hi

Lecture 15, 28.02.2017

Einstein's Theory of Special Relativity, 4

housekeeping

Question about anything?

I'll make a movie for you:

Marie Curie movie anyone?

yes! I'll organize for after break...looks like room available March 15, 7-9pm...?

Midterm...before ~~or after~~ Spring Break. After:

"The midterm will be released on Sunday night, February 26th and close on Tuesday night, February 28. It will cover all of the material through Tuesday, February 21st class."*

See calendar cartoon:

* senior moment...

next few weeks, v2

S	M	T	W	Th	F	Sa
						2/25 HW7 HW6 due
2/26	2/27	2/28 midterm	3/1	3/2	3/3	3/4 HW8 HW7 due
3/5		Spring Break				3/11
3/12	3/13	3/14	3/15	3/16	3/17	3/18 HW8 due

Honors Project

has begun.

Read the first of two sets of instructions:

MinervaInstructions1_2017.pdf in

www.pa.msu.edu/~brock/file_sharing/QSandBB/2017homework/honors_project_2017/

Relativity, to date:

Consequences of the Second Postulate for co-moving frames of reference:

- space and time determinations are relative
- simultaneity is relative
- electric and magnetic fields are relative

} no absolutes

- velocity within frames transforms differently from Newtonian principles

You might wonder if anything is constant?

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- motion within frames transforms differently from Newtonian principles

} no absolutes

EXPERIMENTALLY CONFIRMED MANY TIMES

You might wonder if anything is constant?

Yes! Some important things...

Principle of Relativity

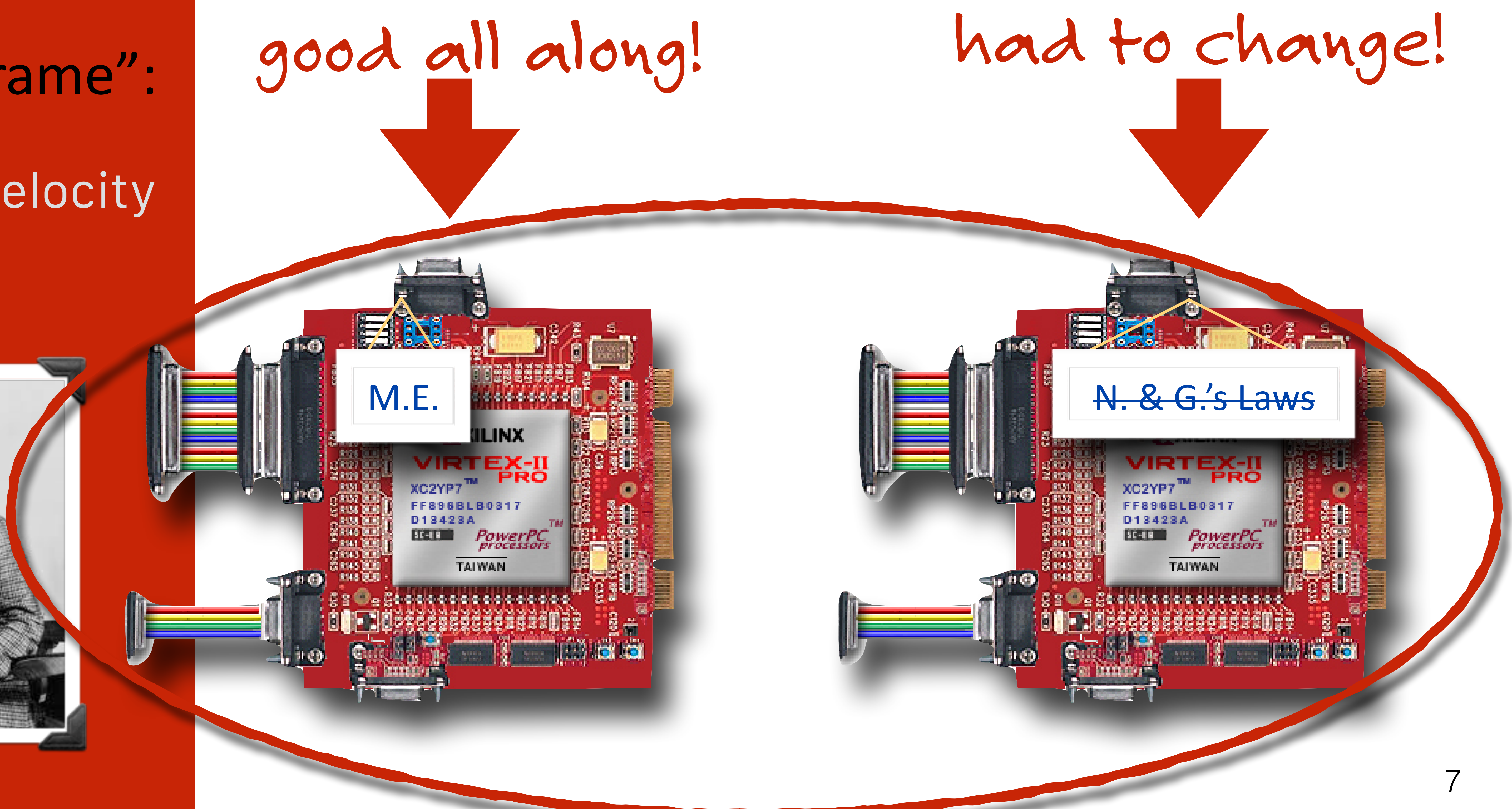
2 Postulates:

1. All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.

2. The speed of light is the same for all inertial observers.

"inertial frame":

constant velocity



the "ether"

"The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space" provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place."

the "Michelson-Morley Experiment" confirmed...or motivated Relativity

? ?

is Relativity

the case?

the Mt Washington experiment and the around the world flight

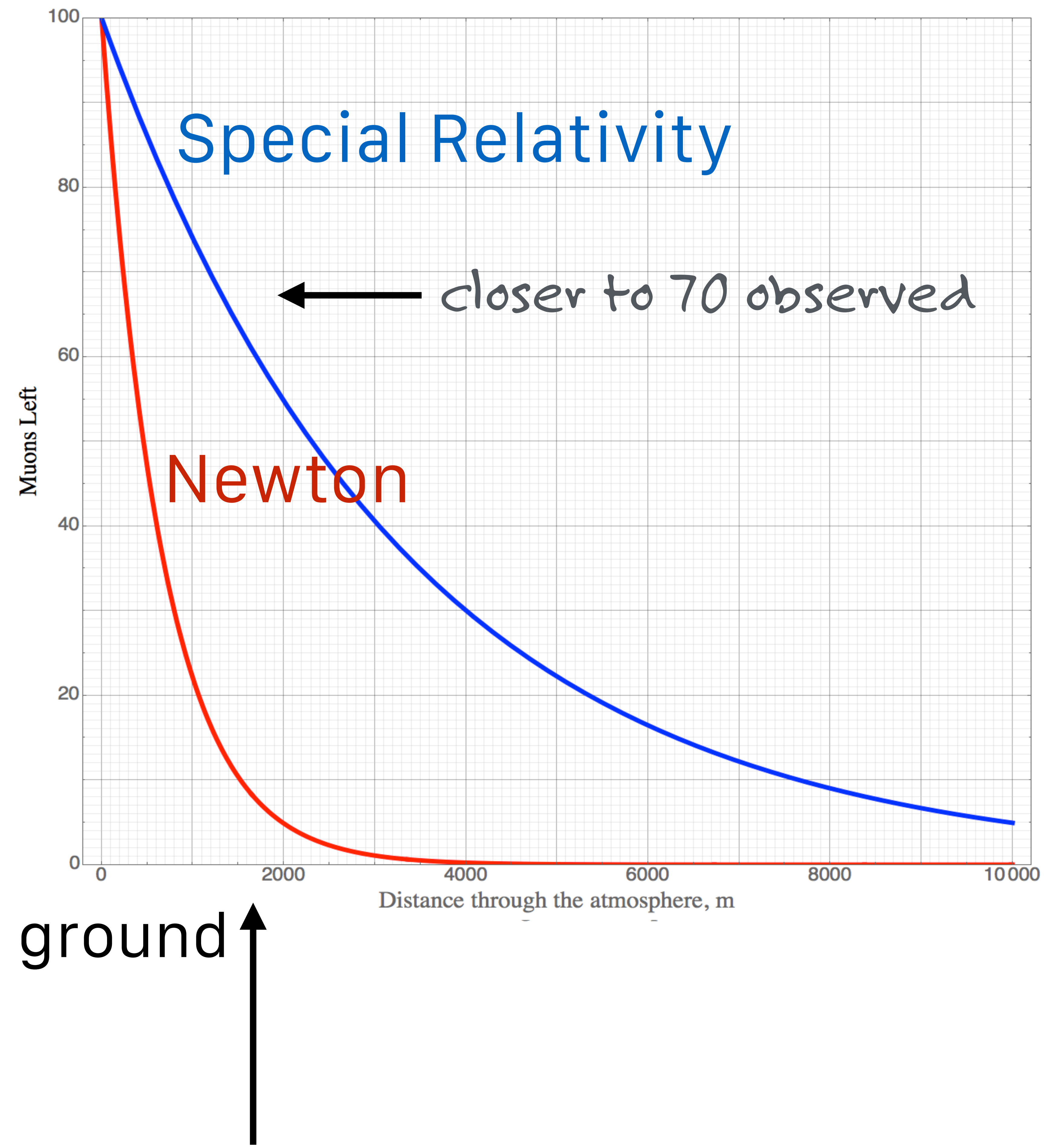
in the muon's rest frame

its "clock" is 1.6 microseconds of life

in the mountain's rest frame

for the muon moving with $\beta = 0.99$

its clock slows to be γ times that, or 7 x 1.6 microseconds

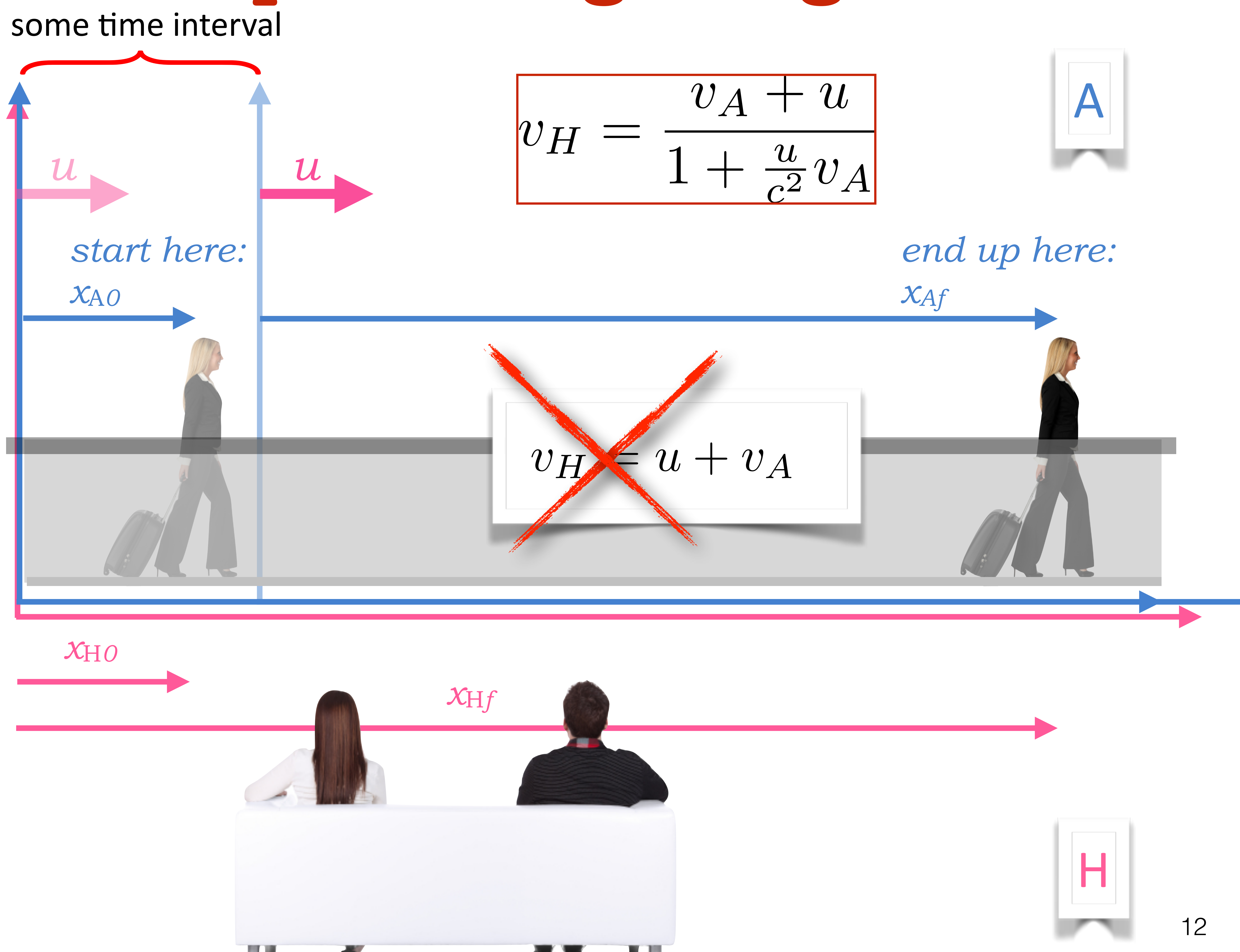


`combine speeds`

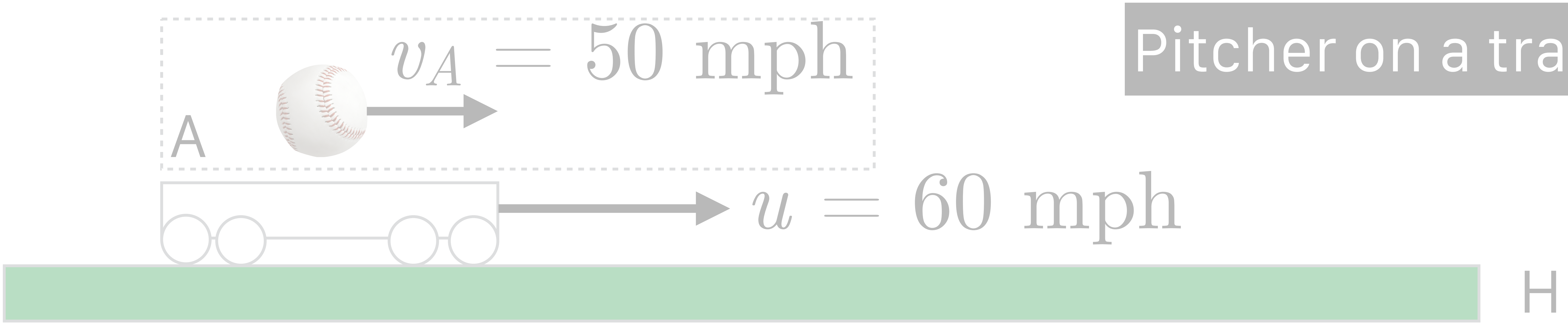
Galileo, nope.

Einstein, yup.

the airport, going fast



two examples



Pitcher on a train

Galilean approach:

What’s the speed of the ball relative to the ground?

a pion decays into a muon

the pion travels at $u = 0.5c$ in the lab (H)

the muon travels right at $v_A = 0.5c$ in the pion’s rest frame

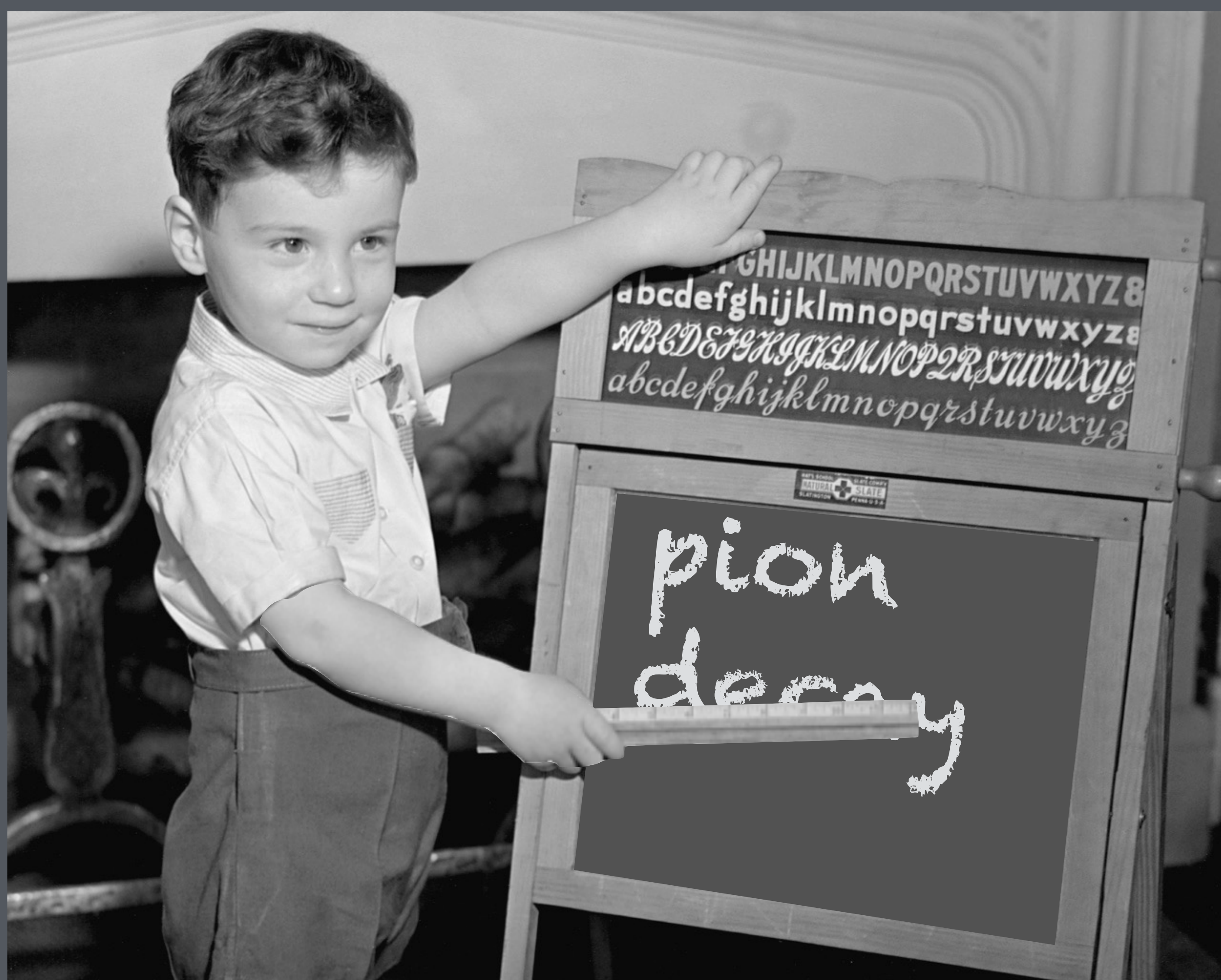
What is the speed of the muon in the lab?

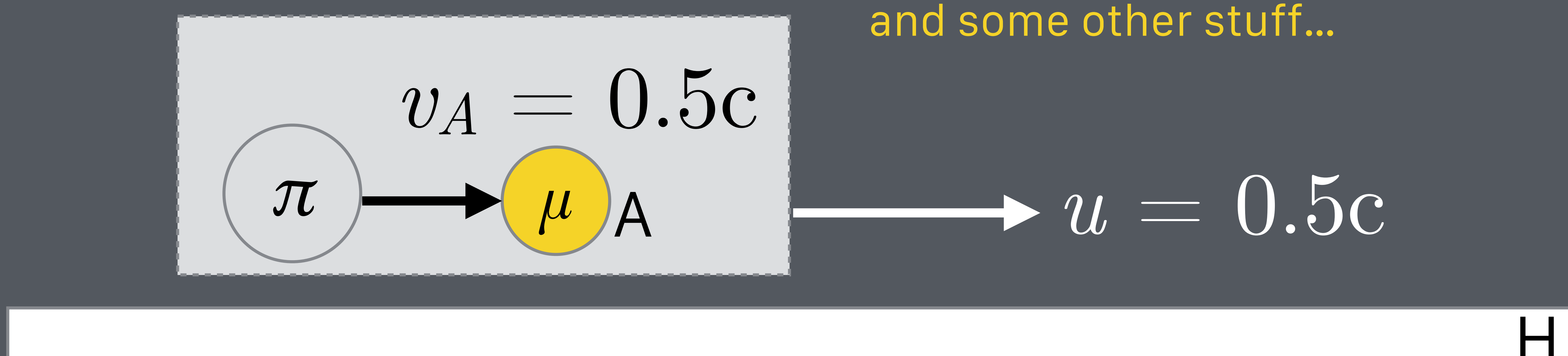
How far does it travel in the lab before decaying?

What is the speed if muon travels left at $v_A = -\,0.5c$ in the pion’s rest frame?

What if the muon travels left at $v_A = -\,0.75c$ in the pion’s rest frame?

and some other stuff…



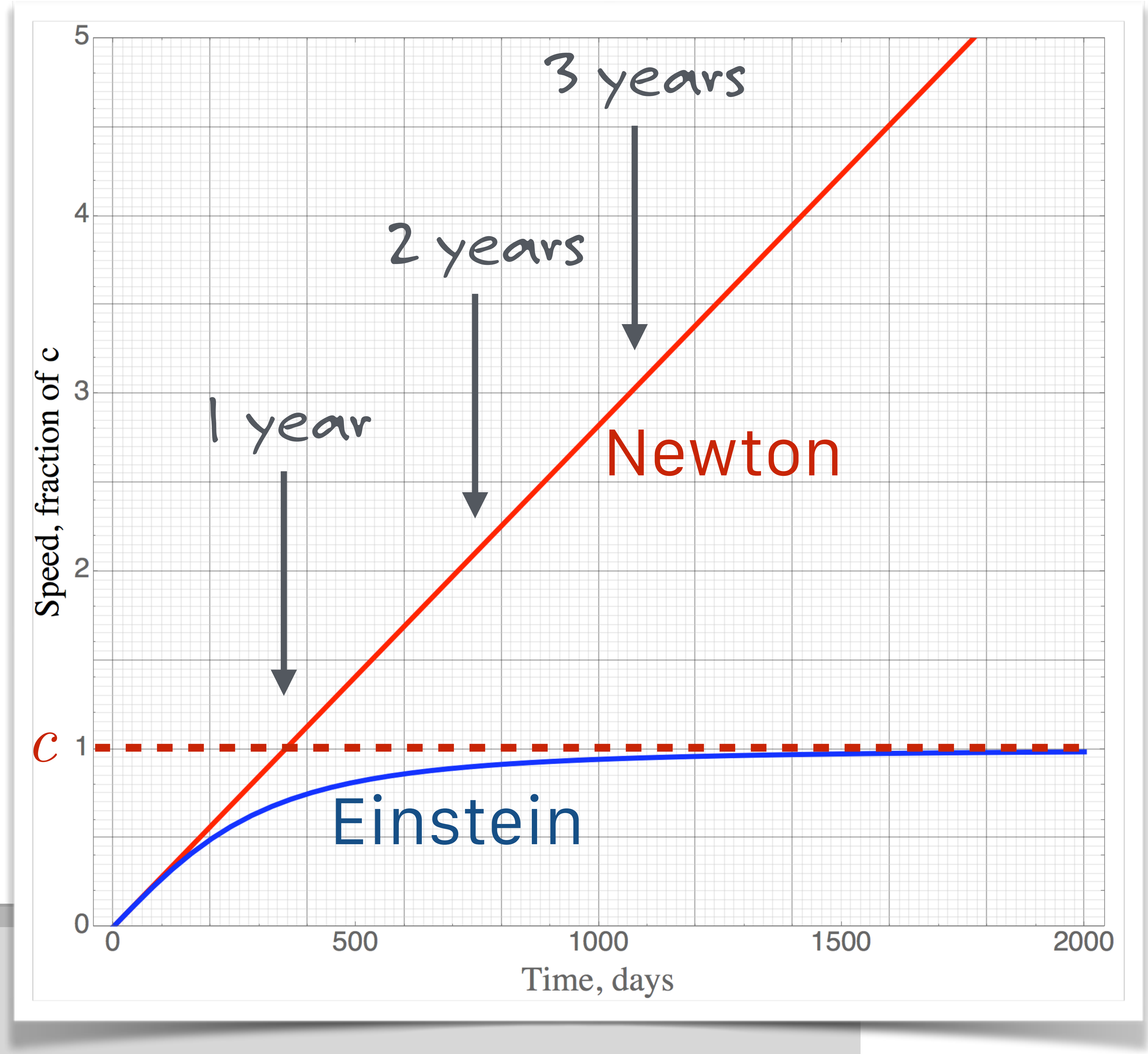


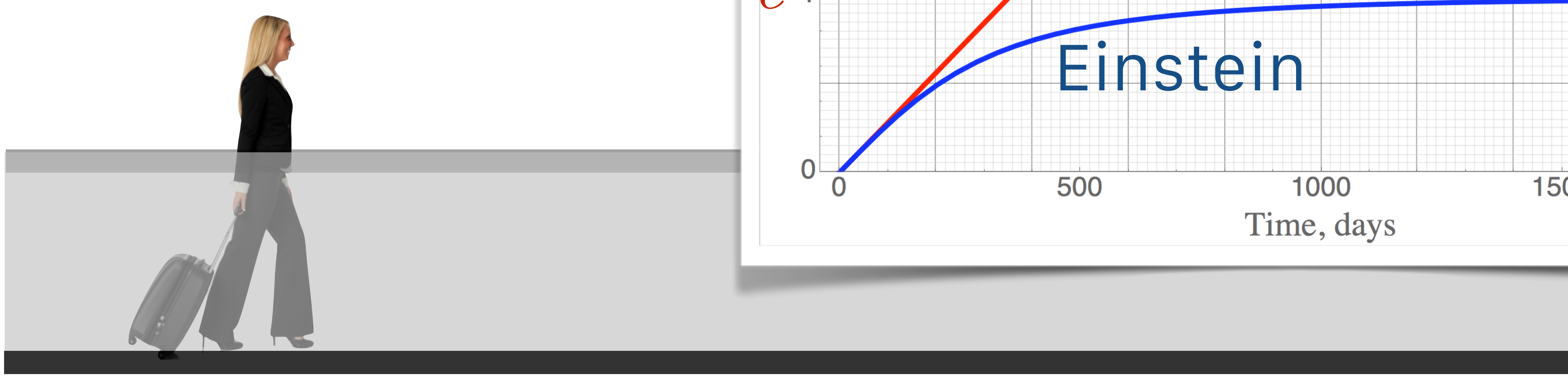
Energy

traveler transformation, 1g acceleration

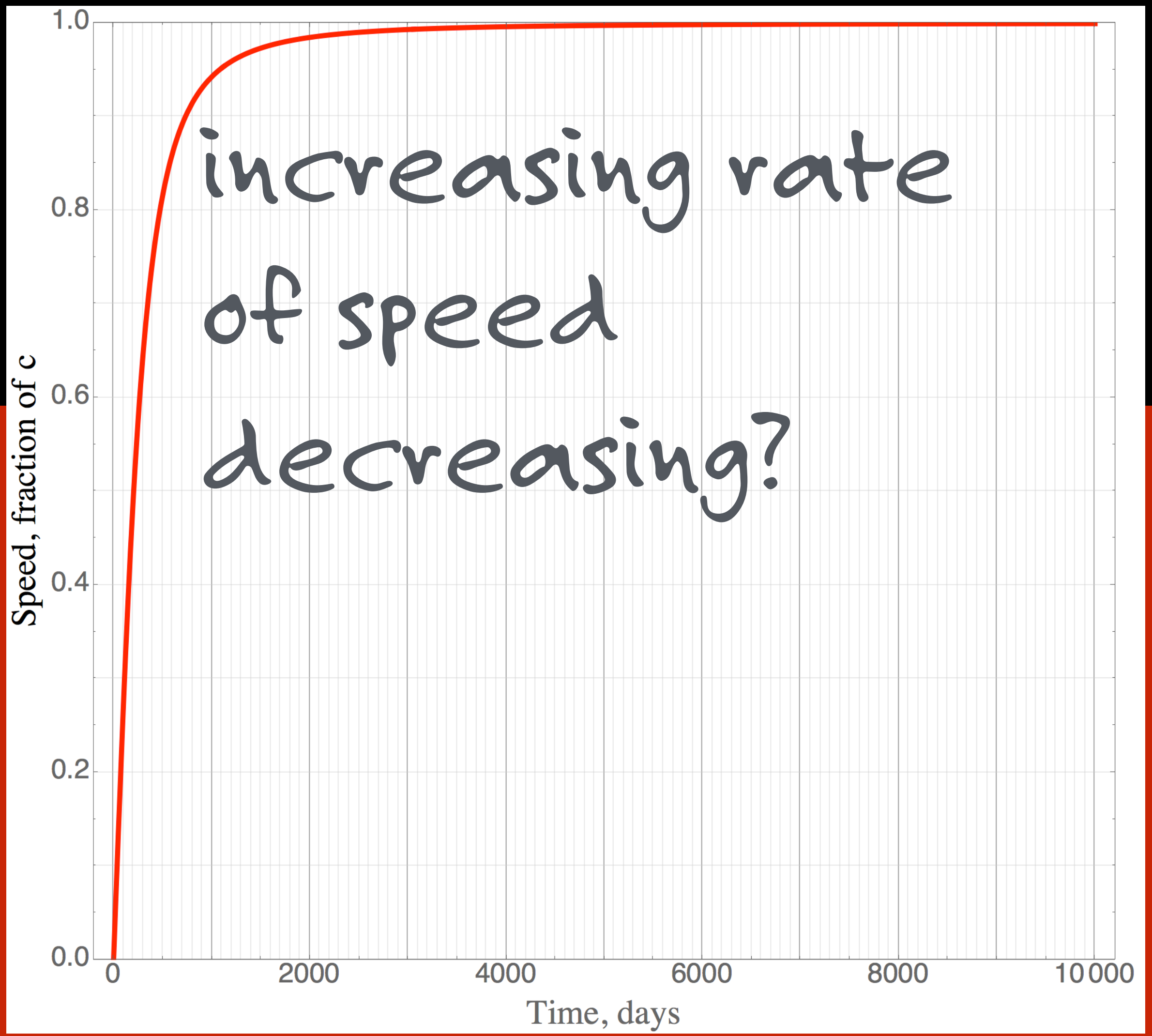
speed not linear

in no frame can she be observed to go above c





doesn’t this look like a reluctance to being accelerated?



Well.

What quantity is a measure of the reluctance to being accelerated?

Inertia.

If this reluctance increases...inertia seems to increase

and…what's the measure of a body's inertia?

mass

classical dynamical quantities

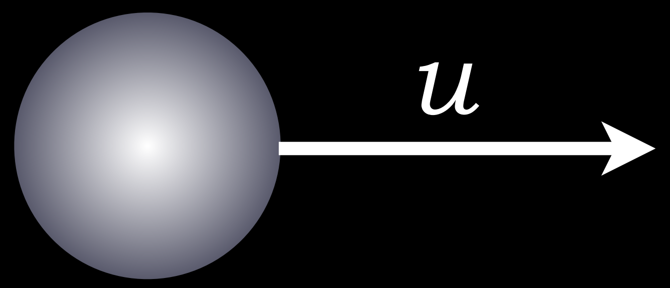
momentum, $p = mv$

Kinetic Energy, $K = 1/2mv^2$

and force $F = ma$

These have to change!

New, relativistic quantities reduce to these when u/c is very small



Momentum in relativity

got to be different from Newton

$$p_H = m\frac{\Delta x_H}{\Delta t_H}$$

want to preserve the idea of momentum conservation

Relativistic Momentum: $$p = m\gamma u$$

relativity and energy

through the back door…

there’s a “real” derivation, but too much mathematics

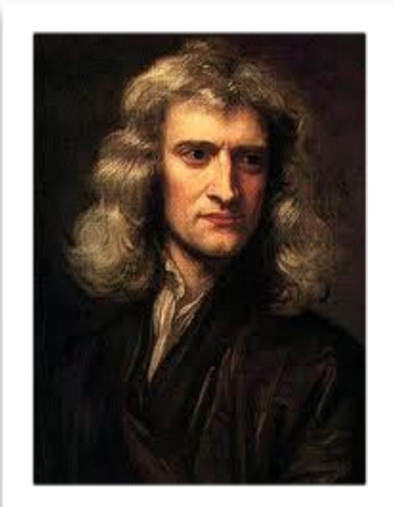
quick aside

approximating functions

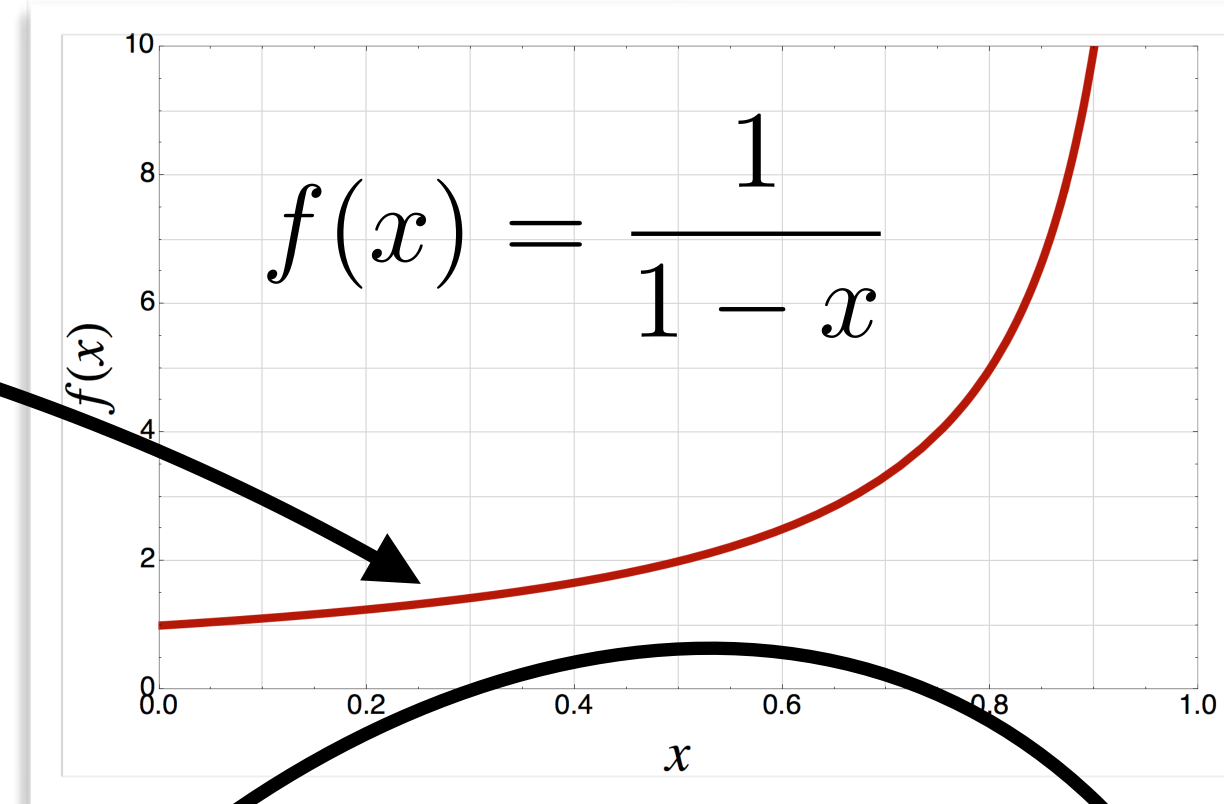
see manuscript math refresher chapter

somewhere in your life: the Binomial Series

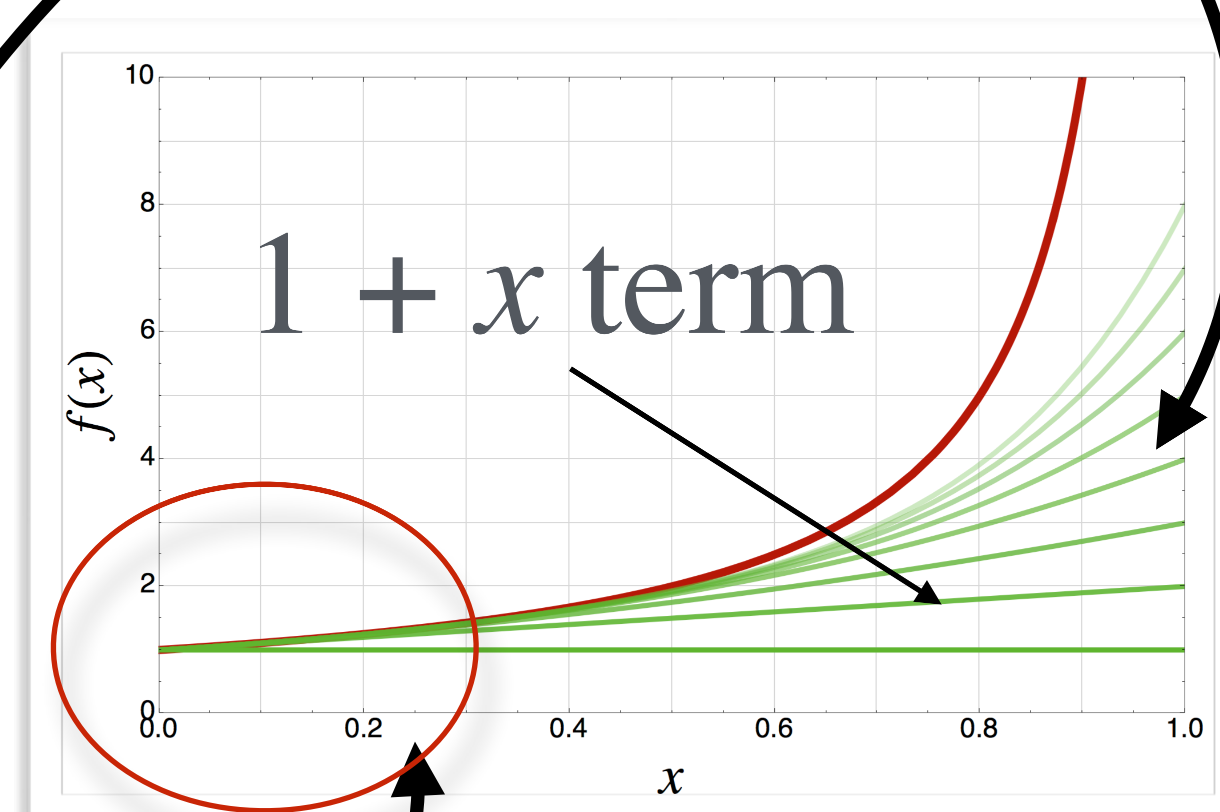
Binomial Series...useful to approximate functions.



$$f(x) = \frac{1}{1-x}$$



$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O\left(x^{11}\right)$$



Suppose that x << 1, then the function could be approximated by a couple of terms...

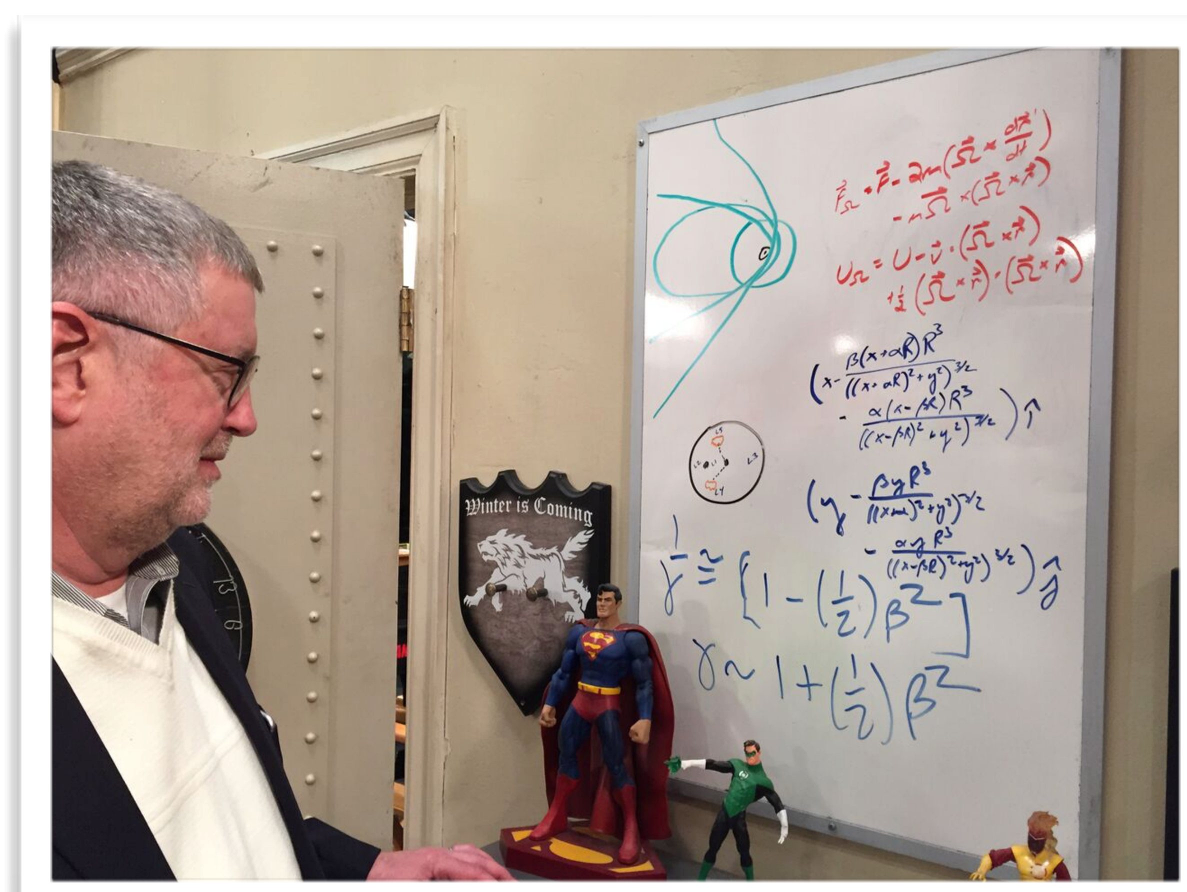
$$f(x) = \frac{1}{1-x} = 1 + x$$

what equation comes to mind?

when you're on the spot?

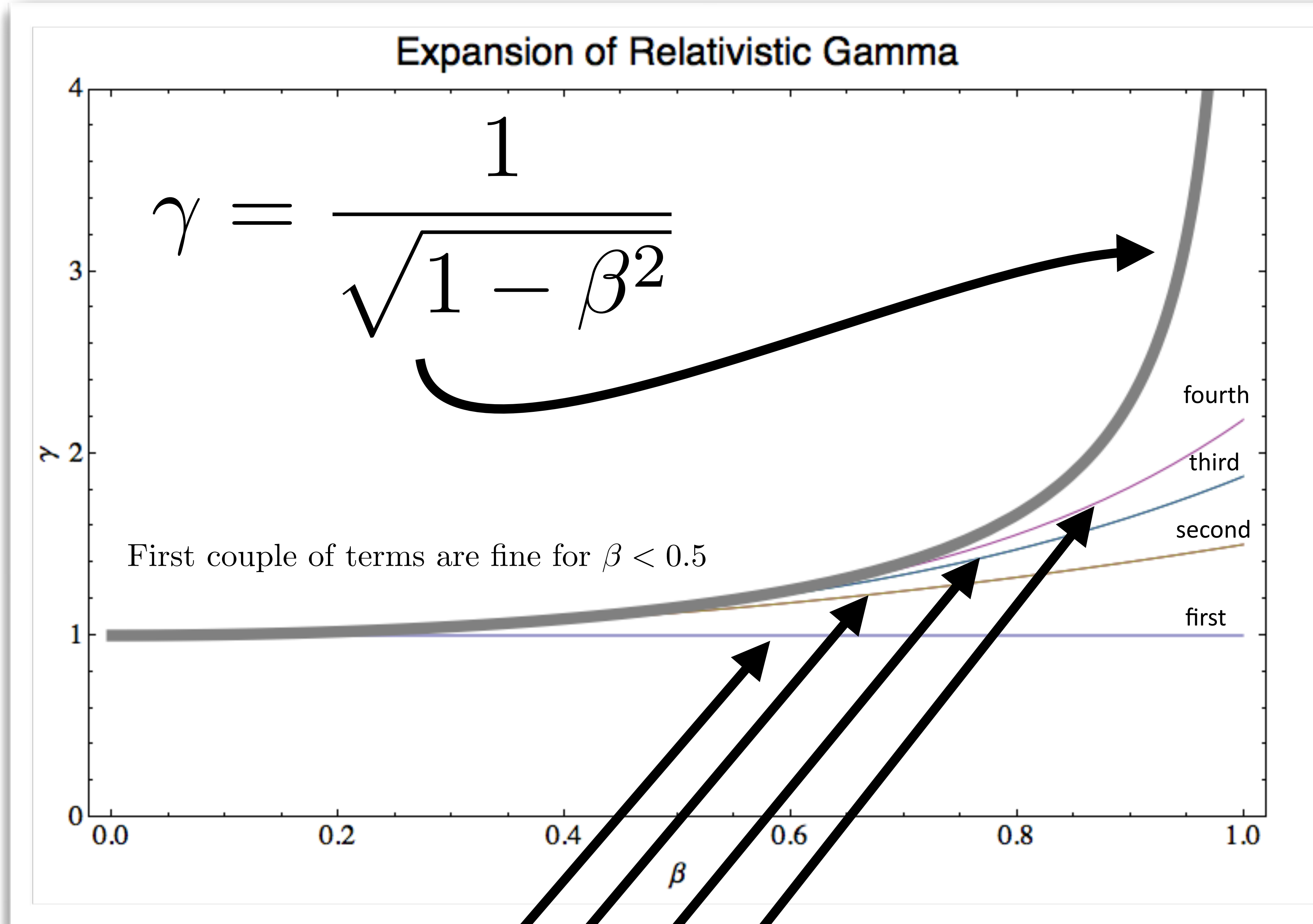
Why the binomial expansion of the relativistic gamma function, of course. Because, Relativity.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$

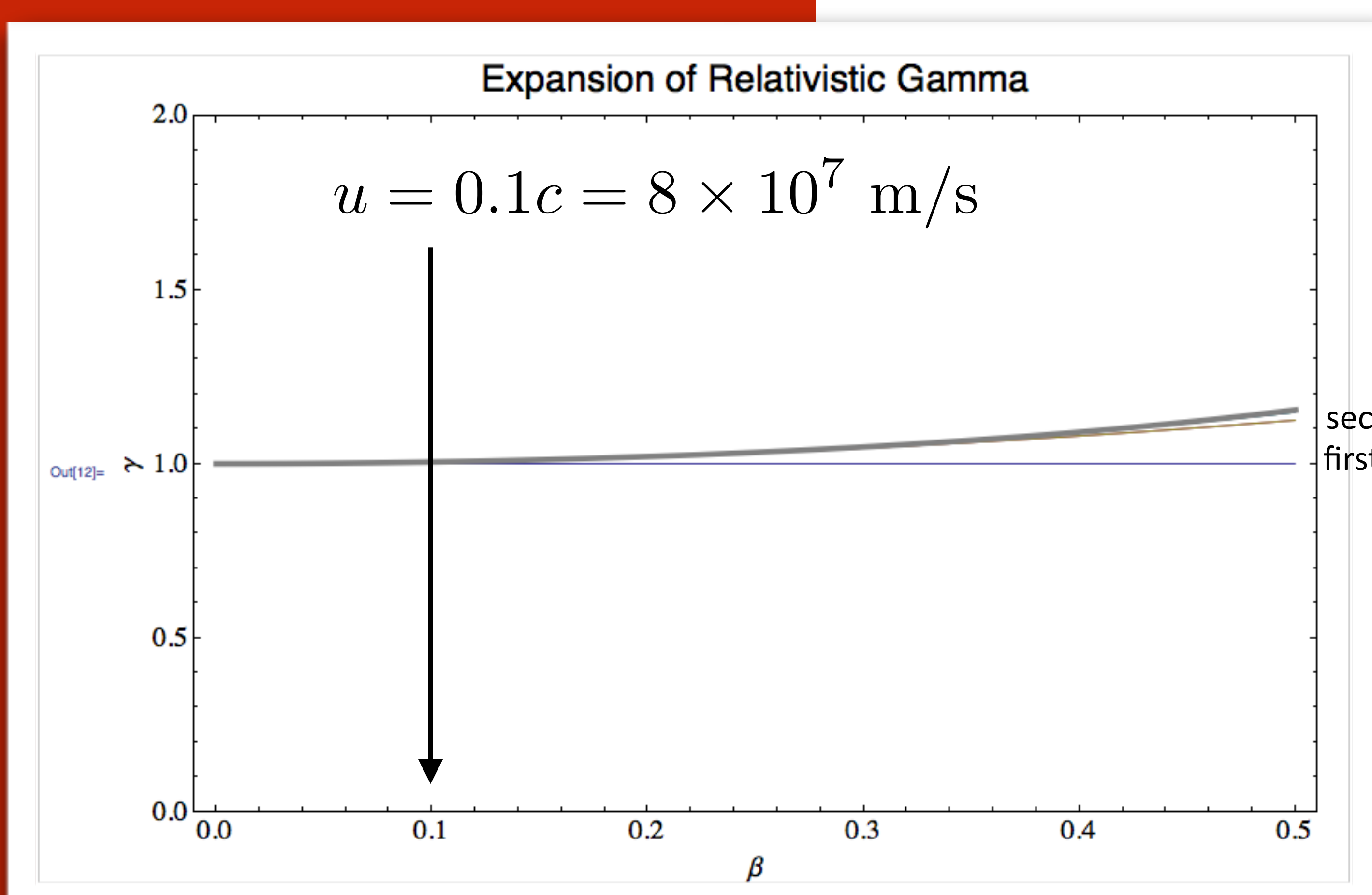


how well?

look at 8 terms in the expansion



$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$



Season 9, Episode 12

The Sales Call Sublimation, January 7, 2016

so let's use this and look for familiar things

slow moving objects but not completely classical

$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\beta^4}{8} + \frac{5\beta^6}{16} + \frac{35\beta^8}{128} + \frac{63\beta^{10}}{256} + \frac{231\beta^{12}}{1024} + \frac{429\beta^{14}}{2048} + O[\beta]^{15}$$

sing along

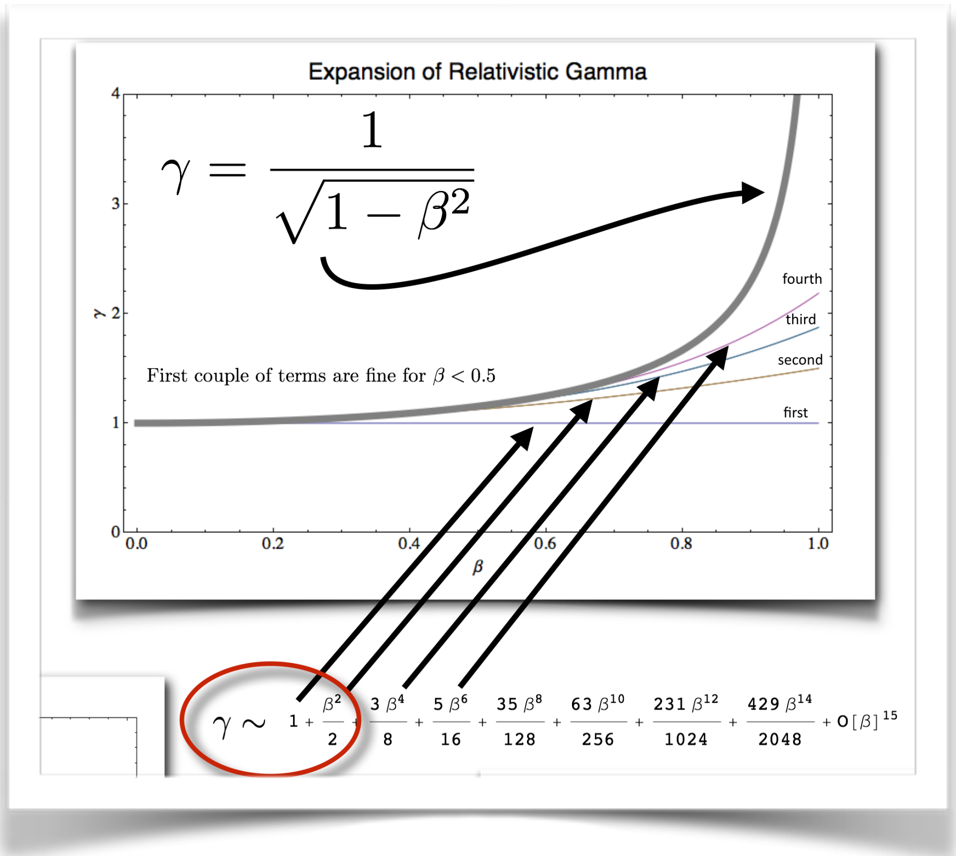
for β small:

now copy the approximate forms, but insert $\beta = u/c$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

now, write along with me:



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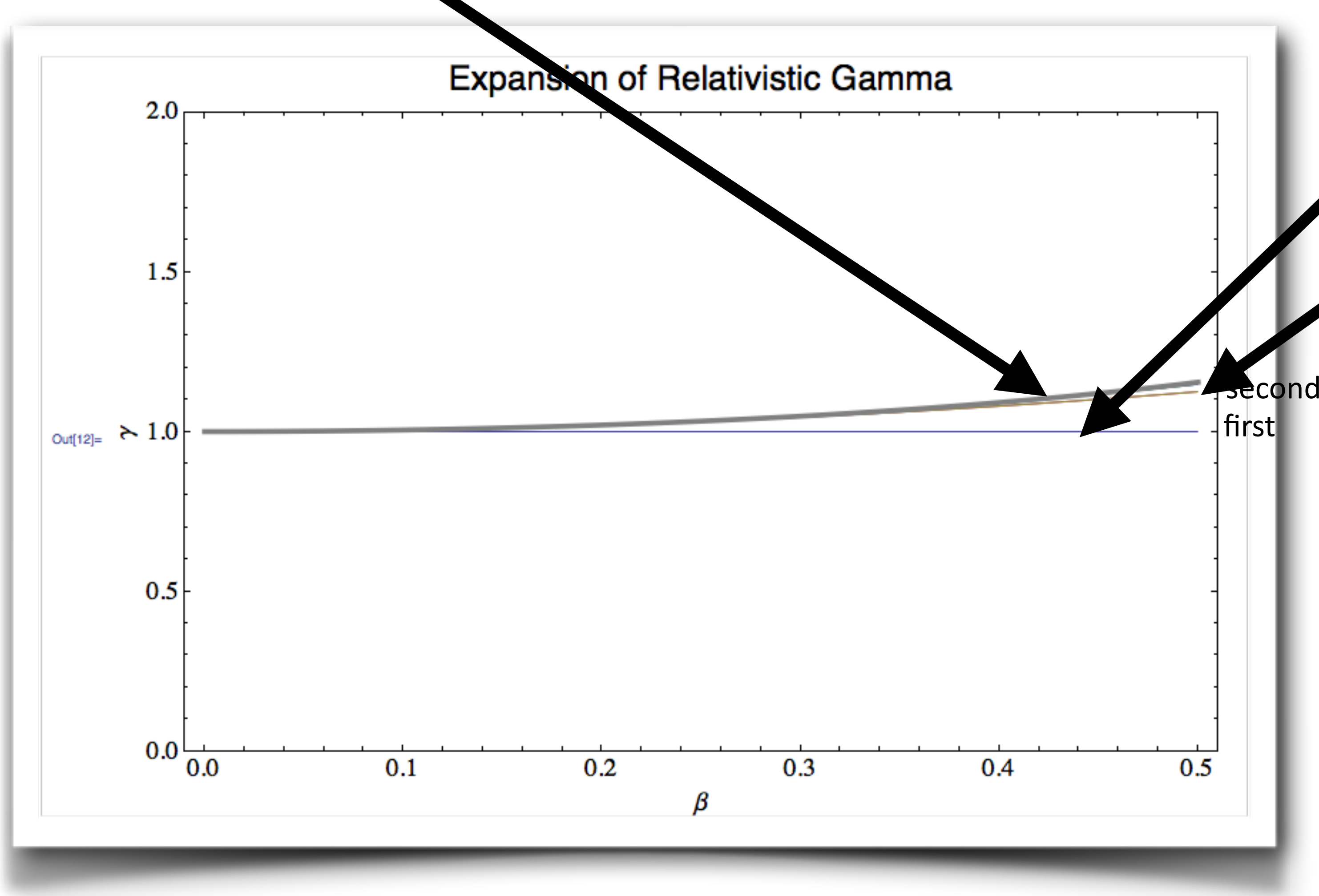
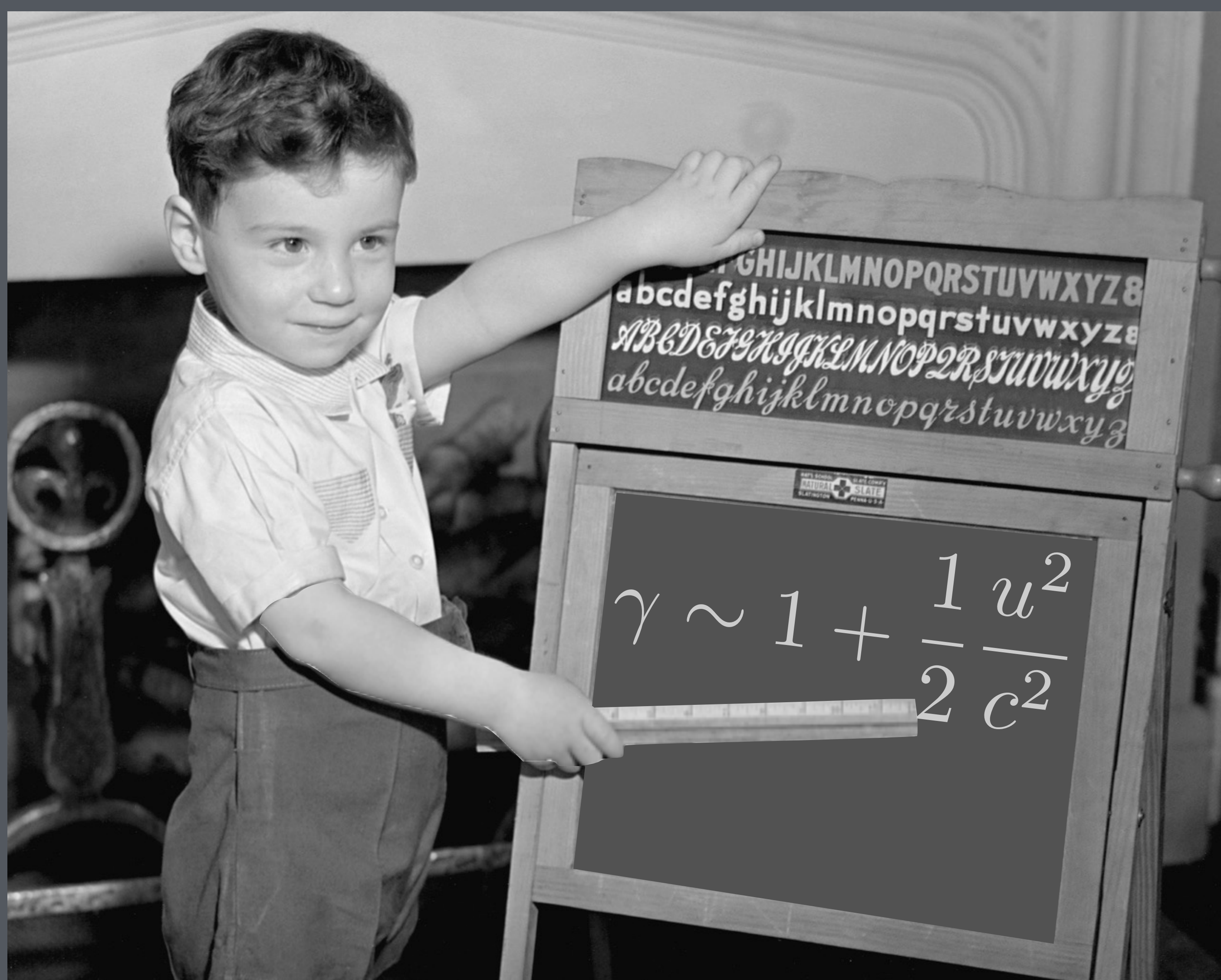
$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\frac{u^2}{c^2}$$





now let's play

Relativistic Energy?

Whatever it is...you want it to reduce to Kinetic Energy for small v

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$K = \frac{1}{2}mu^2$$

Suspicious, though.

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \sim 1 + \frac{1}{2}\frac{u^2}{c^2}$$

by now you expect gamma to be involved.

Kinda looks like K.

dress it up:
nip a little here, tuck a little there...

$$c^2m\times\gamma = \frac{c^2m}{\sqrt{1-\frac{u^2}{c^2}}} \sim (1+\frac{1}{2}\frac{u^2}{c^2})\times c^2m$$

$$c^2m + \frac{1}{2}mu^2 \rightarrow \gamma c^2 m$$

some new kind of energy?

looks like a kinetic energy

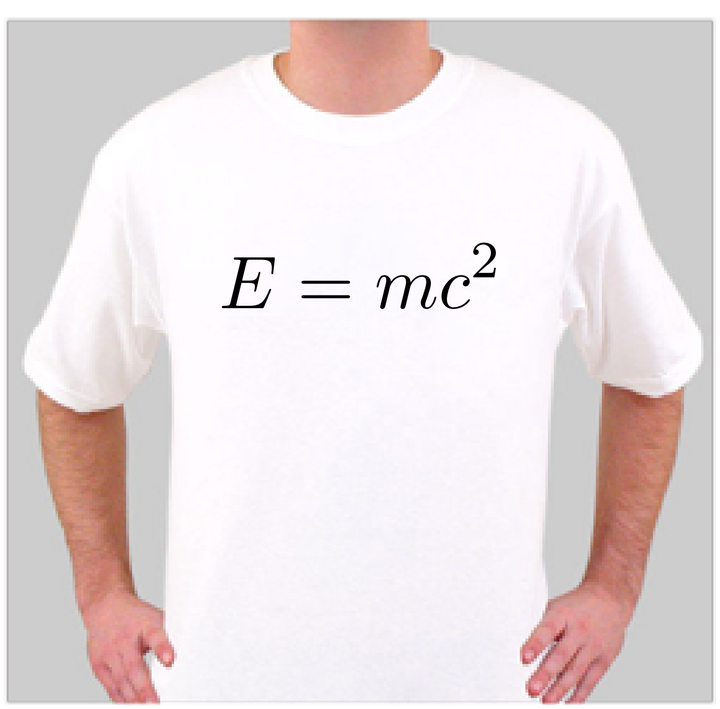
an "energy," that includes Kinetic Energy

$$E_T = m\gamma c^2$$

Relativistic **Total** Energy = new energy + kinetic energy

the new energy

energy related to mass only.



The Tee Shirt Equation!

Energy related to mass, - "**rest mass**" - I'll call E_m

$$E_m = mc^2$$

Energy related to motion, I'll continue to call, "**kinetic**,"

$$K$$

Total energy of anything, I'll call E_T

$$E_T = E_m + K$$

what "mass" really is: "trapped energy"

jargon alert: **rest mass**

refers to: mass of an object in its own rest frame (related to Rest Energy, the mass-energy of an object in its own frame)

entomology: “rest” implying...not moving

example: the rest mass of the electron is 9.109×10^{-31} kg

~~"convert mass into energy"~~

no.

Mass is energy and energy is mass.

$\$ = € * 1.06$ both currency, can both buy stuff

just a conversion factor...

$E_m = mc^2$ both energy, can both do work

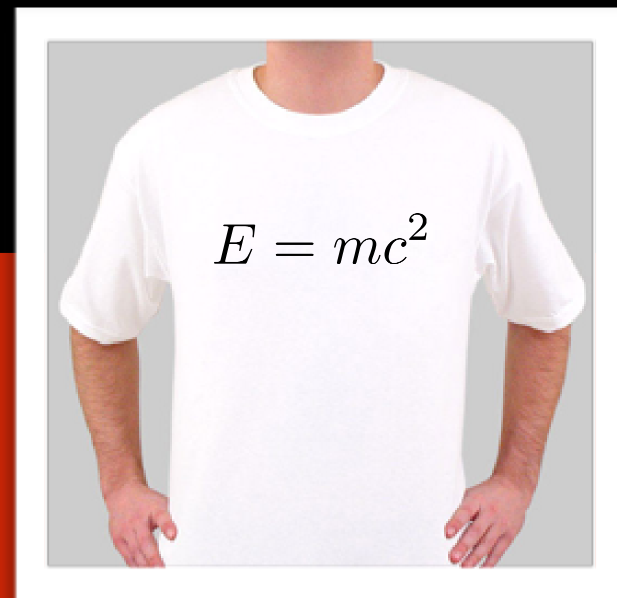
just a conversion factor...

a big conversion factor

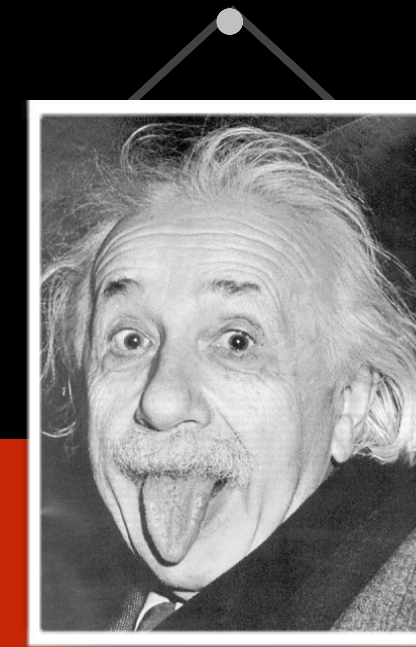
I *could* speak of the energy of mass... and the mass of energy

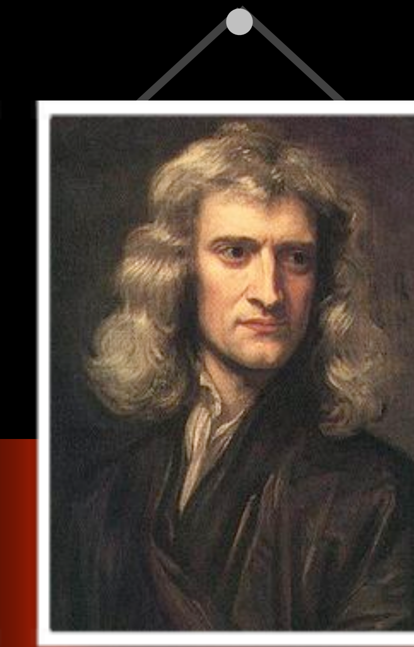
and I will.

Energy



cheat shirt





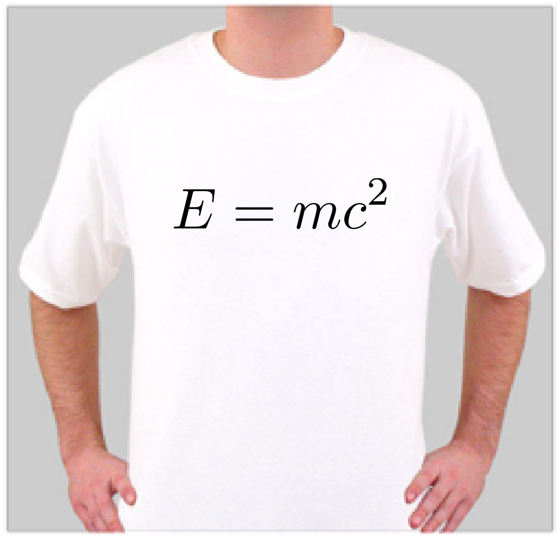
lots of pent-up energy in an apple

mass of the apple = 100 gm = 0.1 kg

$c^2 = 9 \times 10^{16}\ m^2/s^2$

Motion energy = 1 Joule

$$E_m = mc^2$$
$$= (0.1)(3 \times 10^8)^2$$ = Mass energy = 9,000,000,000,000,000 Joules!

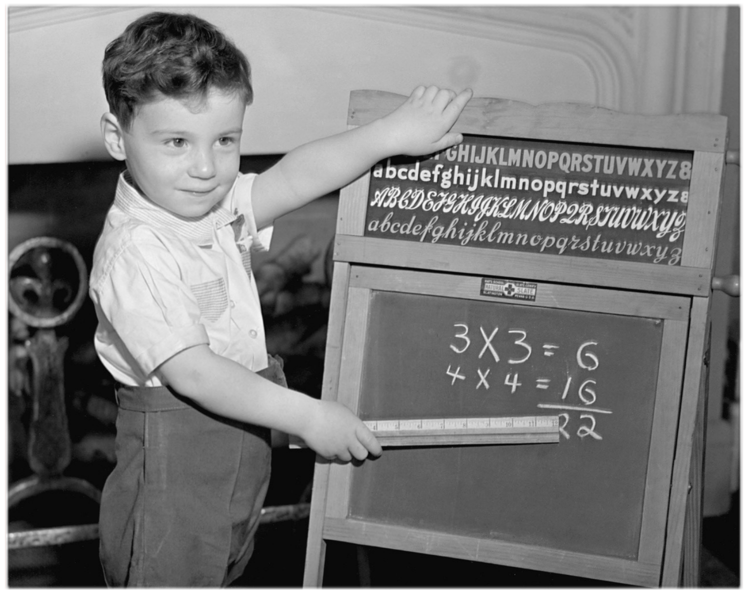


cheat shirt

the mass of a penny is

$$3\text{gm} = 3 \times 10^{-3}\text{kg}$$

The speed of light squared is: $c^2 = 9 \times 10^{16}\text{m}^2/\text{s}^2$



How many Joules of energy is trapped in that mass?

that is...what's the rest energy of a penny?

write it down and show me. talk together.

$$E_m(\text{penny}) = 27 \times 10^{13}\text{J}$$

Aircraft Carrier Nimitz: 91,400 tons at 32 knots:

$$K(\text{Nimitz}) = 1.1 \times 10^{10}\text{J}$$

so: two kinds of energy

energy of motion…Kinetic Energy

\+ energy of rest…associated with mass

Total energy of an object

there aren’t any other kinds of energy

look at the total energy

and increase of mass with velocity

$$E_T = m\gamma c^2$$

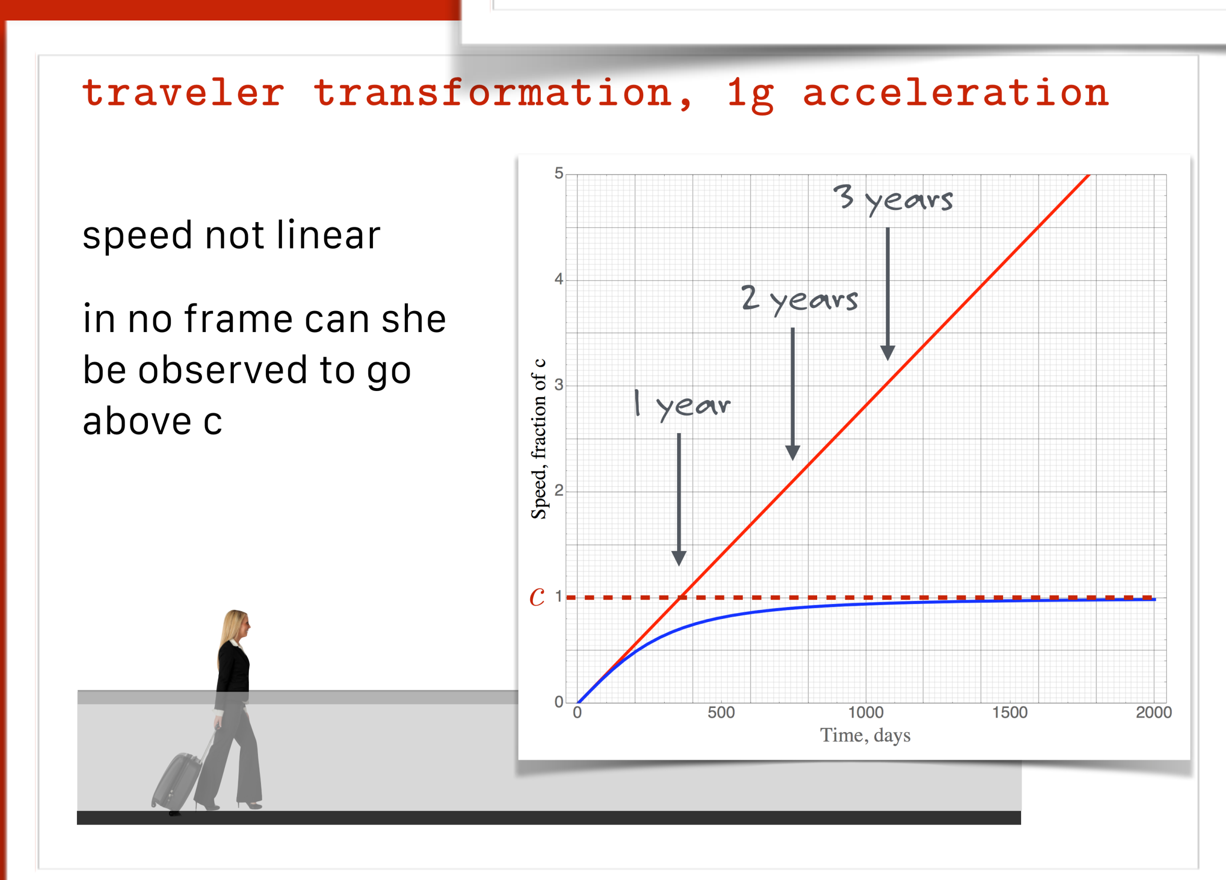
One way to interpret this is to associate gamma and m.

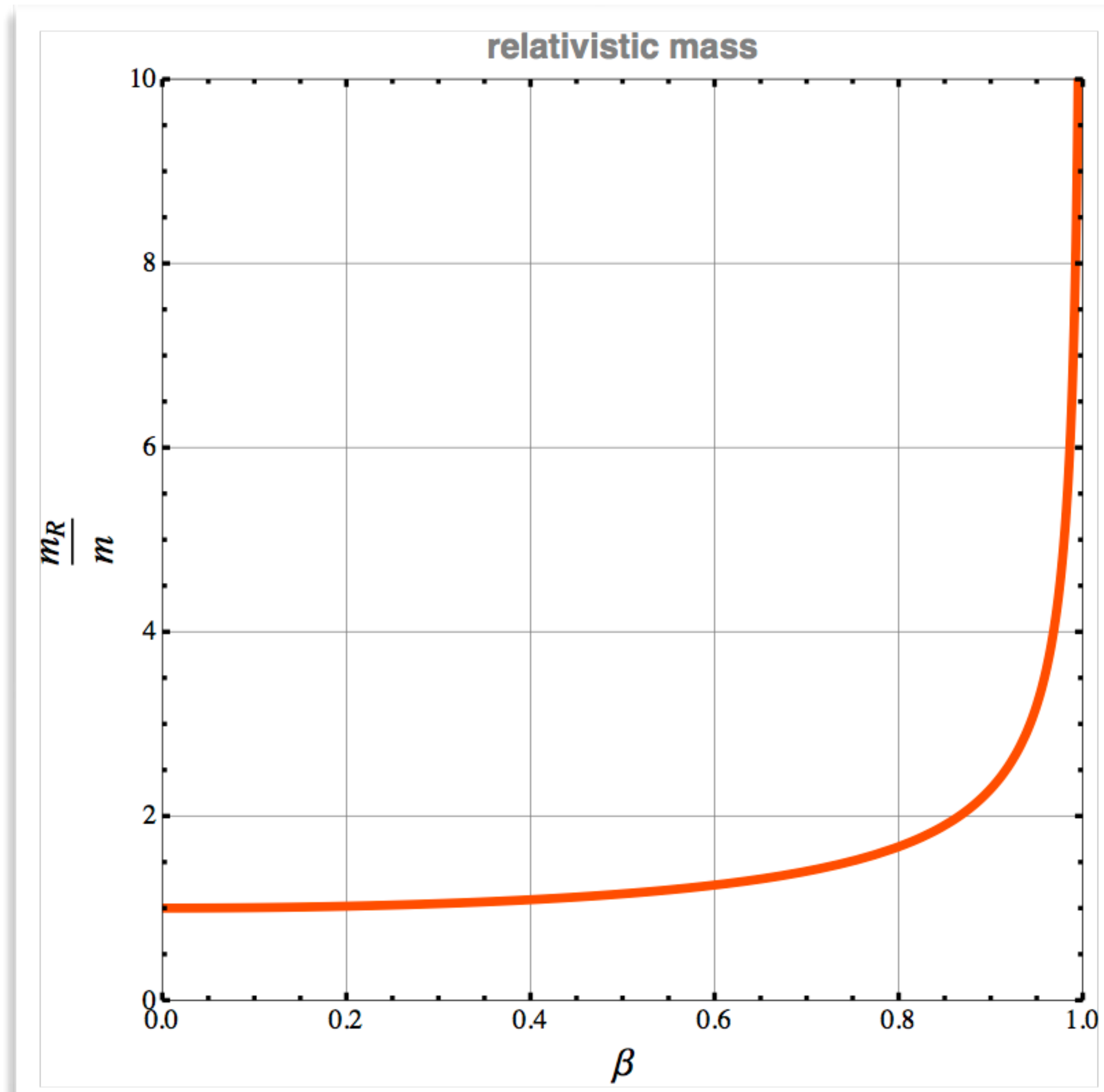
$$E_T = (m\gamma)c^2 \quad \text{so} \quad E_T = m_R c^2$$

...and speak of a speed-dependent "**relativistic mass**."

$$m_R = \gamma m \quad \text{so} \quad \frac{m_R}{m} = \gamma$$

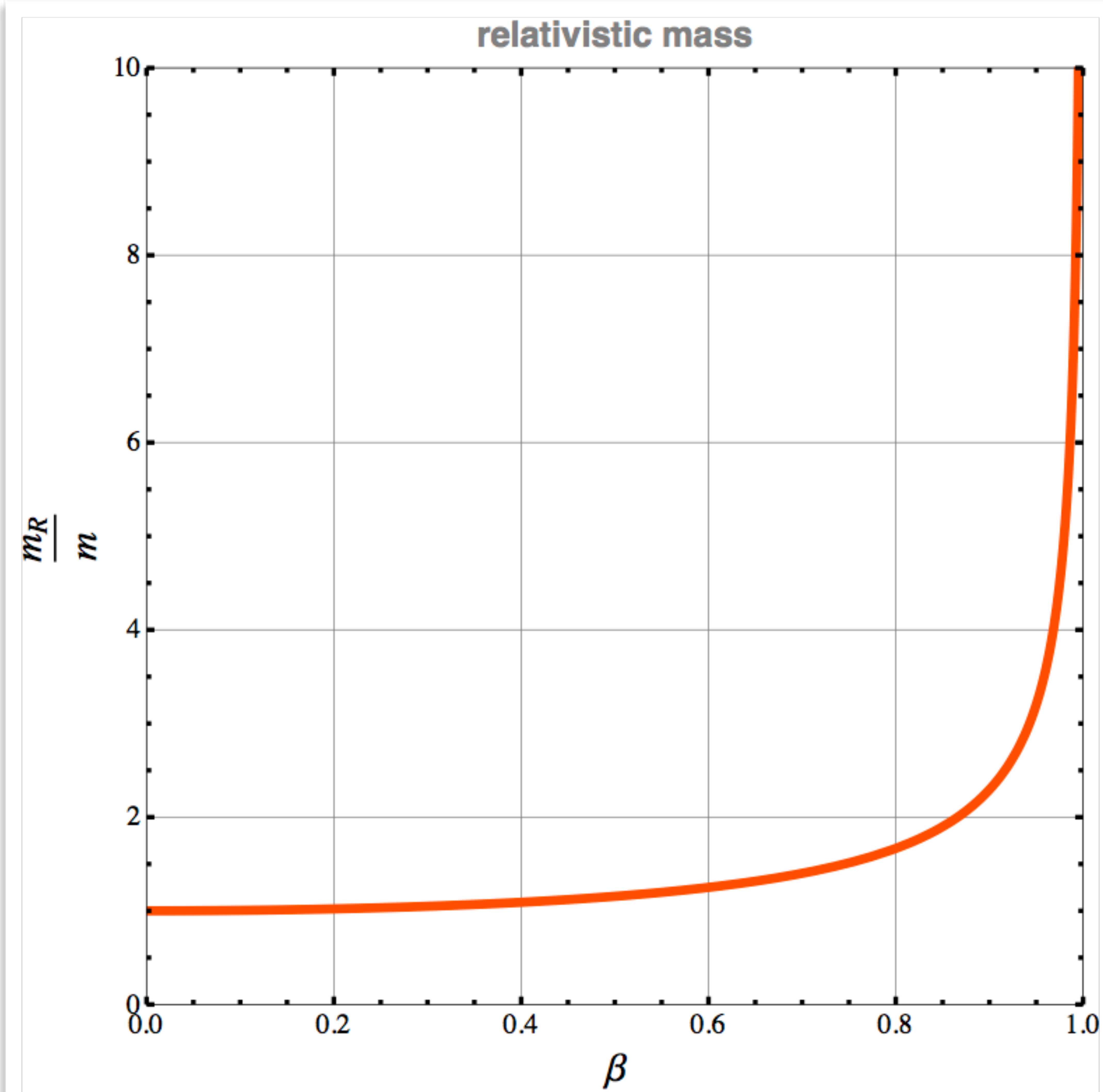
remember the inertia issue?

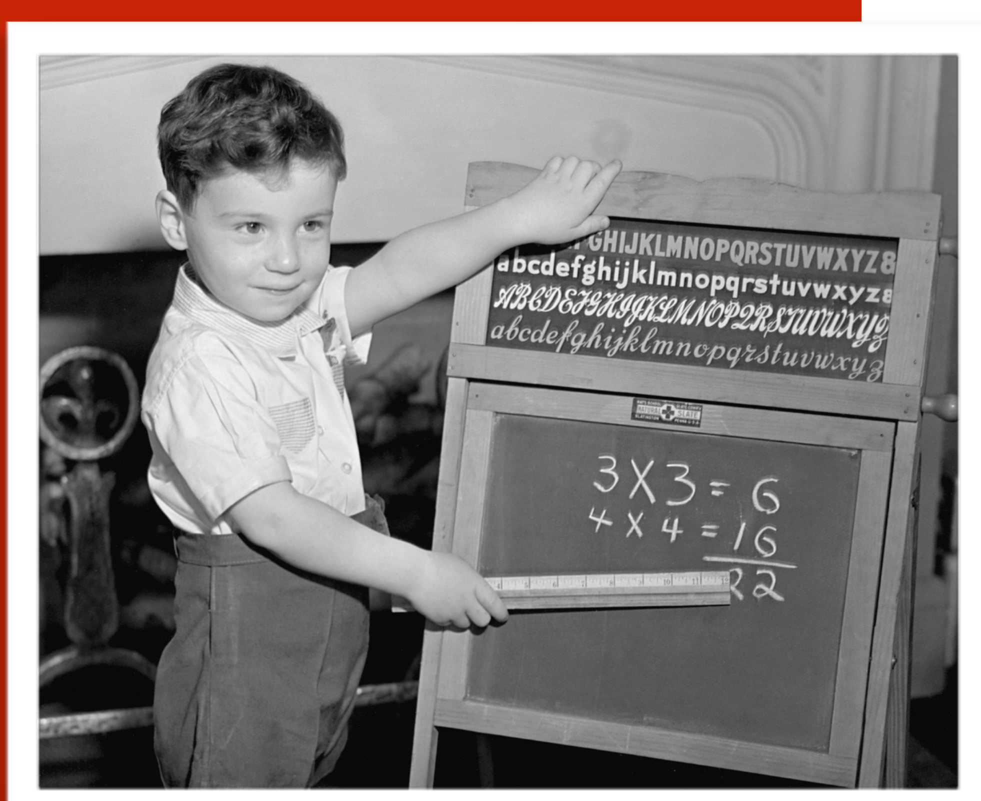




if a proton is going at 0.95% of the speed of light and has mass of 1p

how massive does it appear to be?





let's look at the kinetic energy

energy of motion...Kinetic Energy

\+ energy of rest...associated with mass

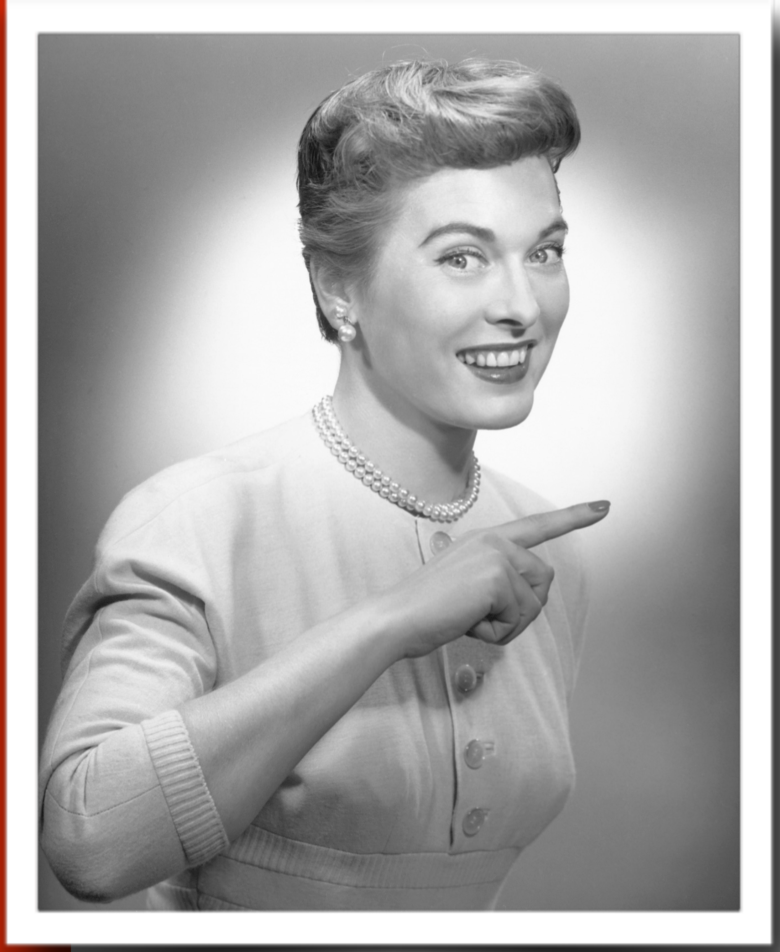
Total energy of an object

mass energy: $E_m = mc^2$

total energy: $E_T = m\gamma c^2$

kinetic energy?

Fully relativistic now



let's look at the kinetic energy

energy of motion...Kinetic Energy

\+ energy of rest...associated with mass

Total energy of an object

$$E_T = E_m + K$$

$$K = E_T - E_m$$

$$K = m\gamma c^2 - mc^2$$

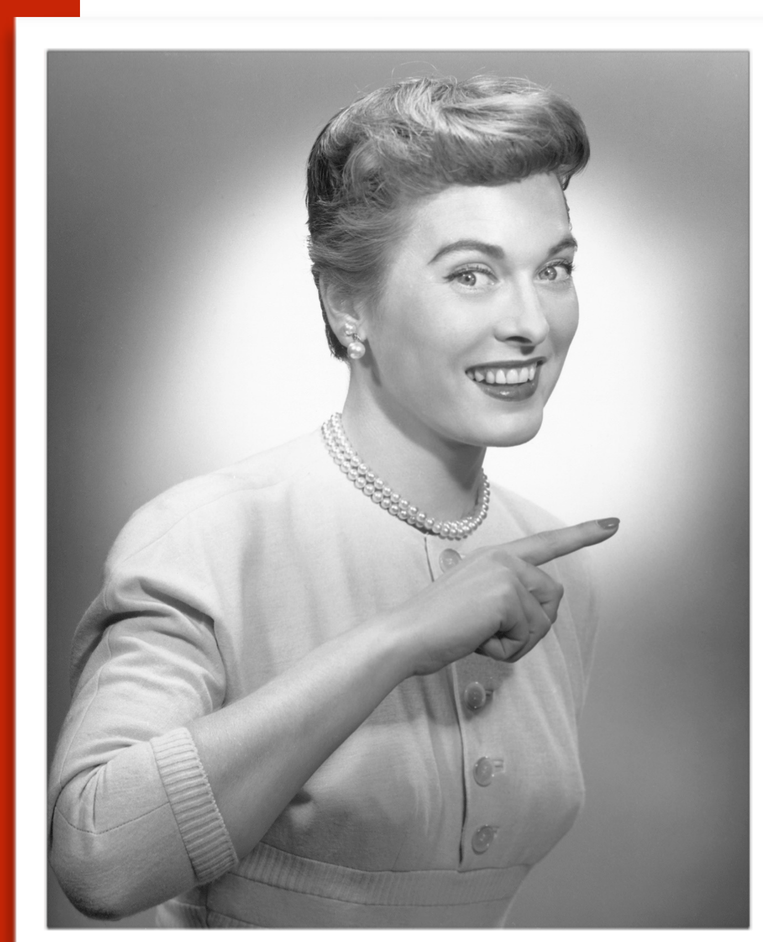
$$K = mc^2(\gamma - 1)$$

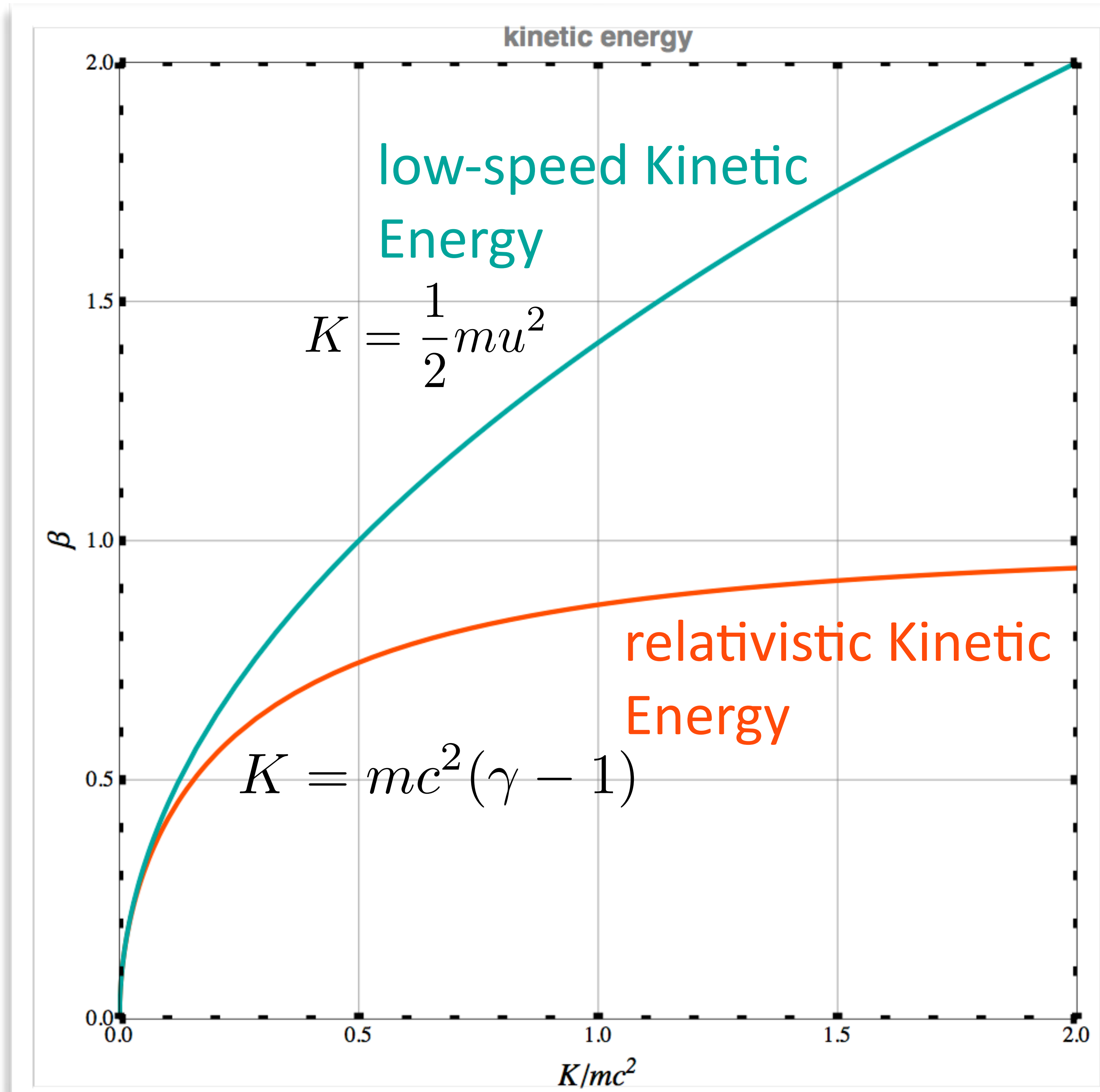
mass energy: $E_m = mc^2$

total energy: $E_T = m\gamma c^2$

kinetic energy?

Fully relativistic now





from this point on:

if I refer to the rest mass*…I'll say so

otherwise, "mass" is this velocity-dependent quantity

*(This is not how we speak in polite particle physics circles…where "mass" is a constant always.)
But, I think for non-specialists this is more clear.

a useful invariant

$$E_m = mc^2$$

$$E_T = m\gamma c^2$$

$$p = m\gamma v$$

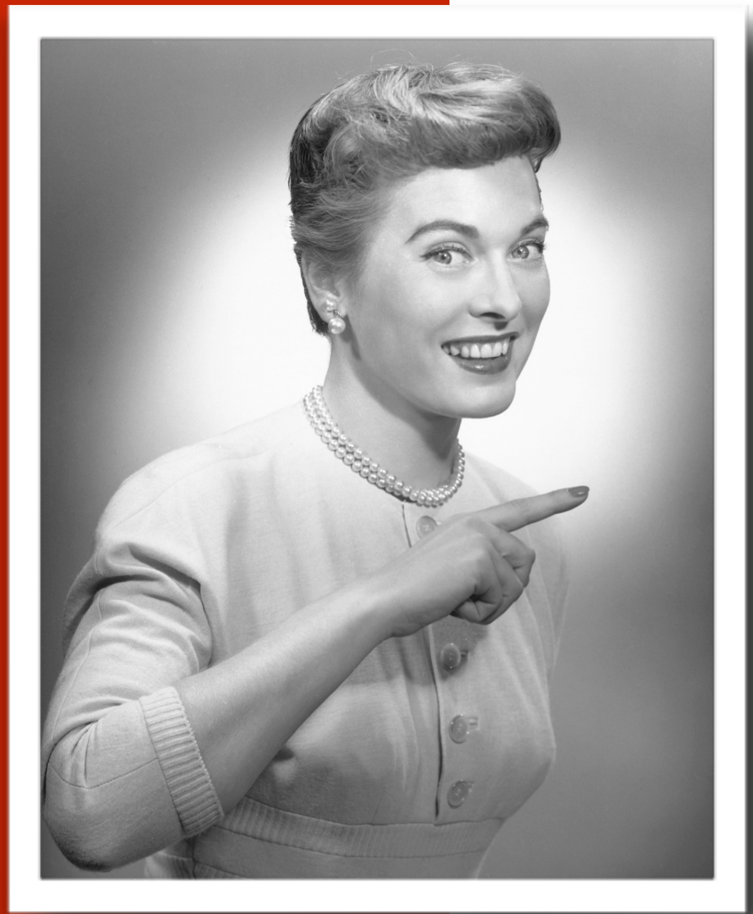
and an important formal linkage

fun fact...just with a little algebra... (video)

$$E_m^2 = E_T^2 - p^2c^2$$

$$m^2c^4 = E_m^2 = E_T^2 - p^2c^2$$

no velocity dependences, just a number...



“Energy-momentum relation”...

$$E_T^2 = (mc^2)^2 + (pc)^2$$

practical

Energy/momentum relations:

"rest mass"... m

the mass of an object in its own frame

"relativistic mass"... $m_R = m\gamma$

the mass of a moving object

"Energy"... $E_T = m\gamma c^2$

the total Energy of a moving object

"rest Energy"... $E = mc^2$

the mass-energy of an object in its own frame

Kinetic Energy... $K = mc^2(\gamma - 1)$

the energy due to motion

Relativistic momentum... $p = m\gamma u$

momentum for each component of space

Energy-momentum relation... $E_T^2 = \left(mc^2\right)^2 + (pc)^2$

an alternative, useful expression

real electrons

HV transmission lines feed substations?

138,000 V is common (BWL for example)

Assume that arc is at 138,000V, so electrons have that energy

...which would be the Kinetic Energy

an exercise in "electron volts"

What's the rest energy?

What's the rest mass?

What's the speed of the electrons?

What's the momentum of one of the electrons?

What's the relativistic mass of one of the electrons?

What's the total energy of one of the electrons?

This will be on video and figure into homework

a collision from earlier

where mechanical energy was not conserved.

1 | 12 | 2

and they stick together

But we certainly would have said: $m_1 + m_2 = M_{12}$

Now...energy conservation is different:

$$E_{\text{(before)}} = E_{(after)}$$

$$[E_{\text{(Object 1)}}] + [E_{\text{(Object 2)}}] = [E_{\text{(Object 12)}}]$$

$$E_{m(1)} + K_1 + E_{m(2)} + K_2 = E_{m12} + K_{12}$$

brand new thing!

a collision from earlier

where mechanical energy was not conserved.

completely inelastic collision

1 | 12 | 2

and they stick together, and stop

system's energy of masses + KE's = system's energy of mass + KE

brand new thing!

$$\begin{pmatrix} K(1)+K(2) \\ + \\ m(1)c^2 + m(2)c^2 \end{pmatrix} = \begin{pmatrix} K(12) \\ + \\ M(12)c^2 \end{pmatrix}$$

$$M(12)c^2 = m(1)c^2 + m(2)c^2 + K(1) + K(2)$$

But now, the mass of the stuck-together system is more than the masses of the projectiles...

But before we certainly would have said:

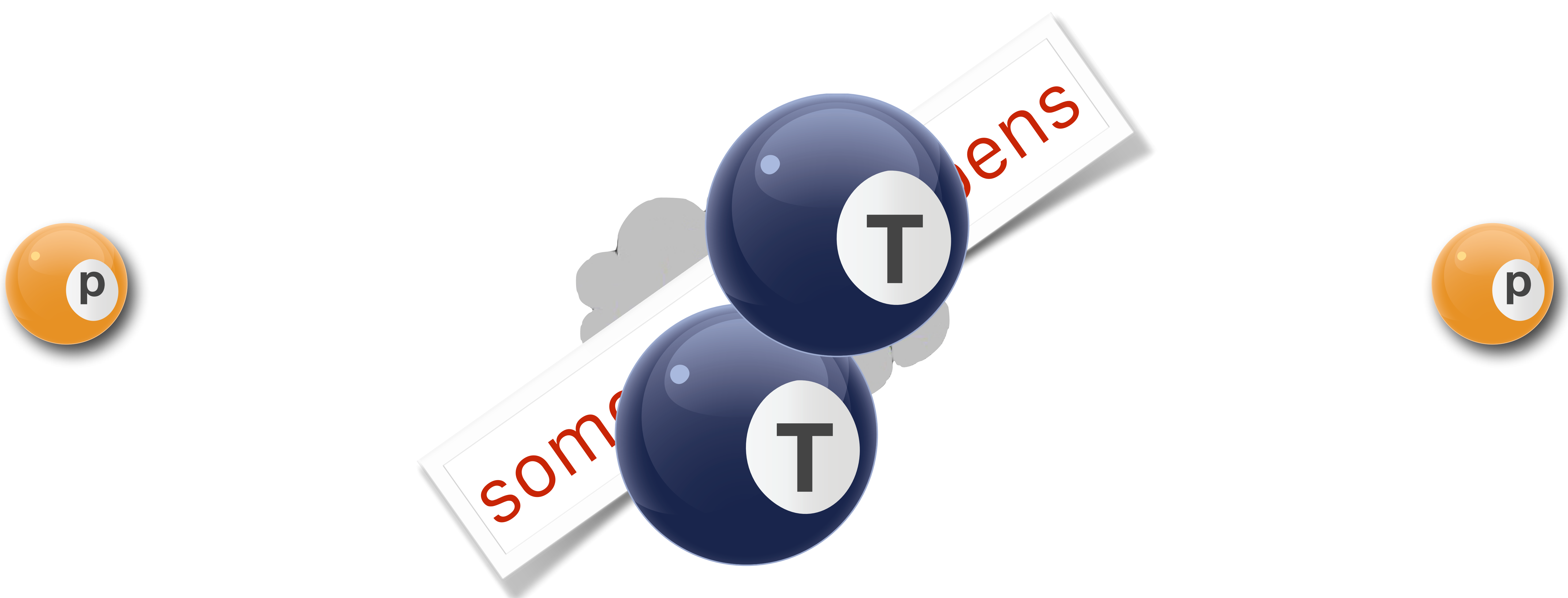
$$m_1 + m_2 = M_{12}$$

The energy of motion has become energy of mass.

this is how

we can take two protons, crash them together, and produce 2 “top quarks”…

each of which has the mass of 170 protons



conserved quantities:

3 of them now:

Energy Conservation:

both mass-energy and kinetic energy are counted

Momentum Conservation

energy-momentum conservation

Energy Conservation in a collision:

$$A + B \rightarrow C + D$$

[MassEnergy$_0$(A) + KE$_0$(A)] + [MassEnergy$_0$(B) + KE$_0$(B)] =

[MassEnergy(C) + KE(C)] + [MassEnergy(D) + KE(D)]

$$[m(A)c^2 + K(A)] + [m(B)c^2 + K(B)] =$$

$$[m(C)c^2 + K(C)] + [m(D)c^2 + K(D)]$$

particle colliding beam

E_T = mass energy + kinetic energy + E_T = mass energy + kinetic energy

1 proton's mass = M_p

1 proton's mass energy = $M_p c^2$

Use head-on collisions to make objects more massive than protons.

Make Two things that each have $M(\text{thing}) = 3.5 \bullet M_p$