

hi

Lecture 13, 21.02.2017

Einstein's Theory of Special Relativity, 2

housekeeping

Question about anything?

I'll make a movie for you:

Marie Curie movie anyone?

yes! I'll organize for after break



Next homework will require today's class content

*you might look at a new manuscript chapter which is in a pre-beta version
so it's due on Saturday midnight, not Thursday midnight.*

Midterm...before or after Spring Break?

*"The midterm will be released on Sunday night, February 26th and close on
Tuesday night, February 28. It will cover all of the material through Tuesday,
February 21st class."*

Honors Project

has begun.

Read the first of two sets of instructions:

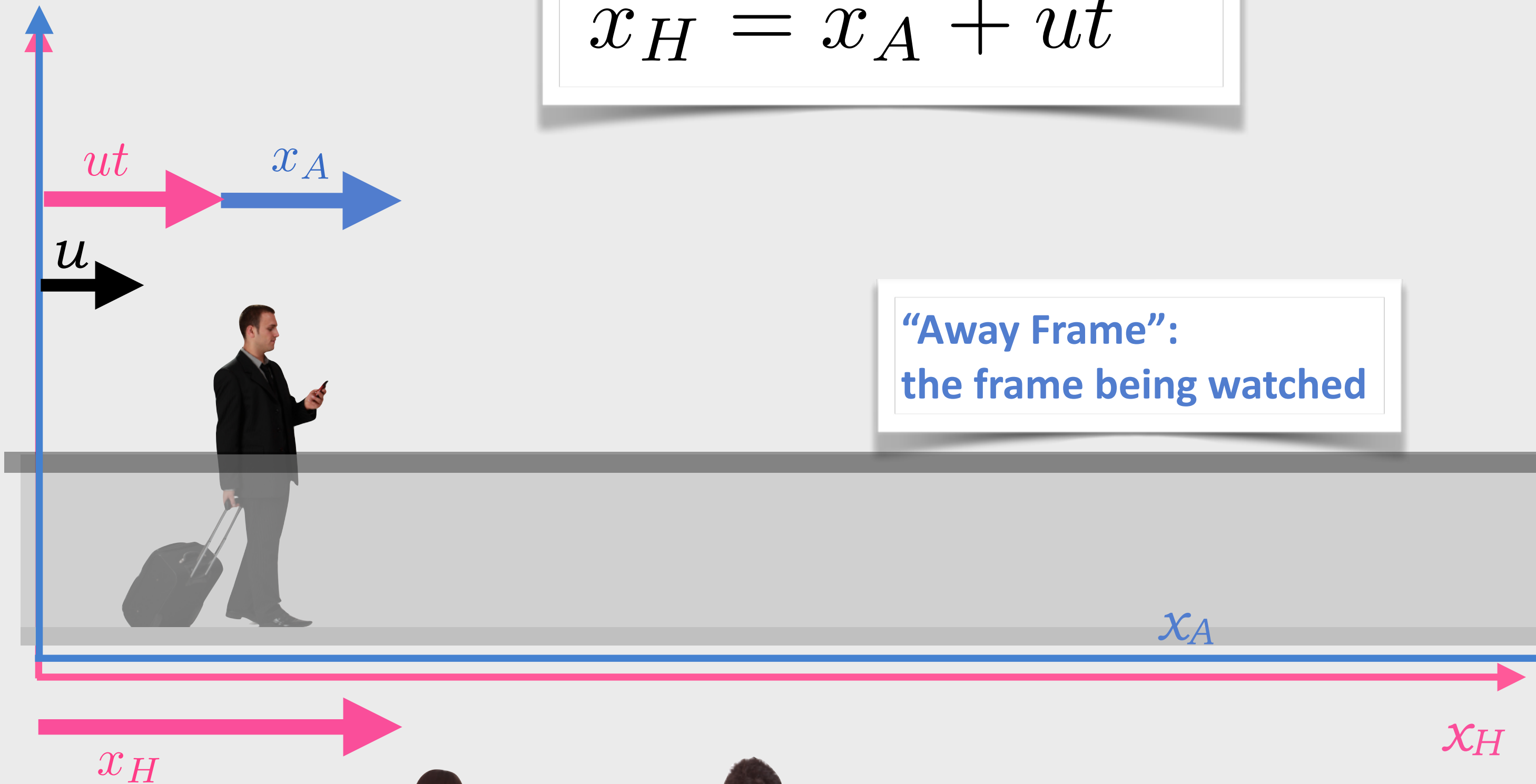
`MinervaInstructions1_2017.pdf` in

www.pa.msu.edu/~brock/file_sharing/QSandBB/2017homework/honors_project_2017/

the airport

“Galilean Transformation”

$$x_H = x_A + ut$$



“Away Frame”:
the frame being watched

“Home Frame”:
watching a moving frame

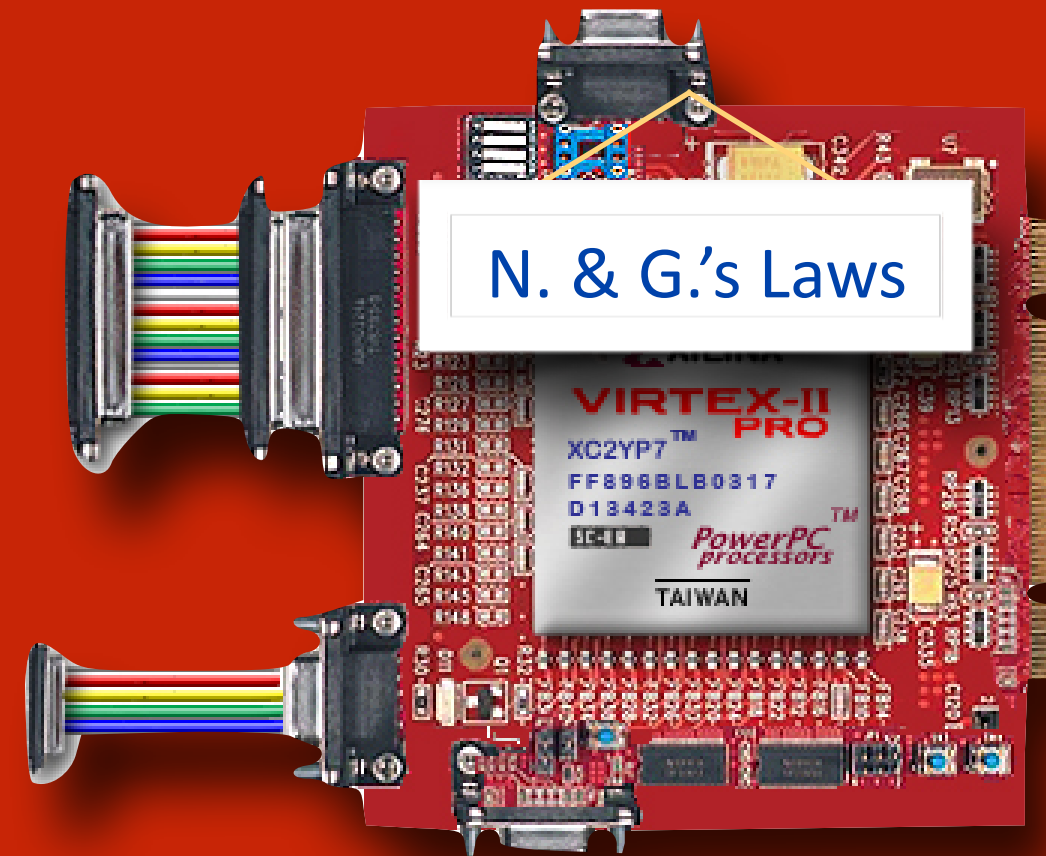
moving at velocity u

Remember: Newton's Laws

seem to work fine

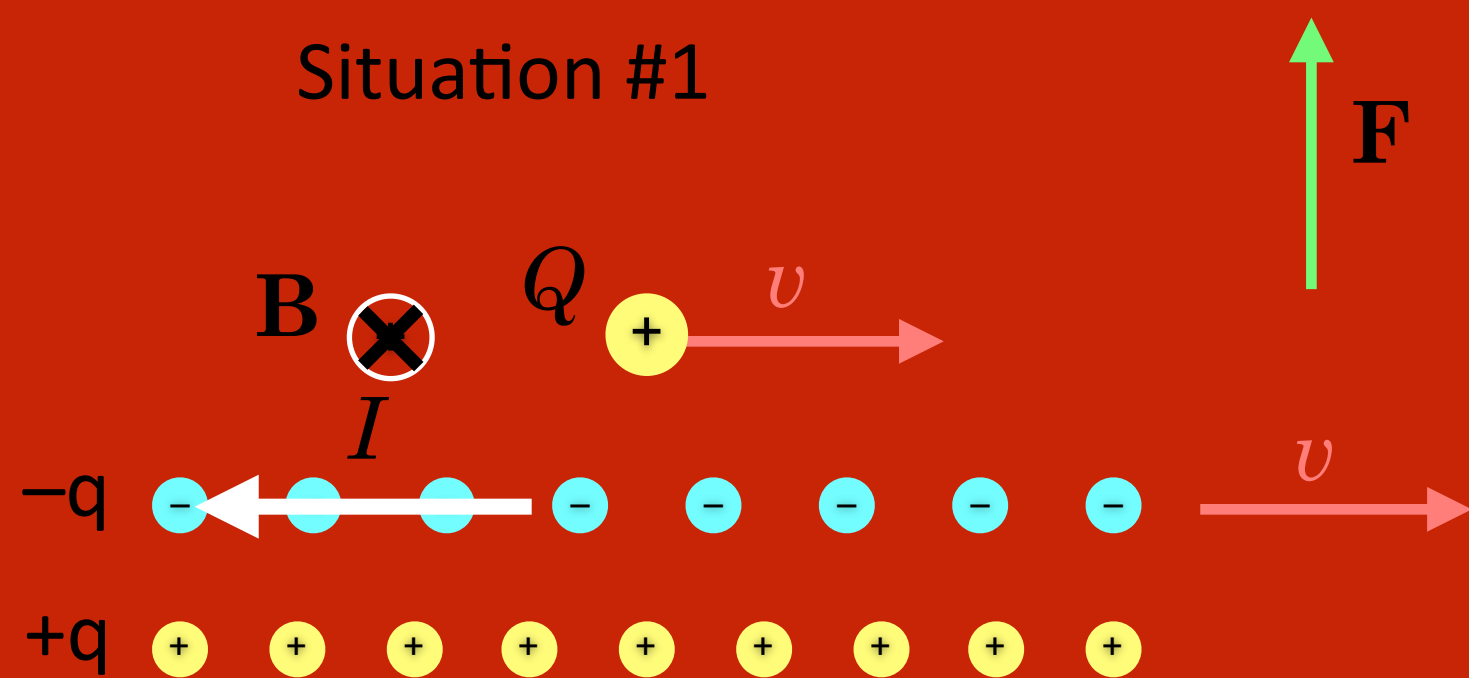
between relatively moving,
constant speed frames

inertial frames



more simple questions

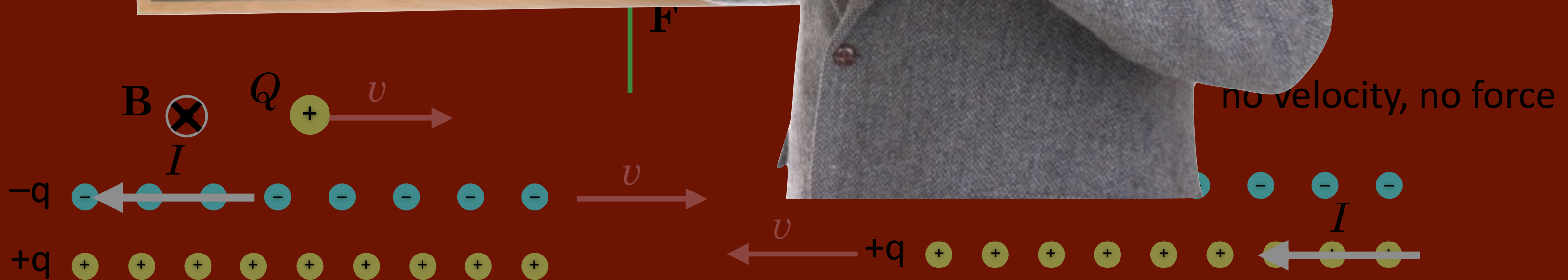
how about a charge next to a current?



These situations differ only in the reference frame...

But, the physical effect – force or no force – is different!

Weird alert #1:
 Two different physical outcomes...
 for situations which differ
 only by the frame of reference



These situations differ only in the reference frame...

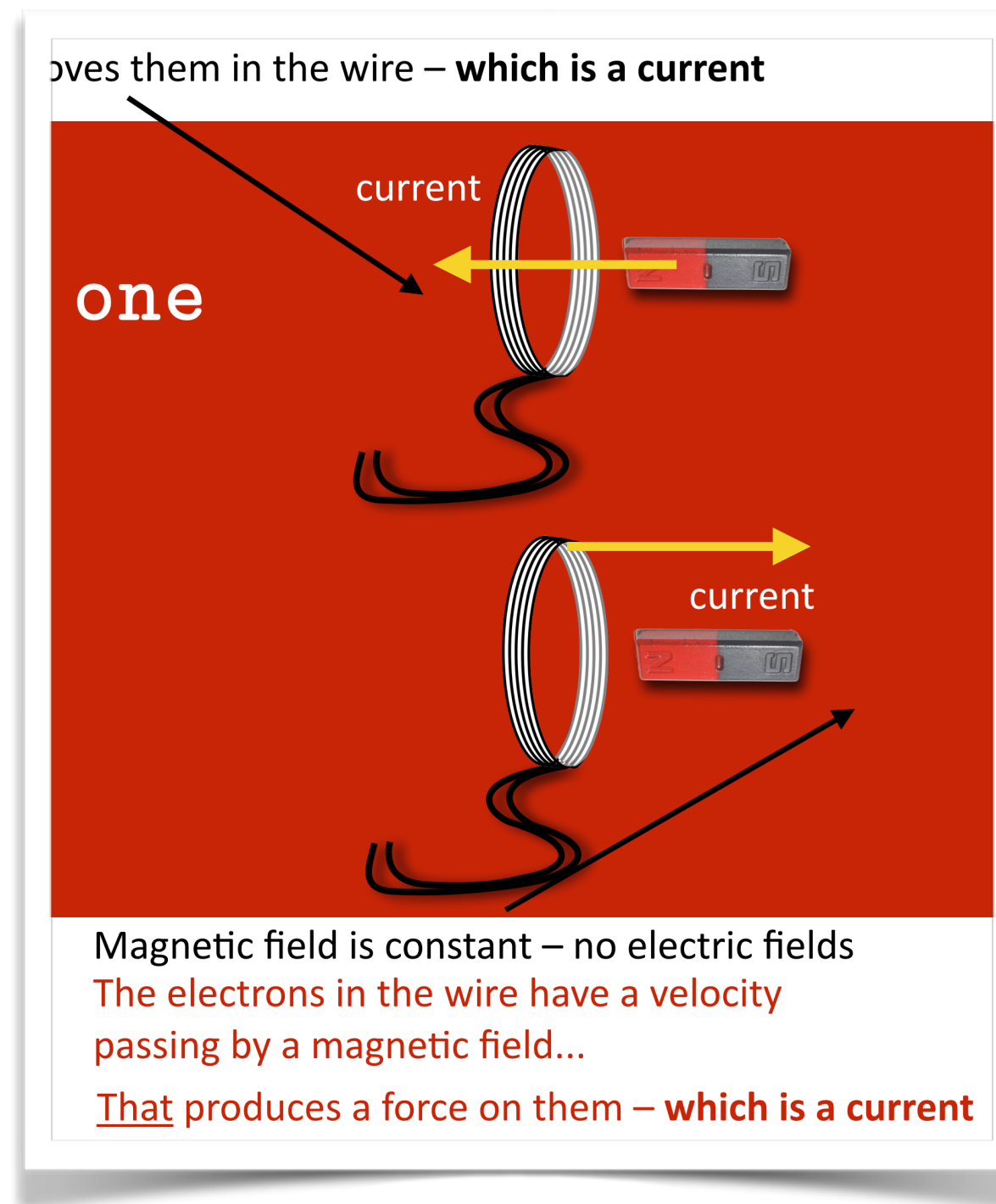
But, the physical effect – force or no force – is different!

and the coil?

move the coil, or move the magnet

one cause

a different
cause



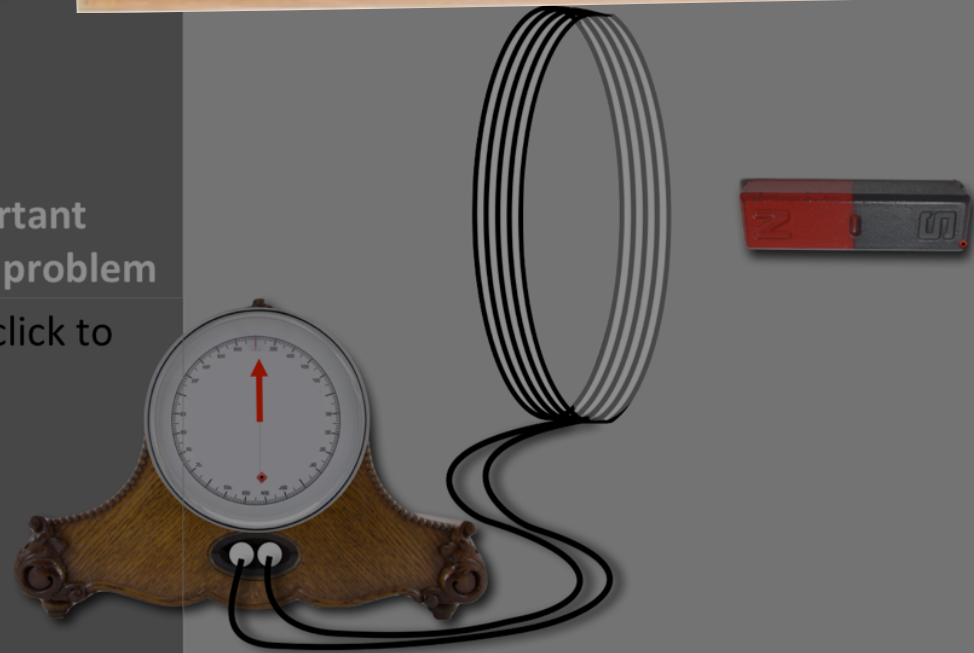
same effect

The changing magnetic field induces an electric field in wire which moves the electrons...

Weird alert #2:
Two identical physical outcomes...
from entirely different physical
causes for situations which differ
only by the frame of reference

an important
thought problem

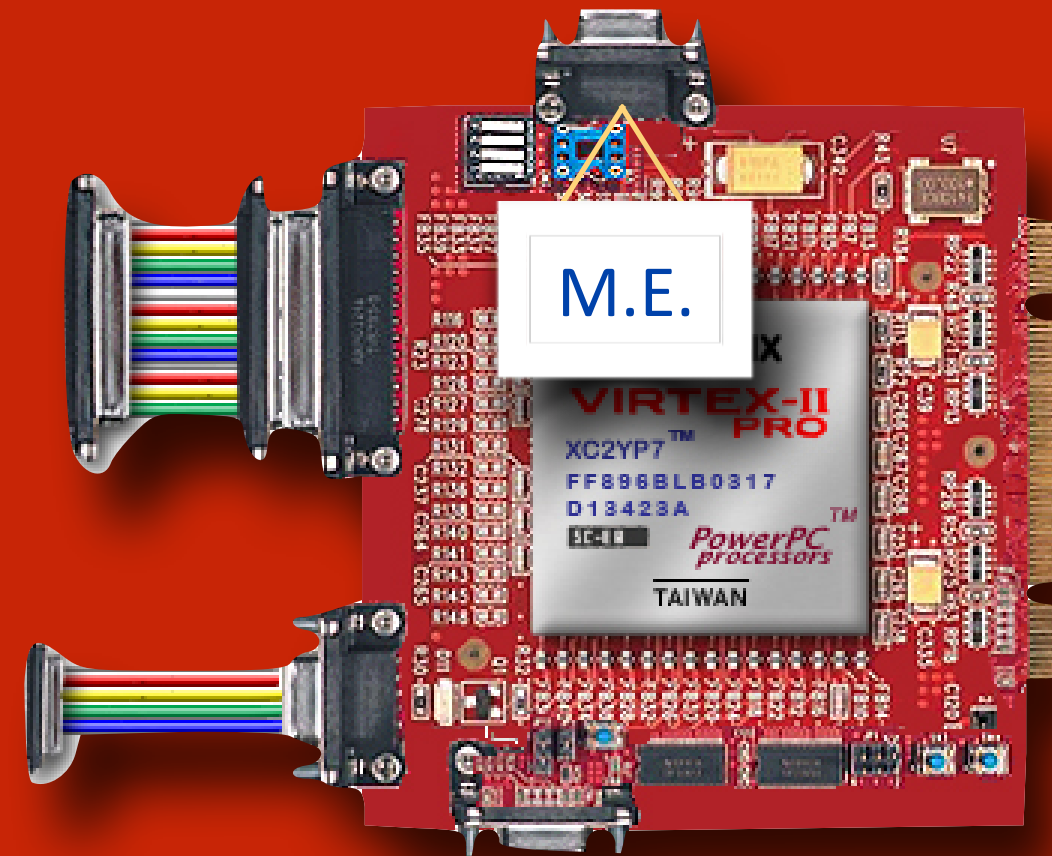
Double-click to
edit



Magnetic field is constant – no electric fields
The electrons in the wire have a velocity
That produces a force on them – a current₉

so Maxwell's Equations

seem to fail between
relatively moving inertial frames



this is crazy! the two models of
the world differ

in their treatment of relatively-moving frames of reference!

Seems to depend on Frame:

Don't appear to depend on Frame:



remember what Maxwell found?

Maxwells aha! moment

$$\text{stuff} \times \vec{E} = 0$$

$$\text{stuff} \times \vec{B} = 0$$

$$\text{stuff} \times \vec{B} = \text{rate of change of } \vec{E}$$

$$\text{stuff} \times \vec{E} = \text{rate of change of } \vec{B}$$

differential equations

$$\text{stuff} \times \vec{E} = 0$$

$$\text{stuff} \times \vec{B} = 0$$

$$\text{stuff} \times \vec{B} = \text{rate of change of } \vec{E}$$

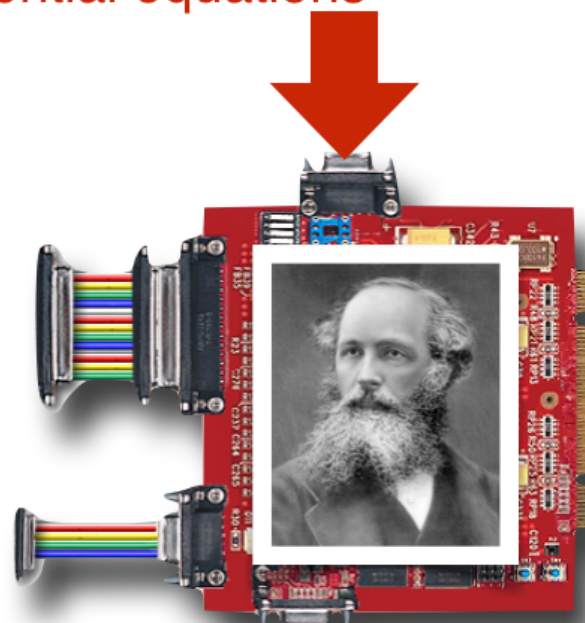
$$\text{stuff} \times \vec{E} = \text{rate of change of } \vec{B}$$

remove the explicit sources, Q & I

Look how the equations are symmetric: $E \leftrightarrow B$

$$\frac{E}{B} = 3 \times 10^8 \text{ m/s}$$

$c!$ the **speed of light!**
Which Maxwell knew.

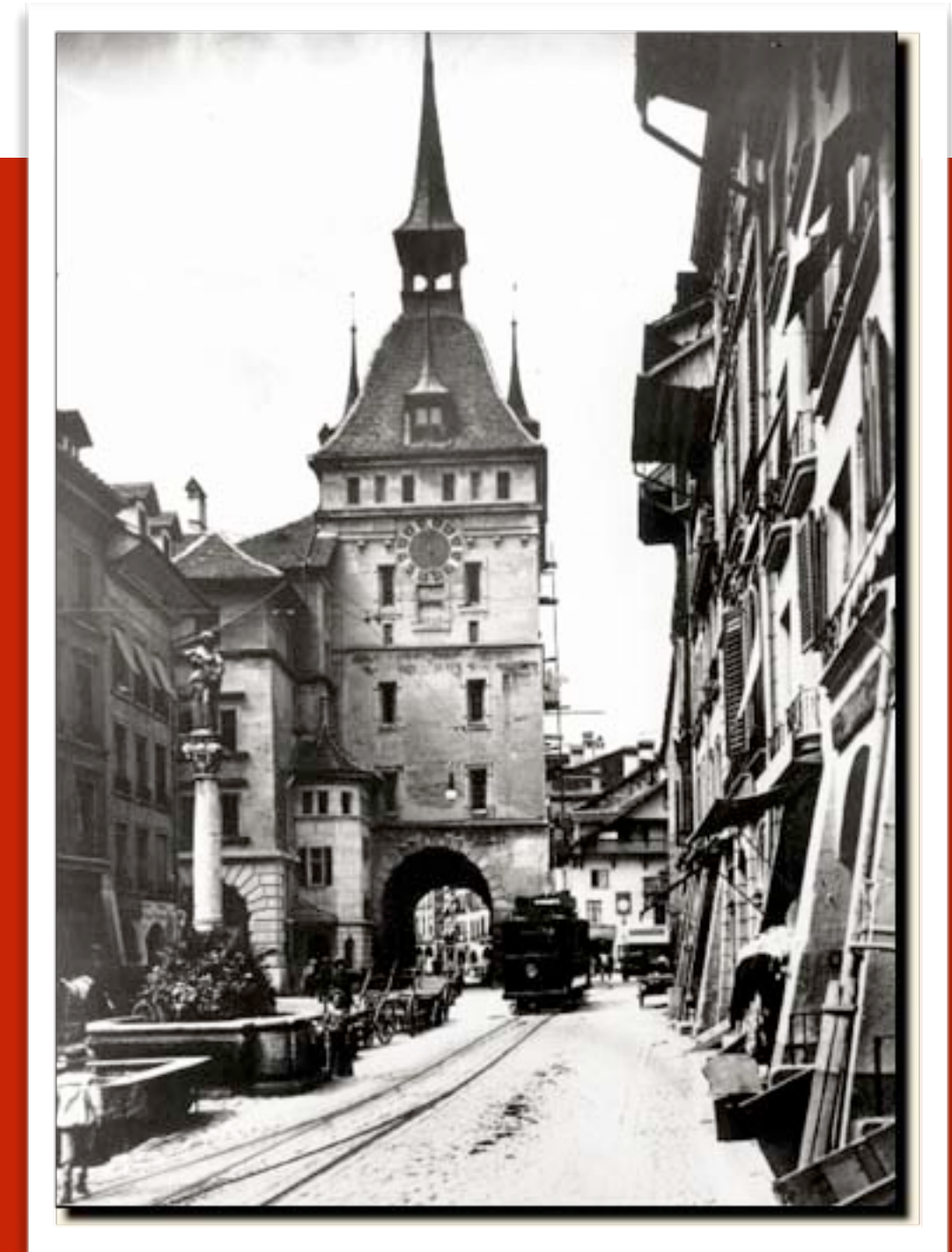


in fact

the faster in space you would travel

time would appear to stop

a light beam from the clock could not keep up



The famous clock tower in Bern, Switzerland that Einstein mused about

This offended the young Einstein.

He took the Maxwell prediction seriously:

light moves at a constant speed

and proposed that c is special

he elevated c to be an **invariant parameter**

Principle of Relativity

1. All laws of physics – mechanical **and electromagnetic** – are identical in co-moving inertial frames.

taking Galileo seriously, and then adding Maxwell

2. The speed of light is the same for all inertial observers.

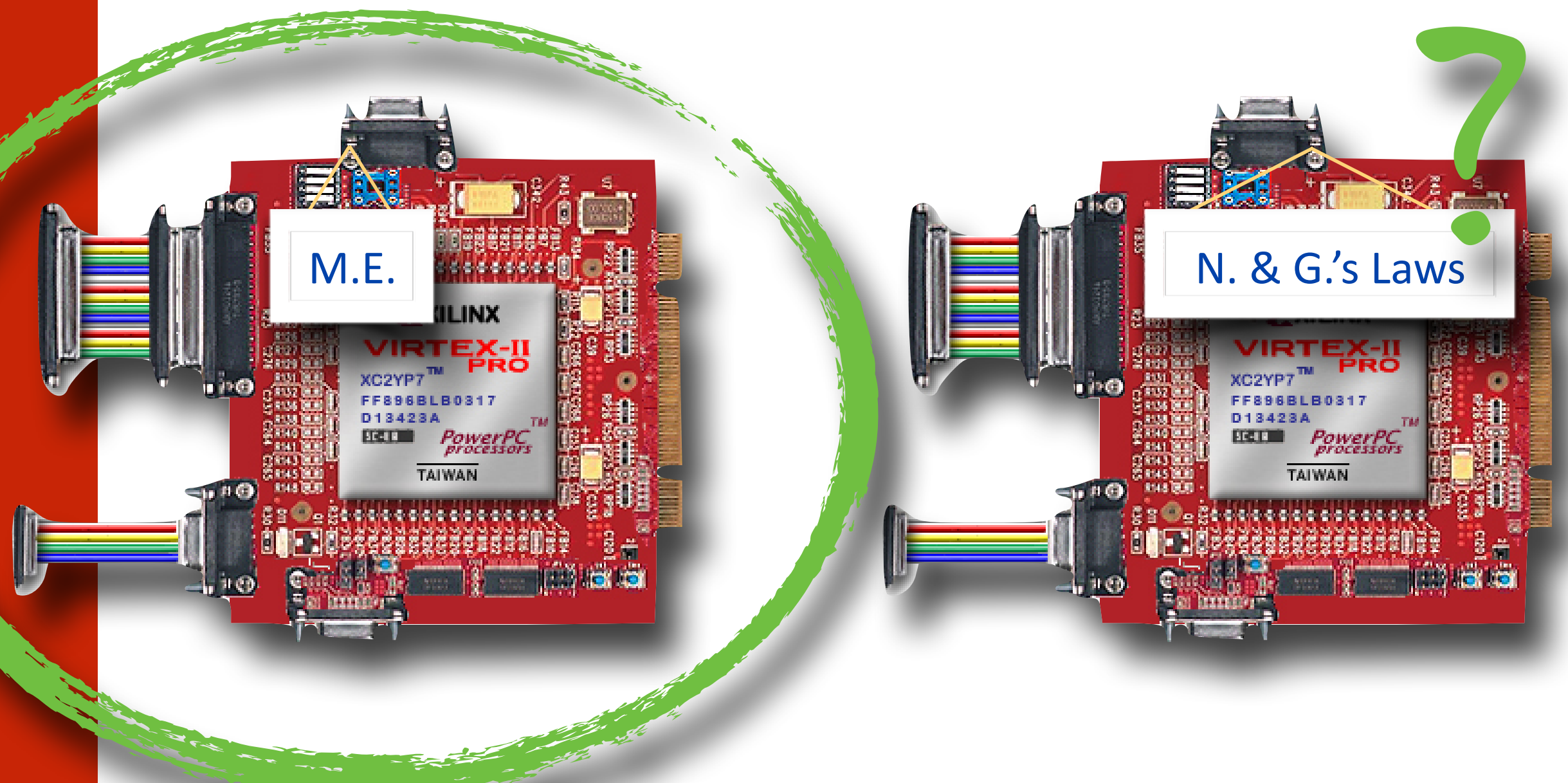
taking Maxwell seriously

2

Postulates:

"inertial frame":

constant
velocity



Einstein writes
very simply

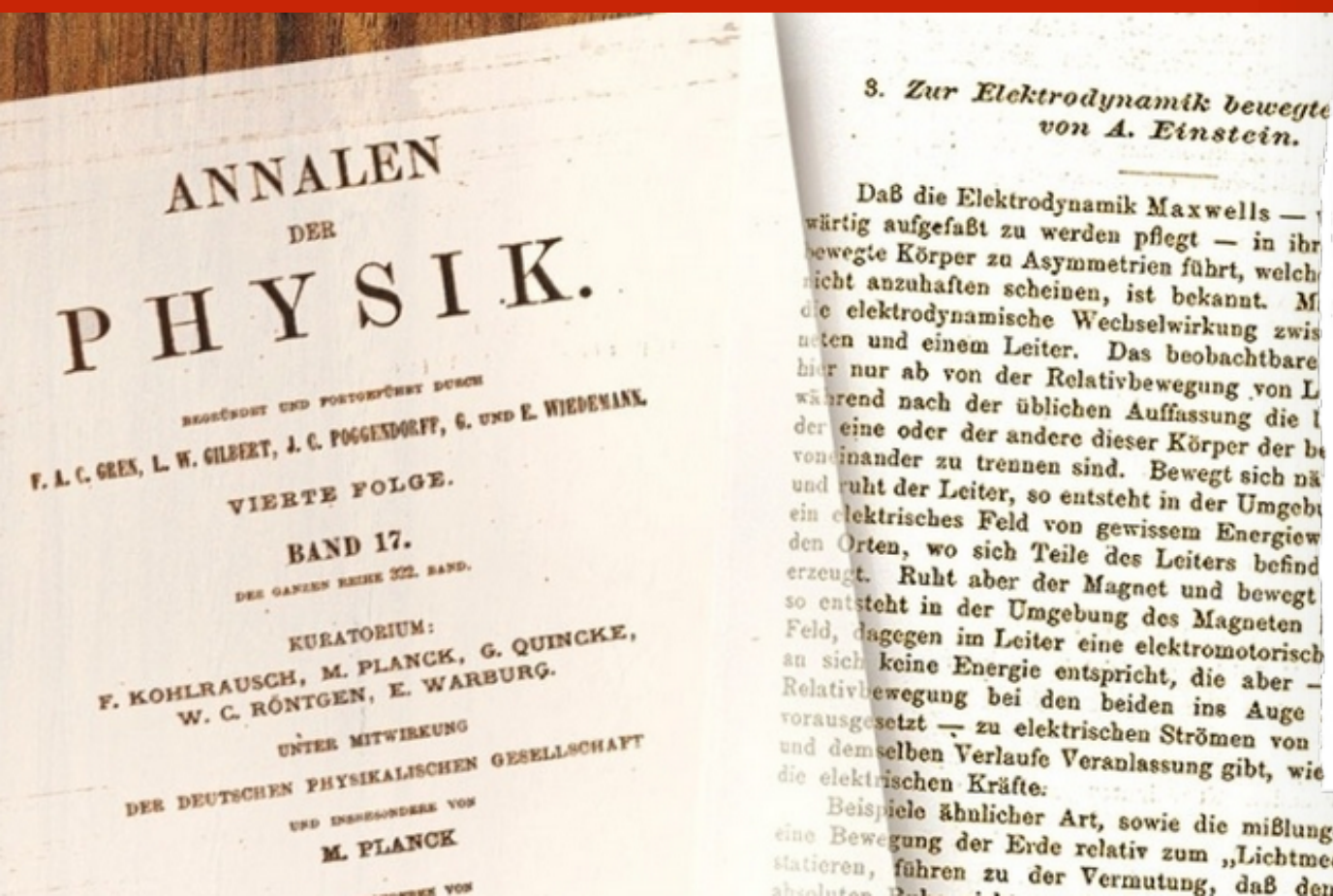
His 1905 Relativity
paper:

"On the Electrodynamics
of Moving Bodies"

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion.

For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated.

But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force...which gives rise...to electric currents of the same path and intensity as those produced by the electric forces in the former case.



Einstein writes
very simply

His 1905 Relativity
paper:

"On the Electrodynamics
of Moving Bodies"

*not your standard physics
journal introduction*

Let us take a system of co-ordinates in which the equations of Newtonian mechanics hold good. In order to render our presentation more precise and to distinguish this system of co-ordinates verbally from others which will be introduced hereafter, we call it the "stationary system."

If a material point is at rest relatively to this system of co-ordinates, its position can be defined relatively thereto by the employment of rigid standards of measurement and the methods of Euclidean geometry, and can be expressed in Cartesian co-ordinates.

If we wish to describe the motion of a material point, we give the values of its co-ordinates as functions of the time. Now we must bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by "time." We have to take into account that all our judgments in which time plays a part are always judgments of simultaneous events. If, for instance, I say, "That train arrives here at 7 o'clock," I mean something like this: "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events."

It might appear possible to overcome all the difficulties attending the definition of "time" by substituting "the position of the small hand of my watch" for "time." And in fact such a definition is satisfactory when we are concerned with defining a time exclusively for the place where the watch is located; but it is no longer satisfactory when we have to connect in time series of events occurring at different places, or-what comes to the same thing-to evaluate the times of events occurring at places remote from the watch.

and then

he played the two postulates out
to see what would result

"A storm broke loose in my mind."

his concern:

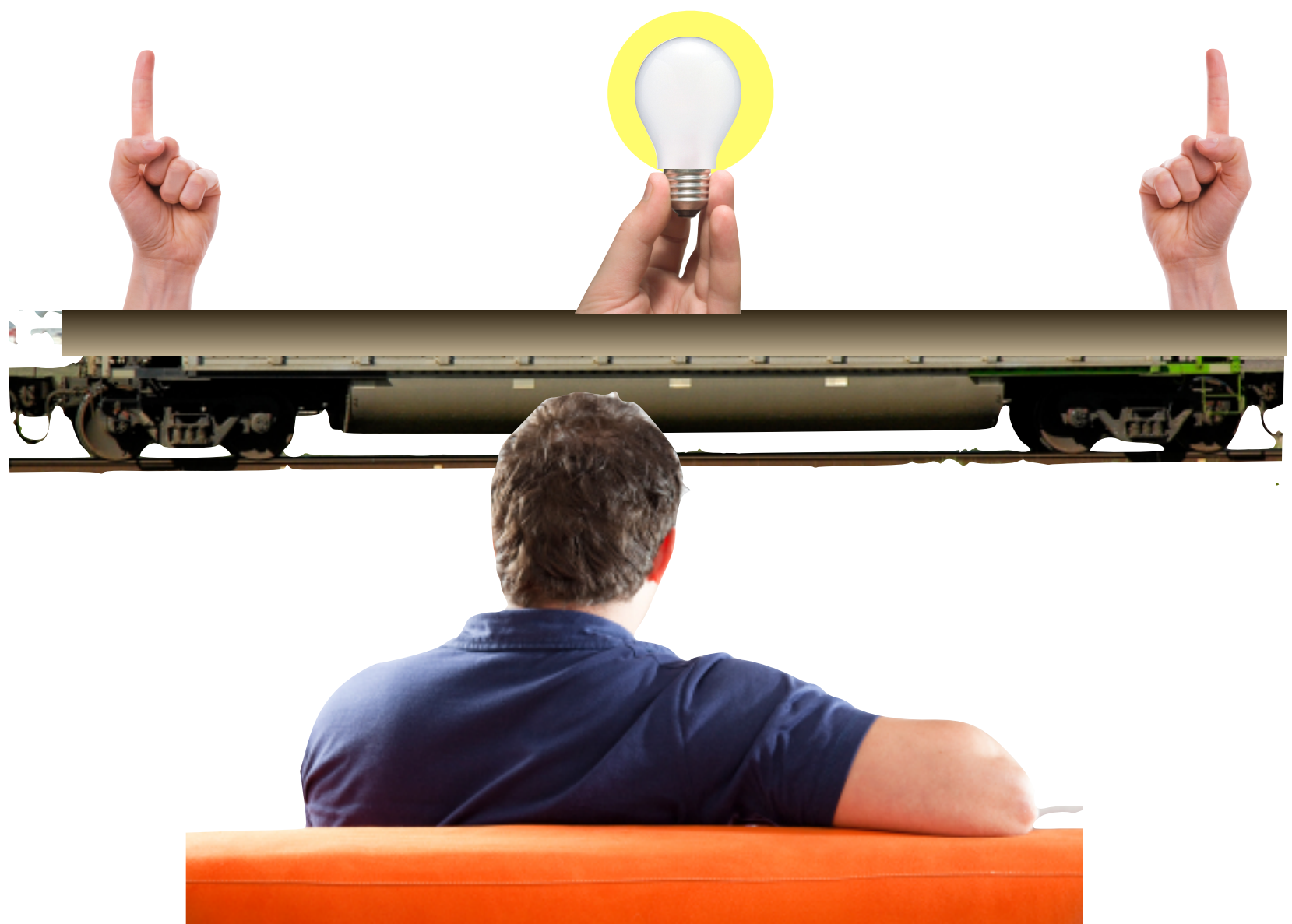
simultaneity

put on
your
seatbelt

philosophical
issues

and

very pragmatic
issues

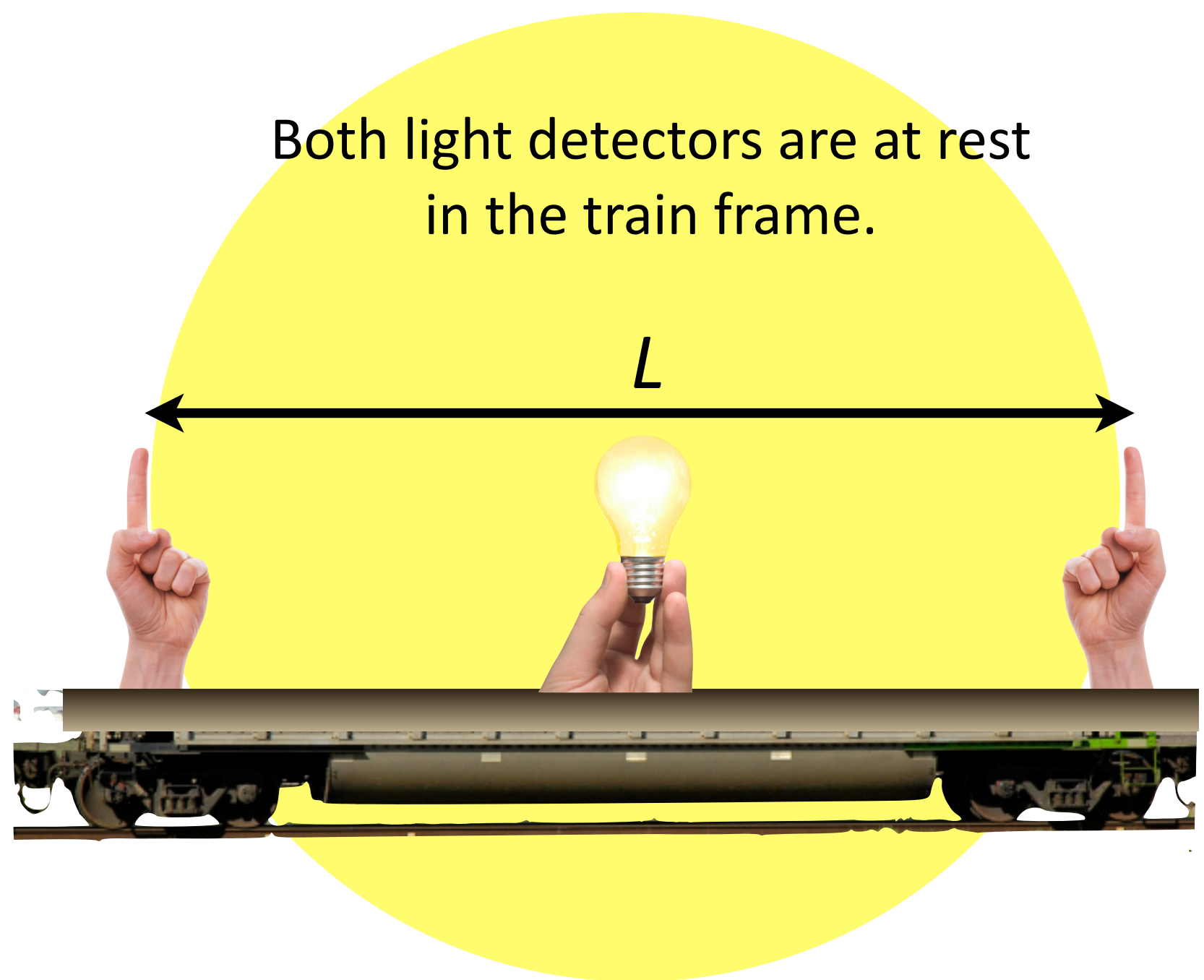


thinking
simple

philosophical
issues

imagine a frame
in which a light
beam is emitted
in the center and
detected in that
frame equal
distances away

**The train observer would declare:
the beams arrived simultaneously**



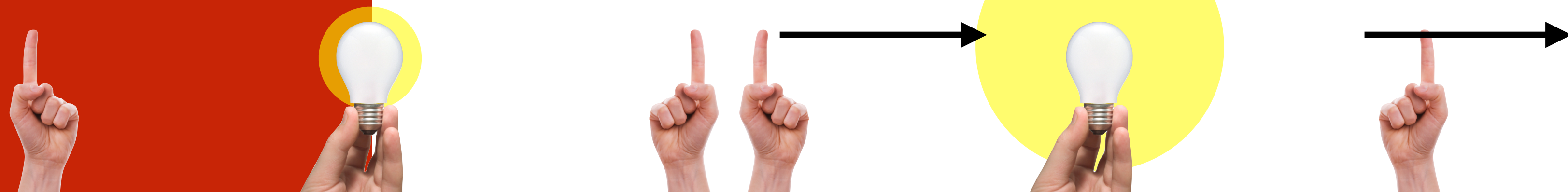
Left and Right hands register receipt of the light beam at the same time.

simple
is hard

the 1st of three
odd things
about space
and time

**The ground observer would declare:
the beams did not arrive
simultaneously**

Notice that the Second Postulate disallows the addition of the train's motion to the light speed.



LH finger
approaching
the beam

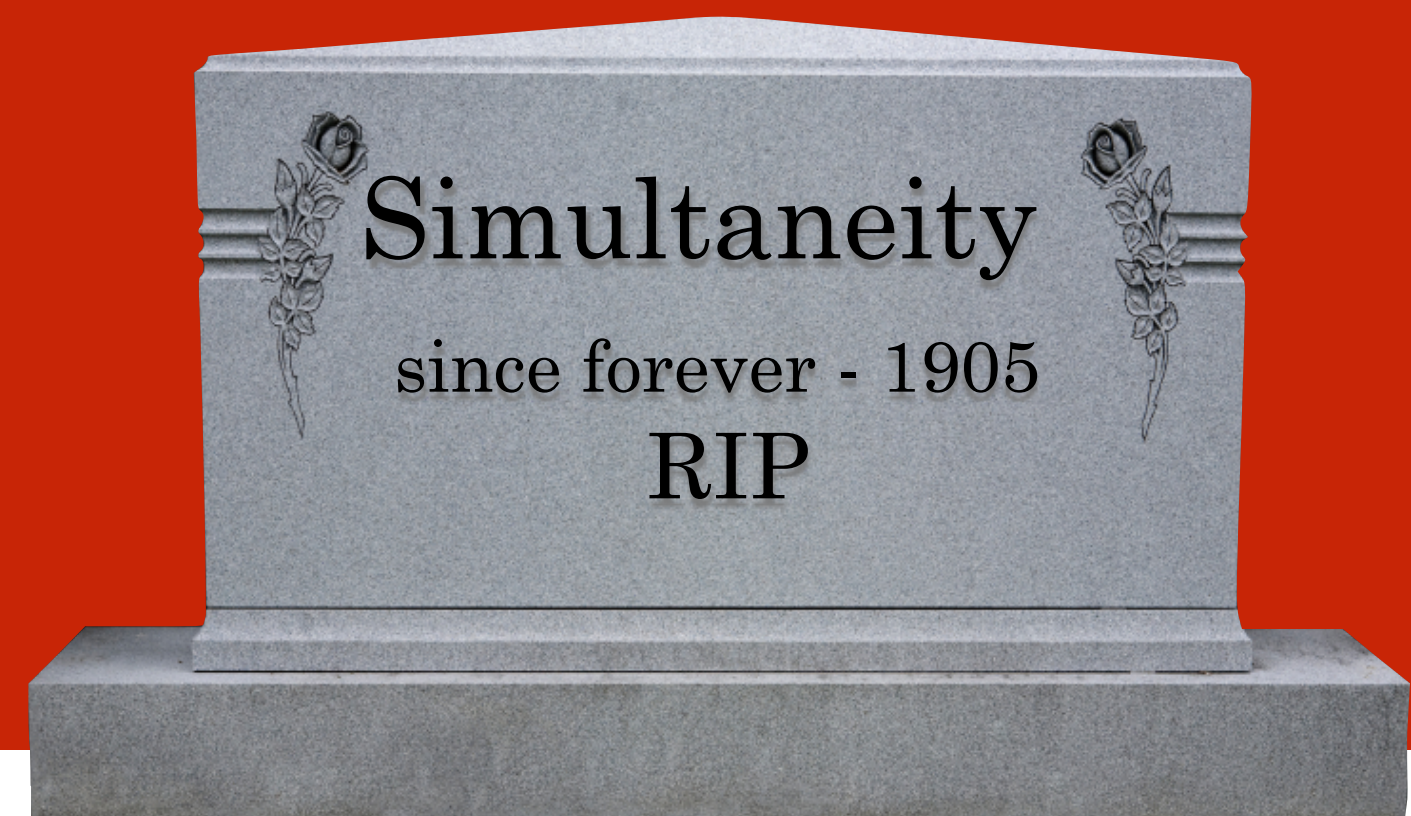
RH finger
moving away
from the beam

What does a H observer see?

**Light hits LH finger
before RH.**

There is no such thing as the *concept of*
simultaneous events

between co-moving frames of reference



two problems with this:

1. Since there is no way to determine that something is simultaneous in one frame and also in another

one can never synchronize clocks between co-moving frames of reference

so no meaningful translation from one frame to another

now?

So .

No inertial frame is special.

All are equivalent.

Why?

because no measurement can be made to tell otherwise

2. “Causality” requires care

Two observers disagree about when events happen
is CAUSALITY

the same time? at different times?

Suppose the hospital order is: first I'm born, then I cry
a casualty!?
would a moving observer believe he first cry, then I'm born?



the 2nd postulate

makes things strange

because c

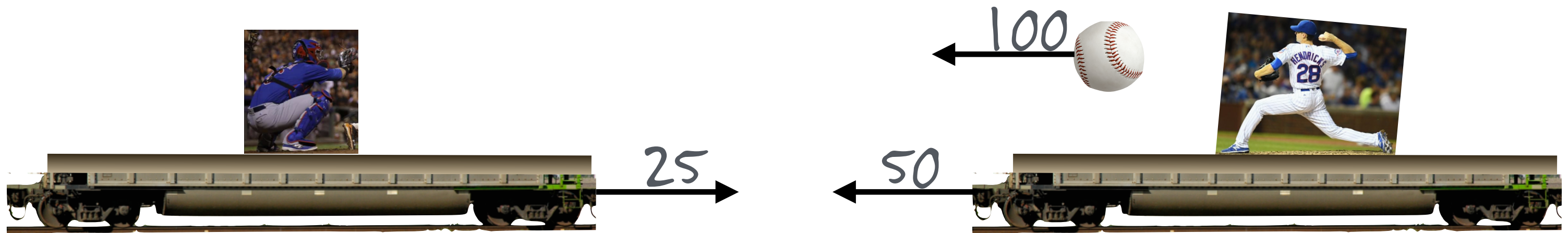
the speed of light is constant in all inertial frames:

$c = 3 \times 10^8 \text{ m/s} = 300 \text{ million m/s} = 1,080 \text{ million km/h}$

$c = 671 \text{ million mph}$

this seems reasonable:

a trap.



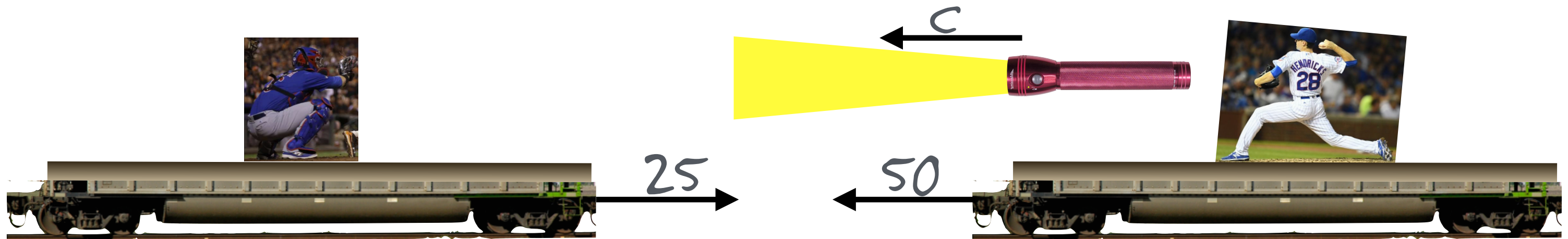
case 1: $v(\text{catcher})=0$

$v(\text{ball})=100$ $v(\text{pitcher})=0$

what's $v(\text{ball})$ that catcher experiences: $v(\text{caught})=100$

this seems strange:

light's different.



case 1: $v(\text{catcher})=0$ $v(\text{light})=c$ $v(\text{pitcher})=0$

what's $v(\text{light})$ that catcher experiences: $v(\text{caught})=c$

case 2: $v(\text{catcher})=0$ $v(\text{light})=c$ $v(\text{pitcher})=50$

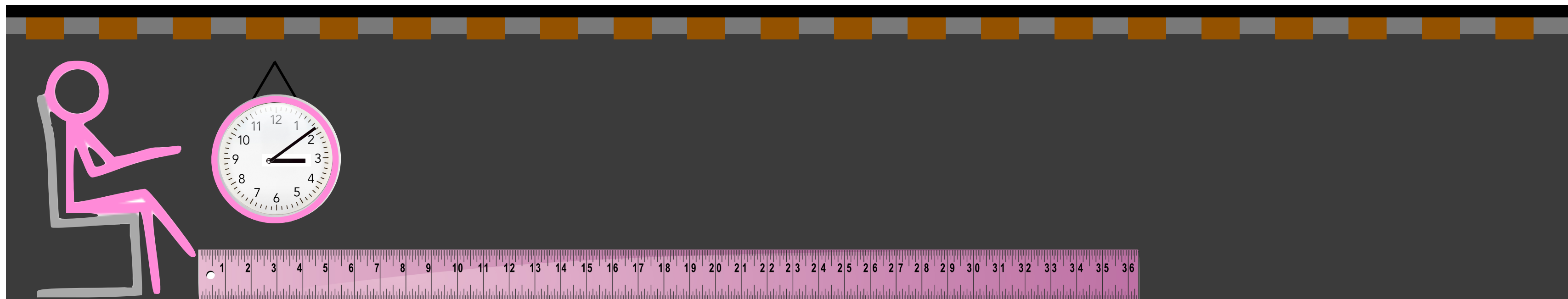
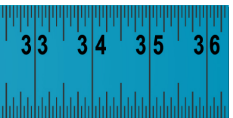
what's $v(\text{light})$ that catcher experiences: $v(\text{caught})=c$

case 3: $v(\text{catcher})=25$ $v(\text{light})=c$ $v(\text{pitcher})=50$

what's $v(\text{light})$ that catcher experiences: $v(\text{caught})=c$

light is constant speed everywhere

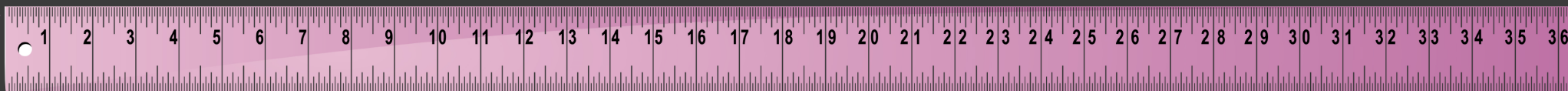
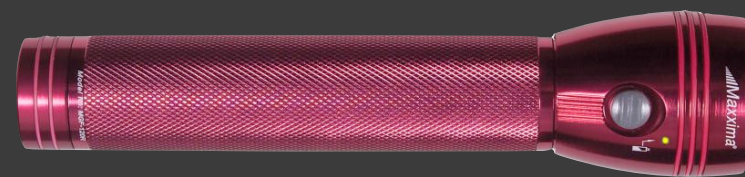
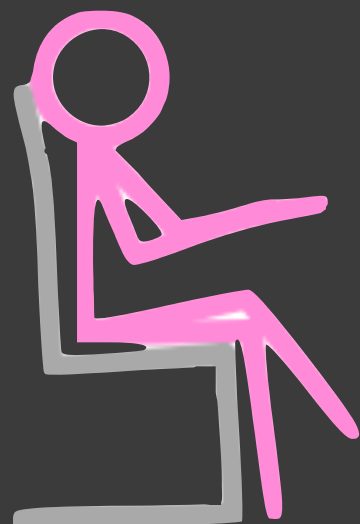
$$v_A = c$$



$$v_H = c!!$$

and the other way as well.

$v_T = c!$
can't catch up!

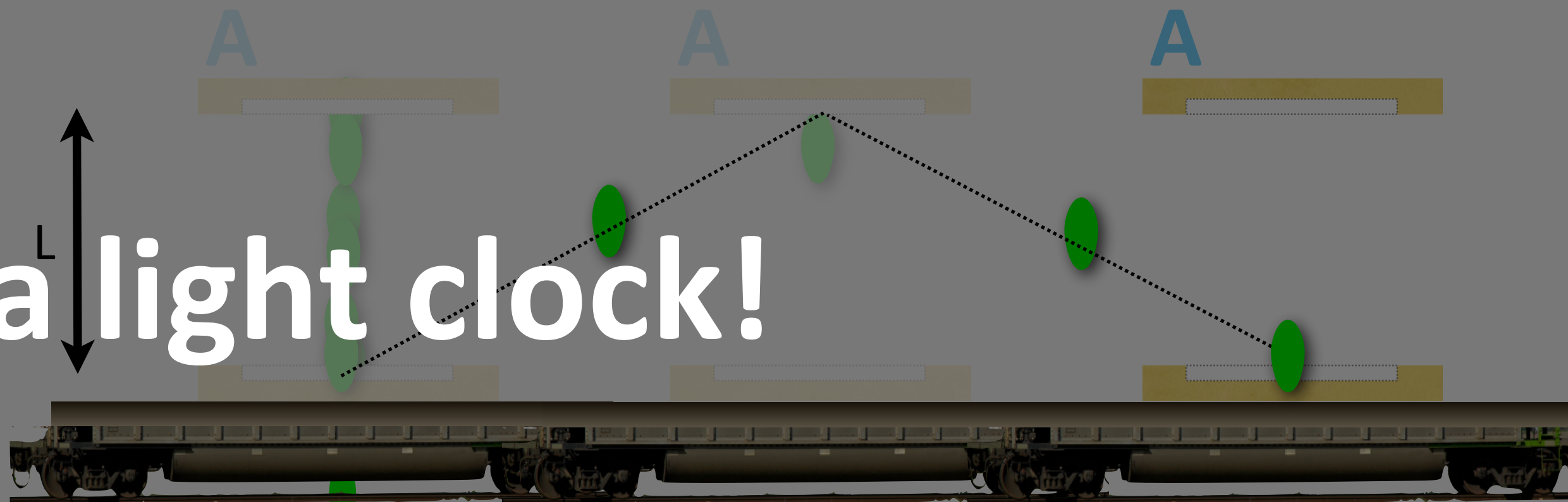


$V_G = c$

there are consequences to this

let's make a light clock
and follow the mathematics

A A A
a light clock!



H

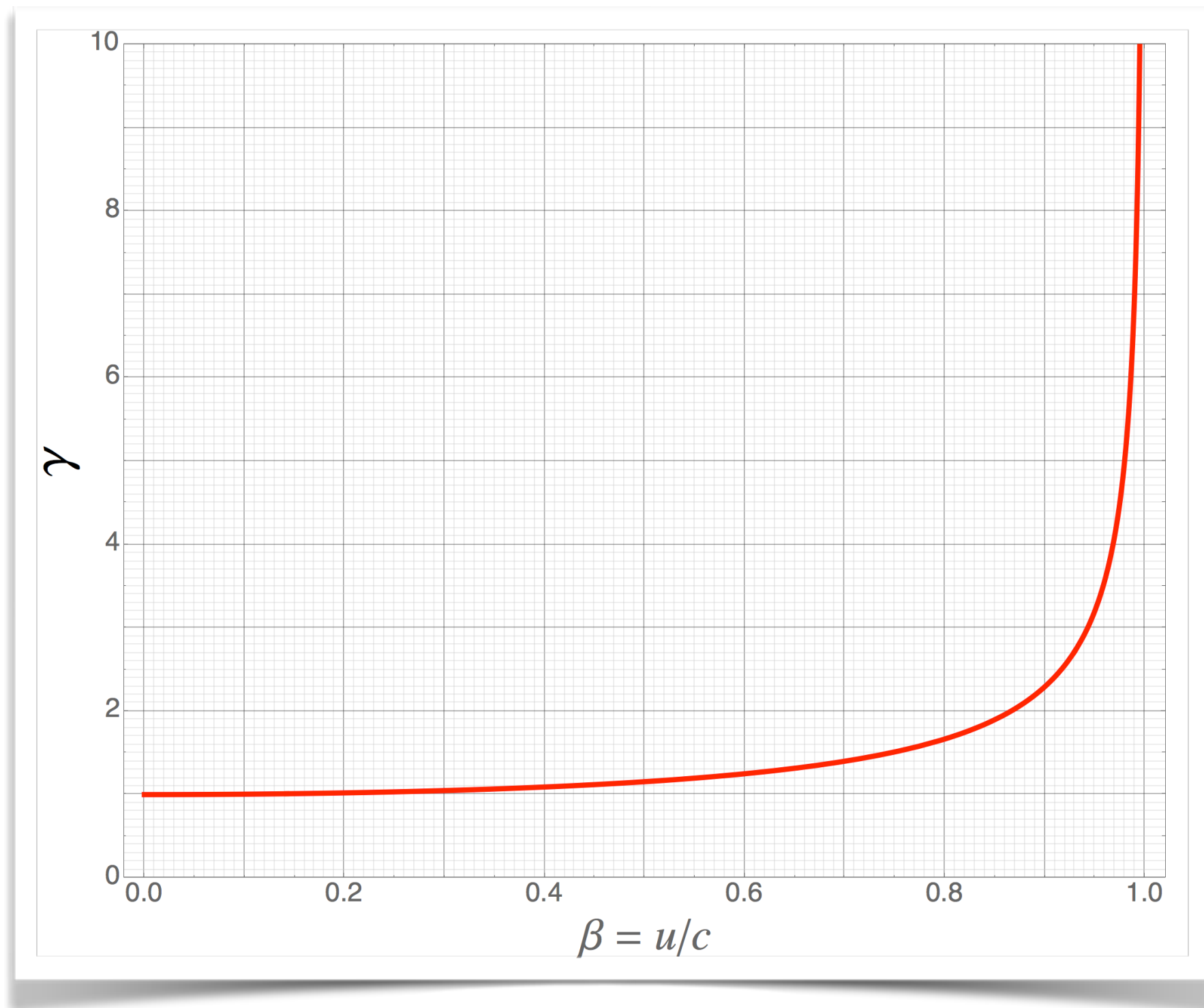


Moving clocks appear to run slower as seen by a relatively stationary observer

$$t_H = \frac{t_A}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$
$$t_H = \gamma t_A$$

time dilation

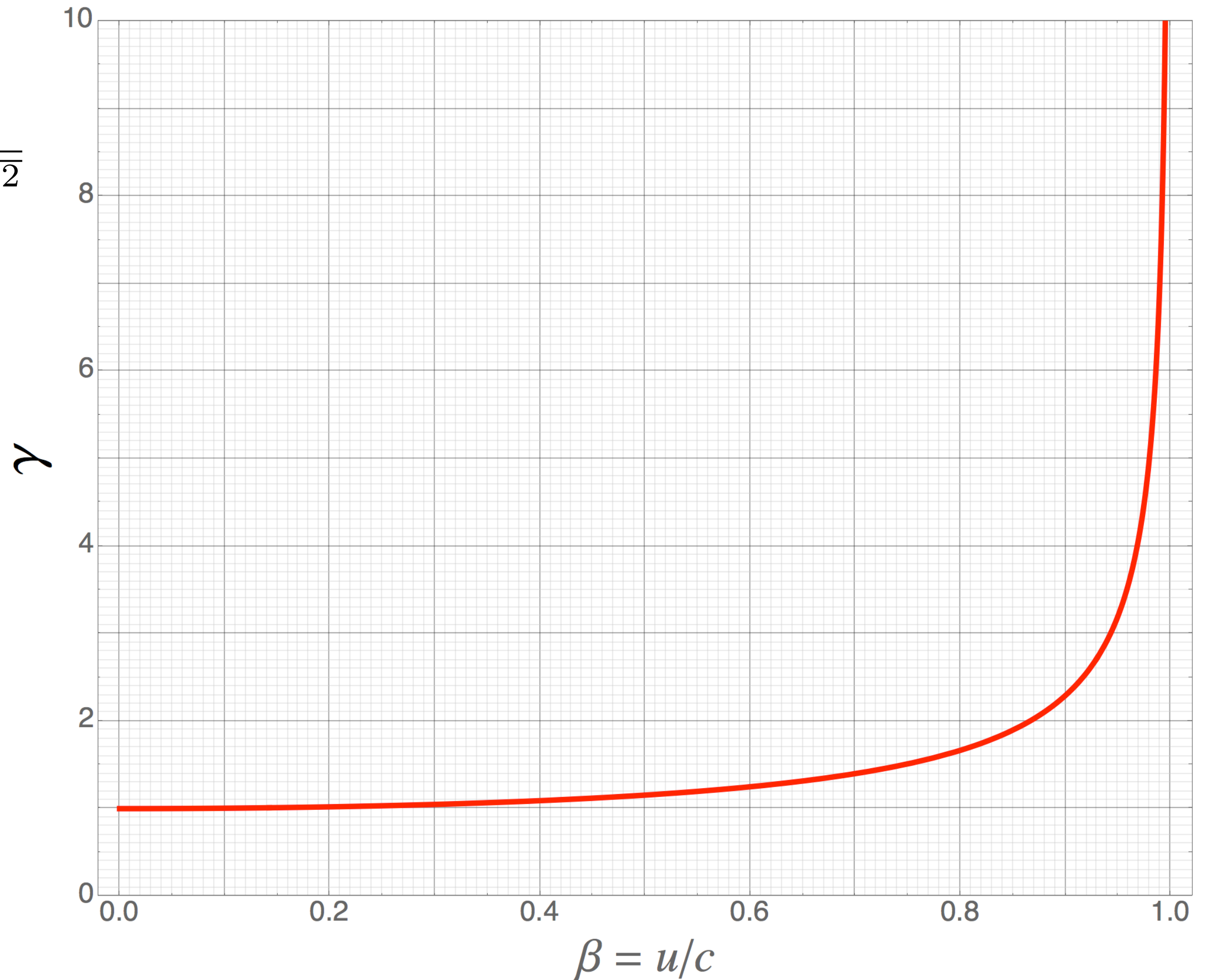
the second of
3 strange
things about
space and
time



“relativistic gamma”

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

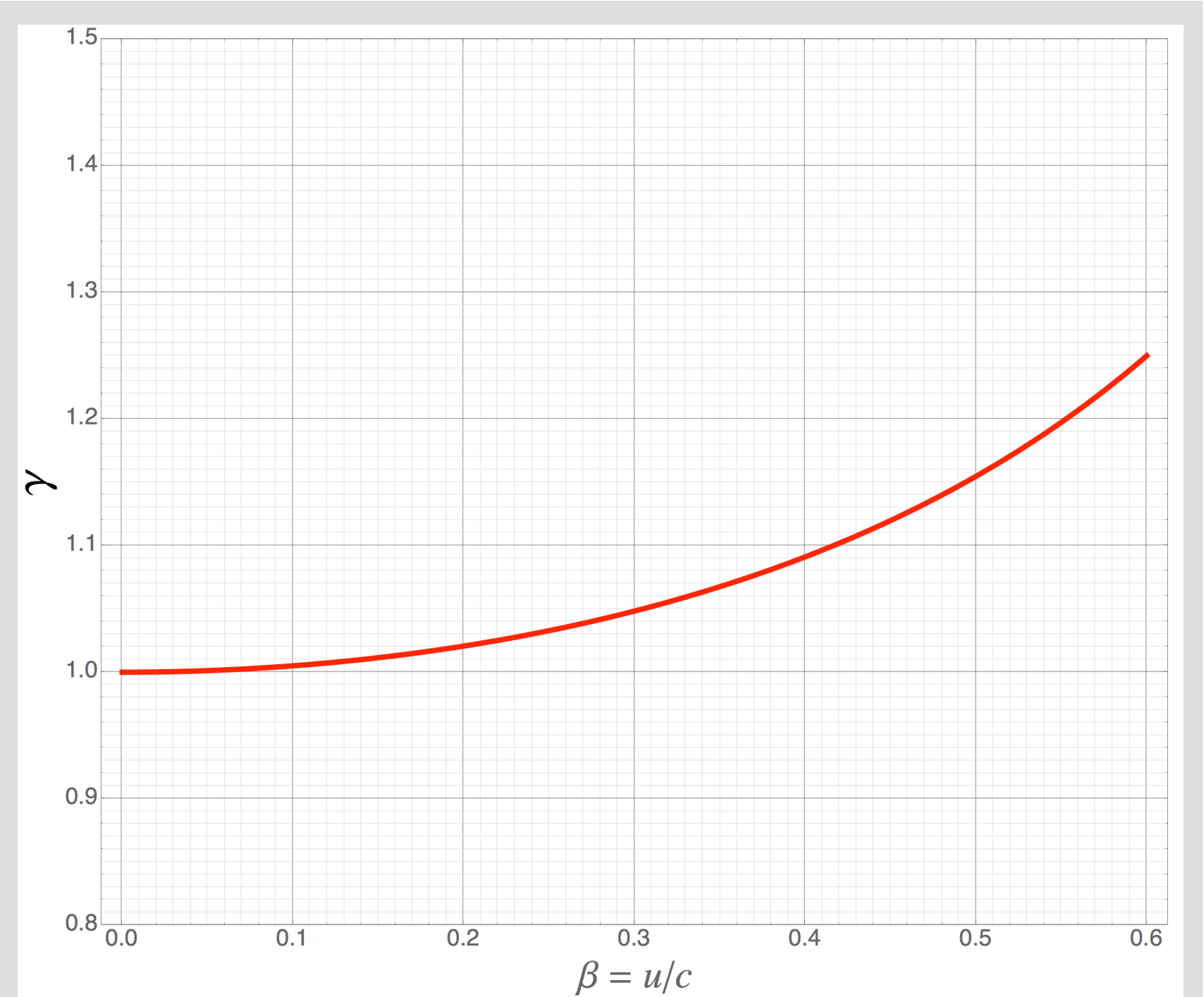
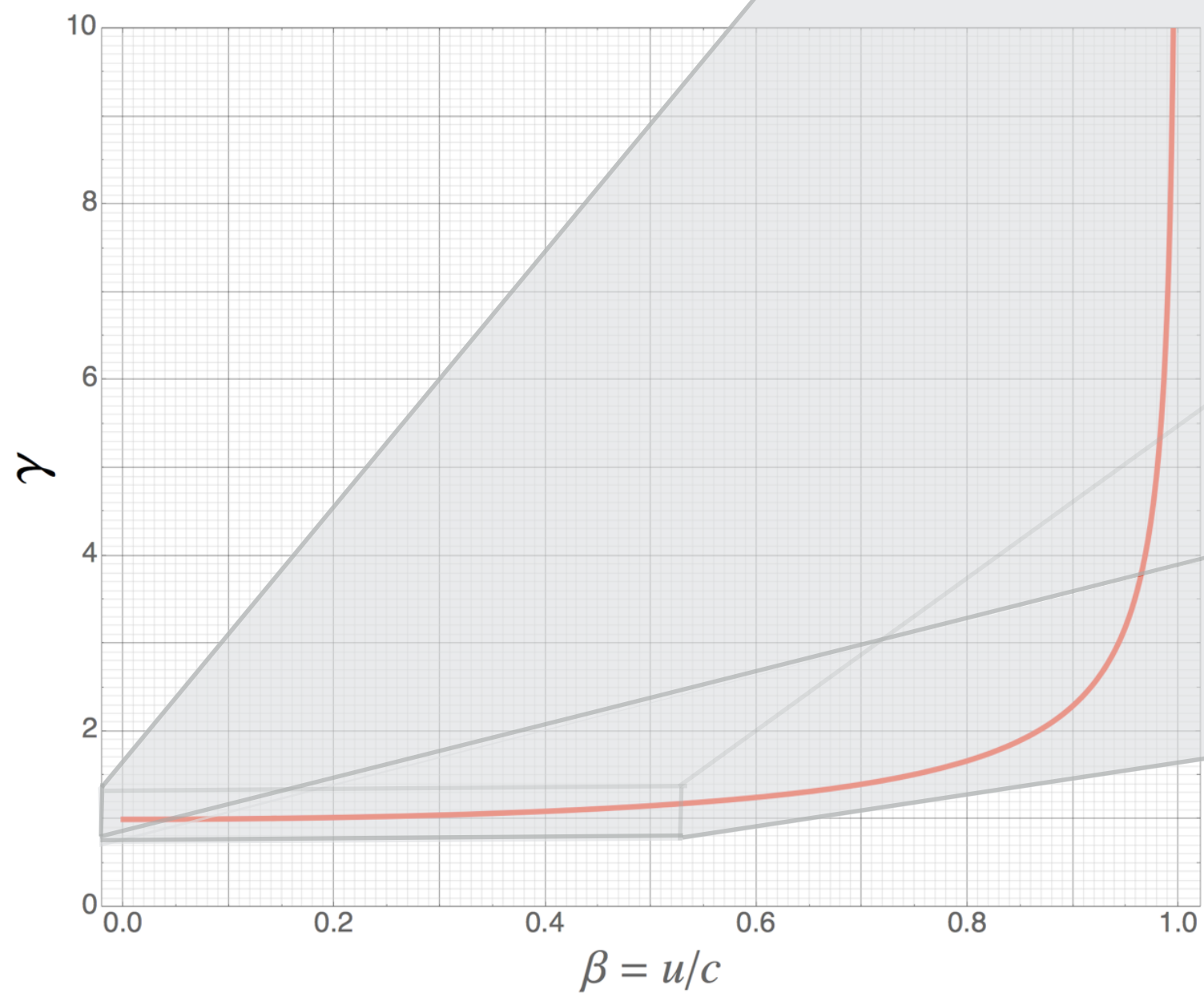
$$\beta = u/c$$

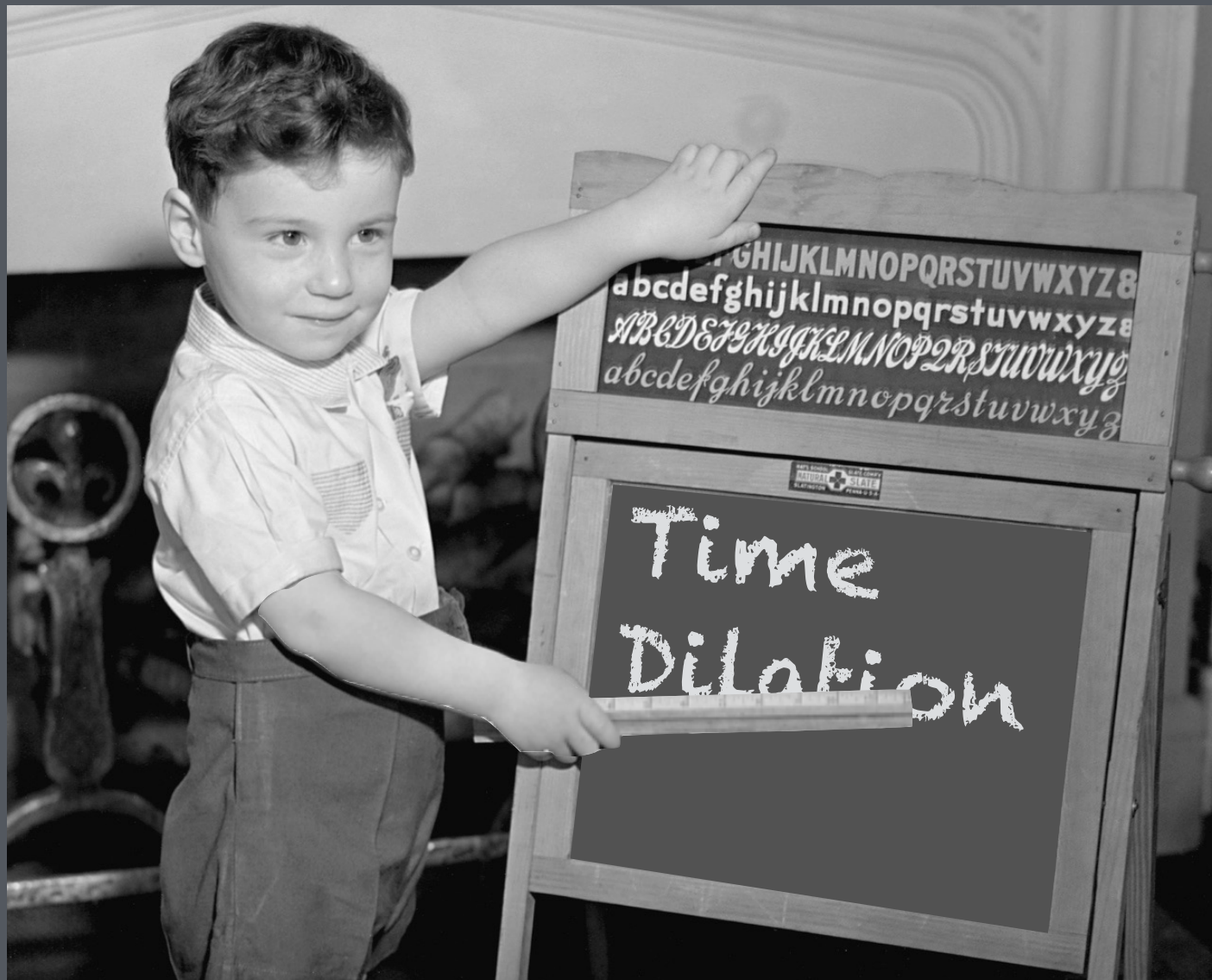


“relativistic gamma”

$$\beta = u/c$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$





You have a clock and I have a clock and they are identical. I observe yours is in an inertial frame of reference moving past my frame of reference.

I also observe that 1 hour on your clock seems to take 2 hours on my clock.

Yours appears to be slower or faster than mine?

How fast is your frame moving relative to mine?

remember what's constant...

The speed of light, ca speed.

$$c = \frac{\text{distance interval}}{\text{time interval}}$$

If clocks are messed with Δt depends on the frame...

and the velocity of light is constant...

Doesn't it stand to reason that lengths are also messed with...

ΔL depends on the frame...?

...shorter as viewed
from the home frame:

$$L_H = \frac{L_A}{\gamma}$$

← a length in the away
frame will seem...

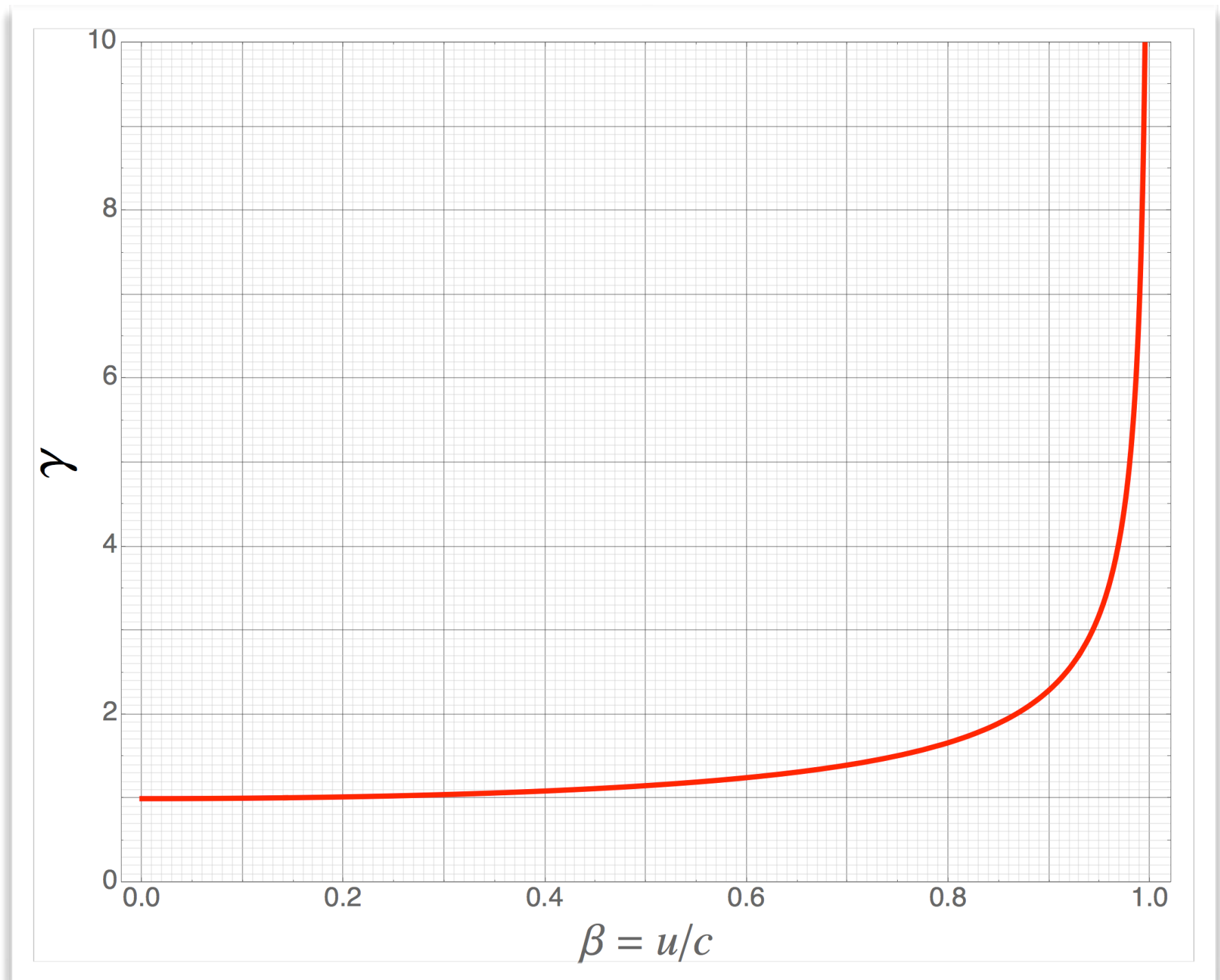
← > 1

Moving lengths appear shorter to a relatively stationary observer

$$L_H = \frac{L_A}{\gamma}$$

length contraction

the third of 3 strange things about space and time



the airport

“Away Frame”:
the frame being watched

x_A
 x_H

“Home Frame”:
watching a moving frame

moving at velocity u



what's he see?





How fast must a meter stick be traveling relative to you in order for its length to appear to be 30 cm as measured by you?

collecting these two consequences

of the two simple postulates

"Time Dilation":

$$t_H = \gamma t_A$$

Moving clocks appear to run slower as seen by a relatively stationary observer

"Length Contraction":

$$L_H = \frac{L_A}{\gamma}$$

Moving lengths appear shorter to a relatively stationary observer

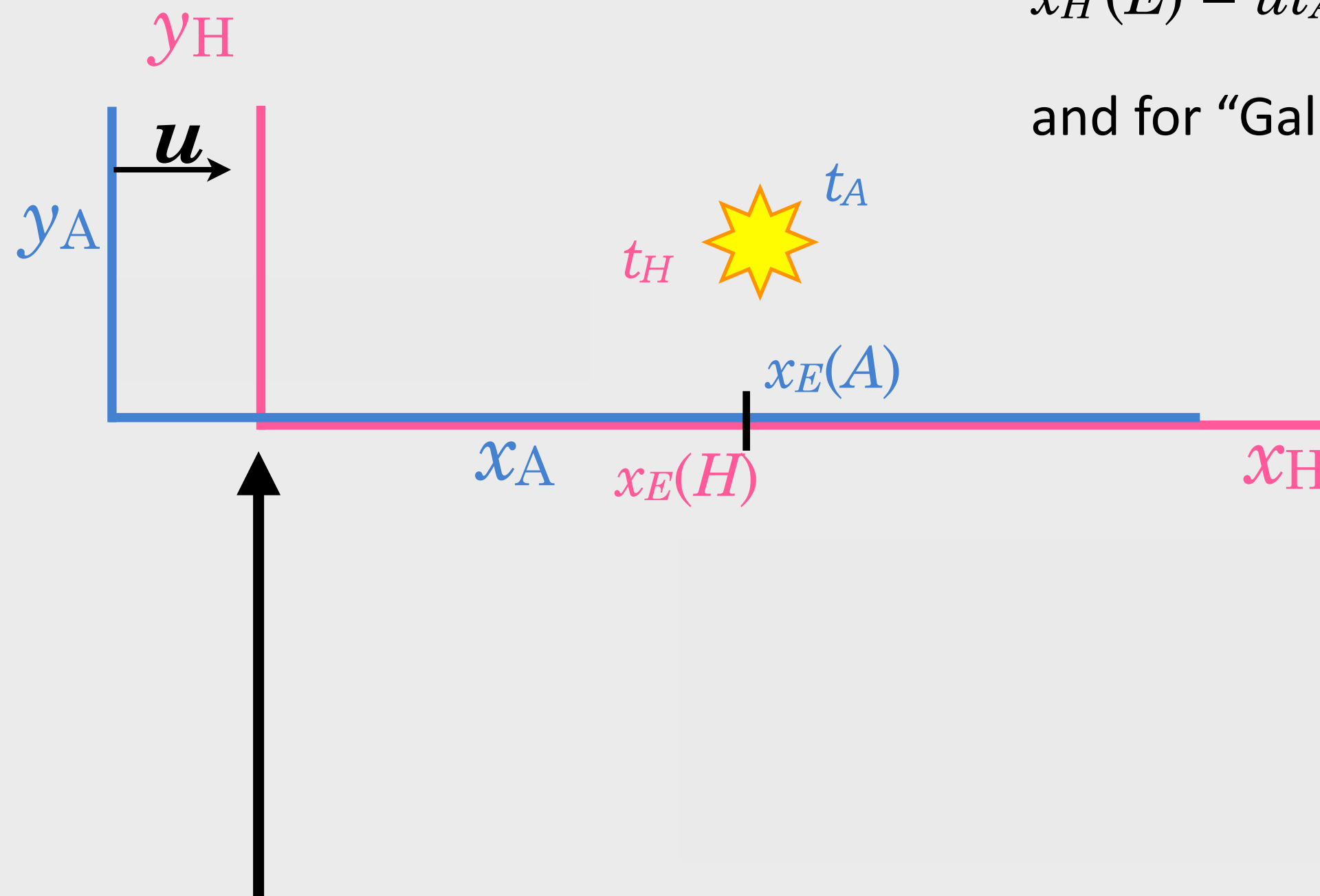
Newton/Galileo?

mixes space coordinates

The Galilean Transformation:

$$x_H(E) = ut_A + x_A(E)$$

and for “Galilean transformations”: $t_H = t_A$



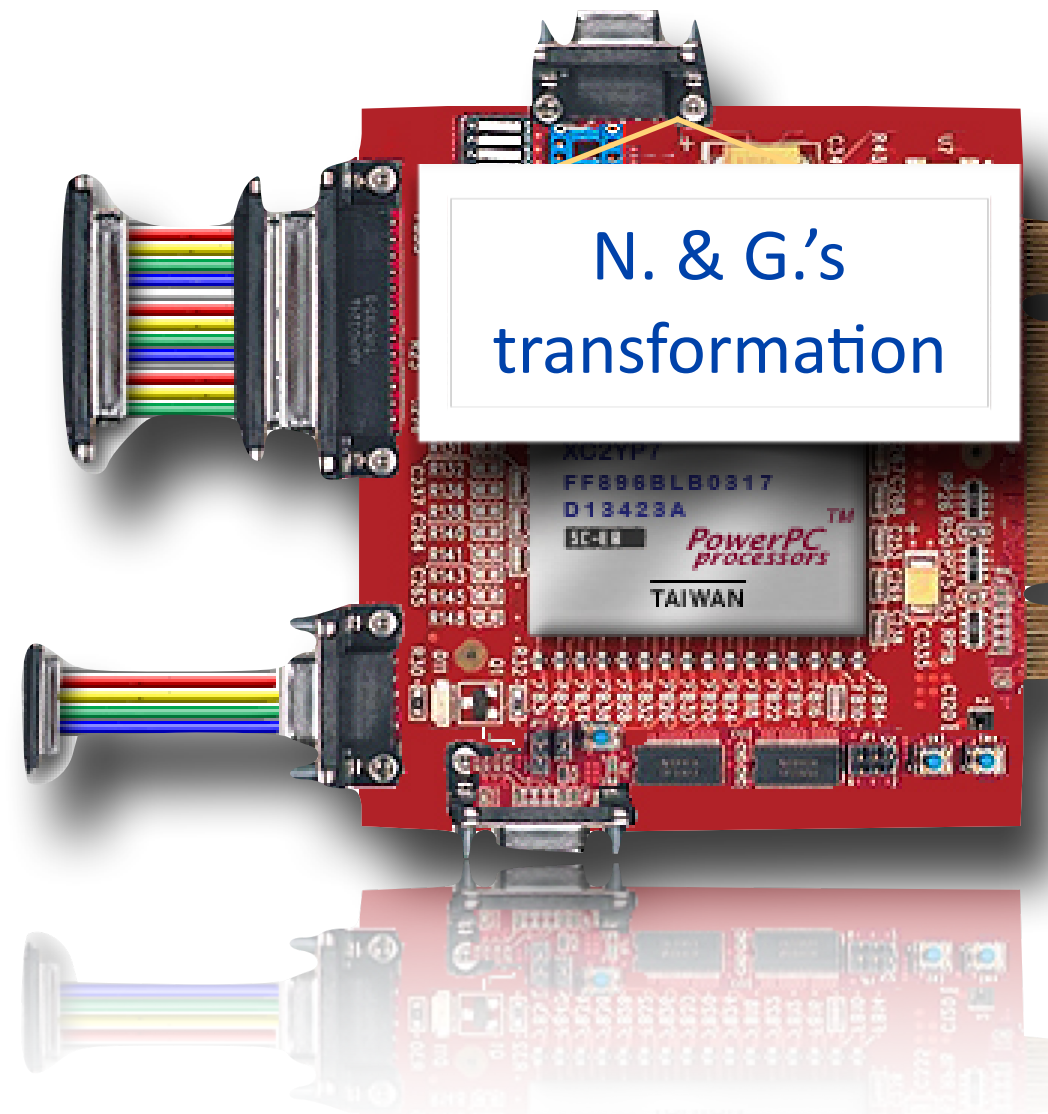
when A's and H's origins cross: start clocks in both.

$t_A(E) = 0$ & $t_H(E) = 0$ when $x_A(E) = 0$ & $x_H(E) = 0$.

Newton/ Galileo?

mixes space
coordinates

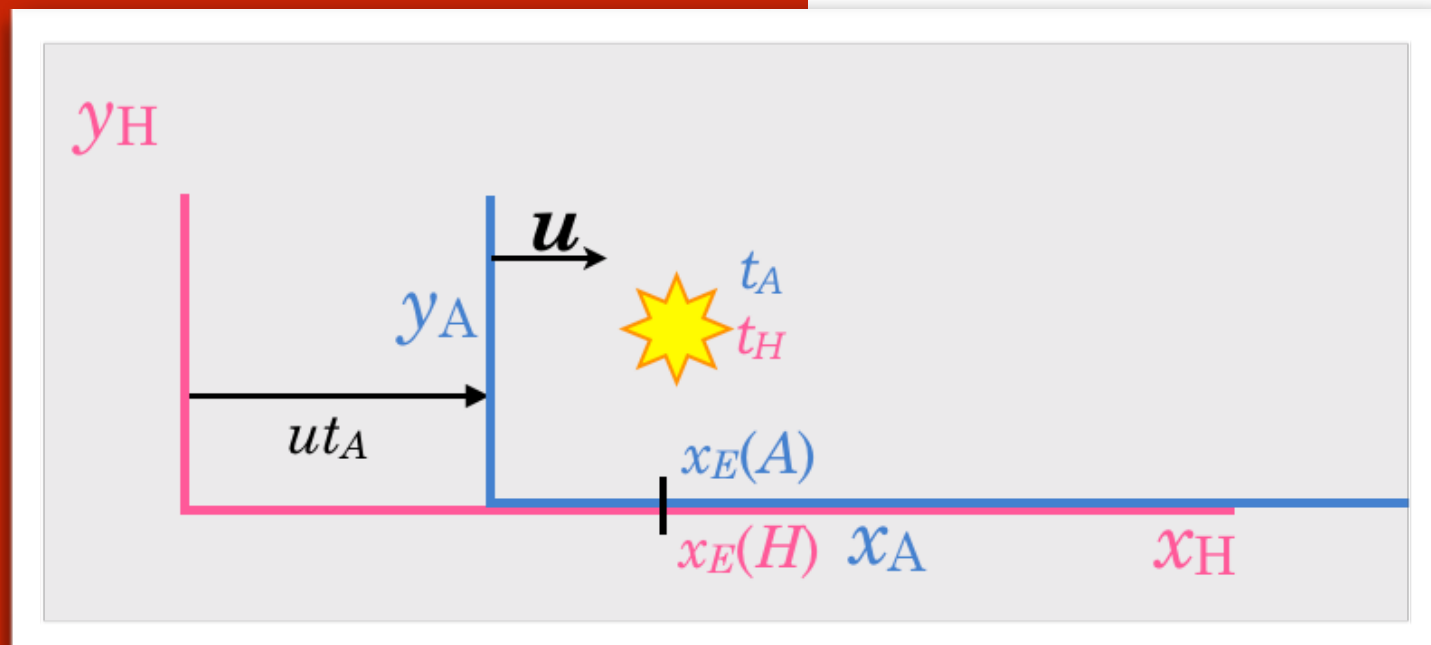
x_A, t_A



Galilean Transformations

$$x_H = x_A + ut$$

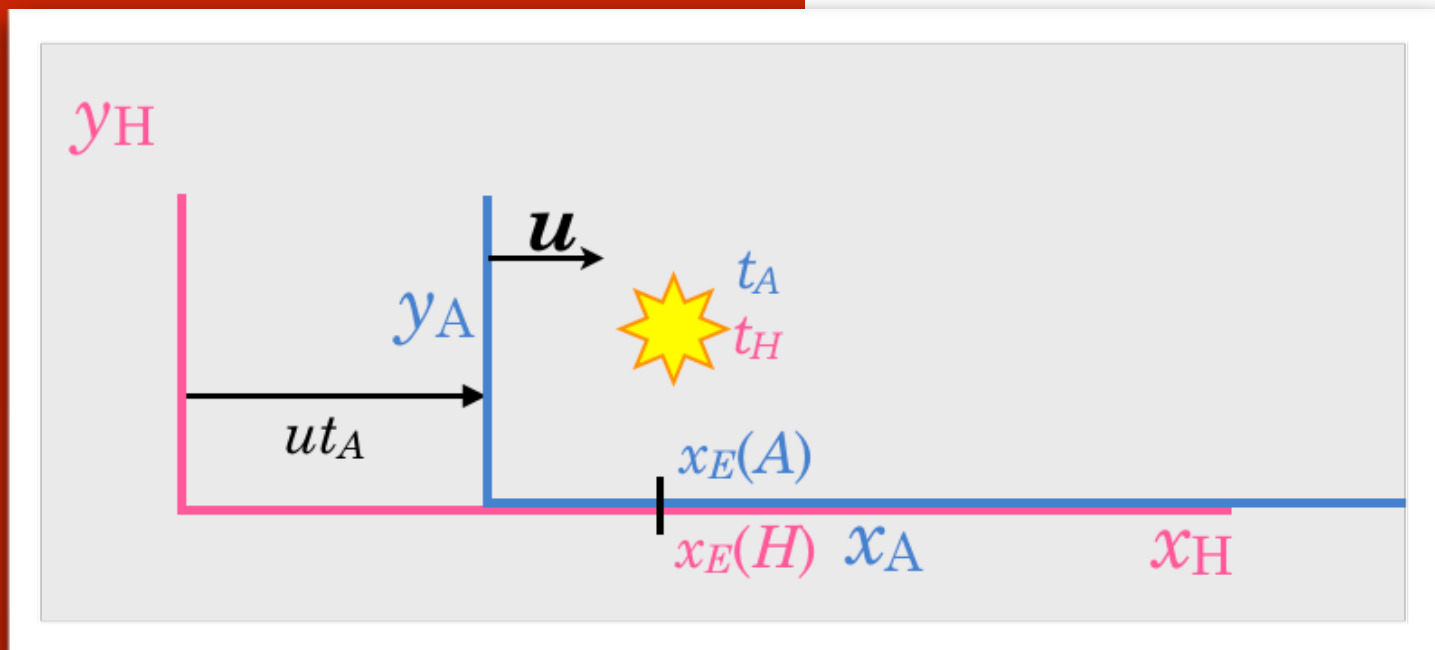
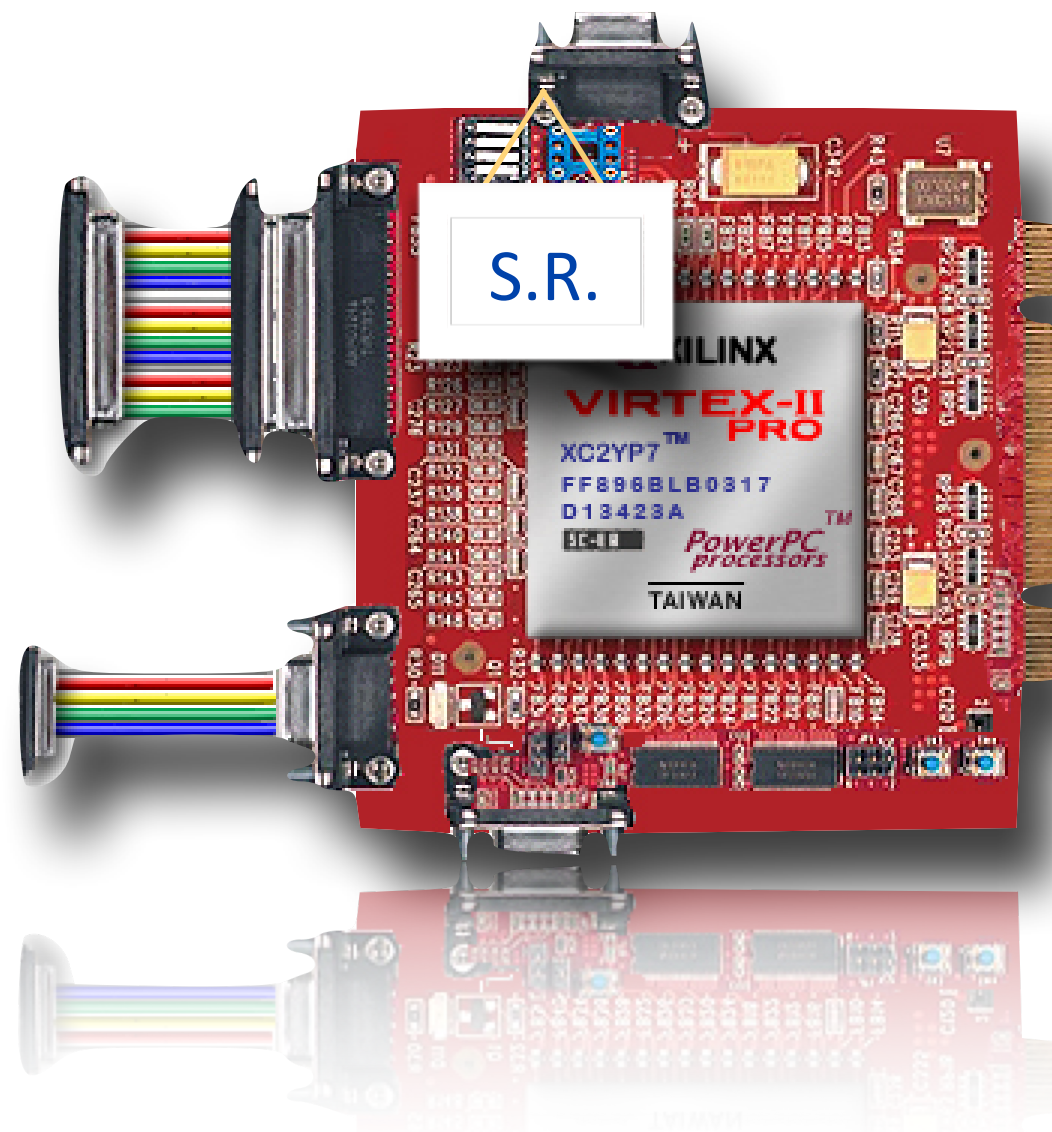
$$t_H = t_A = t$$



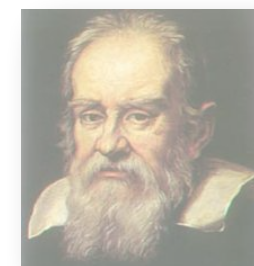
Einstein?

mixes space and time coordinates

x_A, t_A



The prescription is called the **Lorentz Transformations**



$$x_H = \gamma(x_A + ut_A)$$

$$x_H = x_A + ut$$

$$t_H = \gamma\left(t_A + \frac{u}{c^2}x_A\right)$$

$$t_H = t_A = t$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

Let your fingers do this...and show me: 1.

at the top of your board, write the equation for γ

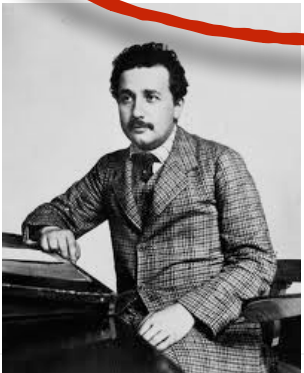
what value does γ approach as $u \ll c$?



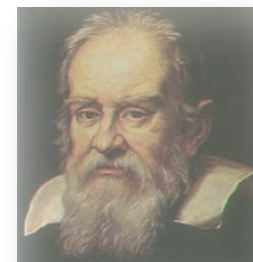
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$$t_H = \gamma\left(t_A + \frac{u}{c^2}x_A\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$



$$x_H = x_A + ut$$



$$t_H = t_A = t$$

Let your fingers do this...and show me, 2.

NOW write the equation for x_H

what value it look like if $u \ll c$?



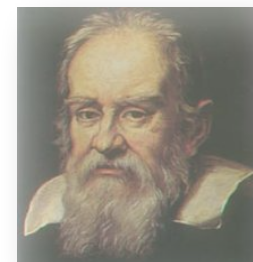
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$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$



$$x_H = x_A + ut$$



$$t_H = t_A = t$$

Let your fingers do this...and show me, 3.

NOW write the equation for t_H

what value it look like if $u \ll c$?



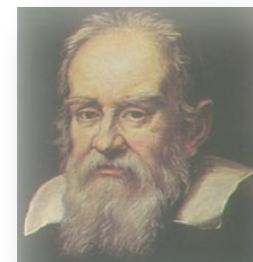
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$$x_H = x_A + ut$$



$$t_H = t_A = t$$