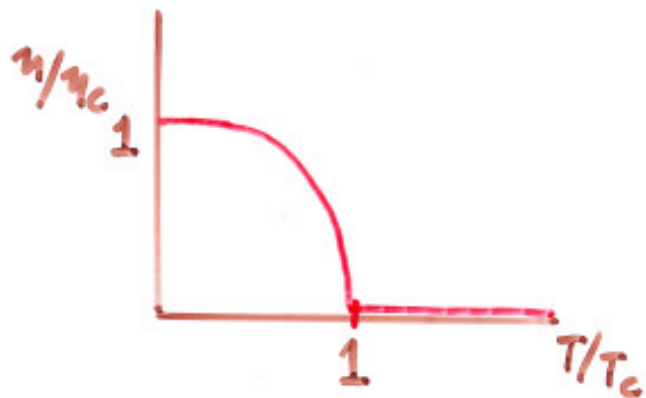
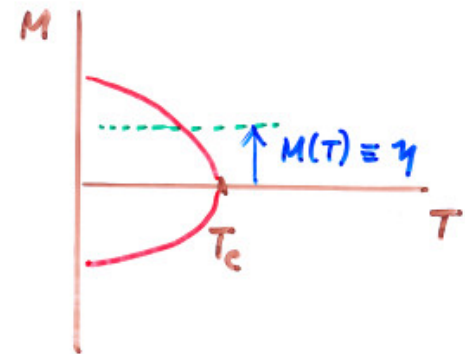
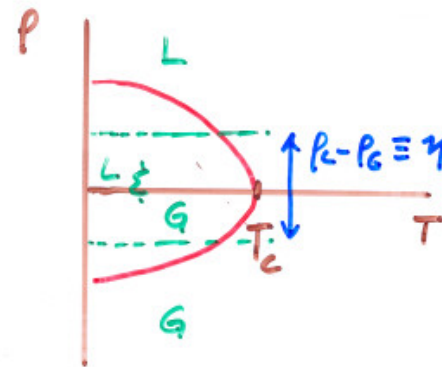
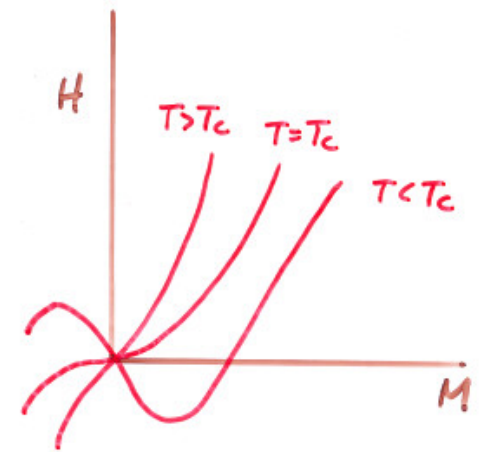
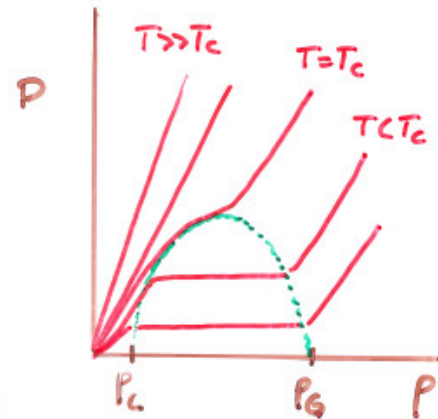
If $\eta = 0$, then the system is in an ordered phase

If $\eta \neq 0$, then the system is in a disordered phase

If $\eta(T) \rightarrow 0$ continuously, the P.T. is second order

here they are:

<u>SYSTEM</u>	<u>η</u>	<u>EXAMPLE</u>	<u>T_c (K)</u>
liquid-gas	$P_c - P_G$	H ₂ O	647
ferromagnet	M	Fe	1044
superfluid	$\psi_{\text{ground state}}$	⁴ He	2
superconductivity	$\psi_{\text{Cooper pairs}}$	Pb	7
ferroelectrics	P	triglycervine sulfate	323
binary alloys	concentration	Cu-Zn	739



near T_C :

Landau postulated:

a function, L (the Landau Free Energy)...related to G

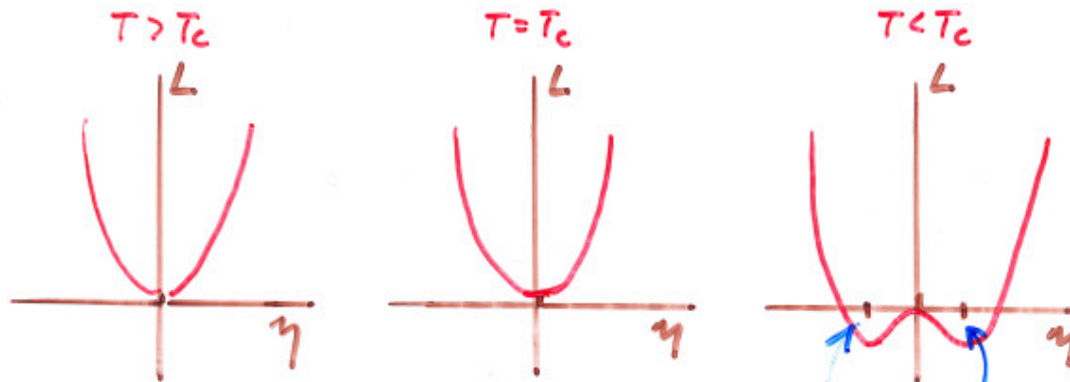
$$L(P, T, \eta) = L_0 + \beta(P, T)\eta^2 + \delta(P, T)\eta^4$$

$$\begin{aligned} \delta &> 0 \\ \beta > 0 &\Rightarrow T \geq T_C \\ \beta < 0 &\Rightarrow T < T_C \end{aligned}$$

flipping above and
below the
transition

ground state? minimize L

$$L = L_0 + b(T - T_c)\eta^2 + \delta\eta^4$$



SIMPLE EXERCISE 3:

SHOW

$$\left\{ \begin{array}{l} \eta = \pm \sqrt{\frac{b}{2\delta} (T - T_c)} \\ L = -\frac{b^2}{2\delta} (T - T_c)^2 \end{array} \right.$$

two important things for $T < T_c$:

the ground state energy is lowered

there are multiple ground state configurations

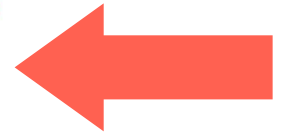
ferromagnetism

$T > T_c$



$$\langle M \rangle = 0$$

- ALL DIRECTIONS EQUALLY PROBABLE... GROUND STATE IS INVARIANT wrt $SO(3)$, U_3
- $[H, G] = 0$
 \Rightarrow HAMILTONIAN INVARIANT wrt $SO(3)$



$T < T_c$



$$\langle M \rangle \neq 0$$

- A SINGLE RANDOM DIRECTION IS SINGLED OUT
- SYMMETRY OF GROUND STATE IS LOWERED
 $SO(3) \rightarrow SO(2)$
- $[H, G] = 0$ still

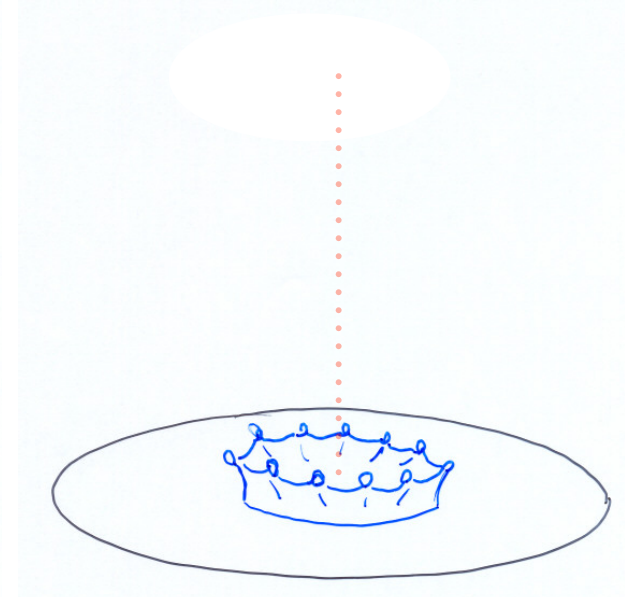
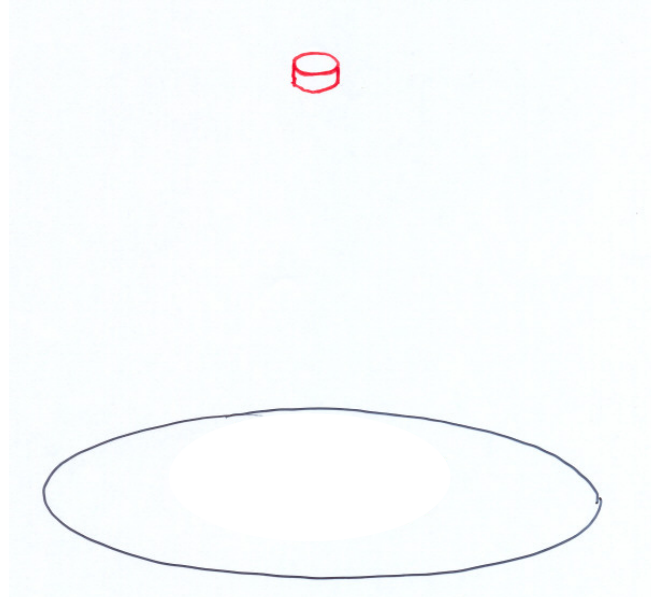
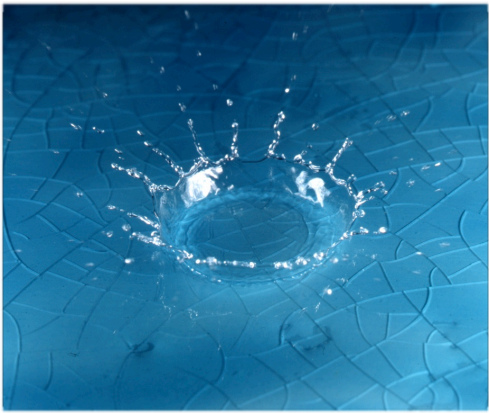


Symmetry said to be "spontaneously broken"
Better: symmetry is "hidden"

beautiful example



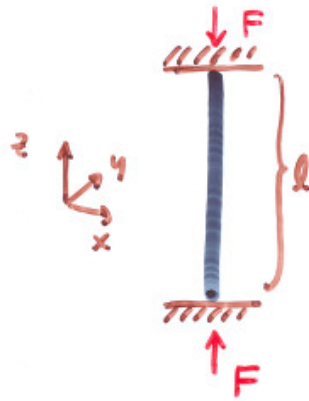
beautiful example



where did the symmetry go?

It's still there: inside of the ensembles of all potential splashes

here's another one: Euler



$$\left. \begin{aligned} EI \frac{d^4 x}{dz^4} + F \frac{d^2 x}{dz^2} &= 0 \\ EI \frac{d^4 y}{dz^4} + F \frac{d^2 y}{dz^2} &= 0 \end{aligned} \right\} x=y=0 \text{ is a solution}$$

BUT, WHEN $F > \frac{4\pi^2 EI}{l^2} \equiv F_c$

$$x \text{ (or } y) = C \sin kz \quad k = \sqrt{|F|/EI}$$



SYMMETRY IS LOST... HIDDEN (same equation of motion)

- ROD COULD HAVE PICKED AN INFINITE NUMBER OF DIRECTIONS TO BULGE... IN ACCORDANCE WITH ORIGINAL SYMMETRY

... just like ferromagnet

CMP theorists were playing

with these ideas...exploring broken symmetries

Steven Weinberg got a whiff of this...

but, he failed to apply it correctly

Because of the dreaded Goldstone Boson.

Goldstone Theorem

A system which has a spontaneously broken symmetry **must** have massless, Bose-like excitations in its spectrum.

There are no spinless, massless particles.

So, Weinberg's initial attempts failed.

Works great in CMP!

eg ferromagnetism

↑ ↑ ↑ ↑ ↑ ↑ ↑

GROUND STATE

↑ ↑ ↑ ↓ ↑ ↑ ↑

1 EXCITED STATE

? no...that's not what magnets do

energetics favor: ↑ ↑ → → ↓ ↓ ↓ ↓ ↓ ↓ ← ← ↑ ↑

long wavelength

spin waves



long wavelength, macroscopic, quantizable, excitations

with an energy dispersion:

$$\epsilon = \hbar^2 S \sum_{\vec{a}} (1 - \cos(\vec{q} \cdot \vec{a}))$$

as $q \rightarrow 0$, the energy goes to 0

$$\epsilon = \sqrt{q^2 c^2 + m^2 c^4}$$

bingo: massless

But...

The Hamiltonian - and the ground state - still respect the original symmetry.

If you lived inside of the magnet,

how would you ever discover that the symmetry of the Hamiltonian is $SO(3)$?

That's our situation.

proof of the Goldstone Theorem

not here...in the handout

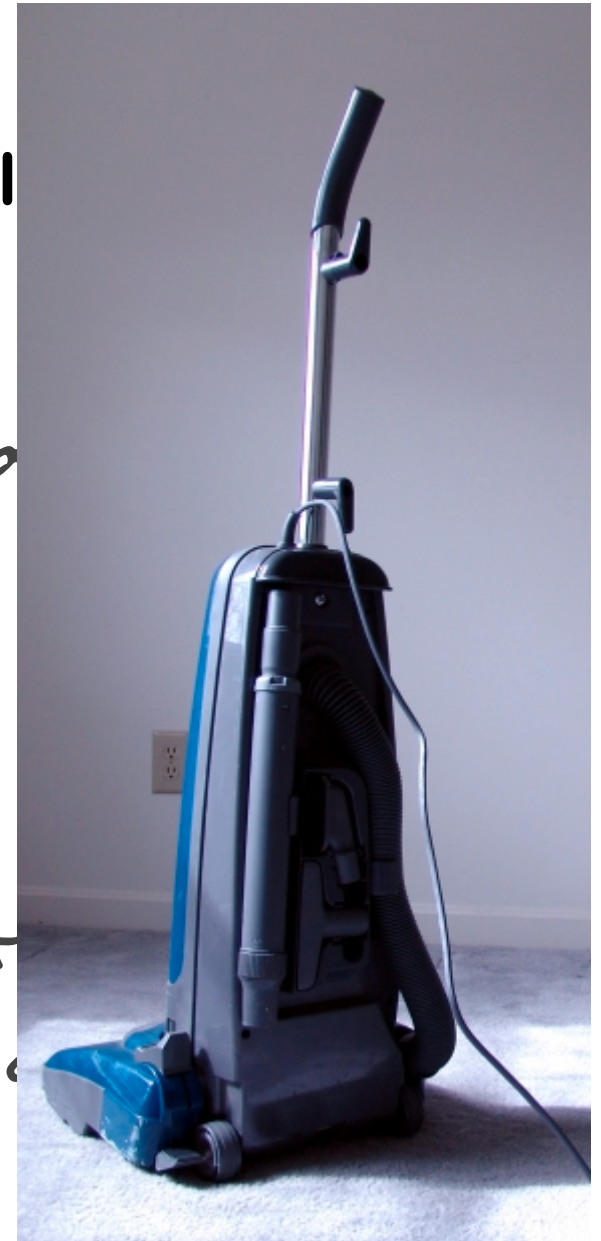
But, the consequences are the following

But:

what's the "ground state" of a quantum field?

The vacuum.

It's typically simple: it carries the trivial representation of the symmetry groups.



response of the vacuum to U :

Two ways:

the normal way:

$$U(Q)|0\rangle = |0\rangle$$

$$U(Q) = e^{-iQ\theta} \implies Q|0\rangle = 0$$

the condensed matter way:

$$U(Q)|0\rangle \neq |0\rangle$$

$$Q|0\rangle \neq 0$$

Remember:

Relativistic quantum fields are operators:

they satisfy an algebra:

$$[Q, \phi(x)] = \phi'(x) \neq 0$$

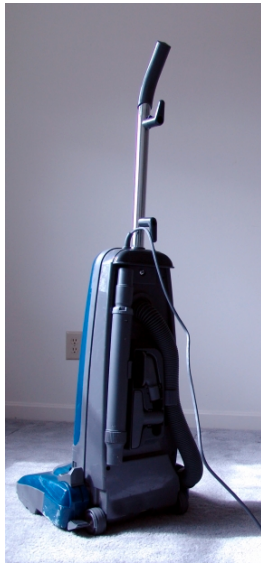
take the “vacuum expectation value”...aka “vev”

$$\langle 0 | [Q, \phi(x)] | 0 \rangle = \langle 0 | \phi'(x) | 0 \rangle \neq 0$$

which says:

the field ϕ in the vacuum is non-vanishing!

at first blush? strange.



is full.



$\phi(x)$



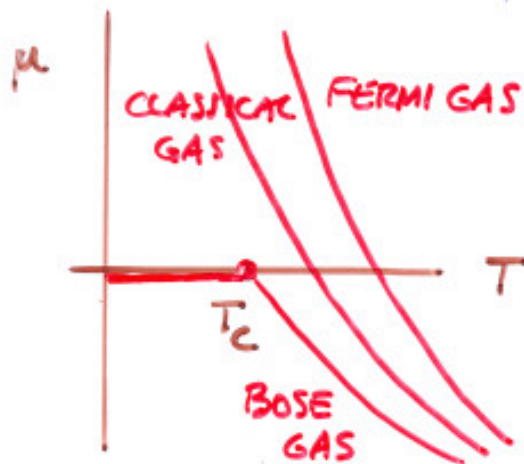
Observation of such a thing is a
trigger for the Goldstone Theorem

Dilute Bose Gas

remember your Stat Mech?

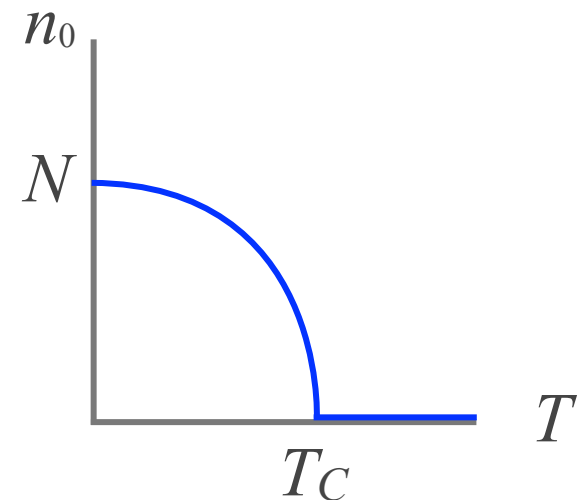
*remember the occupation number for bosons?
you treat the ground state differently*

$$n_i = \frac{g_i}{e^{(\epsilon_i - \mu)/kT} - 1}$$



$$n_0 = N \left[1 - \left(\frac{T}{T_C} \right)^{3/2} \right]$$

Bose-Einstein
Condensate



problem for a field theory

condensing into the ground state was a head-scratcher

*in field theory—relativistic or non-relativistic—
need to build a particle spectrum from an empty vacuum*

$$a^\dagger |0\rangle = |1\rangle \quad \Rightarrow \quad a|0\rangle = 0$$

But, this Bose-Einstein Condensate is a full vacuum!

Bogoliubov trick

the way out.

$$\begin{aligned} H = & \int d^3x \psi^\dagger(x) \left[-\frac{\hbar^2}{2m} \nabla^2 \right] \psi(x) && \text{K.E. term} \\ & + \int d^3x \int d^3x' \psi^\dagger(x) \psi^\dagger(x') v(x, x') \psi(x) \psi(x') && \text{P.E. term } \psi^4 \\ & + \mu \int d^3x \psi^\dagger(x) \psi(x) && \text{C.P. term } \psi^2 \end{aligned}$$

μ is zero in the condensate

the number operator: $N = a^\dagger a \sim \bar{\psi}\psi$

$$a|0\rangle_N = N^{1/2}|0\rangle_{N-1} \sim N^{1/2}|0\rangle_N$$

for large N

$$\text{like: } \sim \langle 0|\psi|0\rangle \neq 0$$

some broken symmetry

Number operator symmetry is broken

$$e^{i\lambda N} \quad U|0\rangle_N \neq |0\rangle_N$$

$$N|0\rangle_N \neq 0$$

shift it away

$$a^\dagger \text{ and } a \quad \textit{almost "c-numbers"} \quad a^\dagger \approx a \approx \sqrt{n_0}$$

$$\psi(x) = \text{vacuum value} + \chi(x)$$

$$\langle 0|\chi|0\rangle = 0$$

substitute into H

$$\chi(x) \sim \sum_{\vec{k} \neq 0} a_{\vec{k}} e^{i\vec{k} \cdot \vec{x}}$$

yadda yadda yadda

$$H = N^2 + \sum_{\vec{k} \neq 0} \omega_{\vec{k}} a_{\vec{k}}^{\dagger} a_{\vec{k}} + N \sum_{\vec{k} \neq 0} (a_{\vec{k}} a_{-\vec{k}} + a_{\vec{k}}^{\dagger} a_{-\vec{k}}^{\dagger})$$

$$\text{where } \omega_{\vec{k}} = \frac{\hbar^2 k^2}{2m} + 2Nf(\vec{k})$$

a mess...diagonalize with a canonical transformation:

$$\left. \begin{aligned} \alpha_{\vec{k}} &\equiv u_{\vec{k}} a_{\vec{k}}^{\dagger} + v_{\vec{k}} a_{-\vec{k}}^{\dagger} \\ \alpha_{-\vec{k}} &\equiv u_{-\vec{k}} a_{-\vec{k}} + v_{-\vec{k}} a_{\vec{k}}^{\dagger} \end{aligned} \right\} \alpha\text{'s have same commutation relations as } a\text{'s.}$$

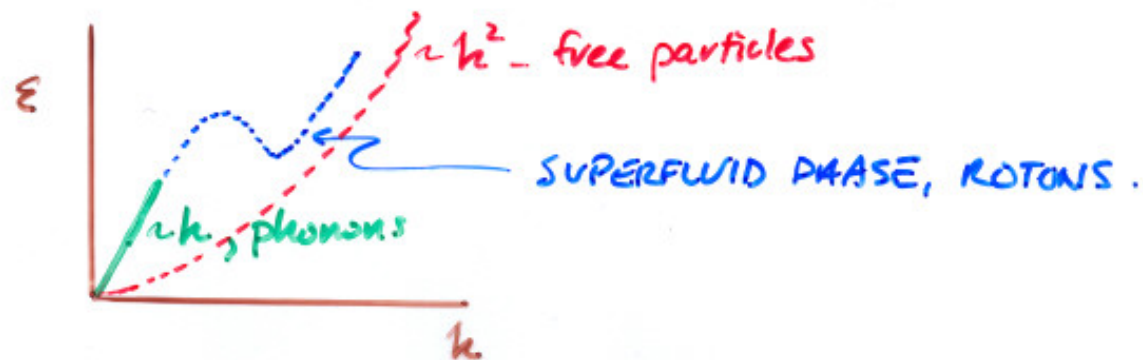
the create and annihilate a new particle spectrum
a quasi particle spectrum

$$H = N^2 - \frac{1}{2} \sum_{\vec{k} \neq 0} (\omega_{\vec{k}} - \epsilon_{\vec{k}}) + \frac{1}{2} \sum_{\vec{k} \neq 0} \epsilon_{\vec{k}} \alpha_{\vec{k}}^{\dagger} \alpha_{\vec{k}}$$

$$\epsilon_{\vec{k}} = \sqrt{\frac{\hbar^4 k^4}{4m^2} + \frac{4\hbar^2 k^2 f(k)}{2m}}$$

phonons

The Goldstone Boson of the Bose Gas



multiple things going on

many phenomena
involve broken
symmetries

The symmetry is “there”:
hidden, not respected by
the ground state.



Ground state is full:

$$\langle 0 | \phi | 0 \rangle = 0$$

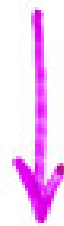
Broken continuous symmetry?

Massless Goldstone Bosons
must appear.



shift field operators:

c-number vacuum term + quasi particle operator term



insert into the model,

stir...out pops a quasi particle spectrum

Ginsburg Landau
phenomenology:

identify order parameter-
induce phase transition

build a toy theory

A relativistic quantum field theory

Jeffery Goldstone, "Field Theories with Superconductor Solutions" 1960

$$\mathcal{L} = \underbrace{\frac{1}{2} \partial_\mu \phi \partial^\mu \phi}_{\text{KE term}} - \underbrace{a \frac{\mu^2}{2} |\phi|^2}_{\text{mass term (like } \mu \text{ in Bose gas)}} - \underbrace{\frac{\lambda}{4} \phi^4}_{\text{self interaction}} \quad \Rightarrow \quad \partial_\mu \partial^\mu \phi + a \mu^2 \phi = \lambda \phi^3 \quad \checkmark$$

ϕ has mass $\sqrt{a} \mu \dots$

The symmetry?

reflection symmetry $\phi \rightarrow -\phi$ LEAVES \mathcal{L} ALONE

- IDENTIFY LANDAU FREE ENERGY WITH PE TERM OF \mathcal{L}

$$V(\phi) = \frac{a \mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4$$

- MINIMIZE TO FIND GROUND STATE:
minimum @ $V(\phi) = 0$

phase transition, ala' L&G

before the phase transition:



induce the phase transition as
Landau-Ginsburg

$$a \rightarrow -|a|$$

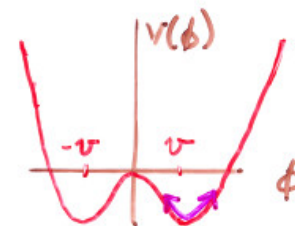
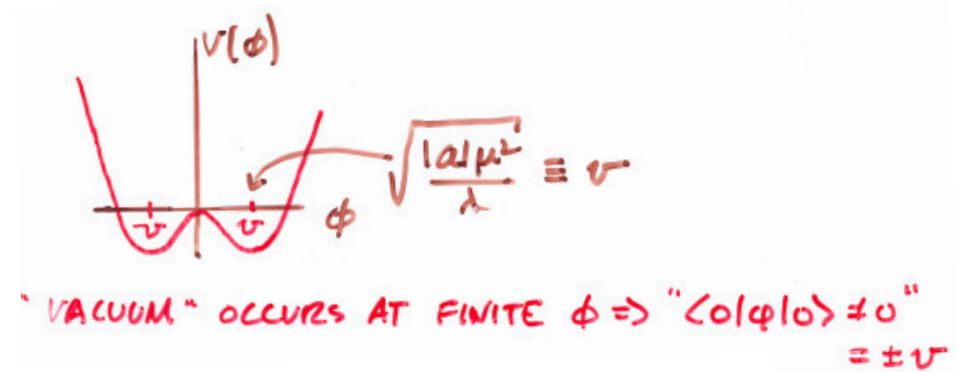
$$V(\phi) = -\frac{a\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4$$

and minimize to find GS:

pick one of the vacua...

shift &

build a particle spectrum



$$\begin{aligned} \phi(x) &= \langle 0 | \phi | 0 \rangle + \chi(x) \\ &= v + \chi(x) \end{aligned}$$

and substitute it back:

$$\mathcal{L}(\chi) = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - |\alpha| \mu^2 \chi^2 + \text{quartic \& cubic self interactions}$$



the correct form for a
massive boson!

Was the Goldstone Theorem violated?

no: this was a discrete symmetry

Goldstone Theorem holds for continuous symmetries

do it again...

a 2-component field

- FOR A CONTINUOUS SYMMETRY.. NEED MORE THAN 1. COMPONENT

OBJECT: $\varphi_1 \neq \varphi_2$ or $\varphi \neq \varphi^\dagger = \frac{\varphi_1 \pm i\varphi_2}{\sqrt{2}}$

$$\mathcal{L}(\varphi) = \frac{1}{2} \partial_\mu \varphi^\dagger \partial^\mu \varphi - \frac{1}{2} a \varphi^\dagger \varphi - \frac{1}{4} \lambda (\varphi^\dagger \varphi)^2$$

SYMMETRY: $\varphi \rightarrow \varphi' = e^{i\theta} \varphi$ LEAVES \mathcal{L} ALONE...

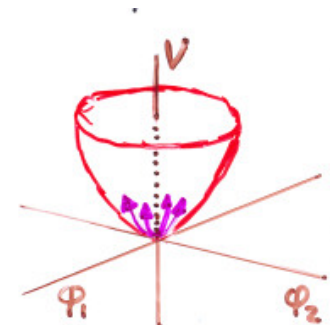
or $\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}' = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$

... Global $U(1)$ or $SO(2)$, which are isomorphic.

$$\mathcal{L}(\varphi) = \frac{1}{2} \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \frac{1}{2} \partial_\mu \varphi_2 \partial^\mu \varphi_2 - \frac{a\mu^2}{2} (\varphi_1^2 + \varphi_2^2) - \frac{\lambda}{4} (\varphi_1^2 + \varphi_2^2)^2$$

$$V(\varphi_1, \varphi_2) = \frac{a\mu^2}{2} (\varphi_1^2 + \varphi_2^2) + \frac{\lambda}{4} (\varphi_1^2 + \varphi_2^2)^2$$

MINIMIZATION LEADS TO:

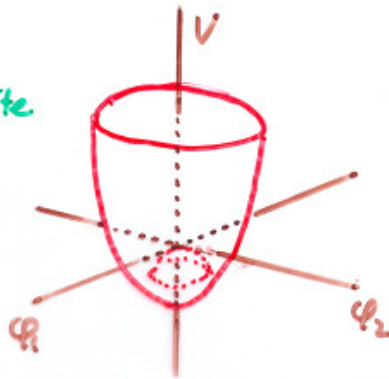


do the L-G thing

$$a \rightarrow -a \text{ in: } V(\varphi_1, \varphi_2) = \frac{a\mu^2}{2} (\varphi_1^2 + \varphi_2^2) + \frac{\lambda}{4} (\varphi_1^2 + \varphi_2^2)^2$$

MINIMIZATION LEADS TO: $\varphi_1^2 + \varphi_2^2 = \frac{|a\mu^2|}{\lambda}$

number of vacua is now infinite
→ CHOICE OF ONE INVOLVES
A SLICE IN $\varphi_1 - \varphi_2$ &
BREAKS THE $SO(2)$
SYMMETRY



a circle
@ RADIUS
 $v = \sqrt{\frac{a\mu^2}{\lambda}}$

LOCUS: $\langle 0 | \varphi | 0 \rangle = v e^{i\alpha} = v \cos \alpha + i v \sin \alpha$

gotta pick a direction

to break the symmetry...and build a Bogoliubov-like spectrum:

CHOOSE TO BREAK SYMMETRY BY $\alpha = 0$

$$\left. \begin{aligned} \langle 0 | \varphi_1 | 0 \rangle &= v \\ \langle 0 | \varphi_2 | 0 \rangle &= 0 \end{aligned} \right\} \text{A } \varphi_2 = 0 \text{ SLICE}$$

$$\langle 0 | \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} | 0 \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

SHIFT FIELDS USING COMPLEX REPRESENTATION...

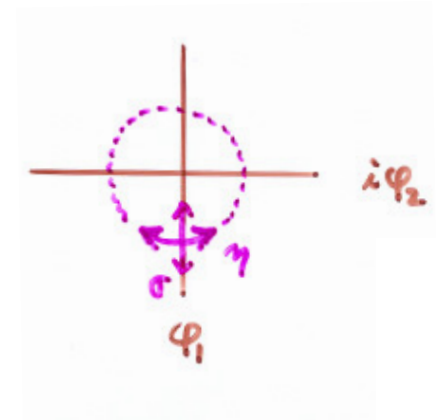
$$\varphi = \underbrace{v + \sigma(x)}_{\varphi_1} + \underbrace{i\eta(x)}_{i\varphi_2} \quad \text{TO QUASI PARTICLE SET,}$$

$$\sigma(x) \neq \eta(x).$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - a |\mu^2| \sigma^2 + \underbrace{\text{cubic \& quartic interactions}}_{\text{no } \eta^2 \text{ term}}$$

φ_2 LOST ITS MASS... η IS MASSLESS (THE GOLDSTONE BOSON)

σ IS MASSIVE, $m_\sigma = \sqrt{2a} \mu$



single loophole

Remember Local U(1) symmetries?

the Goldstone theorem: Global symmetries

Remember the routine:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi^\dagger \partial^\mu \varphi - \frac{a\mu^2}{2} \varphi^\dagger \varphi - \frac{\lambda}{4} (\varphi^\dagger \varphi)^2$$

WE KNOW HOW TO MAKE THIS LOCALLY GAUGE INVARIANT...

$\partial^\mu \rightarrow \partial^\mu + ig a^\mu$ SUBSTITUTION + TRANSFORMATIONS:

$$\varphi \rightarrow \varphi' = e^{ig\theta(x)} \varphi(x) \quad \& \quad a^\mu \rightarrow a^{\mu'} = a^\mu - \partial_\mu \theta(x)$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underbrace{\frac{1}{2} (D_\mu \varphi)^\dagger D^\mu \varphi}_{\text{encryption of } a-\varphi \text{ interaction}} - \frac{a\mu^2}{2} \varphi^\dagger \varphi - \frac{\lambda}{4} (\varphi^\dagger \varphi)^2$$

encryption of a - φ interaction

$$\frac{1}{2} \partial_\mu \varphi^\dagger \partial^\mu \varphi + \frac{1}{2} g^2 a^\mu a_\mu \varphi^\dagger \varphi$$

force a phase transformation

FORCE $a \rightarrow -|a|$ AND SHIFT FIELDS...

$$\langle 0|\varphi_1|0\rangle = v \equiv \frac{a\mu^2}{\lambda} \quad \langle 0|\varphi_2|0\rangle = 0$$

$$\varphi = v + \sigma + i\eta \quad \text{AGAIN}$$

and substitute back...

magic

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ - \frac{1}{2} 2g\nu \partial_\mu \eta a^\mu + g^2 \nu \sigma a^2 + \frac{1}{2} g^2 \nu^2 a^2 - a_\mu^2 \sigma + \text{cubic \& quart.} \\ \text{interactions}$$

LOOK AT TERMS...

$$\frac{1}{2} (\partial_\mu \eta \partial^\mu \eta - 2g\nu \partial_\mu \eta a^\mu + g^2 \nu^2 a^2) \\ = \frac{1}{2} (g\nu a_\mu - \partial_\mu \eta)^2 = \frac{1}{2} g^2 \nu^2 (a_\mu - \frac{1}{g\nu} \partial_\mu \eta)^2$$

(RE)DEFINE $\alpha_\mu \equiv a_\mu - \frac{1}{g\nu} \partial_\mu \eta$

so, $\Phi_{\mu\nu} \equiv \partial_\mu \alpha_\nu - \partial_\nu \alpha_\mu$

(looks like gauge transformation.
-- doesn't affect F's)

-- or Φ 's...

tada

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} + \frac{1}{2} g^2 v^2 \alpha^2 - a\mu^2 \sigma^2 + \text{int terms}$$

The η has disappeared!

no massless bosons

The σ is still there

but has gained a mass!

$$m_\sigma = \sqrt{2a\mu^2}$$

The a_μ has disappeared and replaced by α_μ

but has gained a mass!

$$m_\alpha = \frac{gv}{\sqrt{2}}$$

Higgs Mechanism


the original, massless a_μ

had 2 dof

the original η

existed as a gradient, $\partial_\mu\eta(x)$

*together,
making 3 dof!*



The Goldstone boson was “eaten” by the (gradient of the) spin 1 massless field

to become a spin 1 massive field.

Discovered by:

*Anderson, Nambu, Englert, Brout, Gilbert, Guralnik,
Higgs, Hagen, and Kibble around 1964*

so naturally called the Higgs Mechanism

that's superconductivity

Start out with:

*2 component, degenerative Boson pair
massless spin 1 vector Boson*

so, a local $U(1)$ symmetry is assured-Gauge invariant

Do the Landau-Ginsburg mechanical inducement of a phase transition

End up with:

1 massive spin 0 Boson Higgs Boson

1 massive spin 1 Boson Makes you think of the W



"superconductivity," you say?

In a superconductor, the order parameter:

Cooper Pairs—a Bose-like excitation

A breaking of charge invariance

What happens when a magnetic field impinges on a superconductor?

It's quenched within a skin-depth: Meisner Effect.

LONDON EQUATION

$$\vec{j} = -\frac{n_s e^2}{mc} \vec{A}$$

MANIPULATE:

$$\vec{\nabla} \times \vec{j} = -\frac{e^2 n_s}{mc} \vec{B}$$

WITH Ampere's law

$$\vec{\nabla} \times \vec{B} = \vec{j}$$

$$\nabla^2 \vec{B} = -\frac{e^2 n_s}{mc} \vec{B}$$

$$\nabla^2 \vec{B} + \frac{1}{\lambda} \vec{B} = 0$$

A Klein Gordon Equation
for a photon of mass $1/\lambda$

WITH SOLUTION

$$\vec{B} = \vec{B}_0 e^{-x/\lambda}$$

$$\lambda = \frac{mc}{e^2 n_s}$$

bingo

When an electromagnetic field encounters a superconductor

it gains a mass.

That's where we live: inside a Universal Superconductor.

where some "photons" are massive.

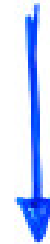
the chain of events:

weak
interactions
need massive,
spin 1 W^\pm bosons



Yang-Mills,
1954

Demand **local**
 $SU(2)$ gauge
invariance



get
massless
spin 1
bosons



and so on...

Bose-Gas-like
doublet scalar
fields

Ginsburg-
Landau,
1950

spontaneously
break that
LOCAL SU(2)
gauge
symmetry

Higgs et al.,
1964

get massive spin 1
field, Higgs boson,
no photon

Steven
Weinberg,
1967



$$SU(2) \otimes U(1)$$

Weinberg, "A Model of Leptons" 1967

less than 3 pages.

¹¹ In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

¹² M. Ademollo and R. Gatto, Nuovo Cimento **44A**, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters **17**, 888 (1966).

¹³ The predicted ratio [eq. (12)] from the current algebra

is slightly larger than that (0.23%) obtained from the ρ -dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\gamma\gamma)$ calculated in Refs. 12 and 14.

¹⁴ L. M. Brown and P. Singer, Phys. Rev. Letters **9**, 460 (1962).

A MODEL OF LEPTONS*

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(Received 17 October 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite¹ these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons.² This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing a Higgs field, and the intermediate boson fields as gauge fields.³ The model may be renormalizable.

We will restrict our attention to symmetry groups that connect the observed electron-type leptons only with each other, i.e., not with muon-type leptons or other unobserved leptons or hadrons. The symmetries then act on a left-handed doublet

$$L = \left[\begin{matrix} \nu_e \\ e \end{matrix} \right] \quad (1)$$

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \bar{R}\gamma^\mu(\partial_\mu - ig'B_\mu)R - L\gamma^\mu(\partial_\mu + ig\vec{T} \cdot \vec{A}_\mu - i\frac{1}{2}g'B_\mu)L$$

$$- \frac{1}{2}[\partial_\mu \varphi - ig\vec{A}_\mu \cdot \vec{T}\varphi + i\frac{1}{2}g'B_\mu\varphi]^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L) - M_1^2\varphi^\dagger\varphi + h(\varphi^\dagger\varphi)^2. \quad (4)$$

We have chosen the phase of the R field to make G_e real, and can also adjust the phase of the L and Q fields to make the vacuum expectation value $\lambda \equiv \langle \varphi^0 \rangle$ real. The "physical" φ fields are then φ^-

and on a right-handed singlet

$$R = \left[\frac{1}{2}(1 - \gamma_5) \right] e. \quad (2)$$

The largest group that leaves invariant the kinetic terms $-\bar{L}\gamma^\mu\partial_\mu L - \bar{R}\gamma^\mu\partial_\mu R$ of the Lagrangian consists of the electronic isospin \vec{T} acting on L , plus the numbers N_L, N_R of left- and right-handed electron-type leptons. As far as we know, two of these symmetries are entirely unbroken: the charge $Q = T_3 - N_R - \frac{1}{2}N_L$, and the electron number $N = N_R + N_L$. But the gauge field corresponding to an unbroken symmetry will have zero mass,⁴ and there is no massless particle coupled to N ,⁵ so we must form our gauge group out of the electronic isospin \vec{T} and the electron charge per change $Y = N_R + \frac{1}{2}N_L$. Therefore, we shall construct our Lagrangian out of L and R , plus gauge fields \vec{A}_μ and B_μ coupled to \vec{T} and Y , plus a spin-zero doublet

$$\varphi = \begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} \quad (3)$$

whose vacuum expectation value will break \vec{T} and Y and give the electron its mass. The only renormalizable Lagrangian which is invariant under \vec{T} and Y gauge transformations is

shift
and
vew

Gauge SU(2) & U(1)

and

$$\varphi_1 = (\varphi^0 + \varphi^- 0^+) / \sqrt{2} \quad \varphi_2 = (\varphi^0 - \varphi^- 0^+) / i\sqrt{2}. \quad (5)$$

The condition that φ_1 have zero vacuum expectation value to all orders of perturbation theory tells us that $\lambda^2 \approx M_1^2 / 2h$, and therefore the field φ_1 has mass M_1 while φ_2 and φ^- have mass zero. But we can easily see that the Goldstone bosons represented by φ_2 and φ^- have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates φ^- and φ_2 everywhere⁶ without changing anything else. We will see that G_e is very small, and in any case M_1 might be very large,⁷ so the φ_1 couplings will also be disregarded in the following.

The effect of all this is just to replace φ everywhere by its vacuum expectation value

$$\langle \varphi \rangle = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (6)$$

The first four terms in \mathcal{L} remain intact, while the rest of the Lagrangian becomes

$$-\frac{1}{2}\lambda^2 g^2 [(A_\mu^1)^2 + (A_\mu^2)^2] - \frac{1}{2}\lambda^2 (gA_\mu^3 + g'B_\mu)^2 - \lambda G_e \bar{e} e. \quad (7)$$

$$\frac{ig}{2\sqrt{2}} \bar{e}\gamma^\mu(1 + \gamma_5)\nu W_\mu + \text{H.c.} + \frac{ig'g'}{(g^2 + g'^2)^{1/2}} \bar{e}\gamma^\mu e A_\mu + \frac{i(g^2 + g'^2)^{1/2}}{4} \left[\frac{3g'^2 - g^2}{g'^2 + g^2} \bar{e}\gamma^\mu e - \bar{e}\gamma^\mu \gamma_5 e + \bar{\nu}\gamma^\mu(1 + \gamma_5)\nu \right] Z_\mu. \quad (14)$$

We see that the rationalized electric charge is

$$e = gg' / (g^2 + g'^2)^{1/2} \quad (15)$$

and, assuming that W_μ couples as usual to hadrons and muons, the usual coupling constant of weak interactions is given by

$$G_W / \sqrt{2} = g^2 / 8M_W^2 = 1/2\lambda^2. \quad (16)$$

Note that then the e - φ coupling constant is

$$G_e = M_e / \lambda = 2^{1/4} M_e G_W^{1/2} = 2.07 \times 10^{-6}.$$

The coupling of φ_1 to muons is stronger by a factor M_μ / M_e , but still very weak. Note also that (14) gives g and g' larger than e , so (16) tells us that $M_W > 40$ BeV, while (12) gives $M_Z > M_W$ and $M_Z > 80$ BeV.

The only unequivocal new predictions made

We see immediately that the electron mass is λG_e . The charged spin-1 field is

$$W_\mu = 2^{-1/2}(A_\mu^1 + iA_\mu^2) \quad (8)$$

and has mass

$$M_W = \frac{1}{2}\lambda g. \quad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu^3 + g'B_\mu), \quad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu^3 + gB_\mu). \quad (11)$$

Their masses are

$$M_Z = \frac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \quad (12)$$

$$M_A = 0, \quad (13)$$

so A_μ is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

by this model have to do with the couplings of the neutral intermediate meson Z_μ . If Z_μ does not couple to hadrons then the best place to look for effects of Z_μ is in electron-neutron scattering. Applying a Fierz transformation to the W -exchange terms, the total effective e - ν interaction is

$$\frac{G_W}{\sqrt{2}} \bar{\nu}\gamma_\mu(1 + \gamma_5)\nu \left\{ \frac{(3g^2 - g'^2)}{2(g^2 + g'^2)} \bar{e}\gamma^\mu e + \frac{1}{2} \bar{e}\gamma^\mu \gamma_5 e \right\}.$$

If $g \gg e$ then $g \gg g'$, and this is just the usual e - ν scattering matrix element times an extra factor $\frac{1}{2}$. If $g \approx e$ then $g \ll g'$, and the vector interaction is multiplied by a factor $-\frac{1}{2}$ rather than $\frac{1}{2}$. Of course our model has too many arbitrary features for these predictions to be

really...2 pages.

taken very seriously, but it is worth keeping in mind that the standard calculation⁸ of the electron-neutrino cross section may well be wrong.

Is this model renormalizable? We usually do not expect non-Abelian gauge theories to be renormalizable if the vector-meson mass is not zero, but our Z_μ and W_μ mesons get their mass from the spontaneous breaking of the symmetry, not from a mass term put in at the beginning. Indeed, the model Lagrangian we start from is probably renormalizable, so the question is whether this renormalizability is lost in the reordering of the perturbation theory implied by our redefinition of the fields. And if this model is renormalizable, then what happens when we extend it to include the couplings of \bar{A}_μ and B_μ to the hadrons?

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†On leave from the University of California, Berkeley, California.

¹The history of attempts to unify weak and electromagnetic interactions is very long, and will not be reviewed here. Possibly the earliest reference is E. Fer-

mi, Z. Physik **88**, 161 (1934). A model similar to ours was discussed by S. Glashow, Nucl. Phys. **22**, 579 (1961); the chief difference is that Glashow introduces symmetry-breaking terms into the Lagrangian, and therefore gets less definite predictions.

²J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

³P. W. Higgs, Phys. Letters **12**, 132 (1964), Phys. Rev. Letters **13**, 508 (1964), and Phys. Rev. **145**, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Letters **13**, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters **13**, 585 (1964).

⁴See particularly T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967). A similar phenomenon occurs in the strong interactions; the ρ -meson mass in zeroth-order perturbation theory is just the bare mass, while the A_1 meson picks up an extra contribution from the spontaneous breaking of chiral symmetry. See S. Weinberg, Phys. Rev. Letters **18**, 507 (1967), especially footnote 7; J. Schwinger, Phys. Letters **24B**, 473 (1967); S. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967), Eq. (13) et seq.

⁵T. D. Lee and C. N. Yang, Phys. Rev. **98**, 101 (1955).

⁶This is the same sort of transformation as that which eliminates the nonderivative $\bar{\pi}$ couplings in the σ model; see S. Weinberg, Phys. Rev. Letters **18**, 188 (1967). The $\bar{\pi}$ reappears with derivative coupling because the strong-interaction Lagrangian is not invariant under chiral gauge transformation.

⁷For a similar argument applied to the σ meson, see Weinberg, Ref. 6.

⁸R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1957).

SPECTRAL-FUNCTION SUM RULES, ω - φ MIXING, AND LEPTON-PAIR
DECAYS OF VECTOR MESONS*

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Within the framework of vector-meson dominance, the current-mixing model is shown to be the only theory of ω - φ mixing consistent with Weinberg's first sum rule as applied to the vector-current spectral functions. Relations among the leptonic decay rates of ρ^0 , ω , and φ are derived, and other related processes are discussed.

We begin by considering Weinberg's first sum rule¹ extended to the (1+8) vector currents of the eightfold way²:

$$\int dm^2 [m^{-2} \rho_{\alpha\beta}^{(1)}(m^2) + \rho_{\alpha\beta}^{(0)}(m^2)] = S \delta_{\alpha\beta} + S' \delta_{\alpha 0} \delta_{\beta 0}, \quad (1)$$

that's it.

Citations in the next 4
years?

*something like 3...all
by Weinberg*

Then, all hell broke
loose in 1979

definitive predictions:

The W exists and the Z exists

The Z would couple everywhere that γ couples

like atoms 1984

like interfering with electron scattering ~1990?

like “weak neutral currents” 1979

The mass of the Z is related to the mass of the W

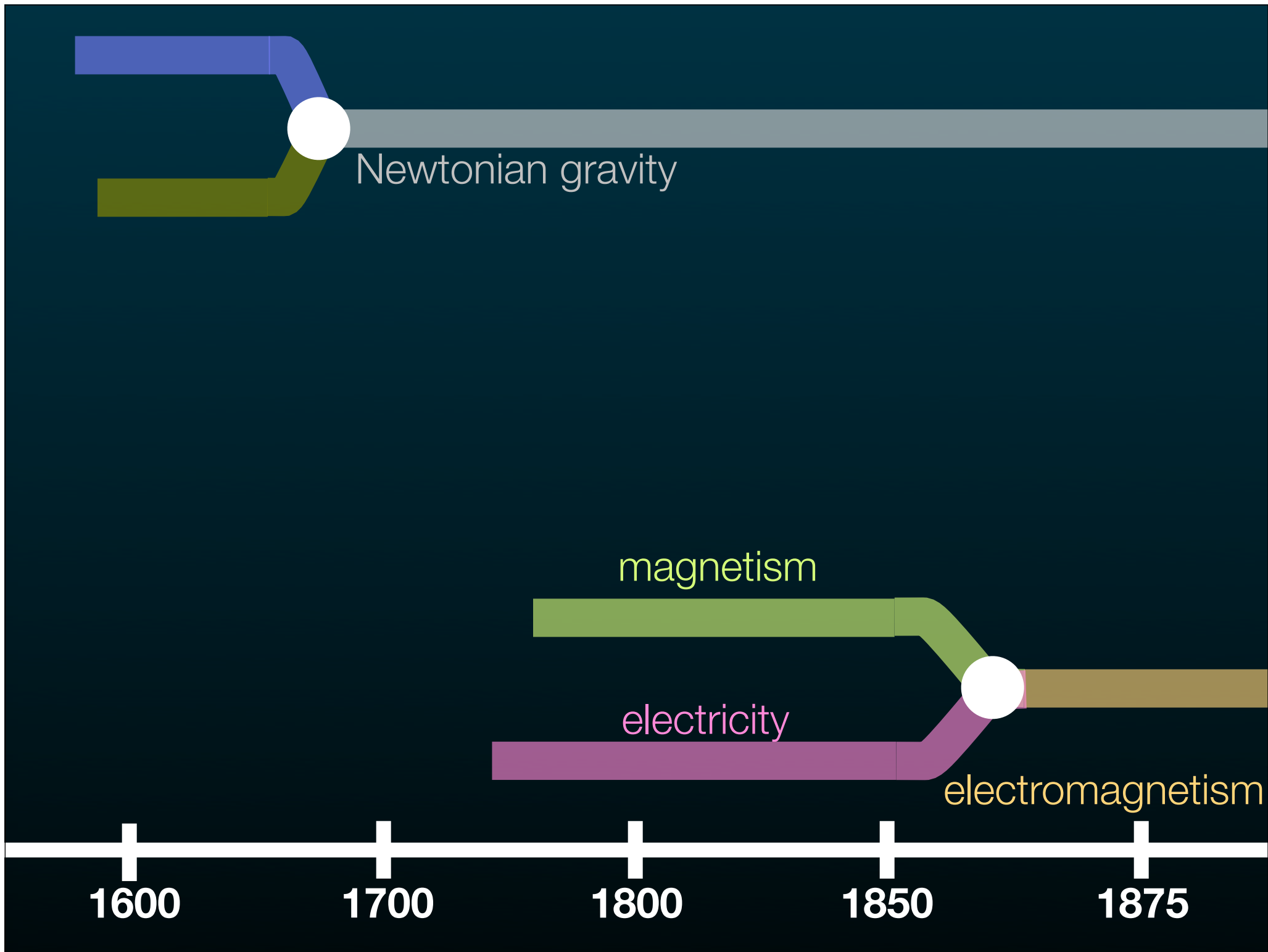
$$\cos\theta_W = \frac{M_W}{M_Z} \quad 1983$$

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

The Standard Model is the most precise theory in the history of physics

pretty damn good.

Quantity	Value	Standard Model	Pull
m_t [GeV]	$172.7 \pm 2.9 \pm 0.6$	172.7 ± 2.8	0.0
M_W [GeV]	80.450 ± 0.058	80.376 ± 0.017	1.3
	80.392 ± 0.039		0.4
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1
Γ_Z [GeV]	2.4952 ± 0.0023	2.4968 ± 0.0011	-0.7
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7434 ± 0.0010	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.65 ± 0.11	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	83.996 ± 0.021	—
σ_{had} [nb]	41.541 ± 0.037	41.467 ± 0.009	2.0
R_e	20.804 ± 0.050	20.756 ± 0.011	1.0
R_μ	20.785 ± 0.033	20.756 ± 0.011	0.9
R_τ	20.764 ± 0.045	20.801 ± 0.011	-0.8
R_b	0.21629 ± 0.00066	0.21578 ± 0.00010	0.8
R_c	0.1721 ± 0.0030	0.17230 ± 0.00004	-0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01622 ± 0.00025	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.5
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1031 ± 0.0008	-2.4
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0737 ± 0.0006	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1032 ± 0.0008	-0.5
$\tilde{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23152 ± 0.00014	0.7
	0.2238 ± 0.0050		-1.5
A_e	0.15138 ± 0.00216	0.1471 ± 0.0011	2.0
	0.1544 ± 0.0060		1.2
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347 ± 0.0001	-0.6
A_c	0.670 ± 0.027	0.6678 ± 0.0005	0.1
A_s	0.895 ± 0.091	0.9356 ± 0.0001	-0.4
$g_{\frac{1}{2}}^2$	0.30005 ± 0.00137	0.30378 ± 0.00021	-2.7
$g_{\frac{3}{2}}^2$	0.03076 ± 0.00110	0.03006 ± 0.00003	0.6
g_V^{pe}	-0.040 ± 0.015	-0.0396 ± 0.0003	0.0
$g_V^{\nu e}$	-0.507 ± 0.014	-0.5064 ± 0.0001	0.0
A_{PV}	-1.31 ± 0.17	-1.53 ± 0.02	1.3
$Q_W(\text{Cs})$	-72.62 ± 0.46	-73.17 ± 0.03	1.2
$Q_W(\text{Tl})$	-116.6 ± 3.7	-116.78 ± 0.05	0.1
$\Gamma(b \rightarrow s\gamma)$	$3.35^{+0.50}_{-0.44} \times 10^{-3}$	$(3.22 \pm 0.09) \times 10^{-3}$	0.3
$\frac{1}{2}(g_\mu - 2 - \frac{\alpha}{\pi})$	4511.07 ± 0.82	4509.82 ± 0.10	1.5
τ_τ [fs]	$290.89^{+0.86}_{-0.98}$	291.87 ± 1.76	-0.4



Newtonian gravity

magnetism

electricity

electromagnetism

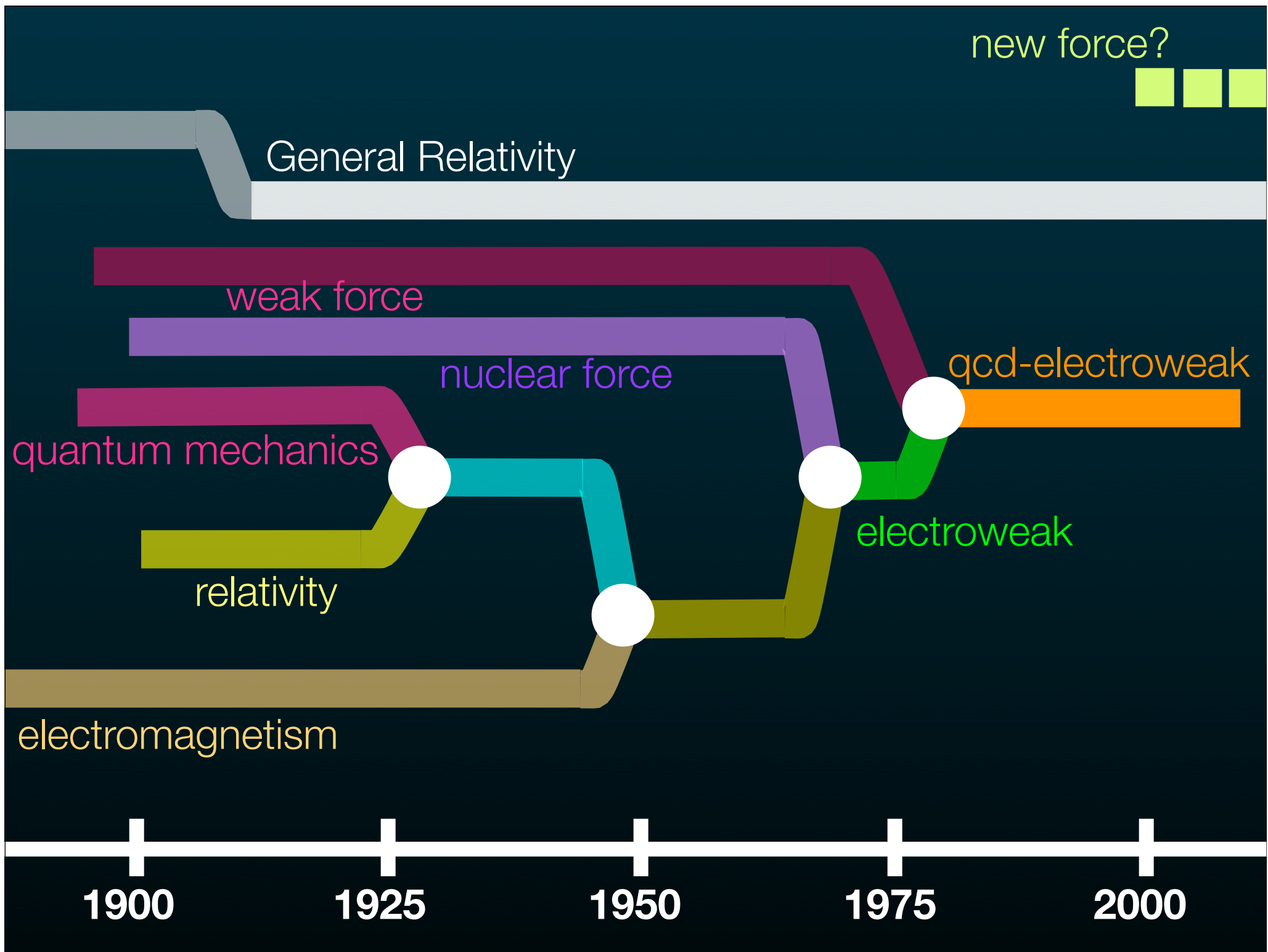
1600

1700

1800

1850

1875



The Standard Model is also a model of the Universe



$$a^0 \quad 0 \text{ } \color{magenta}{\sim\sim\sim\sim\sim}$$

$$B^0 \quad 0 \text{ } \color{cyan}{\sim\sim\sim\sim\sim}$$

$$B^+ \quad + \text{ } \color{cyan}{\sim\sim\sim\sim\sim}$$

$$B^- \quad - \text{ } \color{cyan}{\sim\sim\sim\sim\sim}$$

$$\phi \quad \left(\begin{array}{c} + - - - - \\ 0 - - - - \end{array} \right)$$

$$\phi^* \quad \left(\begin{array}{c} - - - - - \\ 0 - - - - \end{array} \right)$$

γ

Z

W^\pm

H^0

$t = \text{the beginning } 0 \text{ s}$

$t = 10^{-12} \text{ s}$

$t = 10^{+18} \text{ s}$

This...is:

The “Higgs Mechanism”

The remaining primordial
scalar is the **Higgs Field**.

H^0

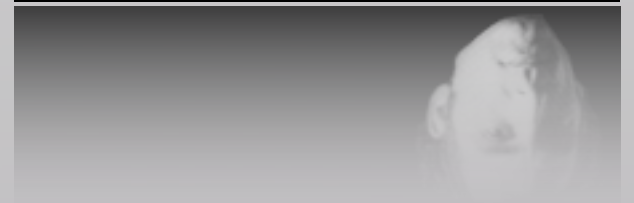


The Higgs Boson creates mass

mass may not be an inherent
property...

but an *acquired* one

about nothing

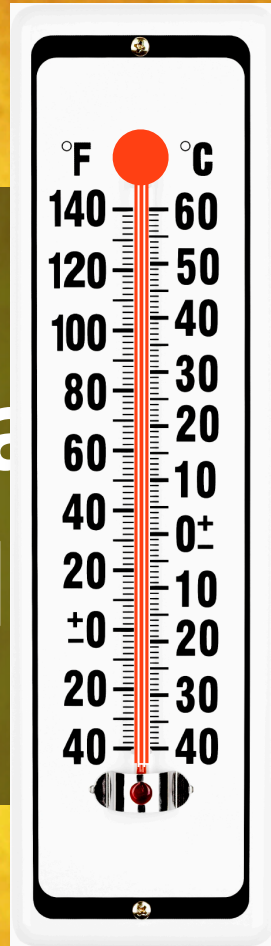


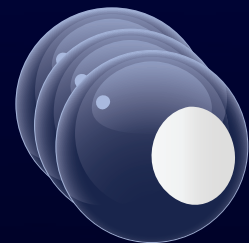
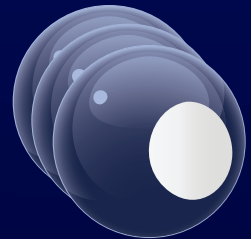
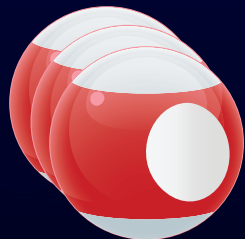
the action

in the
vacuum



a tiny ball of space, time, and energy
13.7 Billion years ago







vacuum

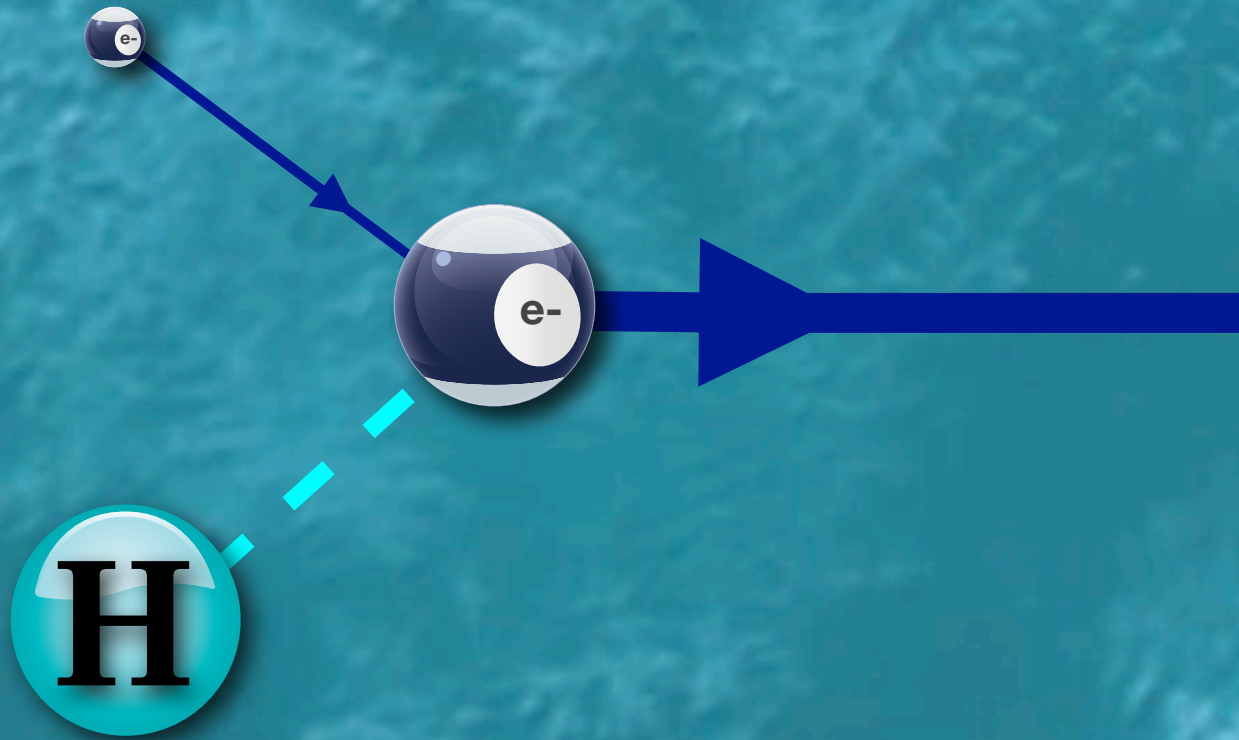
empt



Nature is clumpy



Higgs *Boson*



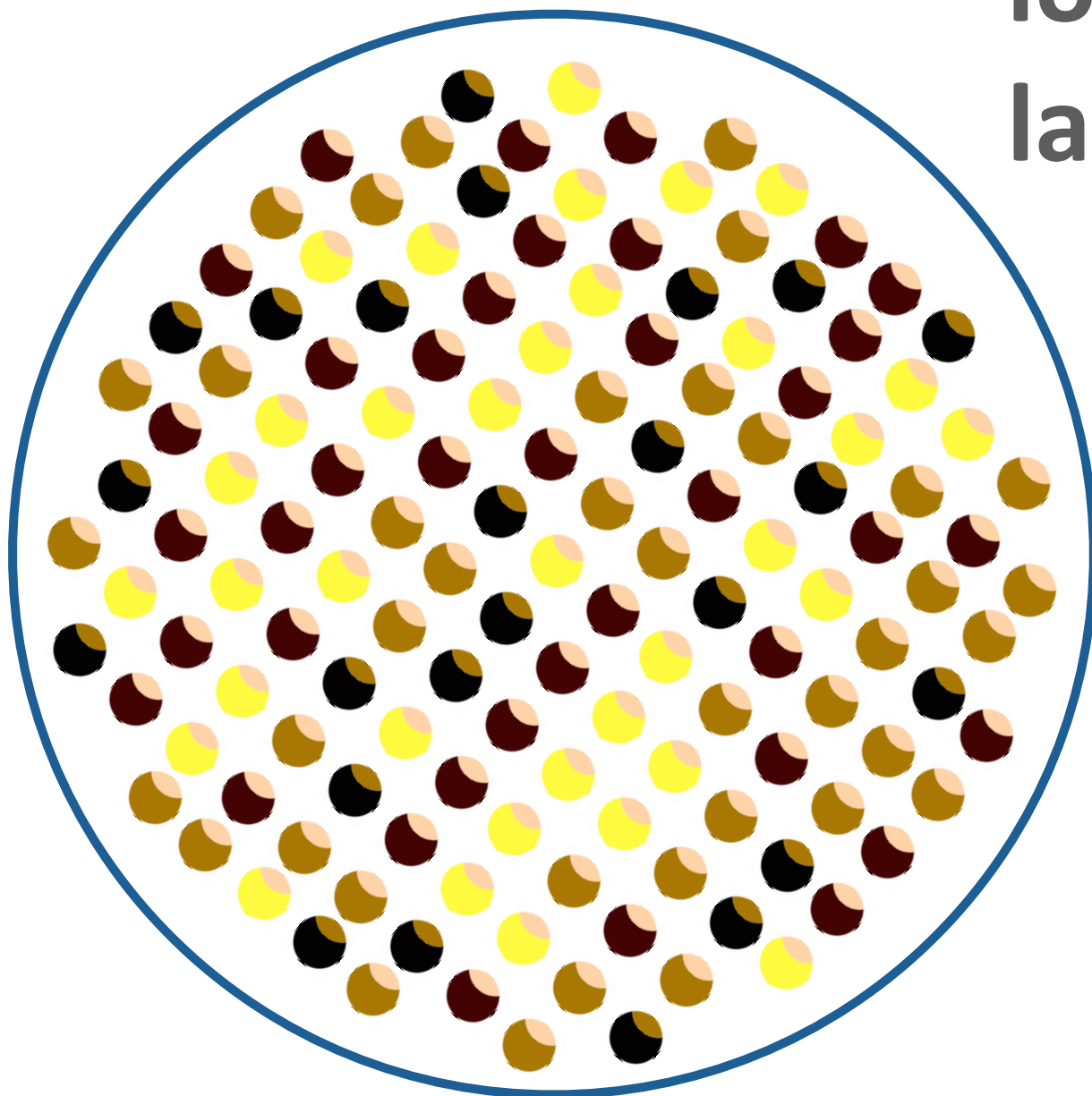
Higgs Boson



a Higgs
metaphor

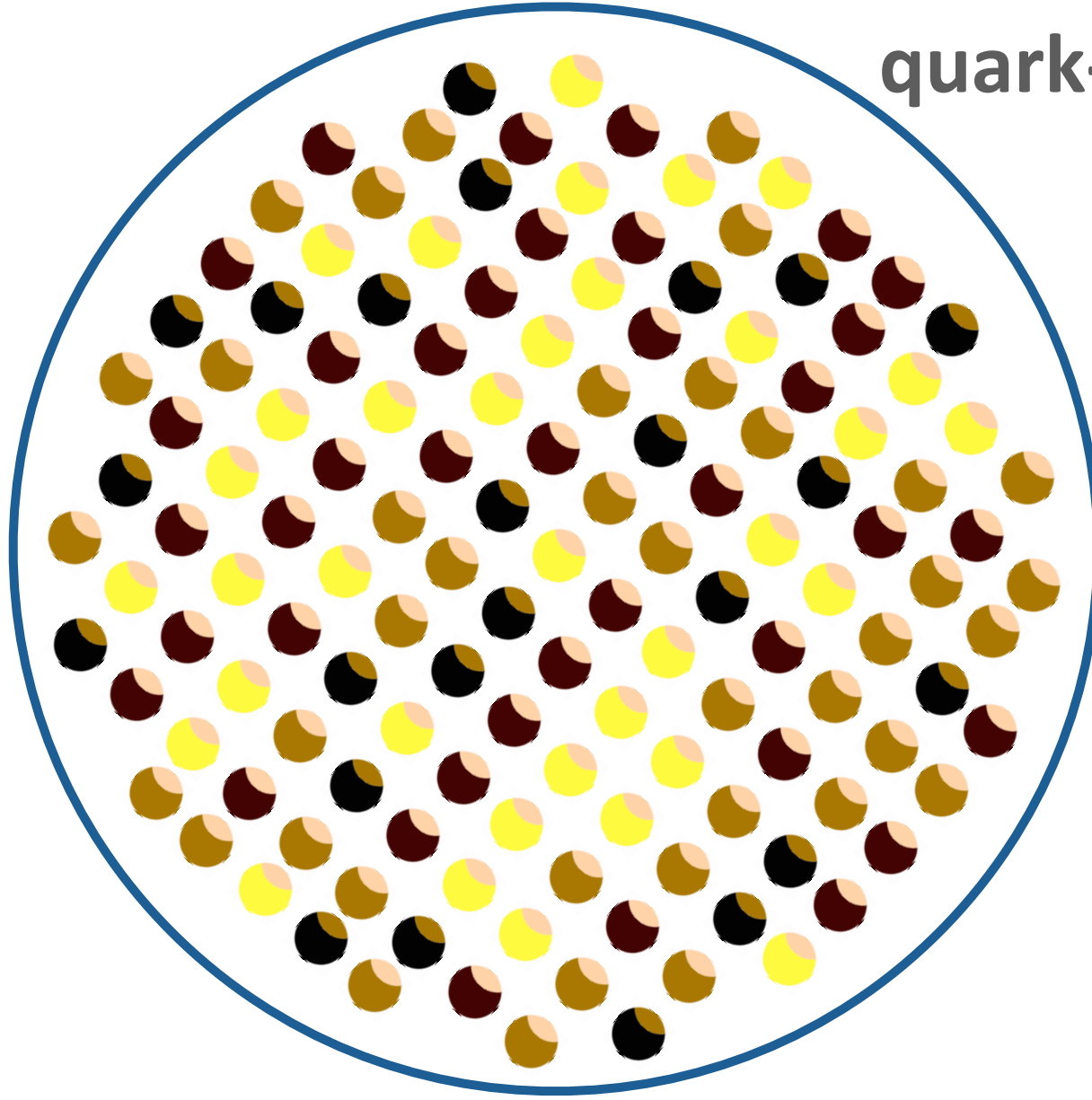
the crowd: Higgs Field

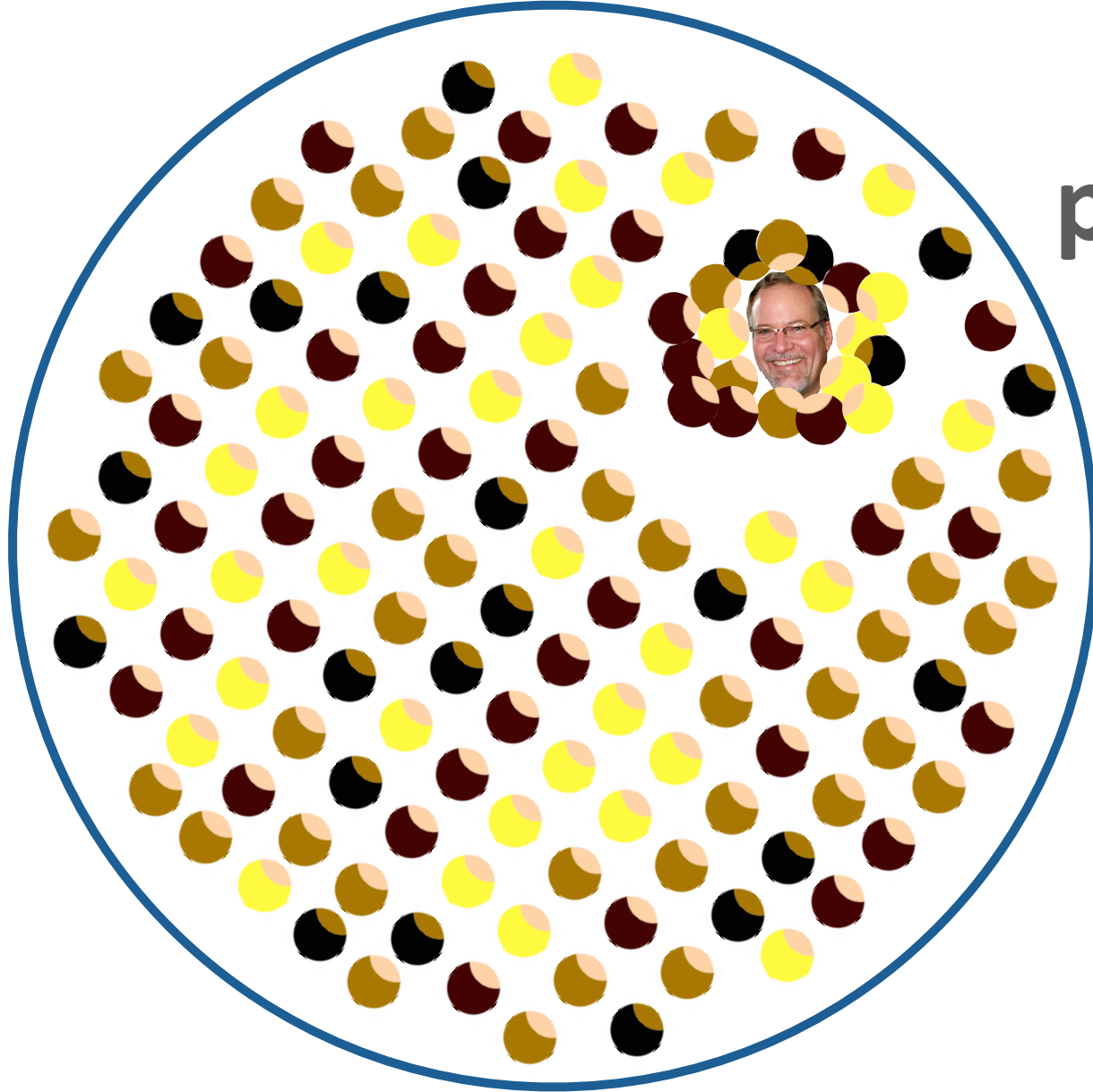
**loud
laughter**



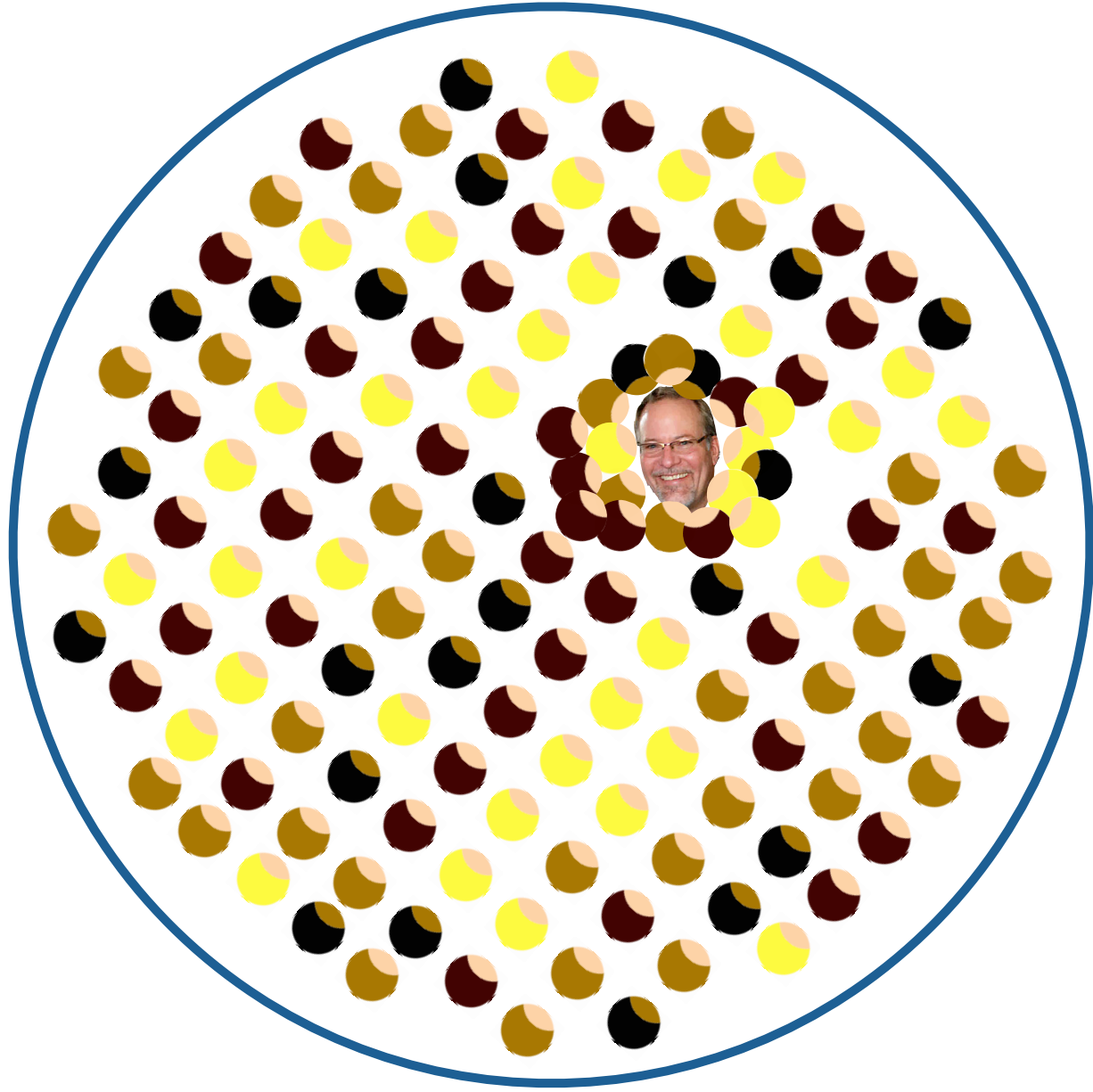


quark-Karl-Gude





popular

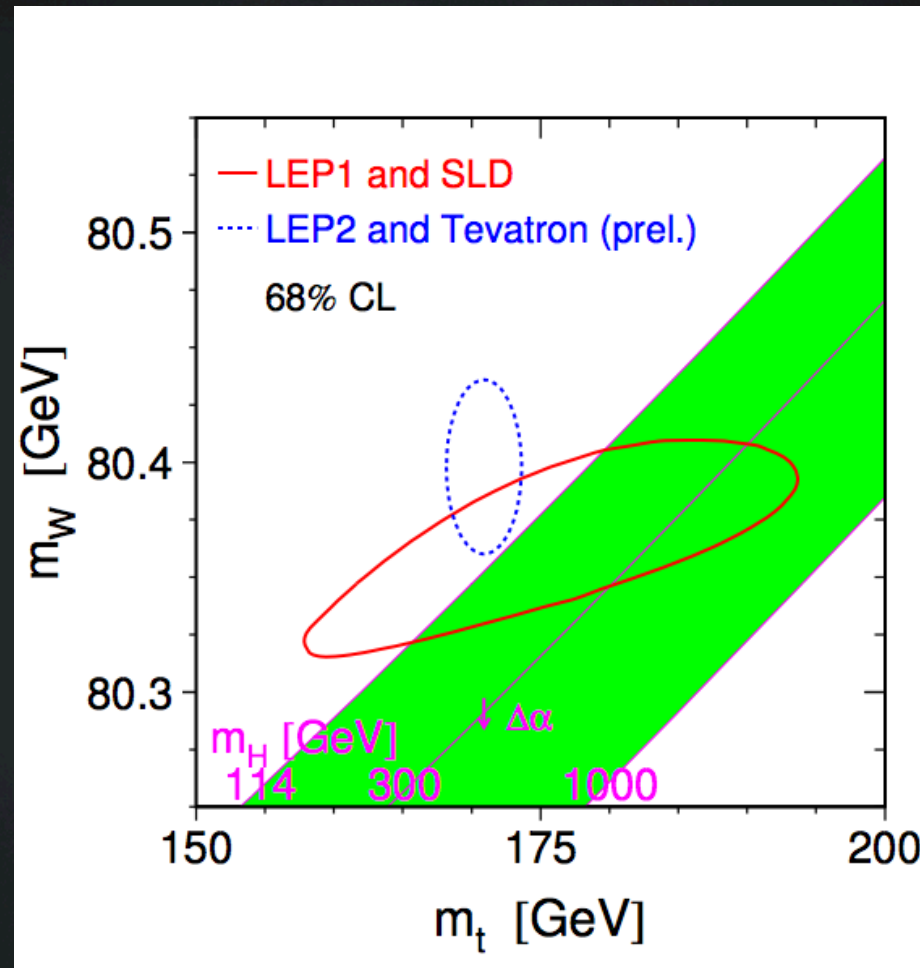


quark-Karl-Gude has gained inertia



quark-Karl-Gude has mass.





By constraining SM measurements:

$$\{M_H < 182 \text{ GeV}/c^2 ; > 114 \text{ GeV}/c^2\} \text{ and: } M_H = 76 +36 -24 \text{ GeV}/c^2$$

SM is a renormalizable theory

with issues... Higgs loops. and Gravity.

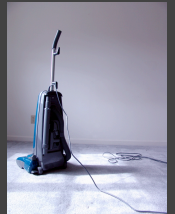
mass corrections

$\sim (10^2 \text{ GeV})^2?$



$$m^2(p^2) = m_0^2 + Cg^2 \int_{p^2}^{\Lambda^2} \text{stuff } dk^2$$

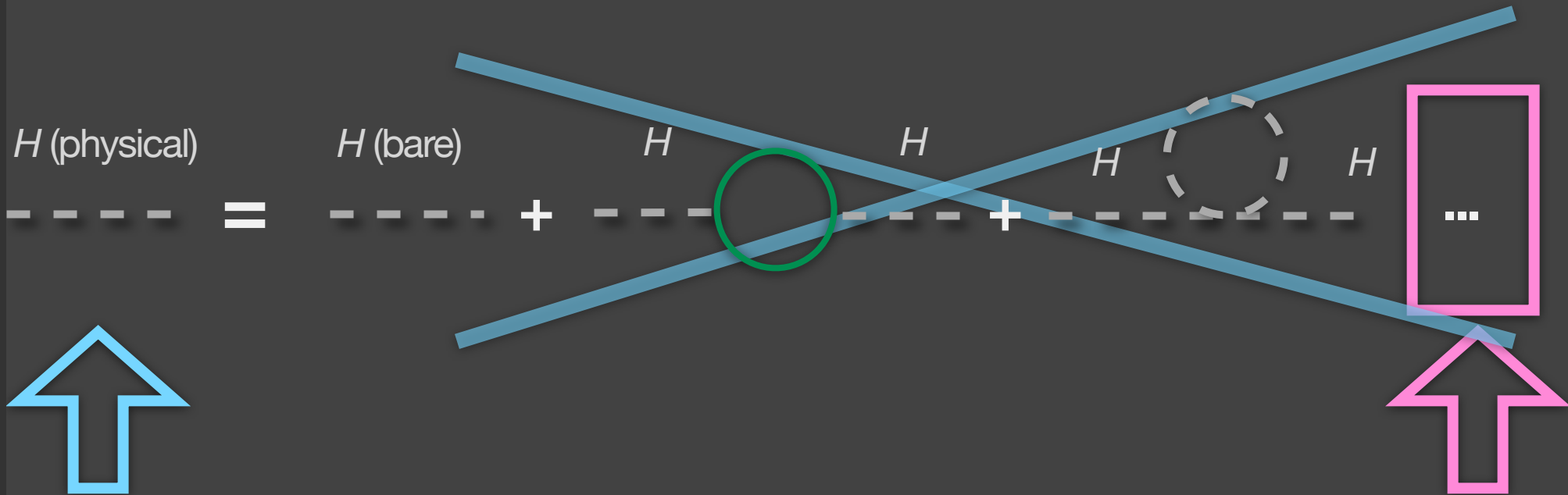
$m^2(p^2) \propto \Lambda^2$ ← That same scale problem as with the



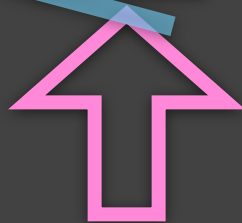
!

$\sim (10^{19} \text{ GeV})^2?$

2 ways out?



Or, the “Higgs” is not an elementary particle after all



New physics causing a cancellation?