hi

Lecture 14, 23.02.2017

Einstein’s Theory of Special Relativity, 3

housekeeping

Question about anything?

I'll make a movie for you:

Marie Curie movie anyone?

yes! I'll organize for after break…looks like room available March 15, 7-9pm

Midterm…before ~~or after~~ Spring Break. After:

"The midterm will be released on Sunday night, February 26th and close on Tuesday night, February 28. It will cover all of the material through Tuesday, February 21st class."

See calendar cartoon:

next few weeks

S	M	T	W	Th	F	Sa
						2/25 HW6 due
2/26	2/27	2/28	3/1	3/2	3/3	3/4 HW7
3/5	midterm					3/11
3/12	3/13	3/14	3/15	3/16	3/17	3/18 HW7 due HW8

Spring Break

Honors Project

has begun.

Read the first of two sets of instructions:

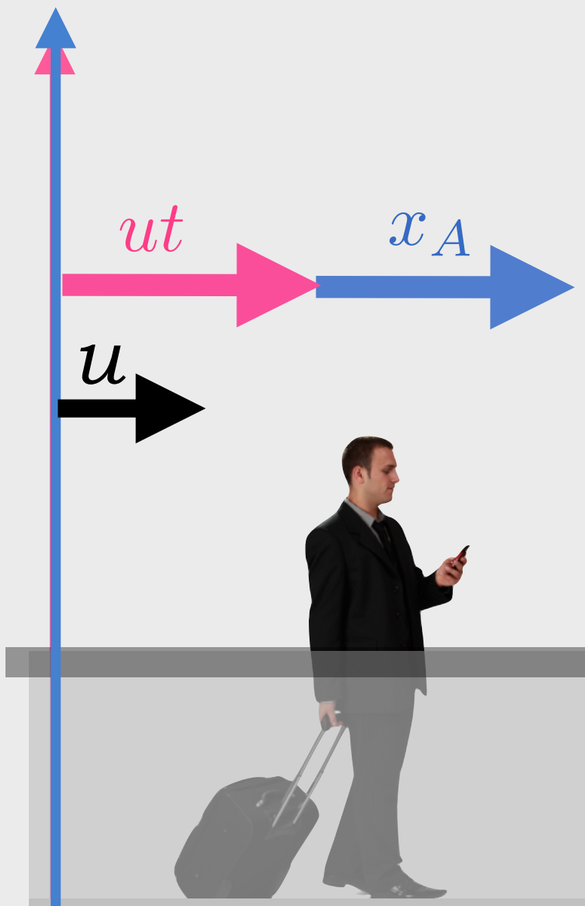
`MinervaInstructions1_2017.pdf` in

www.pa.msu.edu/~brock/file_sharing/QSandBB/2017homework/honors_project_2017/

the airport

"Galilean Transformation"

$$x_H = x_A + ut$$



"Away Frame":
the frame being watched

x_A

x_H

x_H

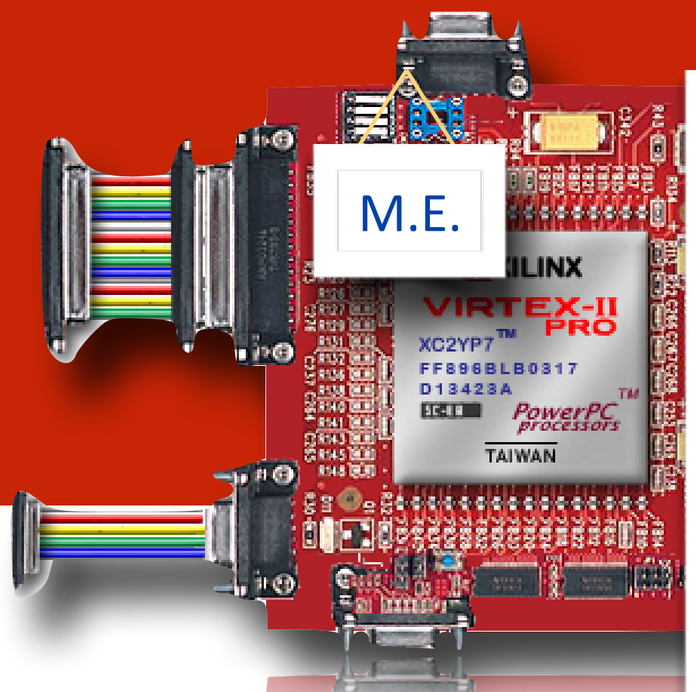
"Home Frame":
watching a moving frame

moving at velocity u

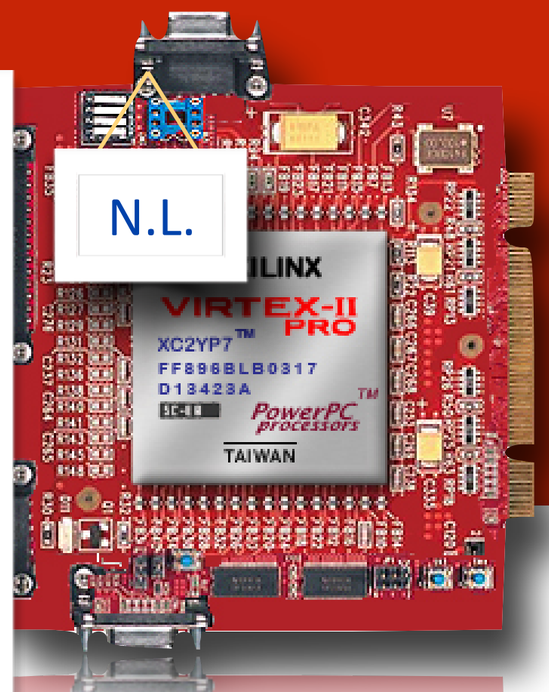
this is crazy! the two models of the world differ

in their treatment of relatively-moving frames of reference!

Seems to depend on Frame:



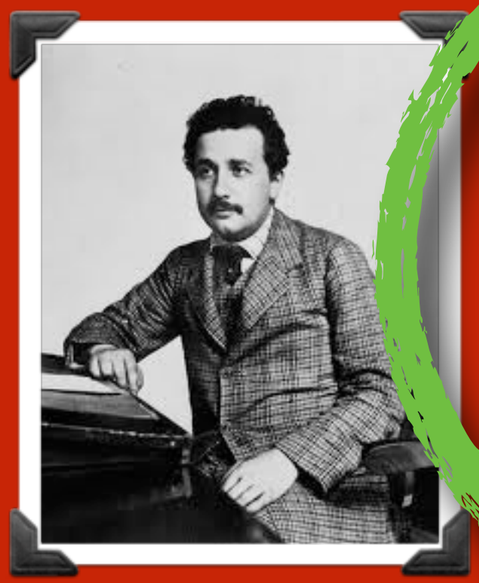
Don’t appear to depend on Frame:



Principle of Relativity

2 Postulates:

"inertial frame": constant velocity

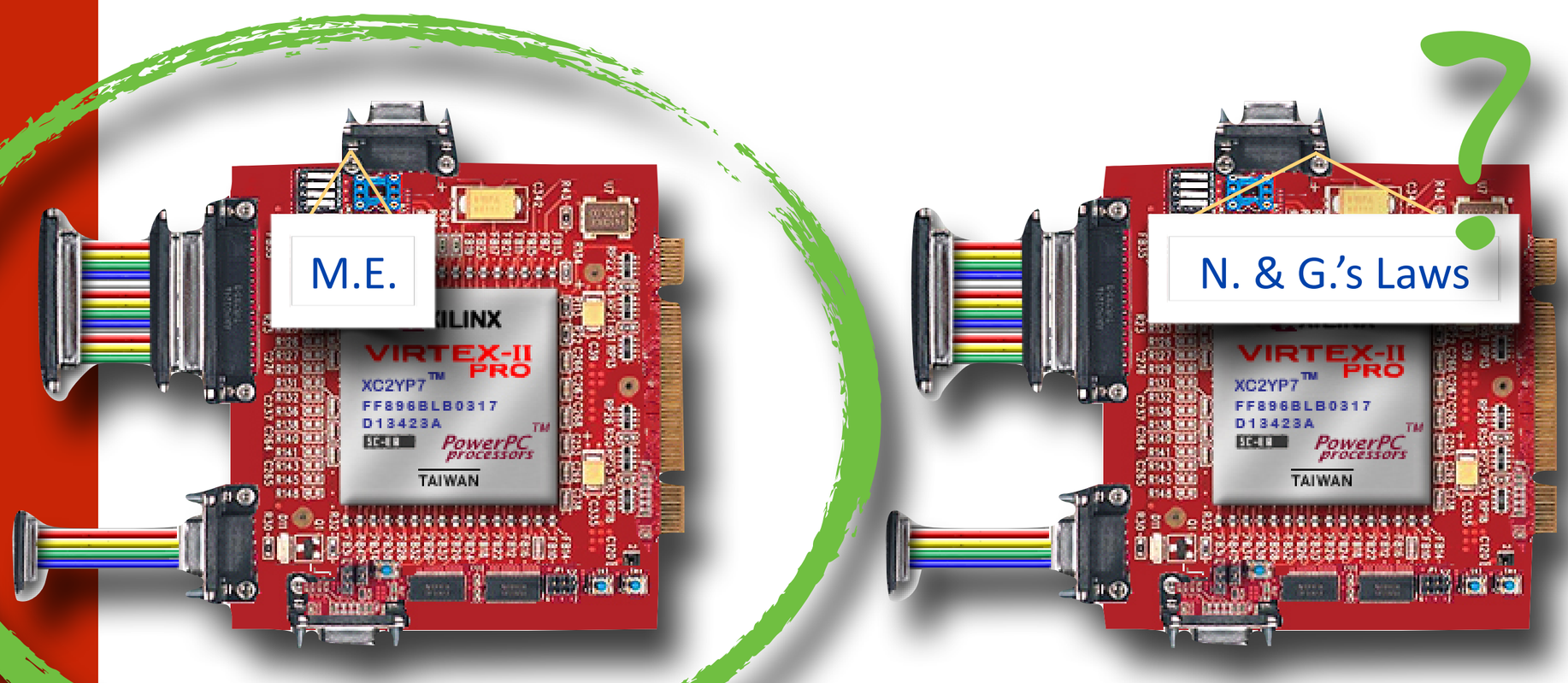


1. All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.

taking Galileo seriously, and then adding Maxwell

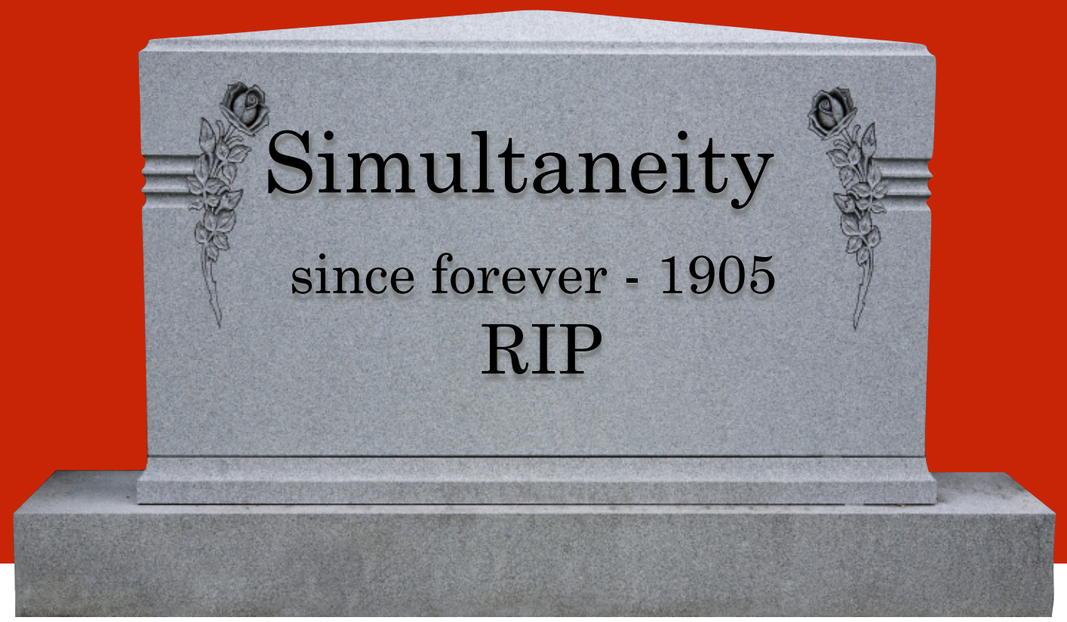
2. The speed of light is the same for all inertial observers.

taking Maxwell seriously



There is no such thing as the *concept of* simultaneous events

between co-moving frames of reference



two problems with this:

1. Since there is no way to determine that something is simultaneous in one frame and also in another

> one can never synchronize clocks between co-moving frames of reference
>
> so one can never confirm or disconfirm the reality of a special frame of reference*

2. The notion that a *cause* always precedes an *effect* seems threatened.

*critical,... queue soapbox:

to science: disconfirmation

not “‘proof”

not “belief”

Unsure about someone’s “scientific” assertion?

Ask what it would take to change your mind.

the 2nd postulate

makes things strange

because c

the speed of light is constant in all inertial frames:

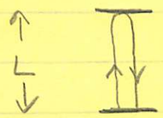
$c = 3 \times 10^8$ m/s = 300 million m/s = 1,080 million km/h

c = 671 million mph

alculation: time dilation 1

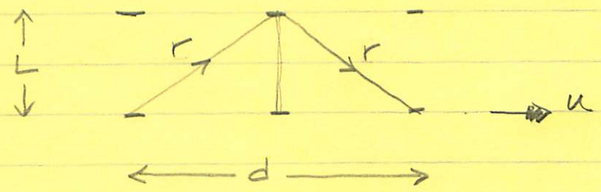
Time Dilation

Away Frame



round trip distance $2L$

Home Frame



round trip distance

$r^2 = L^2 + (d/2)^2$

$r = \sqrt{L^2 + (d/2)^2} \rightarrow 2r = 2\sqrt{L^2 + \left(\frac{d}{2}\right)^2}$

Away - time it takes: t_A

$c_A = \frac{2L}{t_A}$ speed of light in A = c_A

$t_A = \frac{2L}{c_A} \longrightarrow c_A t_A = 2L$

Home - time it takes: t_H

$c_H = \frac{2r}{t_H} = \frac{2\sqrt{L^2 + (d/2)^2}}{t_H}$

$t_H = \frac{2\sqrt{L^2 + (d/2)^2}}{c_H} \rightarrow c_H t_H = 2\sqrt{L^2 + (d/2)^2}$

Square both:

Away	Home
$(c_A t_A)^2 = 4L^2$	$(c_H t_H)^2 = 4L^2 + 4 \cdot \frac{d^2}{4}$
	$(c_H t_H)^2 = 4L^2 + d^2$

rearrange to isolate $4L^2$: $4L^2 = (c_H t_H)^2 - d^2$

$(c_A t_A)^2 = (c_H t_H)^2 - d^2$

Einstein's 2nd Postulate says: $c_H = c_A \equiv c$

$(c t_H)^2 - d^2 = (c t_A)^2$

factor

$t_H^2 \left(c^2 - \frac{d^2}{t_H^2} \right) = c^2 t_A^2$

notice

↑ distance traveled in t_H

$u = \frac{d}{t_H}$

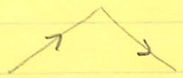
$t_H^2 \left(c^2 - u^2 \right) = c^2 t_H^2$

$t_H^2 \left(1 - u^2/c^2 \right) = t_H^2$

alculation: time dilation 2

and $t_H = \dfrac{t_A}{\sqrt{1 - u^2/c^2}}$

↕ tick-tock observed by A different from



tick-tock observed by H

by $\dfrac{1}{\sqrt{1 - u^2/c^2}} \equiv \gamma$ "gamma factor" "relativistic gamma"

$\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$ notice: if $u = c$

$\gamma \to \dfrac{1}{\sqrt{1 - c^2/c^2}} = \infty$!

so u can't equal c!

if $u > c$

$\gamma \to \dfrac{1}{\sqrt{1 - \text{something bigger than 1}}}$

↑ imaginary

so u can't be > c!

$\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$ so define $\beta \equiv u/c$

↑ less than 1

$\gamma = \dfrac{1}{\sqrt{1 - \beta^2}}$ ← less than 1 so $\boxed{\gamma > 1}$

Time Dilation

$t_H = \gamma t_A$

↑

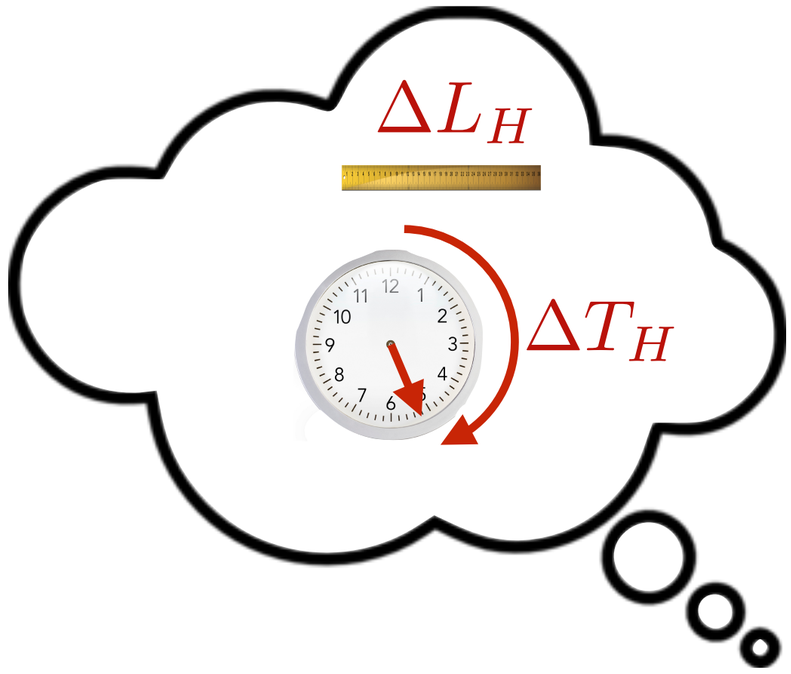
γ • tick-tock in A frame is bigger as seen by H frame

so t_H tick-tock is longer & H observes that A's clock appears to run slower than A does.

Suppose $u = 0.86c \Rightarrow \gamma = 1.96$

a factor of 2. So if 10 minutes passes in A, H says that 20 minutes passes for H appears to be

⇒ A's clock is slower



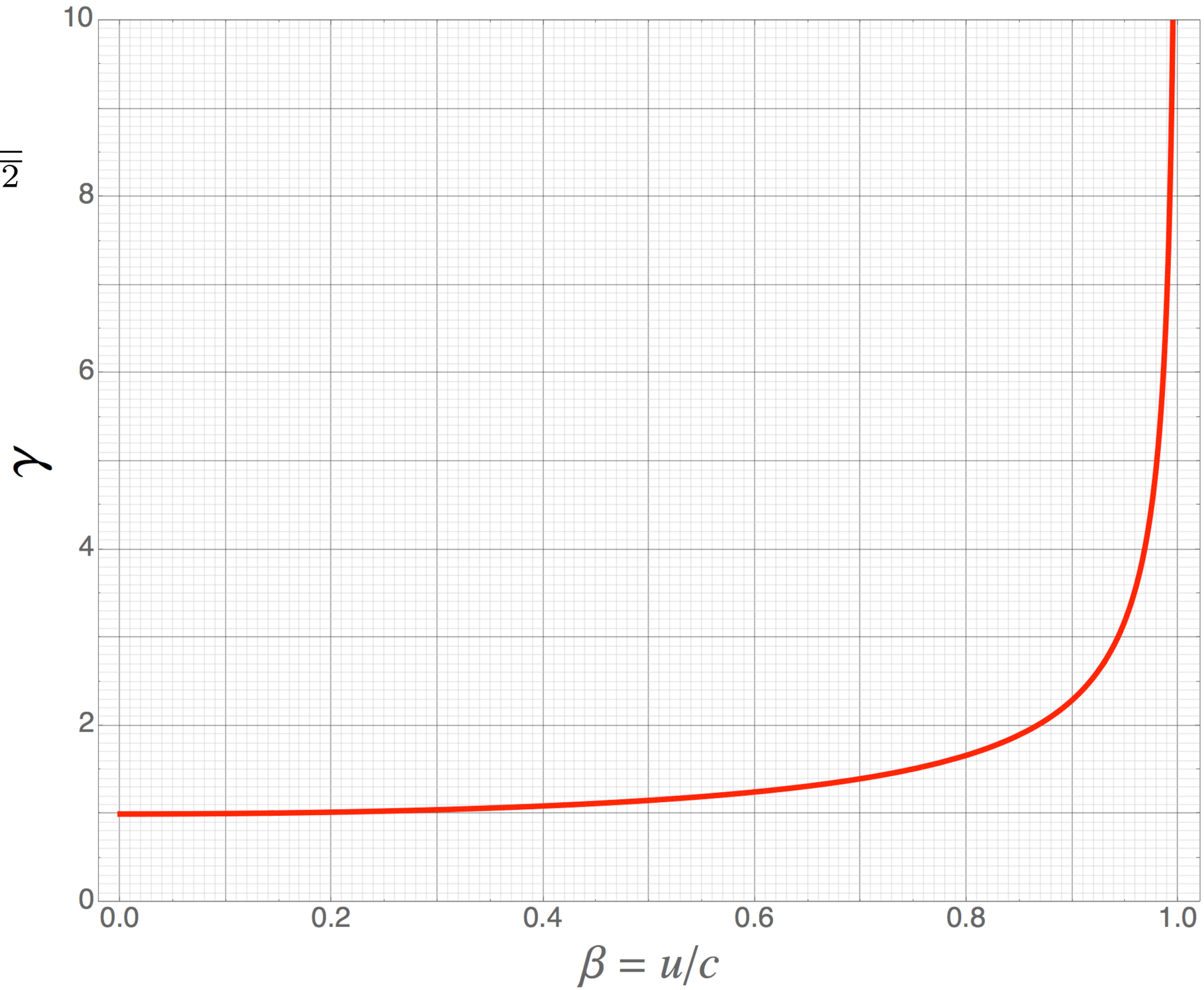
$$\Delta L_H = \frac{\Delta L_A}{\gamma}$$ "length contraction"

$$\Delta T_H = \gamma \Delta T_A$$ "time dilation"

‘‘relativistic gamma’’

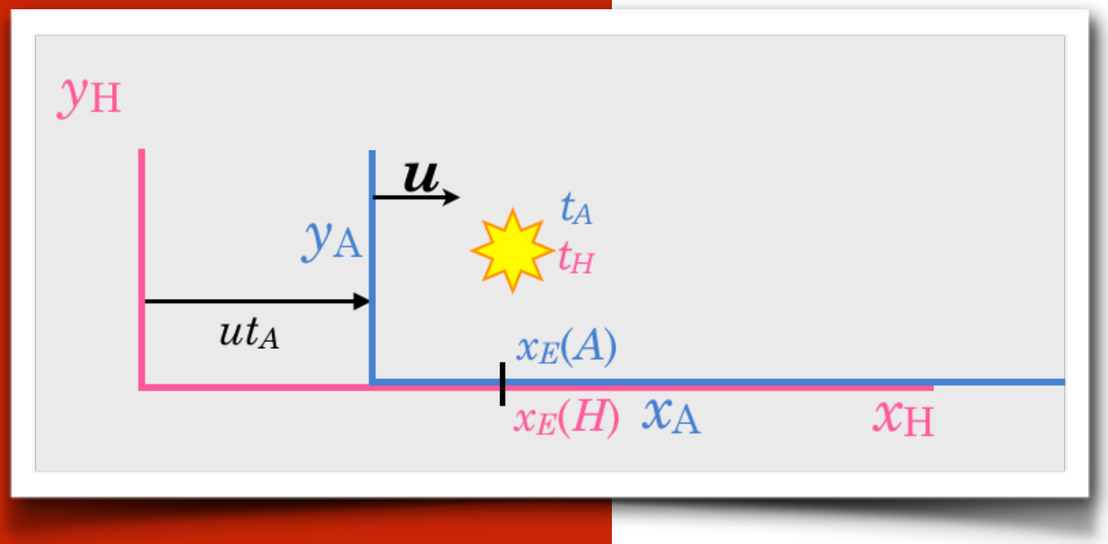
$$\gamma = \frac{1}{\sqrt{1 - (\frac{u}{c})^2}}$$

$$\beta = u/c$$

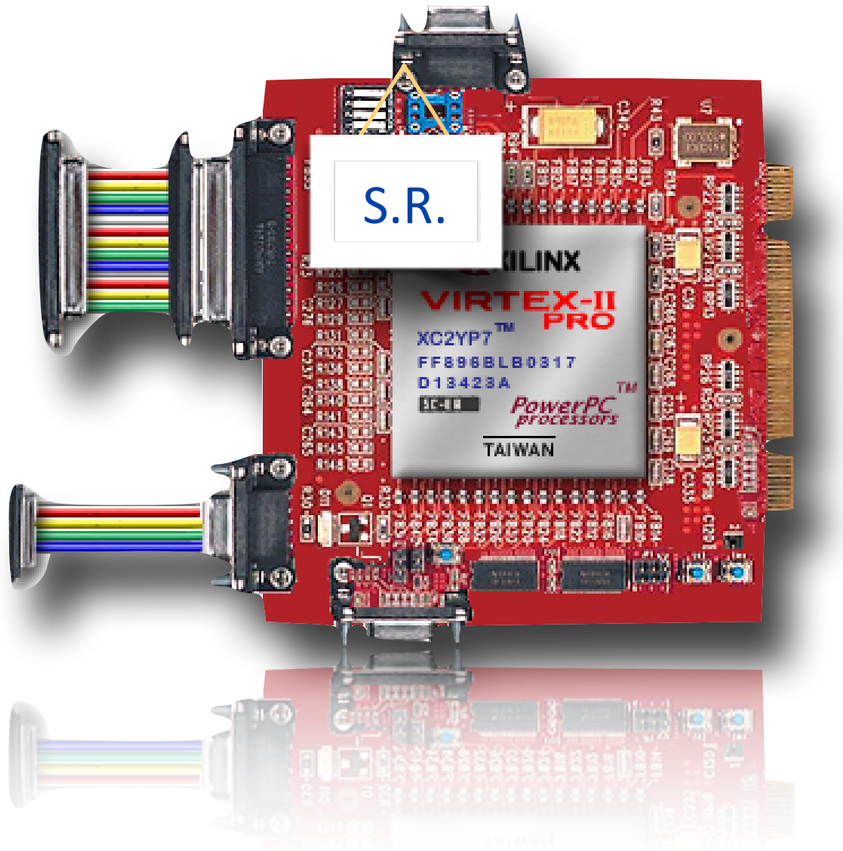


Einstein?

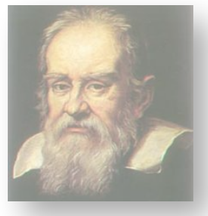
mixes space and time coordinates



x_A, t_A



The prescription is called the **Lorentz Transformations**



$$x_H = \gamma(x_A + ut_A) \qquad x_H = x_A + ut$$

$$t_H = \gamma(t_A + \frac{u}{c^2}x_A) \qquad t_H = t_A = t$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

relatives

this is an electron, e: 

this is a cousin of an electron…the "muon," μ: 

they are exactly alike except that

$$m(\mu) = 209 \times m(e)$$

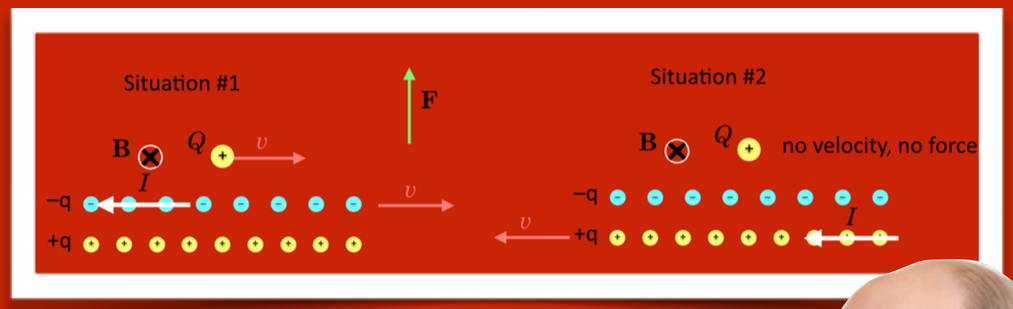
and in about 1.5 microseconds:

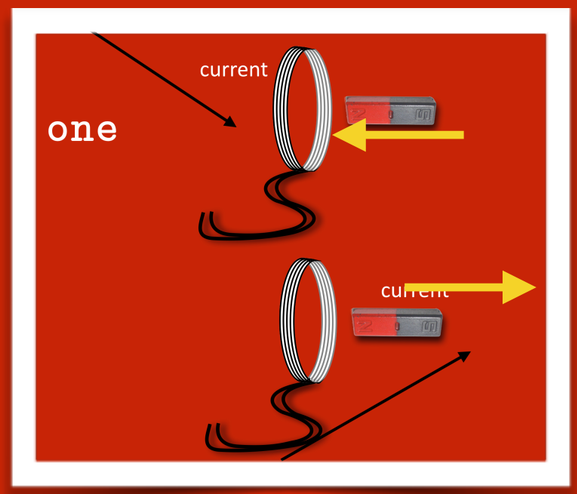
A "muon" has an half-life in it's own rest frame of 1.56 microseconds, 1.56 x 10^{-6} s

suppose it's accelerated in the lab to a β = 0.99,

how long does it appear to live in the lab?

remember?

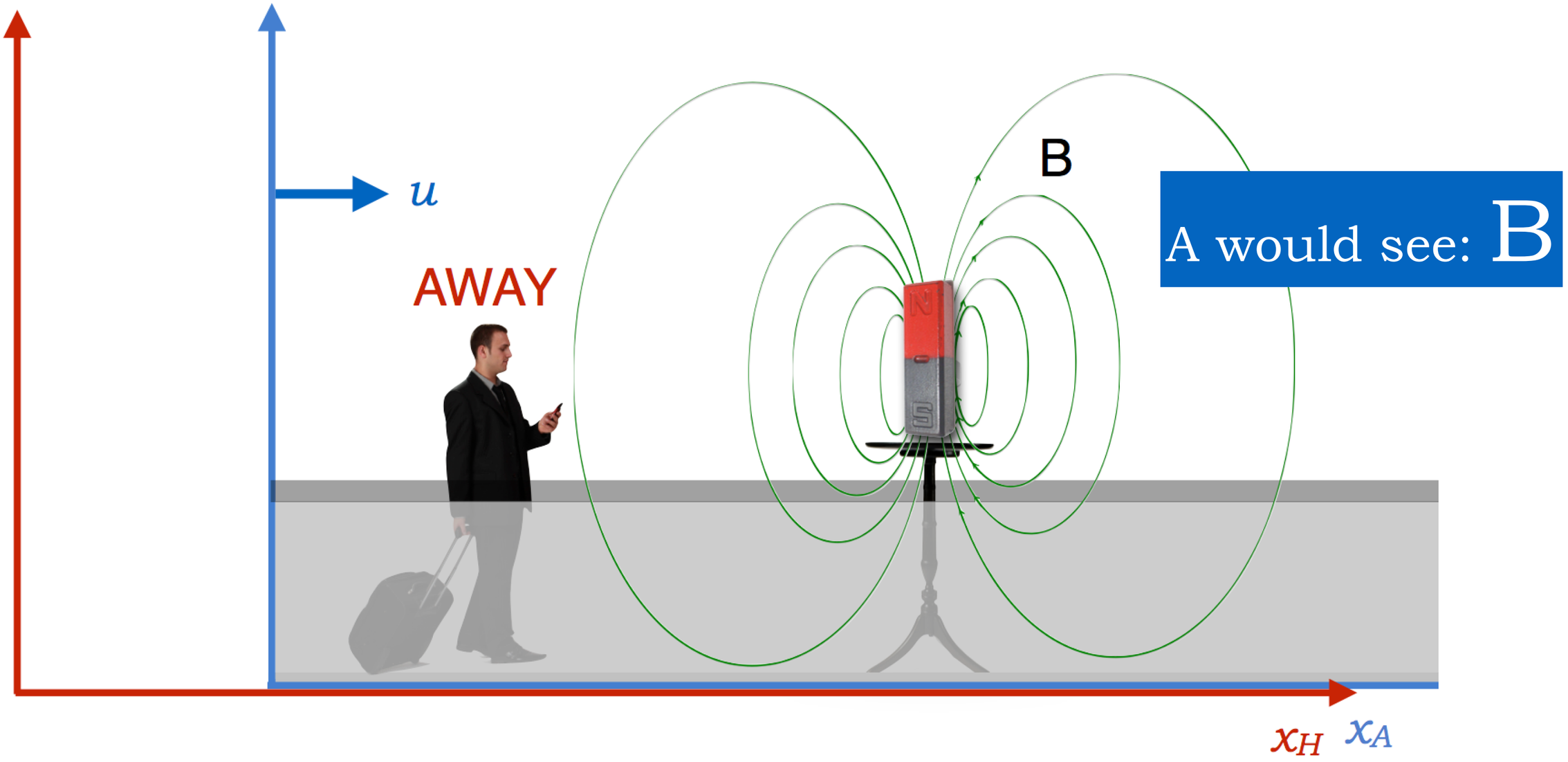




Weird alert #1:
Two different physical outcomes...
for situations which differ only by the frame of reference

Weird alert #2:
Two identical physical outcomes...
from entirely different physical causes for situations which

back to the airport



A would see: B

H would see: E+B!

so the original problems are solved by:

the Lorentz transformations in *x* and *t*

actually **mix** electric and magnetic fields

so

A **magnetic field** in one frame

is a **mixture of magnetic and electric fields** in another frame

An **electric field** in one frame

is a **mixture of electric and magnetic fields** in another frame

so the original problems are solved by:

E and B are two manifestations of one thing:

the Electromagnetic Field

the Lorentz transformations in *x* and *t*

mix electric and magnetic fields

A magnetic field in one frame

is a mixture of magnetic and electric fields in another frame

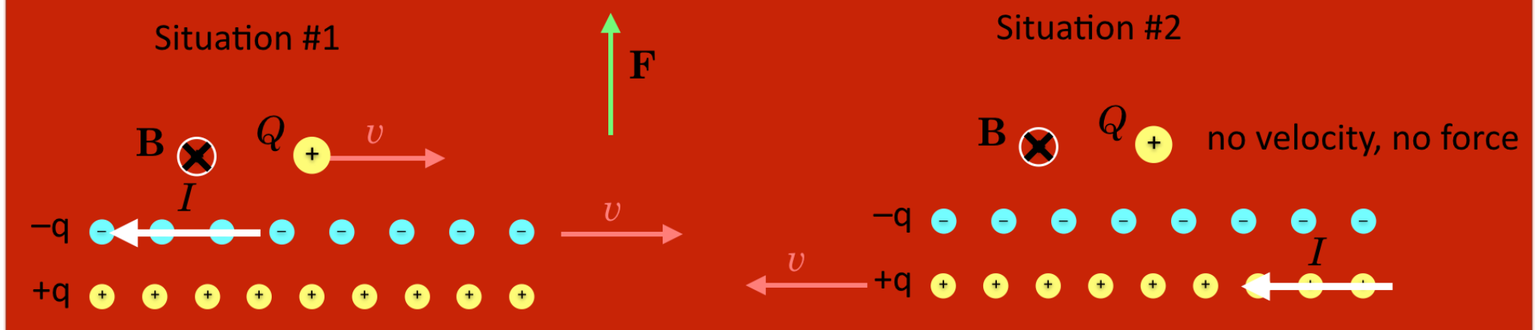
An electric field in one frame

is a mixture of electric and magnetic fields in another frame

remember:

more simple questions

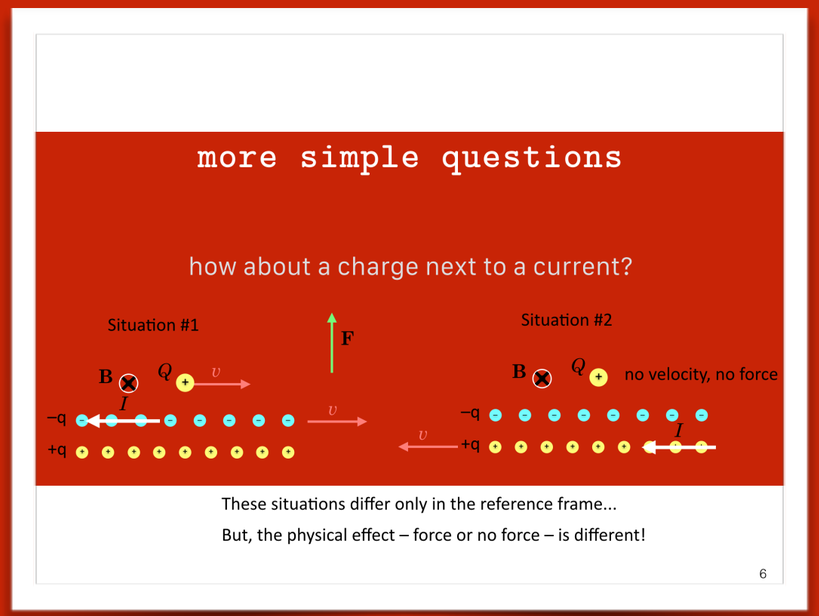
how about a charge next to a current?



These situations differ only in the reference frame...

But, the physical effect – force or no force – is different!

6

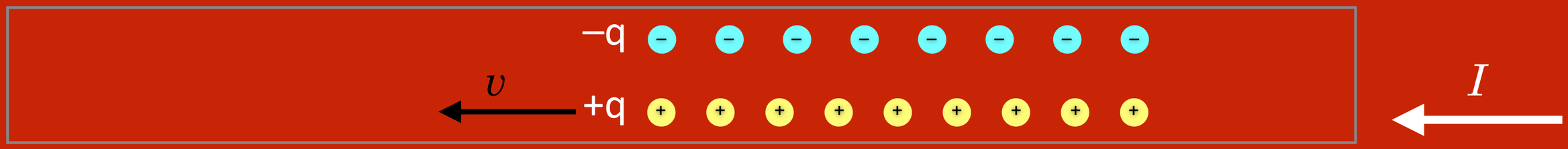


remember?

but there should have been a force!

Situation #2

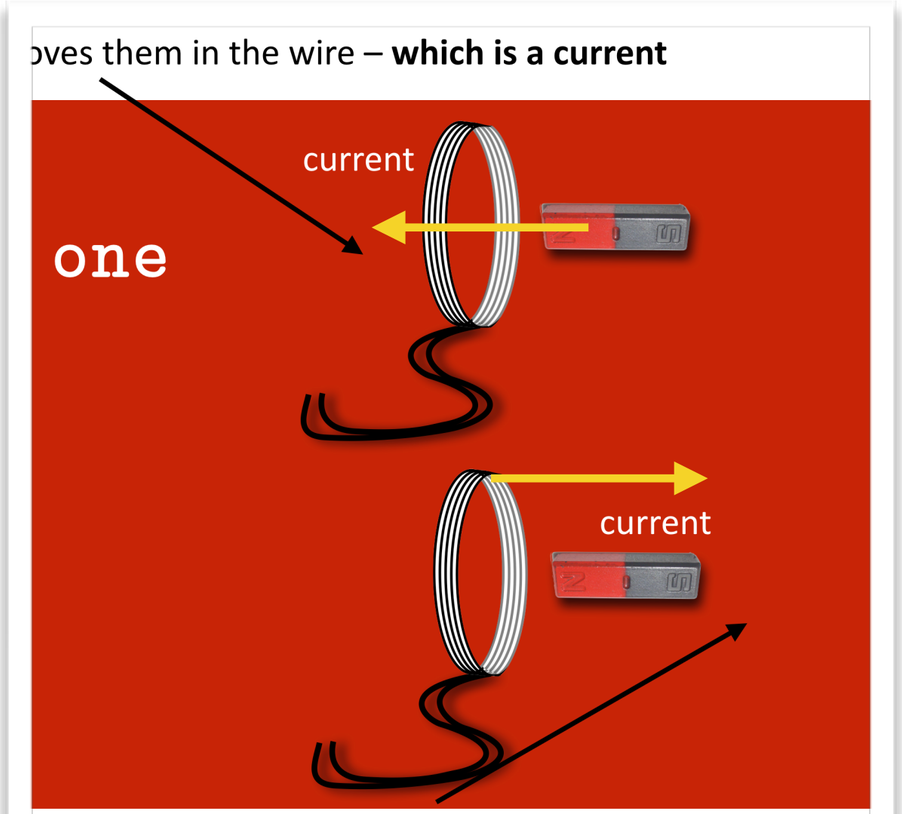
B ⊗ Q + no velocity, no force



and the coil?

yup. right observation all along.

Electric and magnetic fields, depending on the relative frames



the punch line.

Principle of Relativity

2

Postulates:

“inertial frame”:

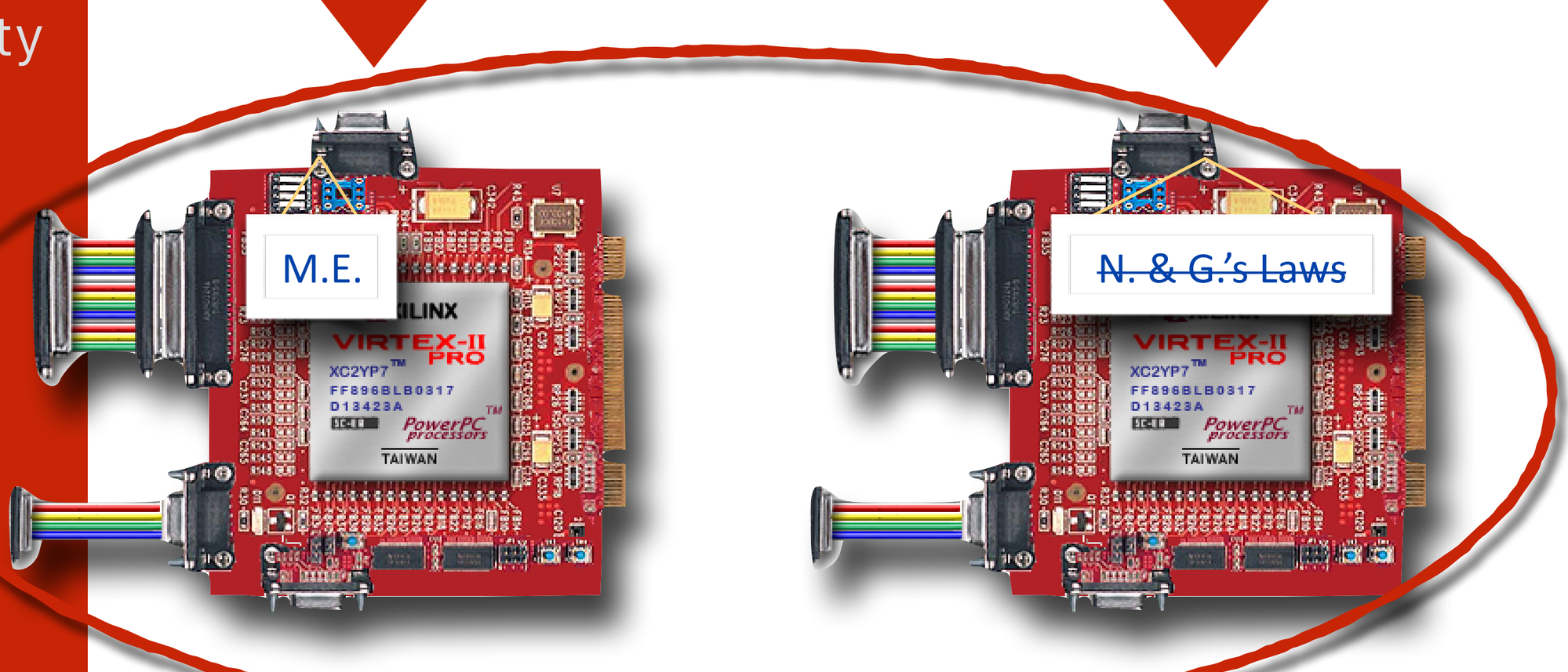
constant velocity

1. All laws of physics –
mechanical and electromagnetic –
are identical in co-moving inertial frames.

2. The speed of light is the same for all inertial observers.

good all along!

had to change!



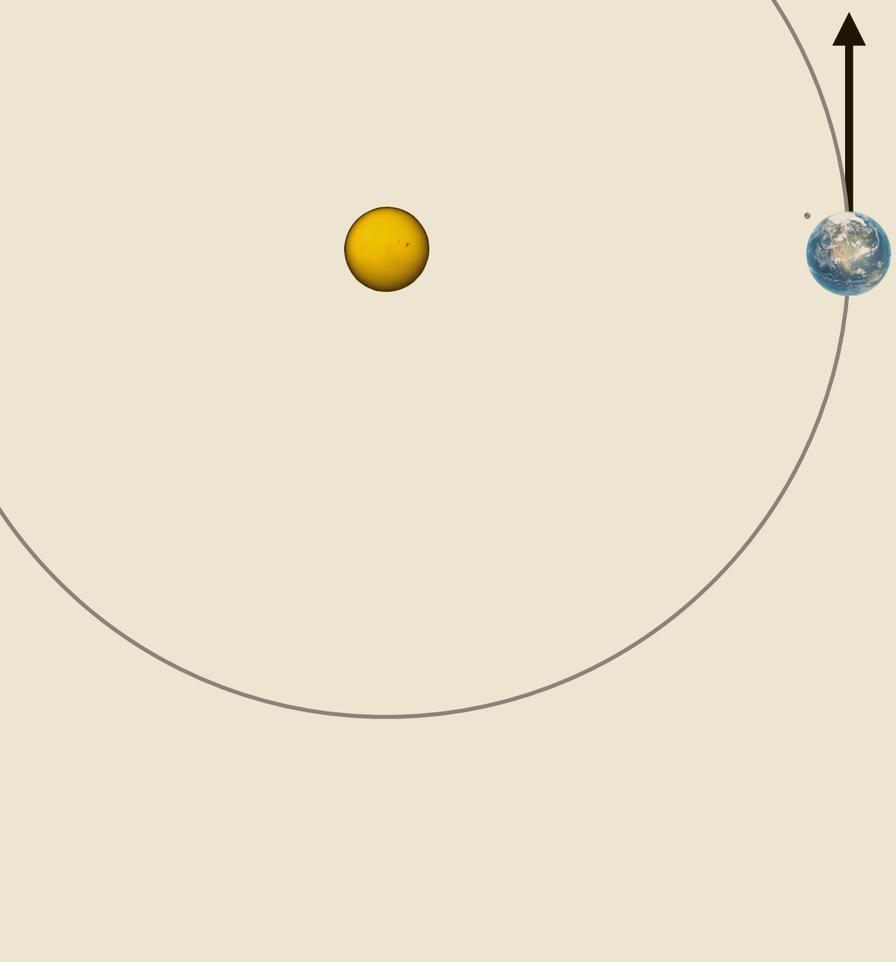
a controversial experiment

not the experiment...but what Einstein knew

moving through the ether?

everyone knew. everyone.

stationary "ether"
a presumed Absolute
rest frame



Earth's Motion
should cause
an "ether wind"

Since the ether
is what light propagates in:

light going upstream
should take a
different time to go a
distance than going
downstream

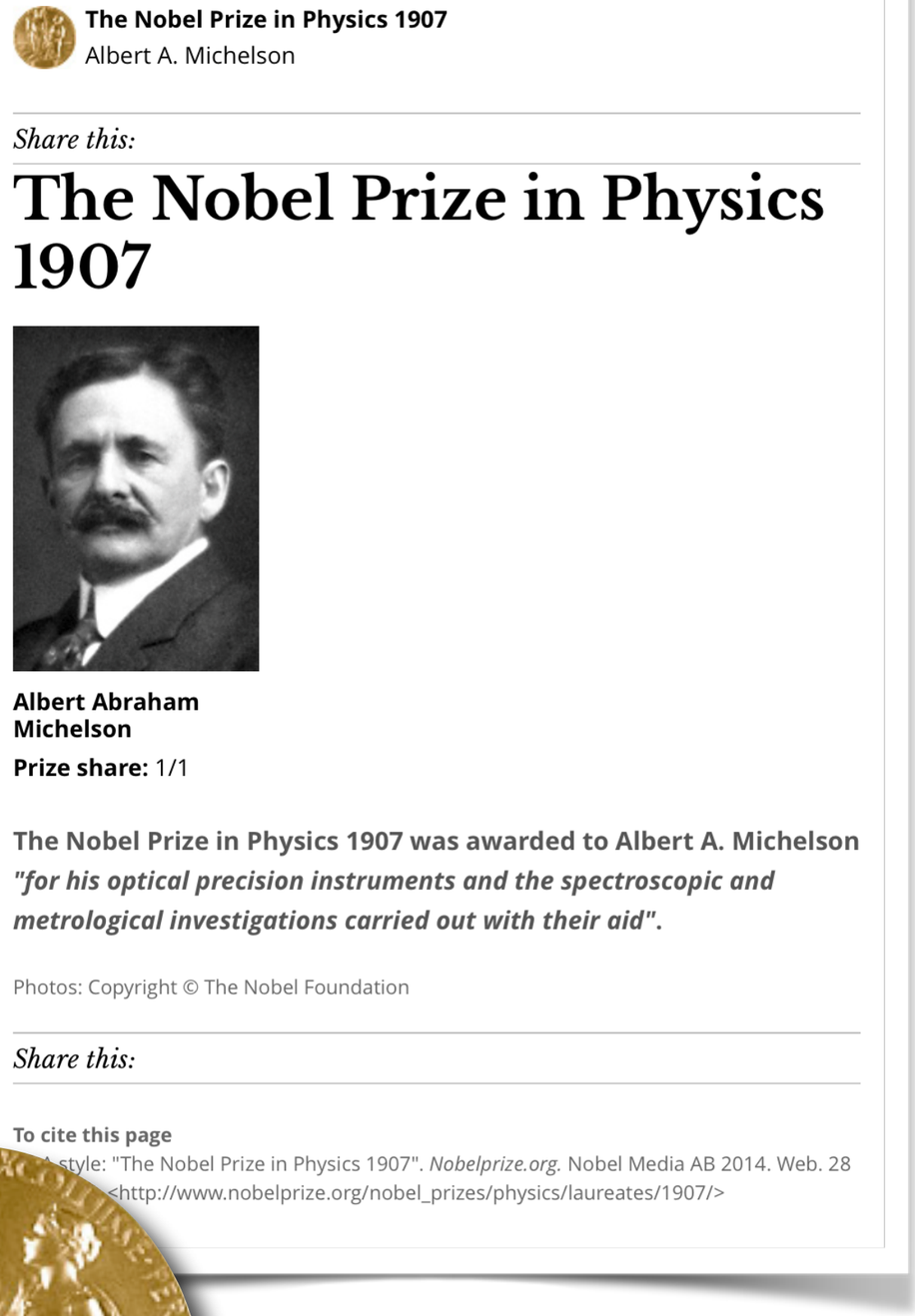
Other measurements showed that the earth doesn't "drag" the ether with it...
ask Mr Google about "stellar aberration"

1887

Albert Michelson (1852-1931)

and

Edward Morley



The Nobel Prize in Physics 1907
Albert A. Michelson

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The Nobel Prize in Physics 1907

Albert Abraham Michelson
Prize share: 1/1

The Nobel Prize in Physics 1907 was awarded to Albert A. Michelson *"for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid".*

Photos: Copyright © The Nobel Foundation

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style: "The Nobel Prize in Physics 1907". *Nobelprize.org.* Nobel Media AB 2014. Web. 28 <http://www.nobelprize.org/nobel_prizes/physics/laureates/1907/>

The prize went to Michelson for the instrument: the Michelson interferometer

the same idea as the gravitational wave instrument

We remember him for the most important null measurement in 200 years

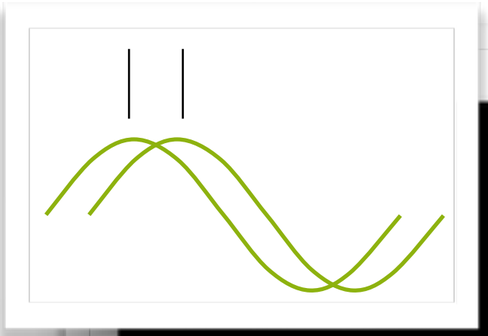
‘‘Michelson Morley Experiments’’

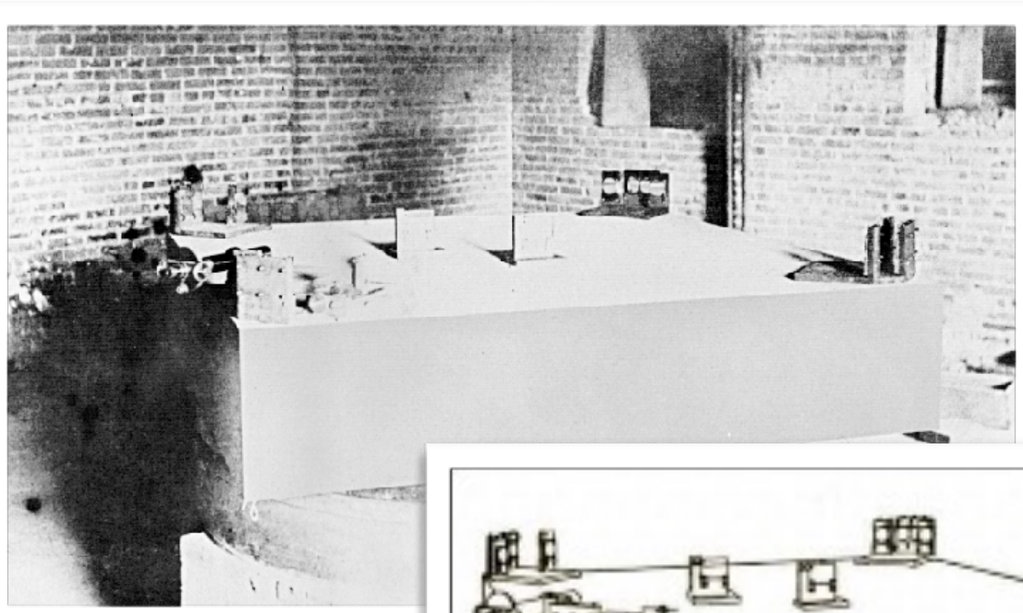
trying to measure the speed of Earth relative to Ether

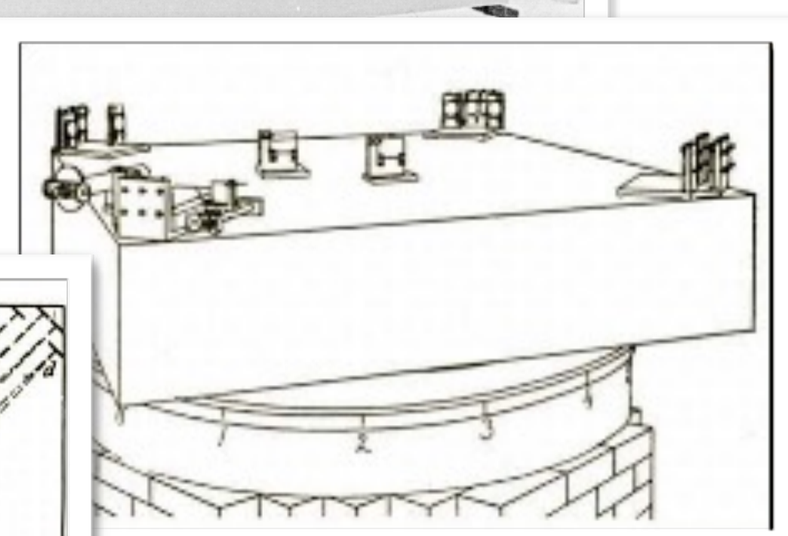
This technique was perfected by cowboy, Albert Michelson and eventually his sidekick, Edward Morley at Case Western Reserve in Cleveland between 1880 and 1888

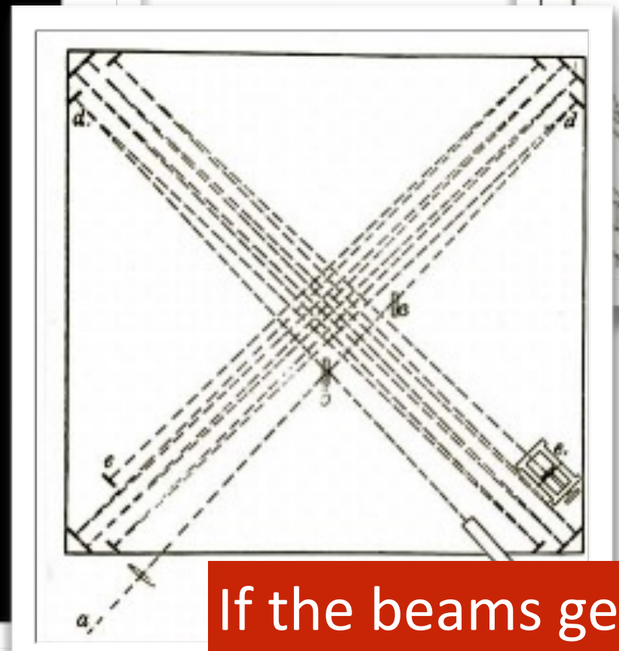
measure the fringes in light interfering from the two paths...then rotate the instrument 90 degrees - and do it again.

The differences between the two configurations is related to the time difference









If the beams get back out of phase...one traveled through the ether differently from the other.

repeated results for Earth-ether speed:

0

zero. zip. nada. nothing. uh-uh. zilch. naught. diddly-squat.

The earth did not appear to be moving through an Ether.
The question: did Einstein know of the MM experiment?
He always said “no.”

but

this is what Einstein declared to be the case.

an attack on cherished notions

one cannot make a measurement to show that one rest frame is more privileged than another

so one cannot speak of the reality of such a frame

the ether cannot be real.

"…[the] phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest."

a new criterion for what can be said to be "real"

If a phenomenon cannot be detected,

it cannot be claimed to exist.

is Relativity

the case?

‘‘muons’’ again: μ

are unstable particles which are easily made in an accelerator lab and shown to have a half life of 1.56 μs…

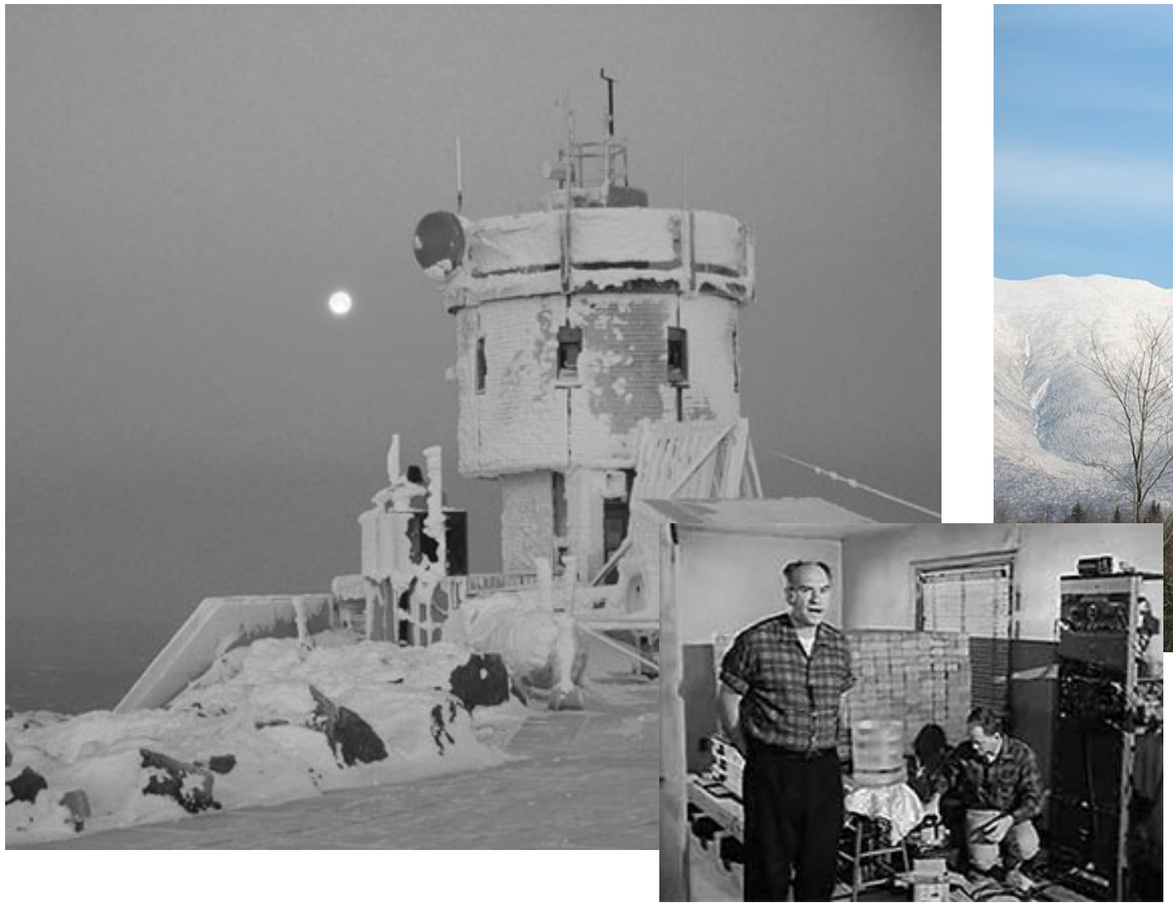
1.56×10^{-6} seconds

Under Newton’s view, even if the muon goes at the speed of light, then it lives for only $(3 \times 10^{8}) \times (1.5 \times 10^{-6}$ seconds$) = 450$ m

stand-up cosmic

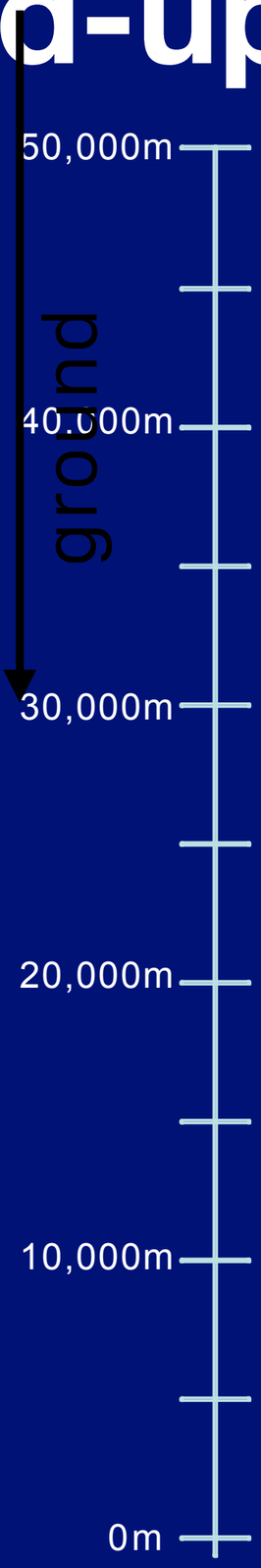
~20 particles/cm/s

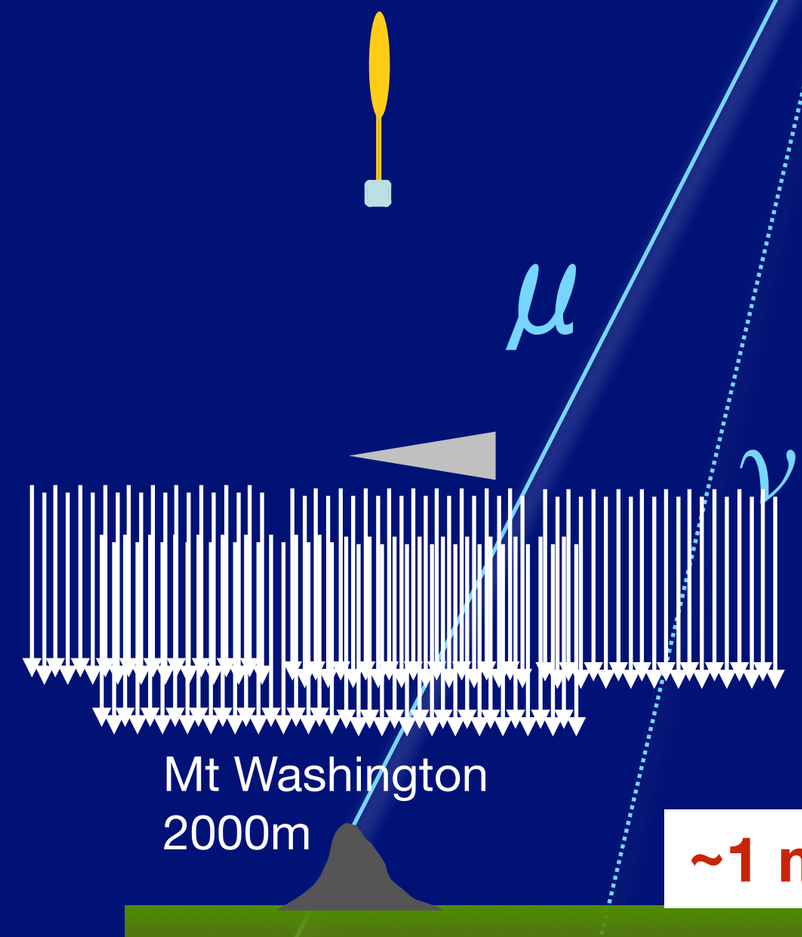
Mount Washington Observatory
New Hampshire

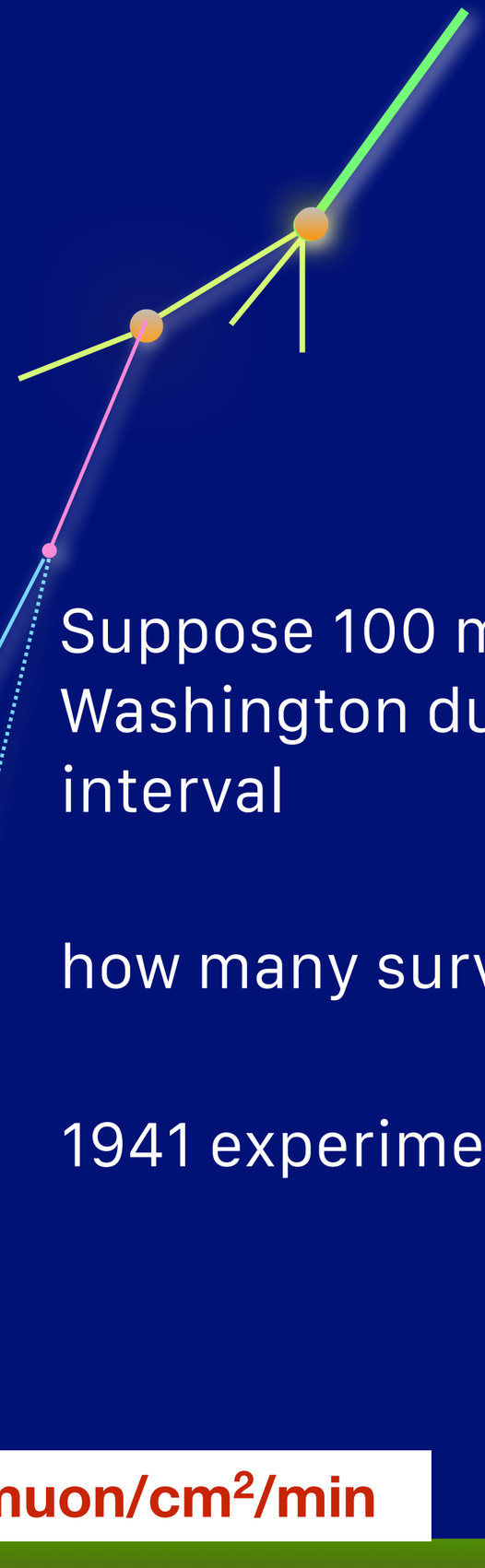


D. H. Frisch and J. H. Smith, “Measurement of the Relativistic Time Dilation Using μ-Mesons,” American Journal of Physics, 31 (5): 342–355, 1963).

stand-up cosmic







Suppose 100 muons pass Mt Washington during some time interval

how many survive to the ground?

1941 experiment

~1 muon/cm^2/min

home and away

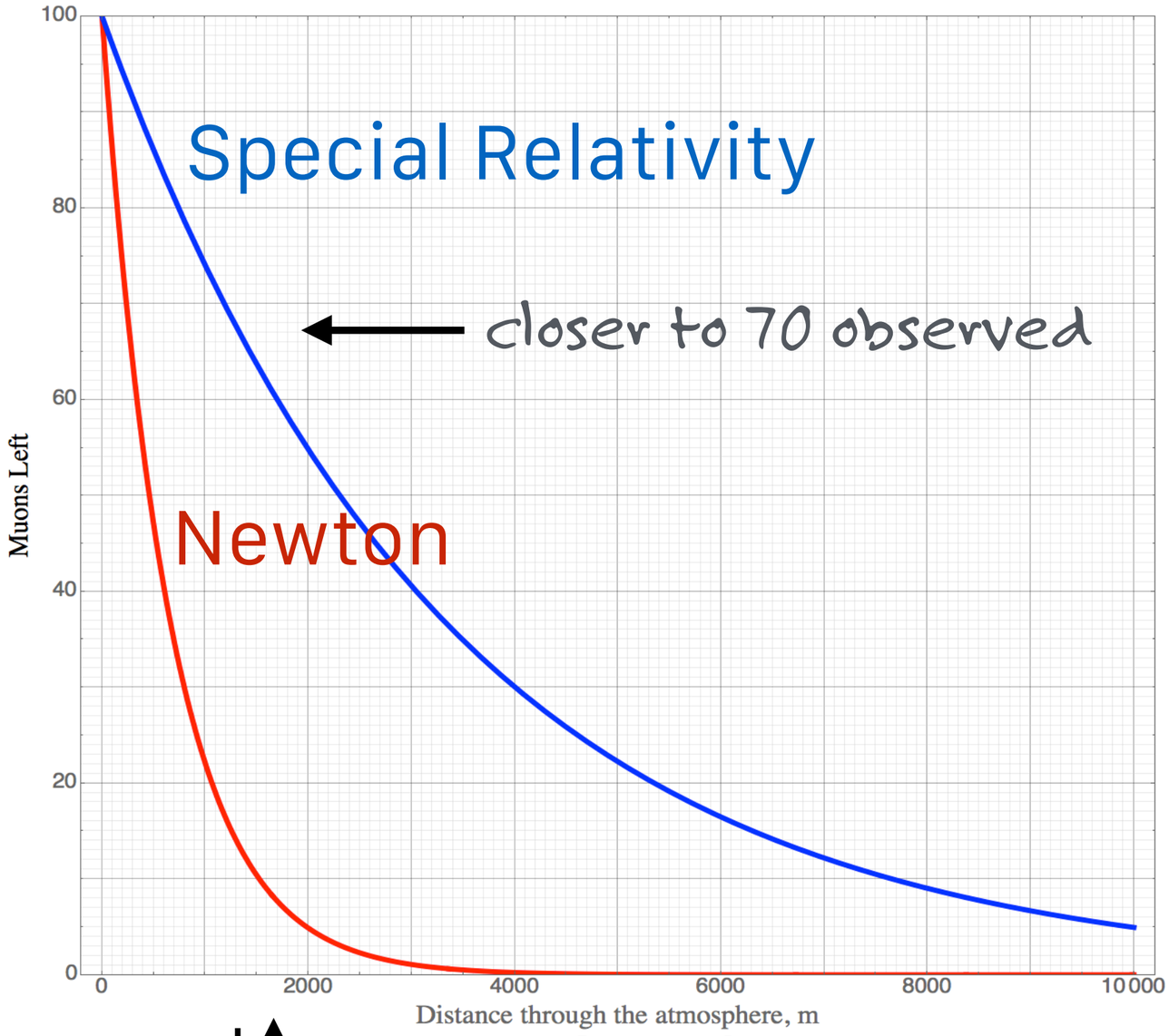
in the muon's rest frame

its "clock" is 1.6 microseconds of life

in the mountain's rest frame

for the muon moving with $\beta = 0.99$

its clock slows to be γ times that, or 7 x 1.6 microseconds



ground

how can it decay and not decay?

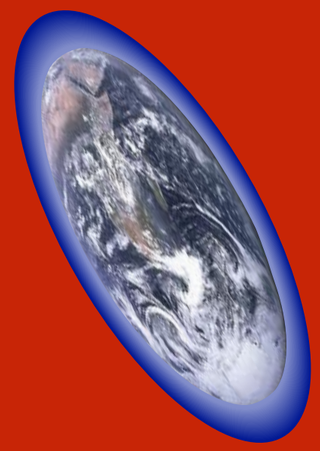
reciprocity

while it decays in 1.5μs in its rest frame...

it sees the atmosphere coming toward it at nearly c

which, to the muon, is Length Contracted

shorter by the same factor that the lifetimes differed



This has been measured many times:

an atomic clock was carefully carried around the world in 1972 and carefully calibrated and compared with ground-based clocks

J. Hafele and R. Keating

There are a number of corrections: accelerations, decelerations, the rotation of the orbit, the fact that the earth is not inertial - but relativity was absolutely correct

Predicted Effect	Flying East	Flying West
GTR (Gravitation)	+ 144 ± 14 ns	+ 179 ± 18 ns
STR (Velocity)	- 184 ± 18 ns	+ 96 ± 18 ns
Total	- 40 ± 23 ns	+ 275 ± 21 ns
measured:	- 59 ± 10 ns	+273 ± 7 ns

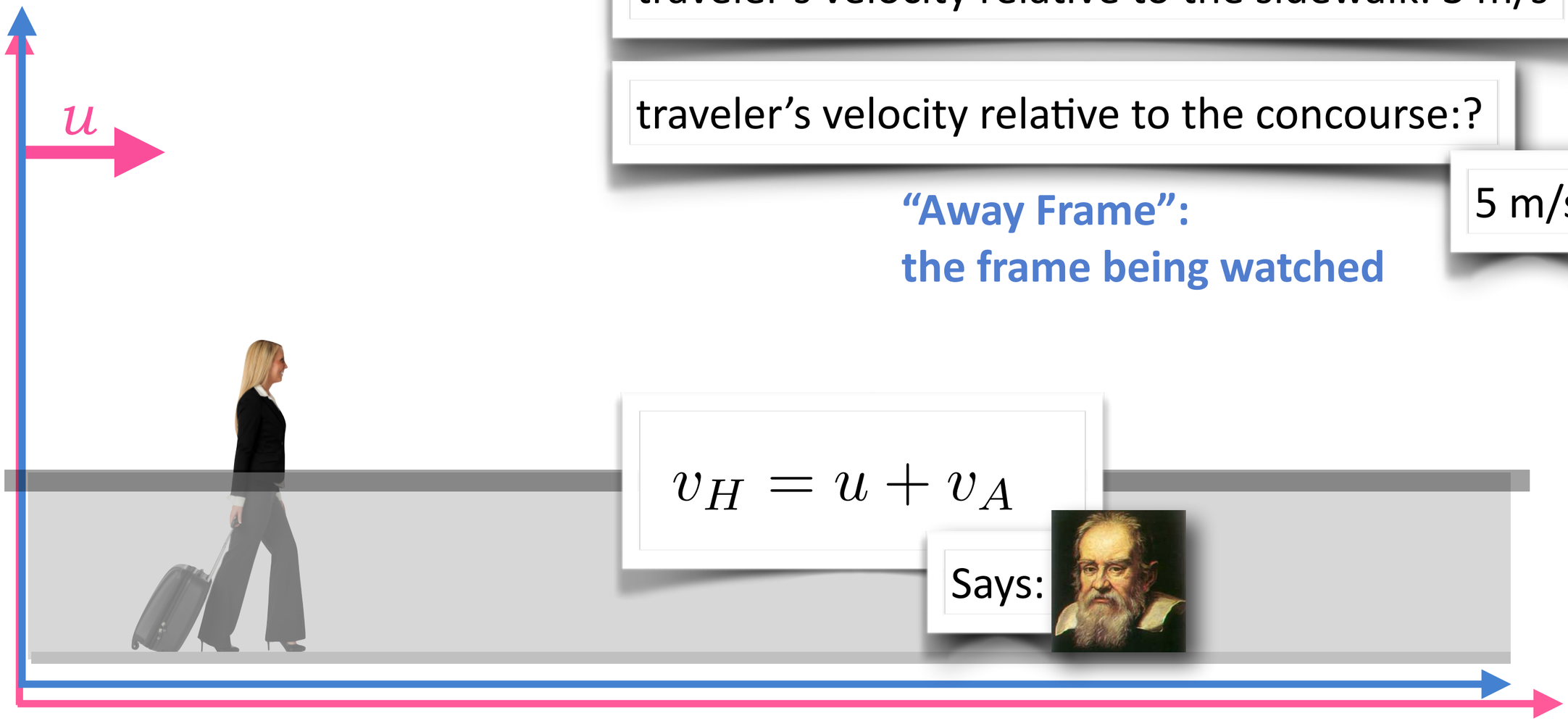
redone twice more in airplanes and rockets/satellites

combine speeds

Galileo, nope.

Einstein, yup.

the airport, redux



sidewalk velocity: 2 m/s

traveler's velocity relative to the sidewalk: 3 m/s

traveler's velocity relative to the concourse:?

5 m/s

"Away Frame":
the frame being watched

$$v_H = u + v_A$$

Says:

"Home Frame":
watching a moving frame

the airport, redux

using v_A quantities?

$$v_H = \frac{\Delta x_H}{\Delta t_H}$$

this is altered by the Postulates of Relativity

ditto!

u

u

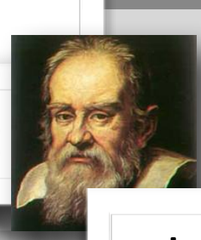
x_{A0}

x_{Af}

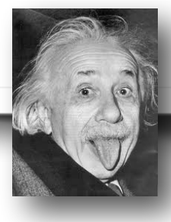
so at high u...

$$v_H \neq u + v_A$$

Says:



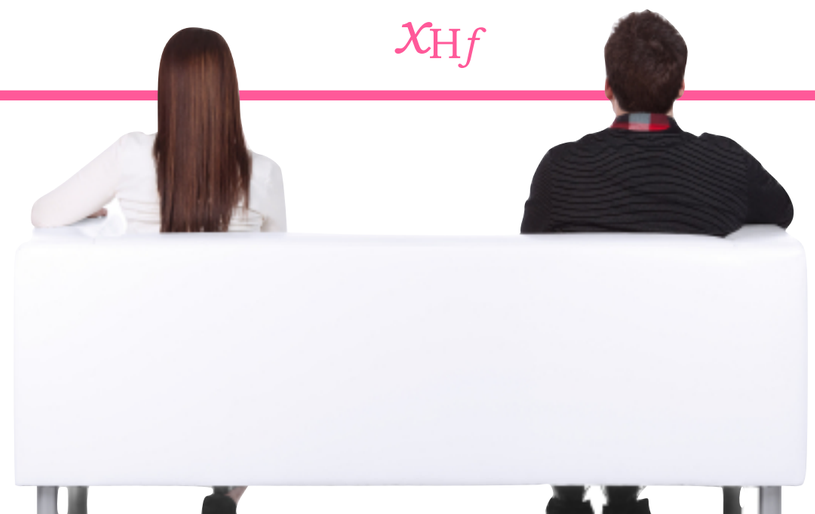
what about:



?

x_{H0}

x_{Hf}



the airport, going fast

some time interval

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

A

u u

start here:
x_{A0}

end up here:
x_{Af}

$$v_H = u + v_A$$

x_{H0}

x_{Hf}

H

write it down.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

relativistic velocity transformation

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

Look at this formula carefully...

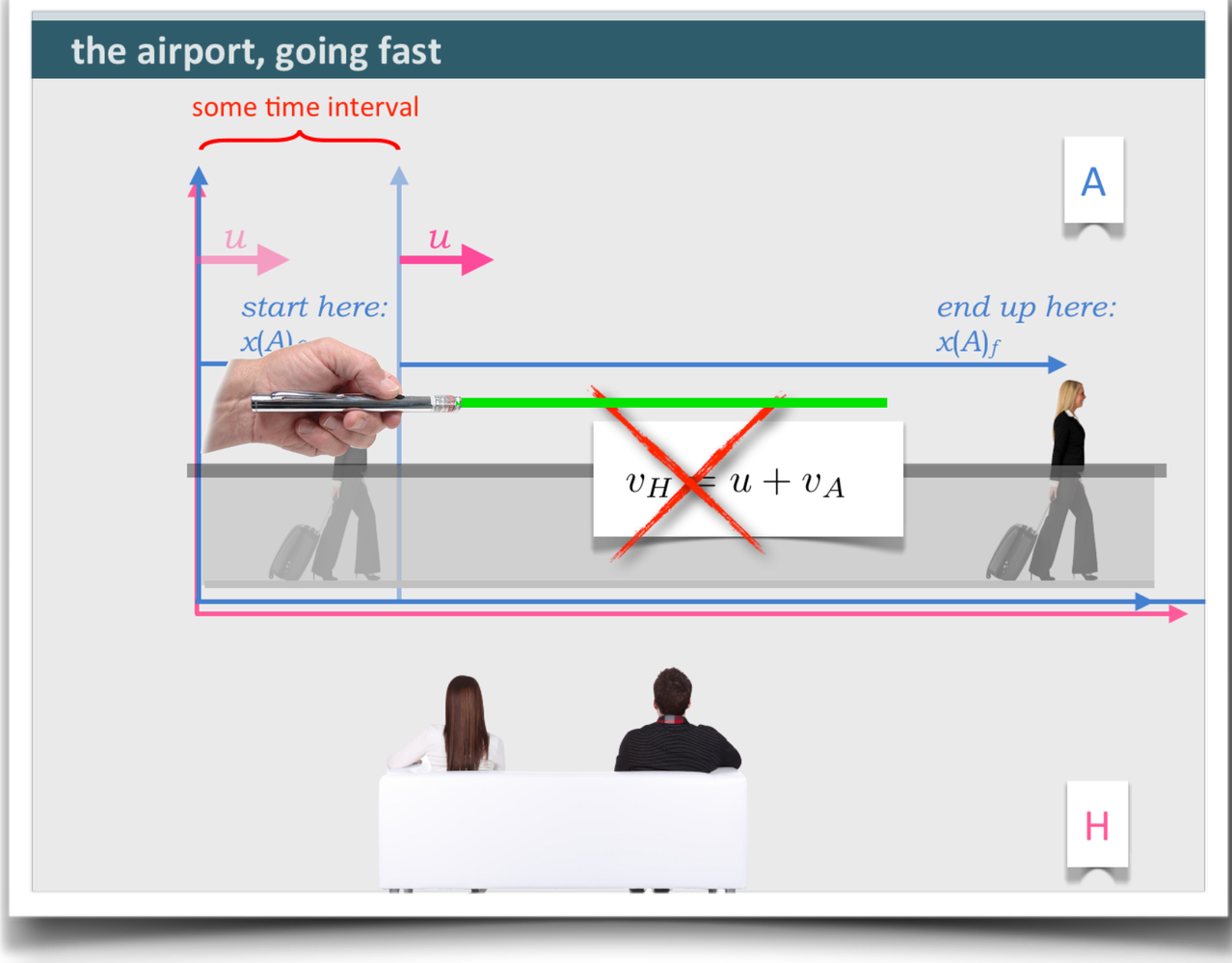
Suppose u/c is very small...like normal life.

work it out

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

$<<1$...so

$v_H \longrightarrow u + v_A$ and the old-time, non-relativistic airport sidewalk formula emerges



Suppose it's not a traveler, but light.

$v_A = c$ work it out

$$v_H = \frac{c + u}{1 + \frac{u}{c^2} c} = \frac{c + u}{(c + u)} c = c$$

The Second Postulate is preserved!

nothing

can accelerate to a speed faster than that of light

be careful

There are 3 velocities going on here.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

u is the frame velocity

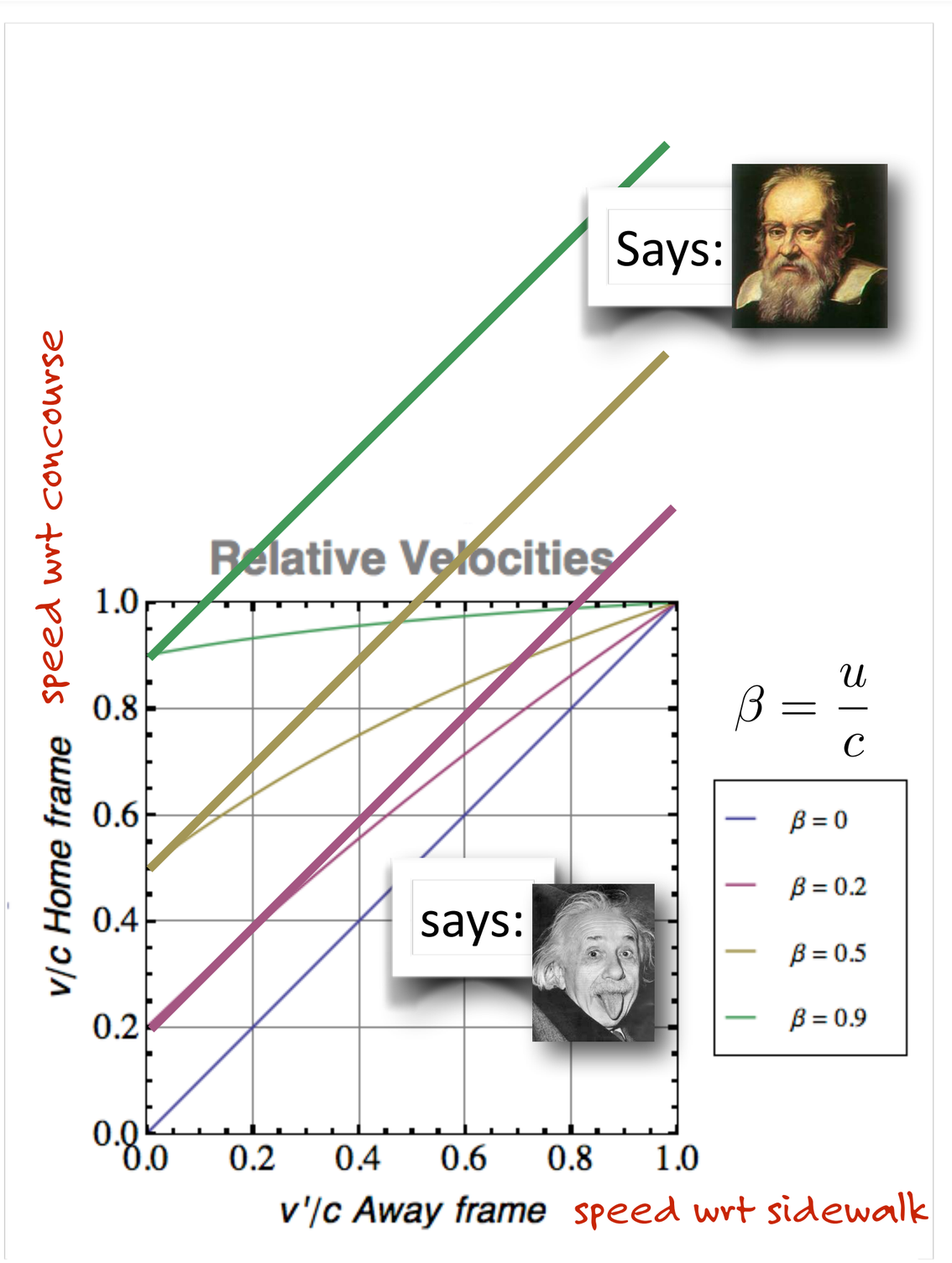
...same, A relative to H or H relative to A (sidewalk)

v_A is the velocity (traveler)

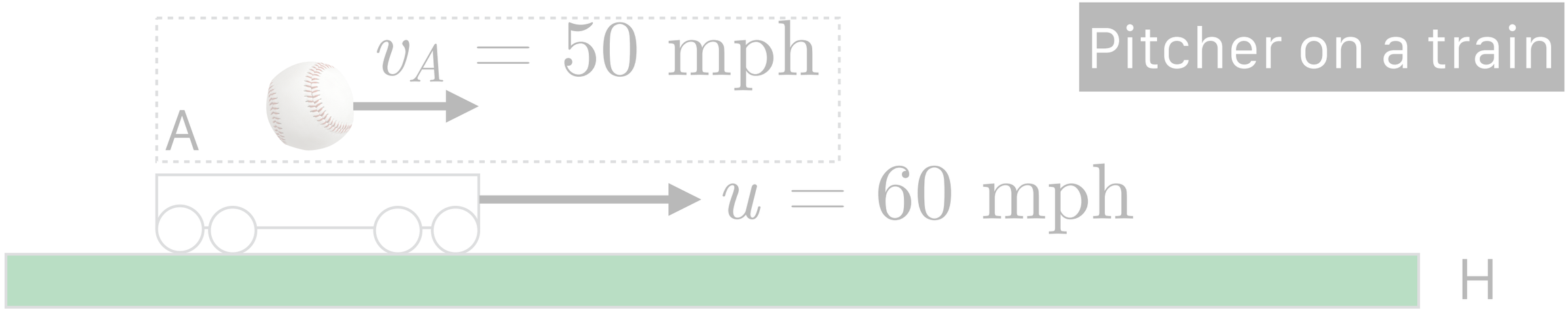
of something measured relative to the A frame

v_H is the velocity (traveler)

of something measured relative to the H frame



two examples



Pitcher on a train

Galilean approach:

What's the speed of the ball relative to the ground?

a pion decays into a muon

the pion travels at $u = 0.5\mathrm{c}$ in the lab (H)

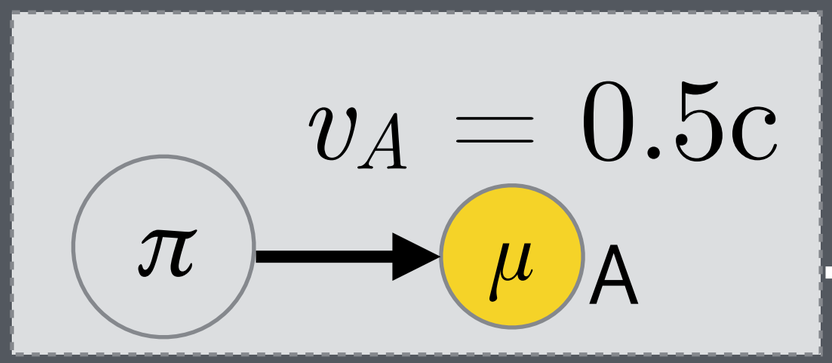
the muon travels right at $v_\mathrm{A} = 0.5\mathrm{c}$ in the pion's rest frame

What is the speed of the muon in the lab?

How far does it travel in the lab before decaying?

What is the speed if muon travels left at $v_\mathrm{A} = -0.5\mathrm{c}$ in the pion's rest frame?

What if the muon travels left at $v_\mathrm{A} = -0.75\mathrm{c}$ in the pion's rest frame?



H

constant of nature:

speed of light

value: $c = 2.99792458 \times 10^8$ m/s

units: m/s or ft/s or km/h

usage: Speed of light in relativity

or approximately $c = 3.0 \times 10^8$ m/s

Energy

push on something

Einstein said:

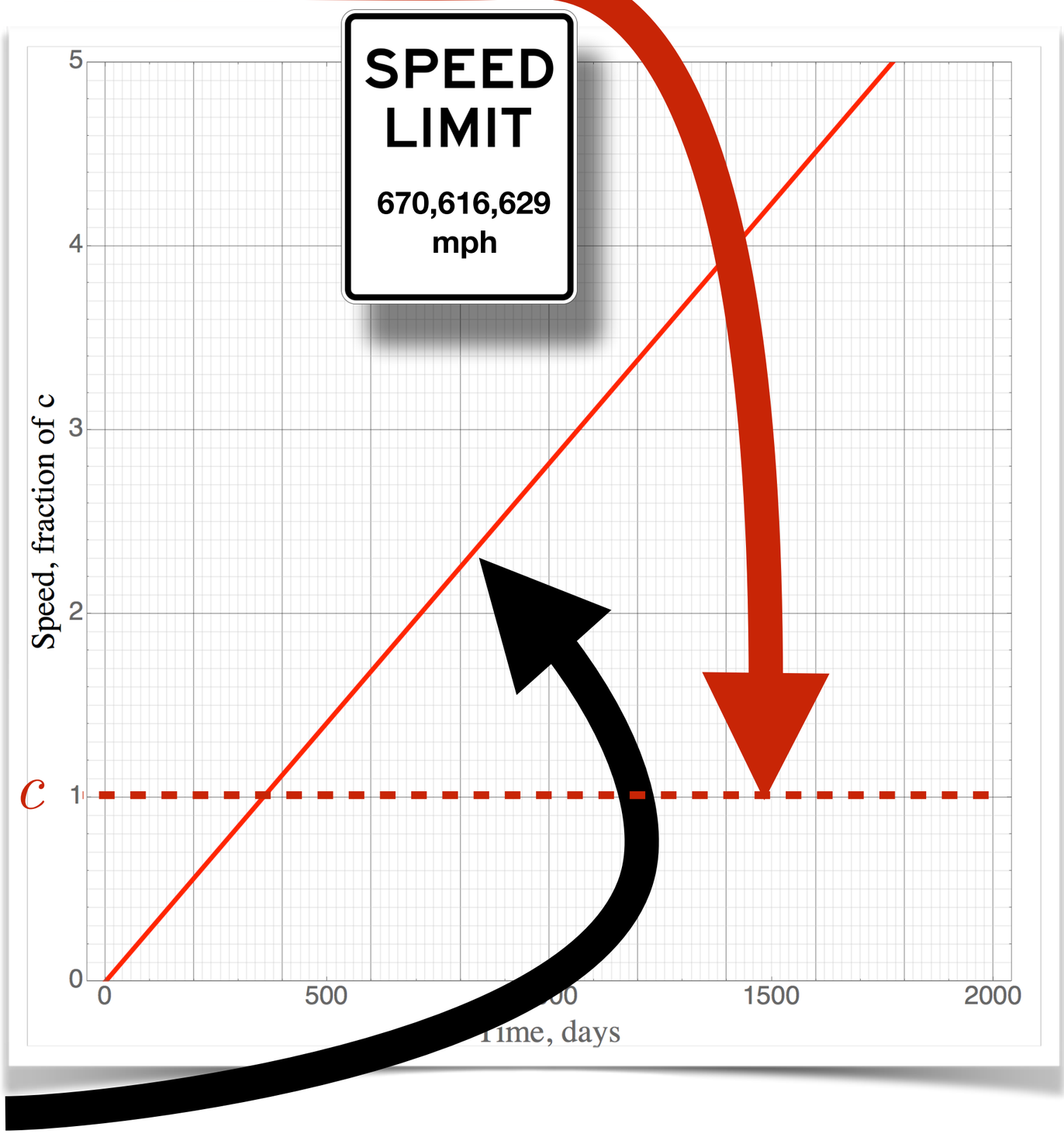
constant force to create a constant acceleration of

$1g$

Galileo/Newton said

speed increases:

$$v = gt$$

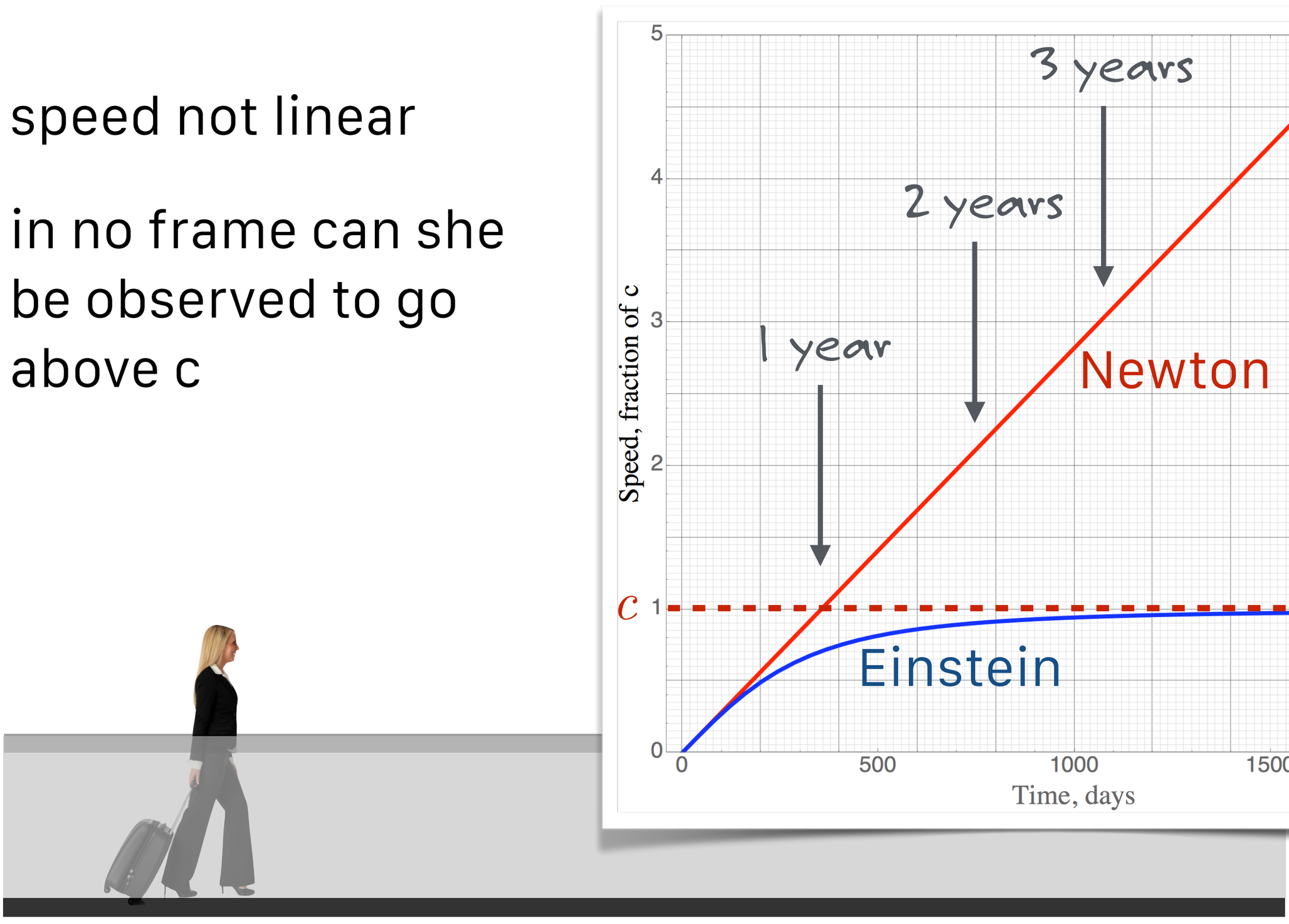


Newton said:

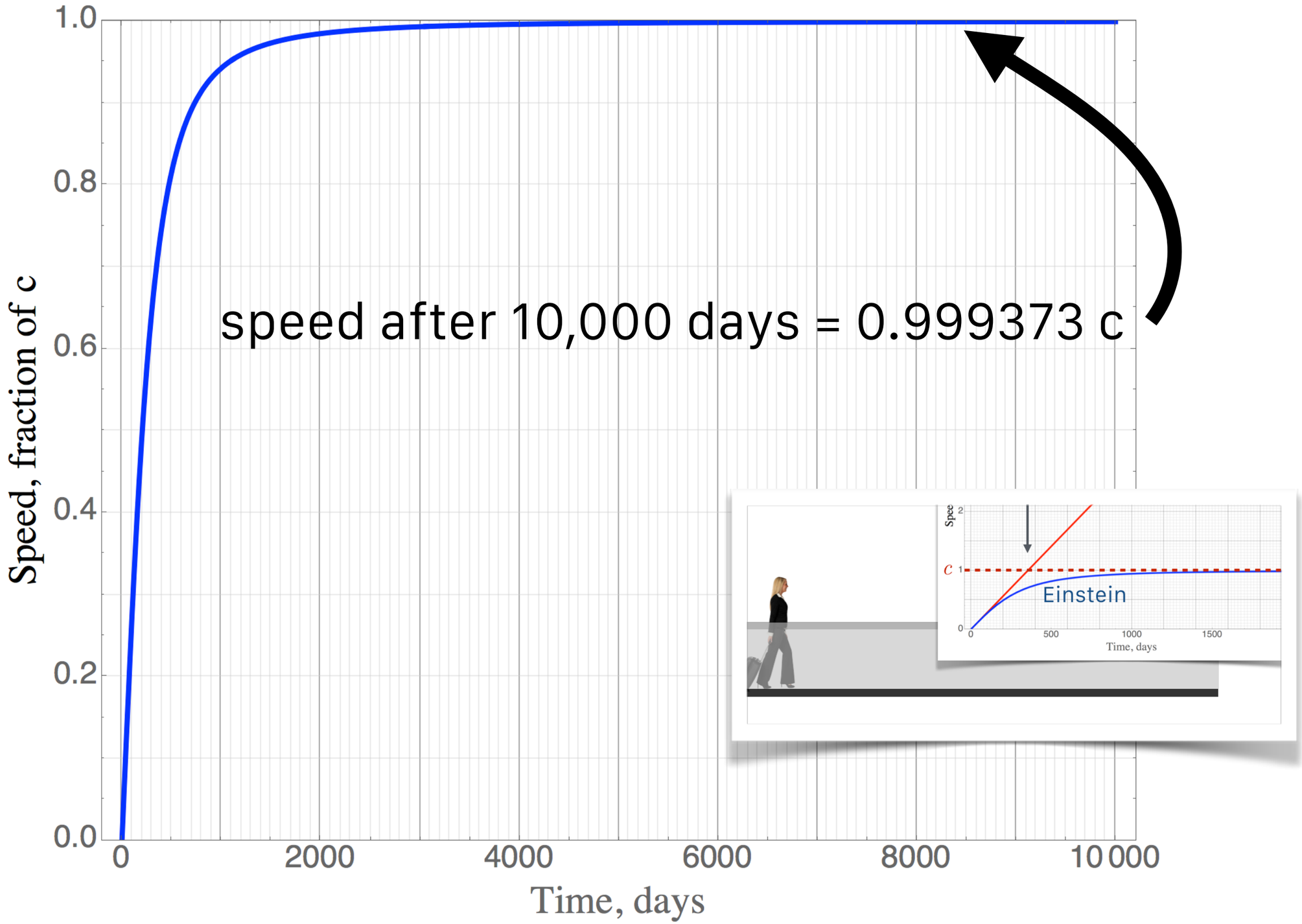
traveler transformation, 1g acceleration

speed not linear

in no frame can she be observed to go above c



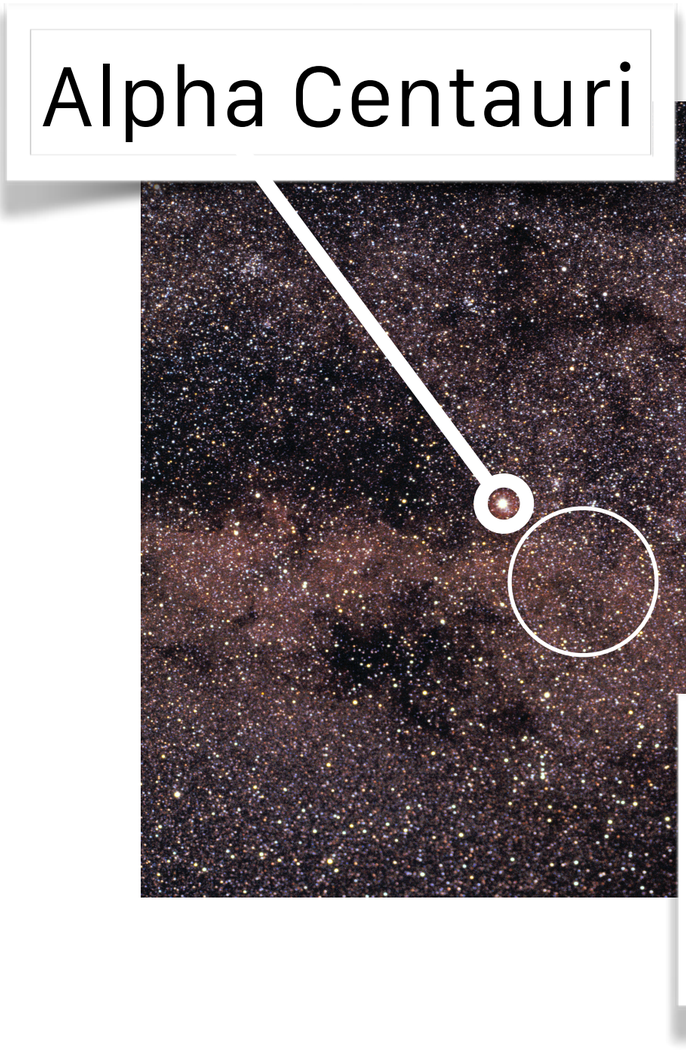
never get there

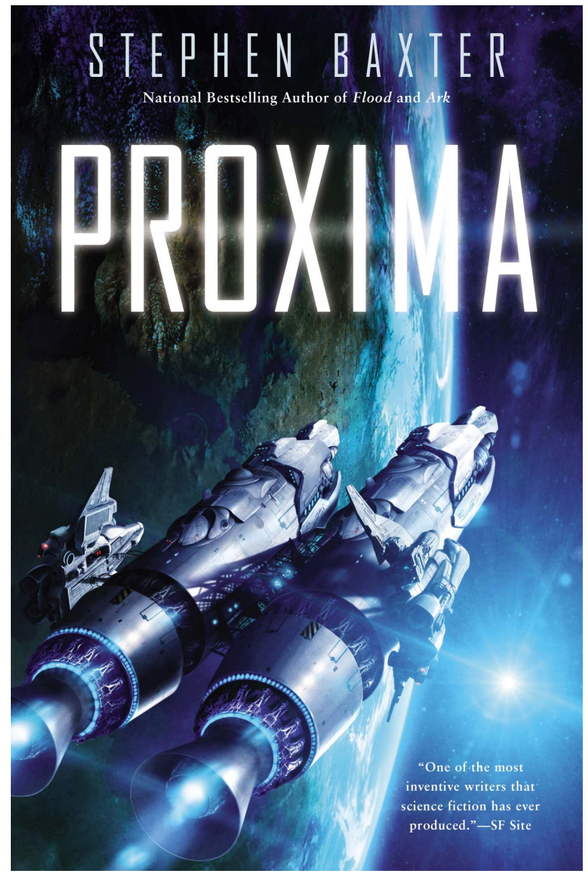


BTW

nearly every science fiction story ever

Closest star to Earth: Proxima Centauri: 4.23 light years



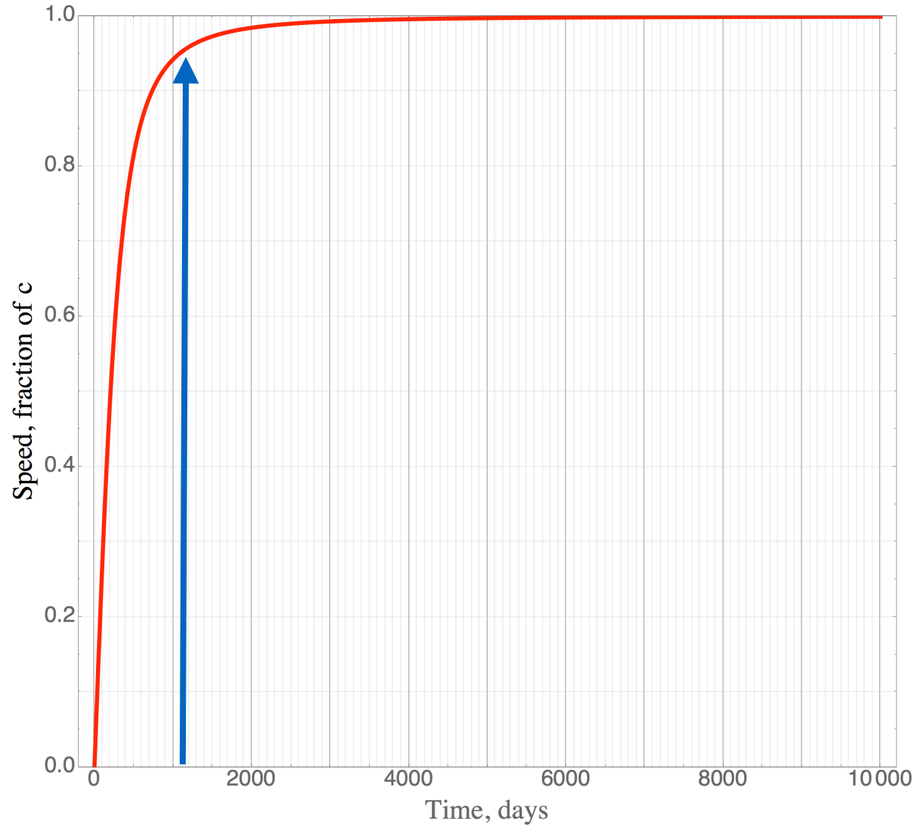


let's go

accelerate at 1g for 2 light years

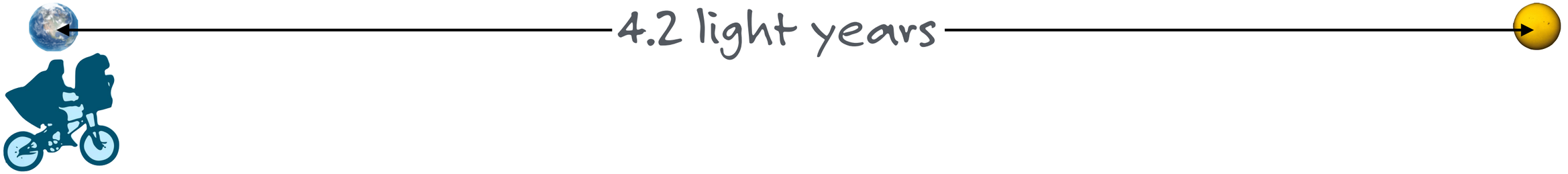
cruise for 0.2 light years

decelerate at -1g for 2 light years



home

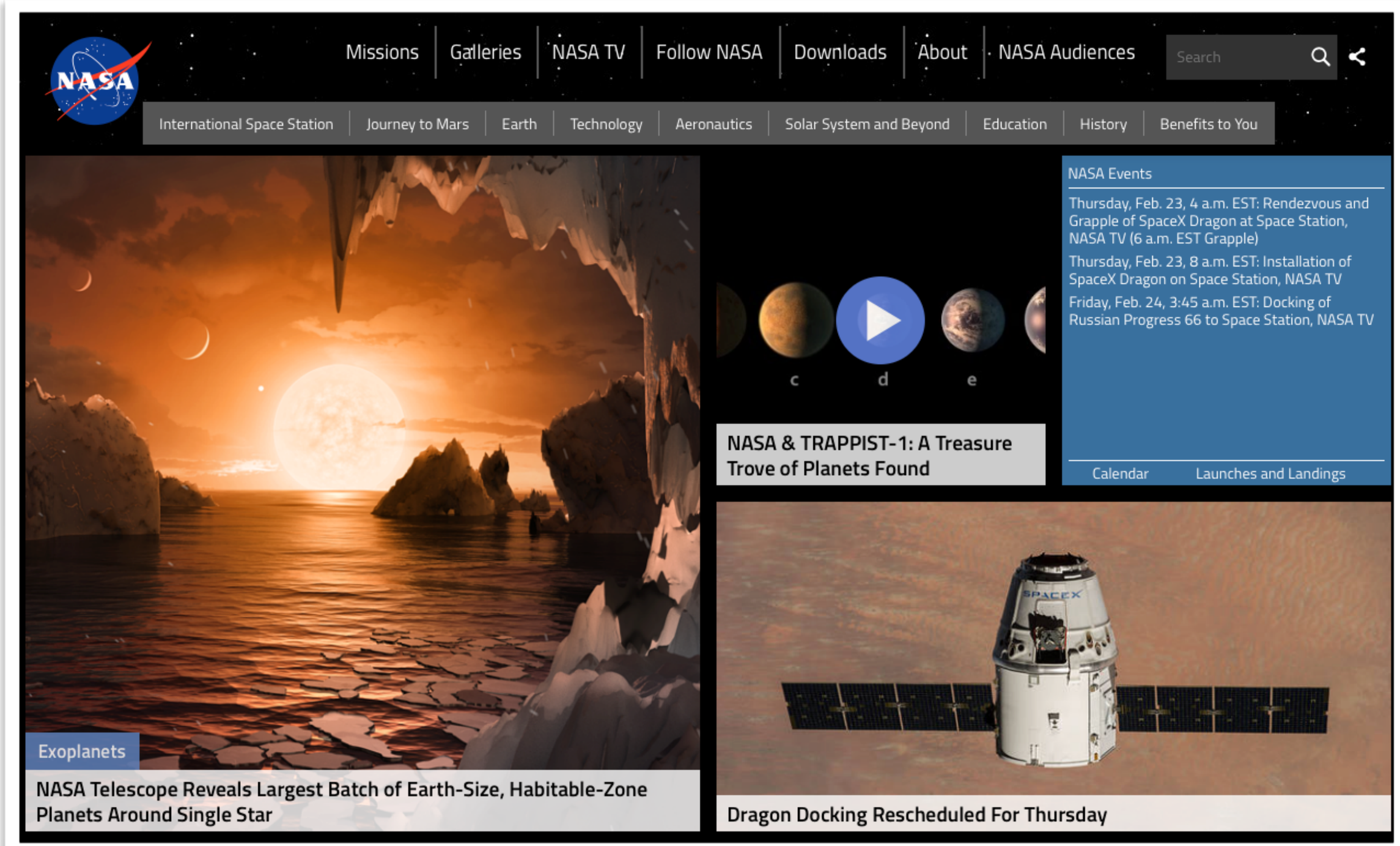
Proxima Centauri



acceleration time, relative to Earth	2.8 years
top speed, relative to Earth	0.9453 c
acceleration time, relative to ship	1.7295 years
whole trip time, relative to Earth	5.8695 years
whole trip time, relative to ship	3.5428 years

day before yesterday

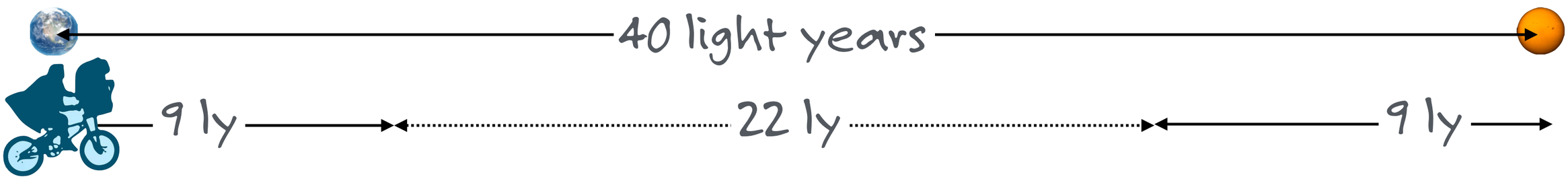
https://www.nytimes.com/2017/02/22/science/trappist-1-exoplanets-nasa.html?emc=edit_na_20170222&nl=breaking-news&nlid=26413858&ref=cta&_r=0



let's go there

40 light years away...star is "Trappist-1" which is a dwarf

How about traveling there? Again, assume 1g acceleration



acceleration time, relative to Earth	9.9 years
top speed, relative to Earth	0.9953 c
acceleration time, relative to ship	2.97 years
whole trip time, relative to Earth	41.9 years
whole trip time, relative to ship	8 years