hi

Lecture 16, 02.03.2017

Einstein's Special Theory of Relativity - the end

Einstein's Theory of General Relativity - the beginning

housekeeping

Question about anything?

I'll make a movie for you:

Marie Curie movie anyone?

new FB poll of 3/15 and 3/17. trying to find a room still…

Blog read-reflect project will start soon.

See calendar cartoon:

next few weeks, v2

S	M	T	W	Th	F	Sa
						2/25 HW7 HW6 due
2/26	2/27 ← midterm	2/28 →	3/1	3/2	3/3	3/4 HW8 HW7 due
3/5 ←			Spring Break			3/11 →
3/12	3/13	3/14	3/15	3/16	3/17	3/18 HW8 due

Honors Project

has begun.

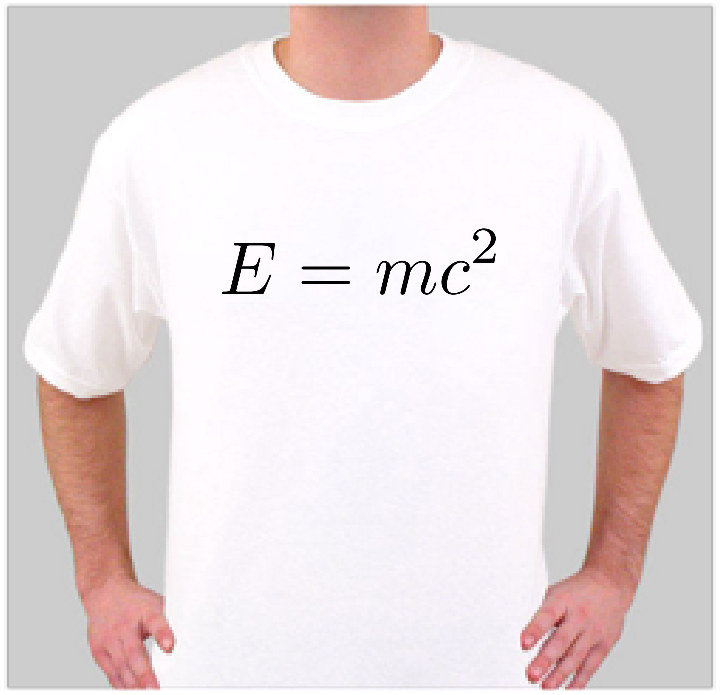
Read the first of two sets of instructions:

MinervaInstructions1_2017.pdf in

www.pa.msu.edu/~brock/file_sharing/QSandBB/2017homework/honors_project_2017/

the new energy

energy related to mass only.



The Tee Shirt Equation!

Energy related to mass, - "**rest mass**" - I'll call E_m

$$E_m = mc^2$$

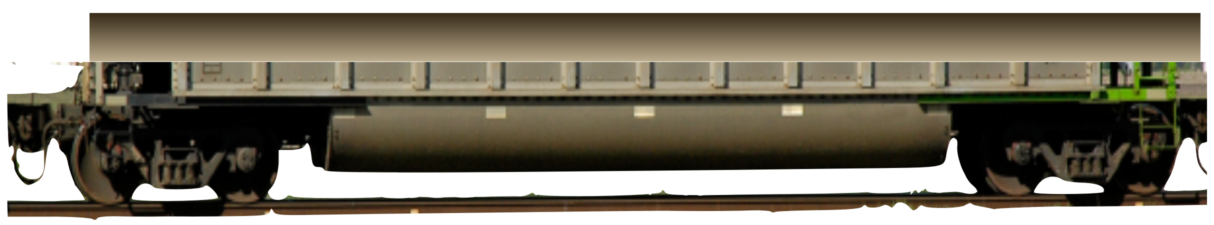
Total energy of anything, I'll call E_T

$$E_T = E_m + K$$

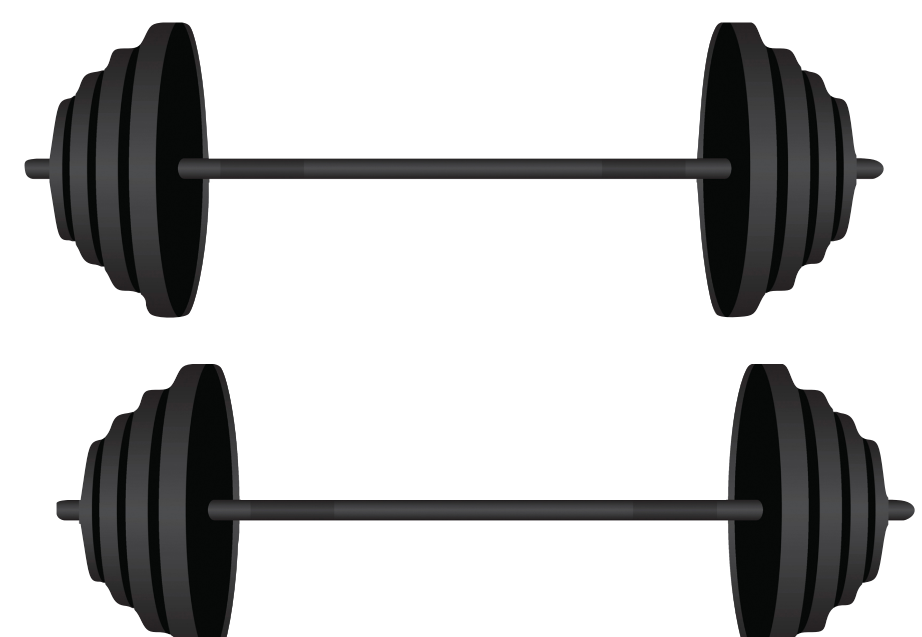
$$E_T = (m\gamma)c^2 \qquad m_R = \gamma m$$

Einstein's 4th 1905 paper: $m = \dfrac{E}{c^2}$

$m_A = 100\ \text{kg}$



100 kg



$m_H = 100\ \text{kg} \times \gamma$

Relativity, to date:

Consequences of the Second Postulate for co-moving frames of reference:

- space and time determinations are relative
- simultaneity is relative
- electric and magnetic fields are relative
- masses (inertia) is relative

} no absolutes

- velocity within frames transforms differently from Newtonian principles
- Newton's Second Law is re-written...obscure, actually

You might wonder if anything is constant?

Yes! Some important things...

Energy/momentum relations:

"rest mass"... m

the mass of an object in its own frame

"relativistic mass"... $m_R = m\gamma$

the mass of a moving object

"Energy"... $E_T = m\gamma c^2$

the total Energy of a moving object

"rest Energy"... $E_m = mc^2$

the mass-energy of an object in its own frame

Kinetic Energy... $K = mc^2(\gamma - 1)$

the energy due to motion

Relativistic momentum... $p = m\gamma u$

momentum for each component of space

Energy-momentum relation... $E_T^2 = \left(mc^2\right)^2 + (pc)^2$

an alternative, useful expression

real electrons

HV transmission lines feed substations?

138,000 V is common (BWL for example)

Assume that arc is at 138,000V, so electrons have that energy

...which would be the Kinetic Energy

an exercise in "electron volts"

What's the rest energy?

What's the rest mass?

What's the speed of the electrons?

What's the momentum of one of the electrons?

What's the relativistic mass of one of the electrons?

What's the total energy of one of the electrons?

This will be on video and figure into homework

Energy Conservation in a collision:

$$A + B \to C + D$$

$[\text{MassEnergy}_0(A) + \text{KE}_0(A)] + [\text{MassEnergy}_0(B) + \text{KE}_0(B)] =$

$[\text{MassEnergy}(C) + \text{KE}(C)] + [\text{MassEnergy}(D) + \text{KE}(D)]$

$$[m(A)c^2 + K(A)] + [m(B)c^2 + K(B)] =$$

$$[m(C)c^2 + K(C)] + [m(D)c^2 + K(D)]$$

invent a new unit of energy

call the mass energy, E_m, of a proton "1"

so if m(proton) = m(neutron) and He nucleus is 2p & 2n

what's E_m(helium)?

"4" right?

particle colliding beam

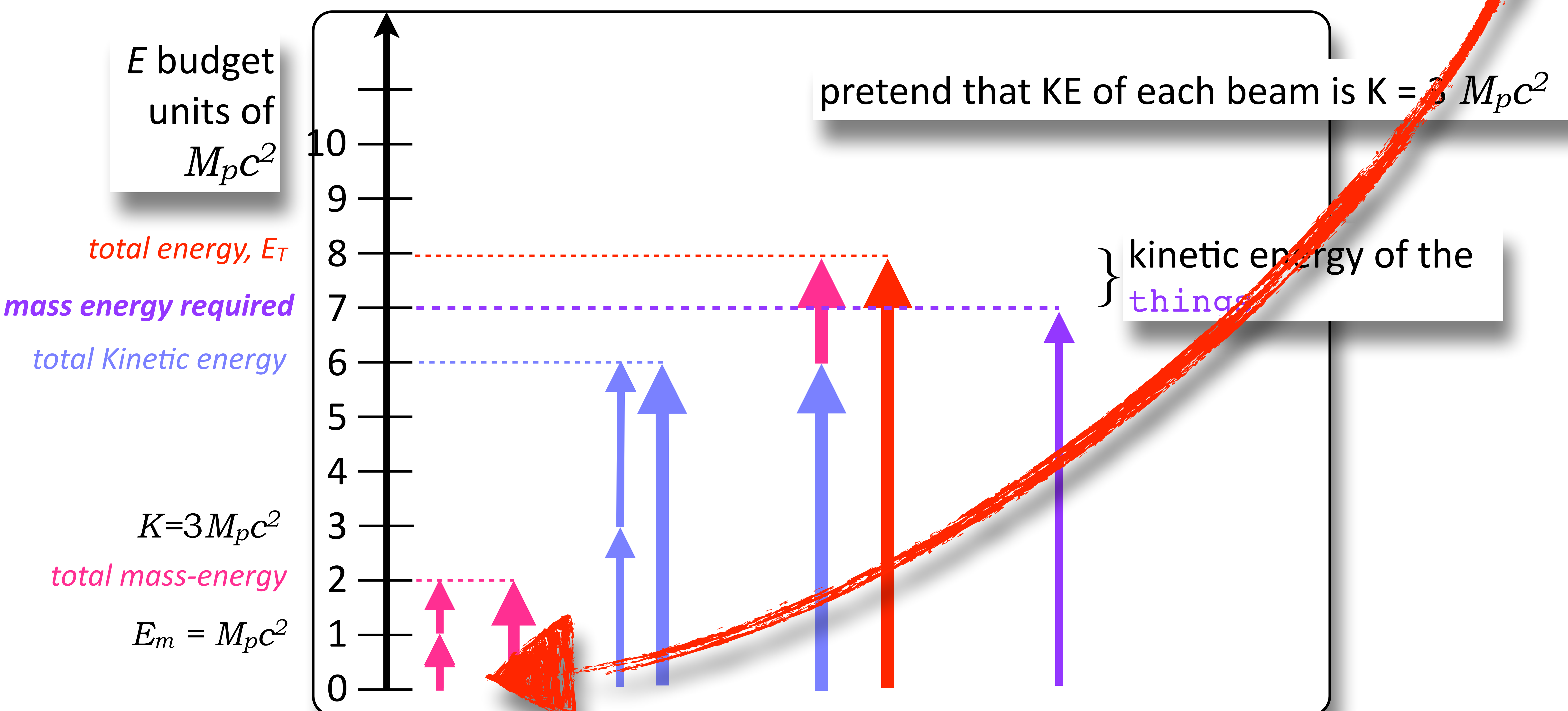
E_T = mass energy + kinetic energy + E_T = mass energy + kinetic energy

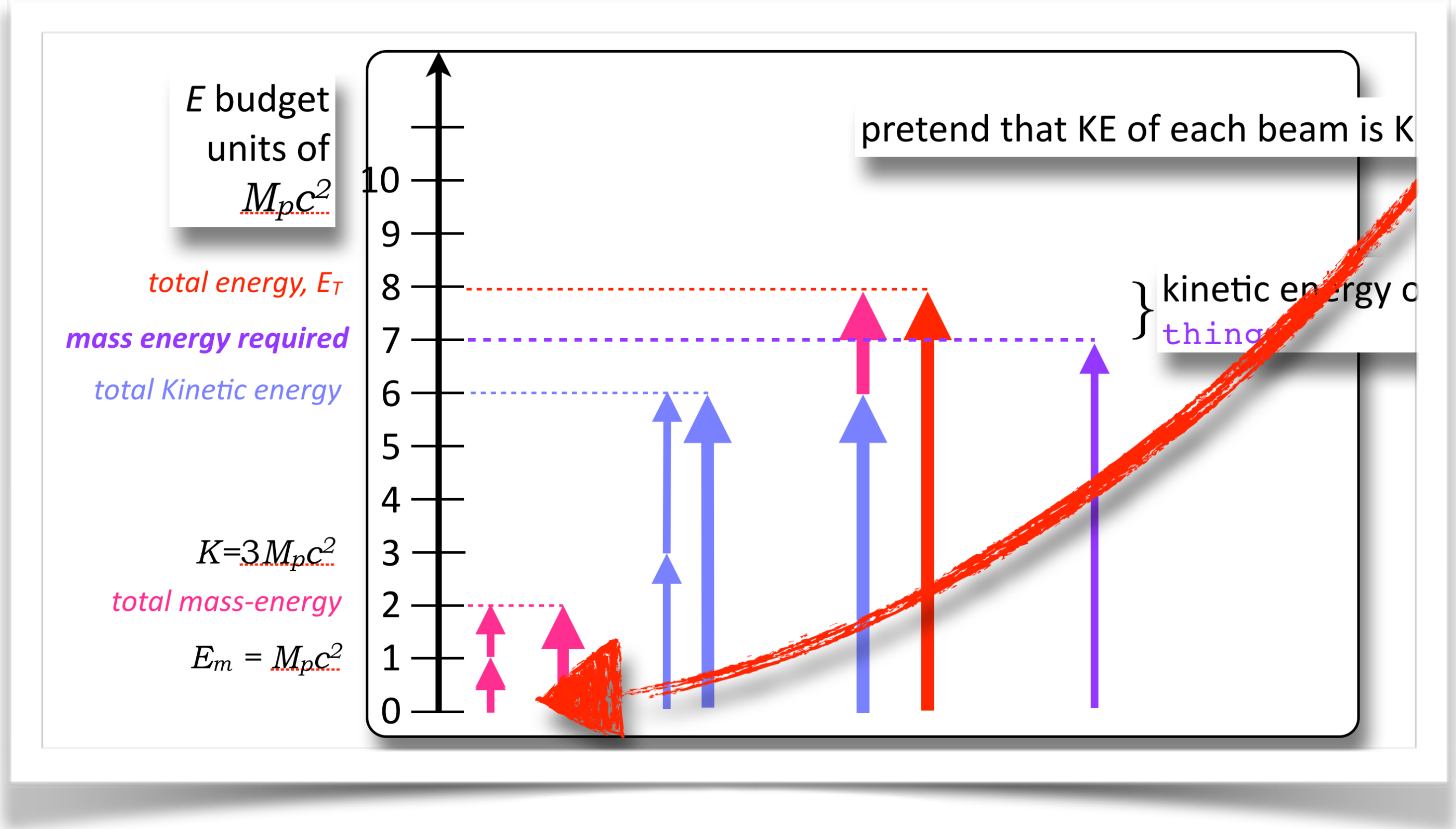
1 proton's mass = M_p

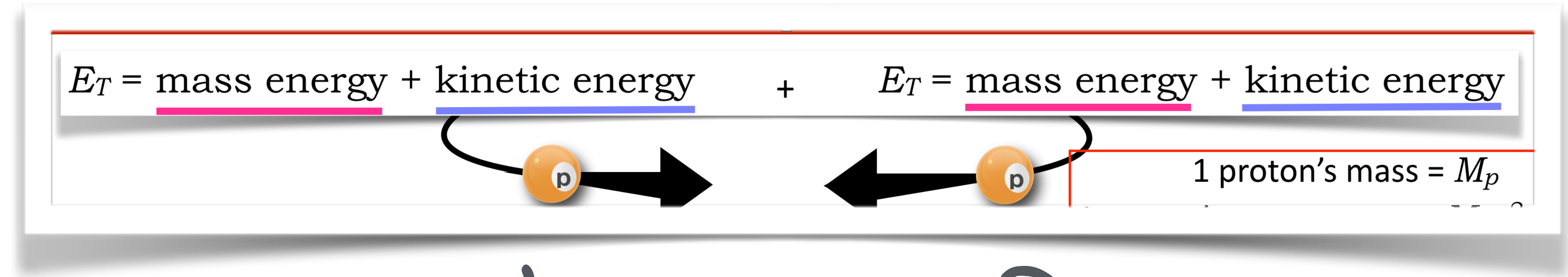
1 proton's mass energy = M_pc^2

Use head-on collisions to make objects more massive than protons.

Make Two things that each have $M(\text{thing}) = 3.5 \bullet M_p$



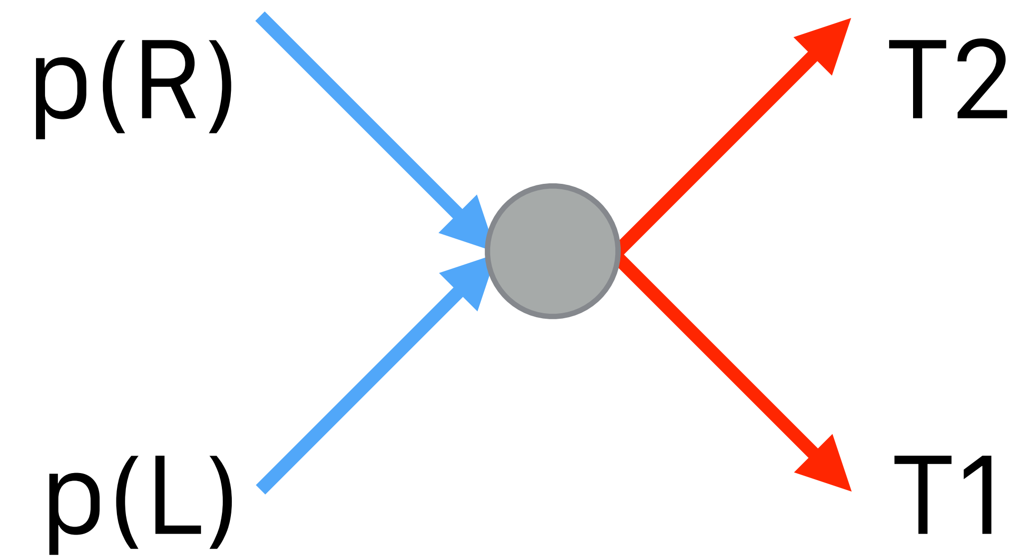




L R

the reaction p(L) + p(R) → T1 + T2

the Feynman Diagram



Energy equation:

$$E_{T,0}(L) + E_{T,0}(R) = E_T(\text{thing1}) + E_T(\text{thing2})$$

1 + 3 + 1 + 3 = 3.5 + 3.5 + ?

8 = 7 + 1

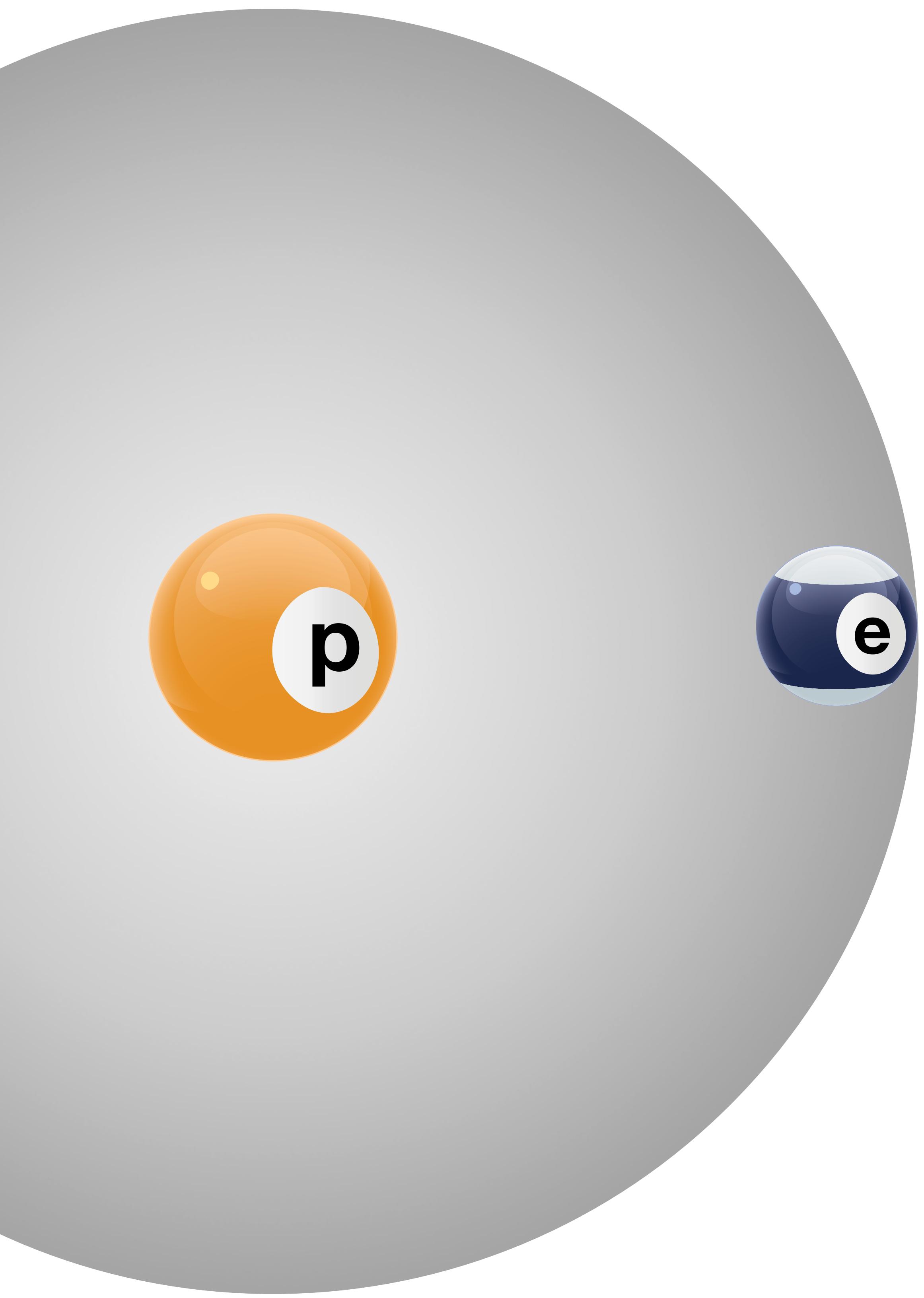
what about the

"energy of mass" and "mass of energy" crack?

suppose we have a bound system

What holds the electron to the proton?

Hydrogen Atom



Last week:

the electrostatic force, or the Electric field, right?

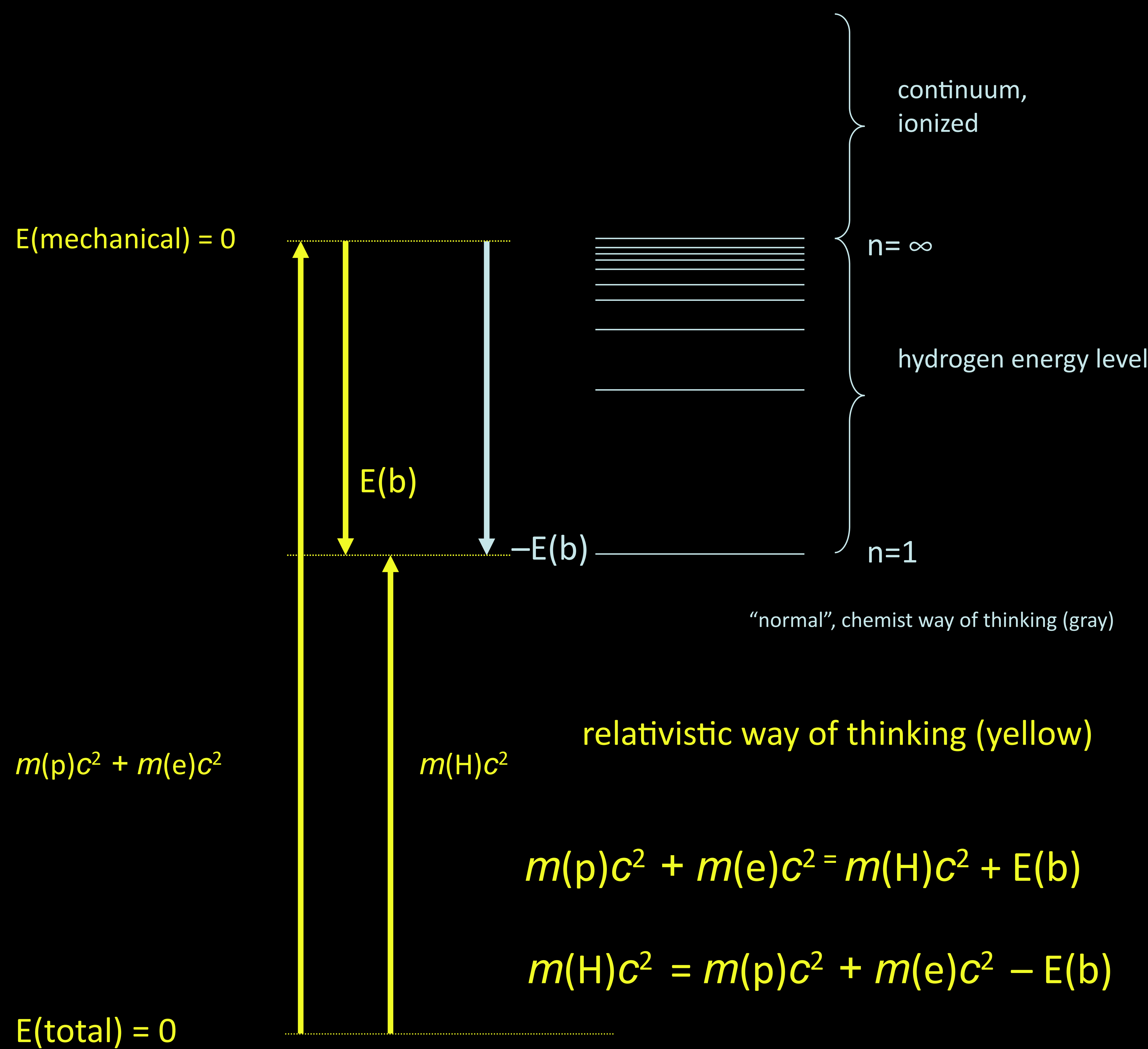
Remember from Chemistry:

What's it take to ionize* Hydrogen?

You must supply 13.6 eV

*make the electron free of the proton's influence

energy diagram for H



$$m(\mathrm{p})c^2 + m(\mathrm{e})c^2 = m(\mathrm{H})c^2 + E(\mathrm{b})$$

$$m(\mathrm{H})c^2 = m(\mathrm{p})c^2 + m(\mathrm{e})c^2 - E(\mathrm{b})$$

The mass of a hydrogen atom is LESS than the sum of $m_p + m_e$
No negative binding energy...just a "mass deficit" in the attraction of the P and e.
The energy is in the field.

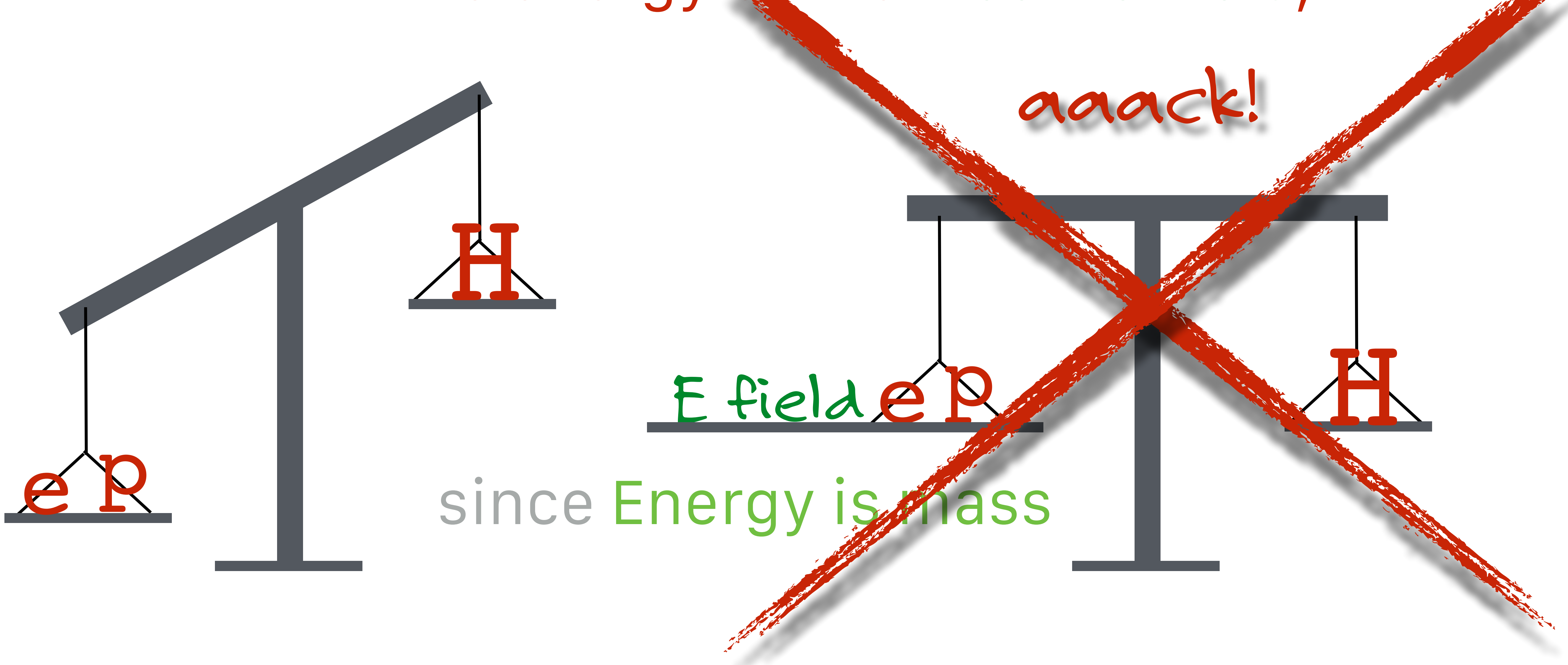
a hydrogen atom, take 1

weighs less than the components of a hydrogen atom

so it can't fall apart into its components

where is that "missing mass"?

in the energy of the Electric Field,



since Energy is mass

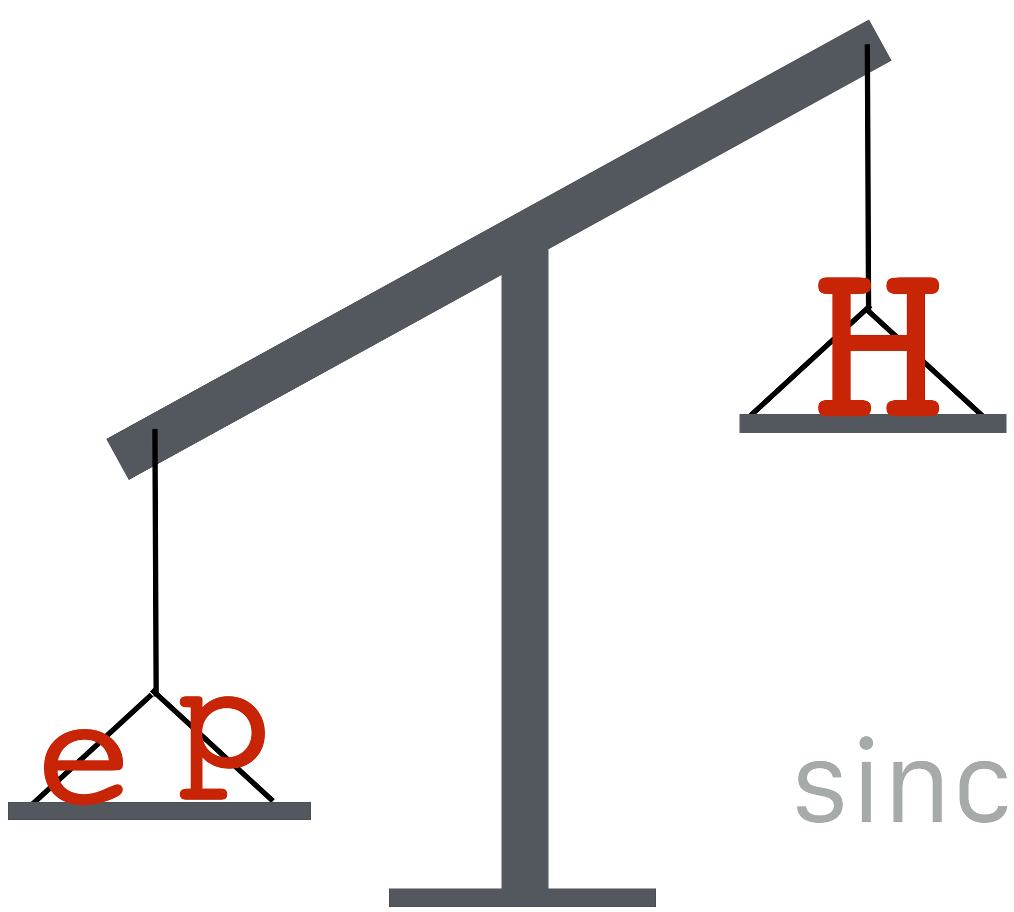
a hydrogen atom, take 2

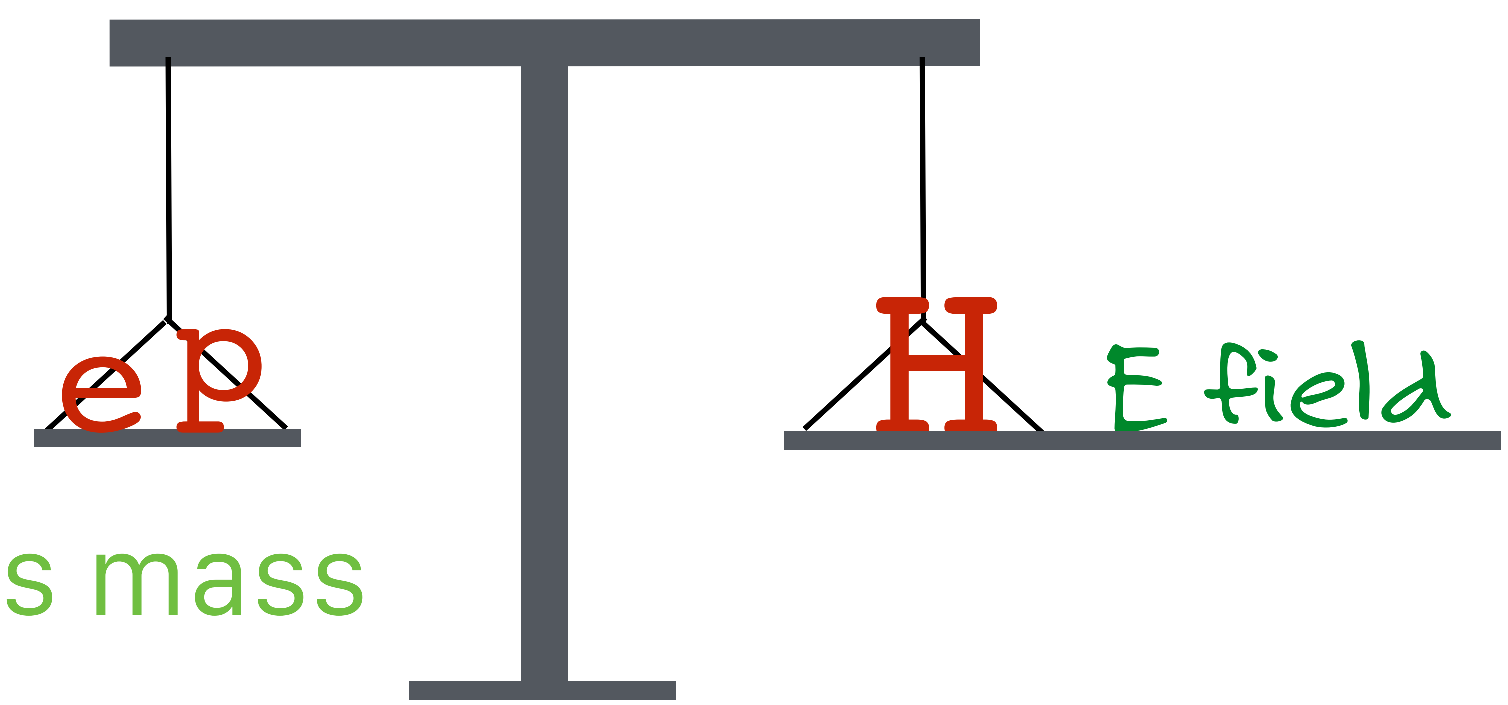
weighs less than the components of a hydrogen atom

so it can’t fall apart into its components

where is that “missing mass”?

in the energy of the Electric Field,





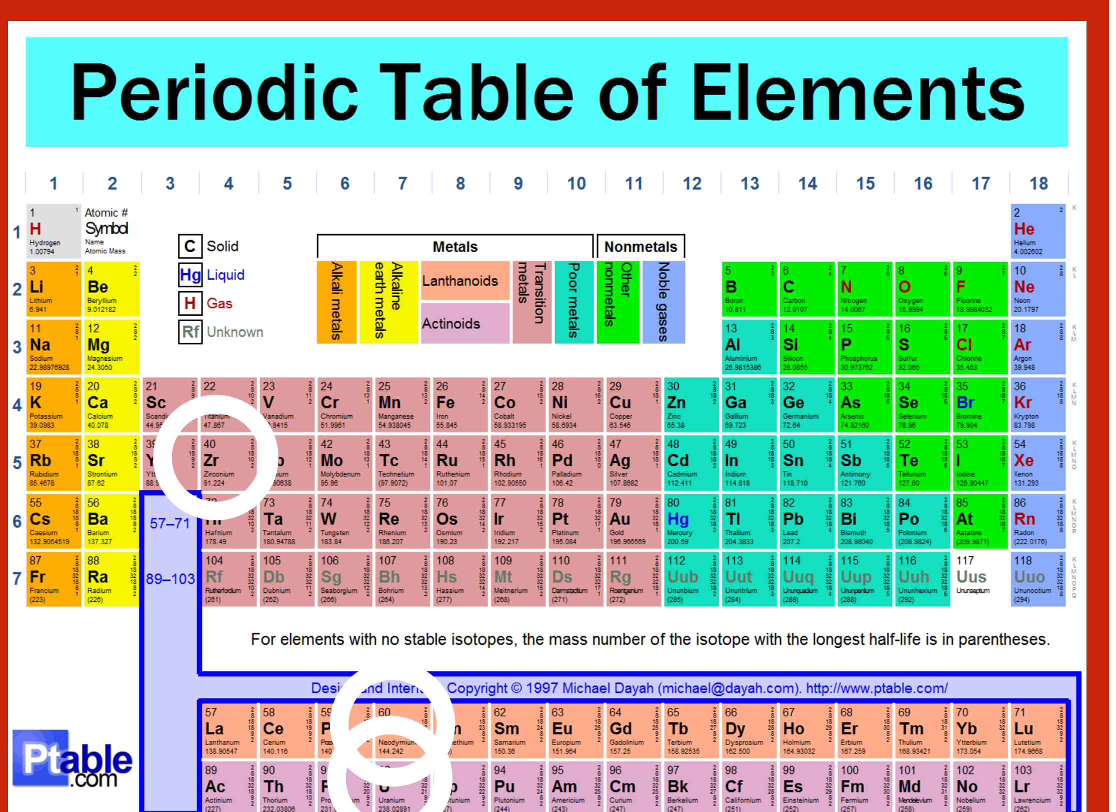
since Energy is mass

the ‘‘mass deficit’’ in nuclei

is observable and works for good and for ill.

a bound system like an atom

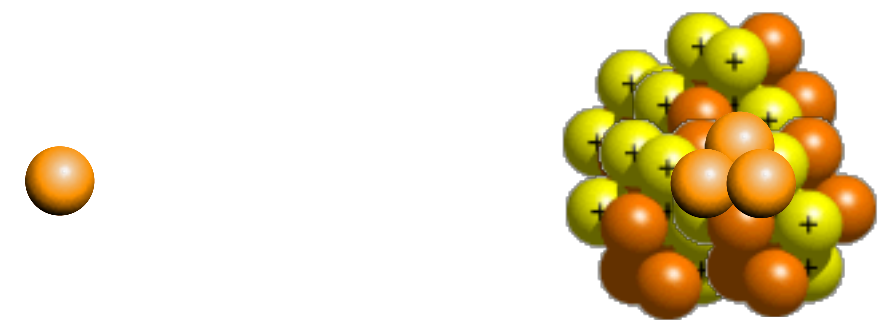
but much stronger!



It happens many ways, here is one:

"Uranium 235" is a big nucleus of 92 protons and 143 neutrons

^{143}Nd



^{90}Zr

the M(^{235}U) < 143 x M(neutron) + 92 x M(proton)

so, it's "bound" like Hydrogen

But when a neutron tickles it...
the mass deficit in binding energy is released as K...which becomes heat in nuclear reactors

M(^{235}U) + M(neutron) > M(^{143}Nd) + M(^{90}Zr) + 3 x M(neutron)

by 200 MeV

1 gm ^{235}U releases 23,000 kW-h

about 25 households' energy needs

looky here...

two things to worry about

$$E_T^2 = \left(mc^2\right)^2 + (pc)^2$$

Energy and momentum are related for massless objects...

$$E = pc$$

What about the negative solution?

$$E_T = \pm\sqrt{(mc^2)^2 + (pc)^2}$$

with all of this relativity stuff,

is anything permanent?

jargon alert: **invariant**

refers to:	something that is unchanging under some transformation
entomology:	*not*-variant
example:	the spacetime interval

Can’t we agree on anything?

Is EVERYTHING RELATIVE?

no.

“The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

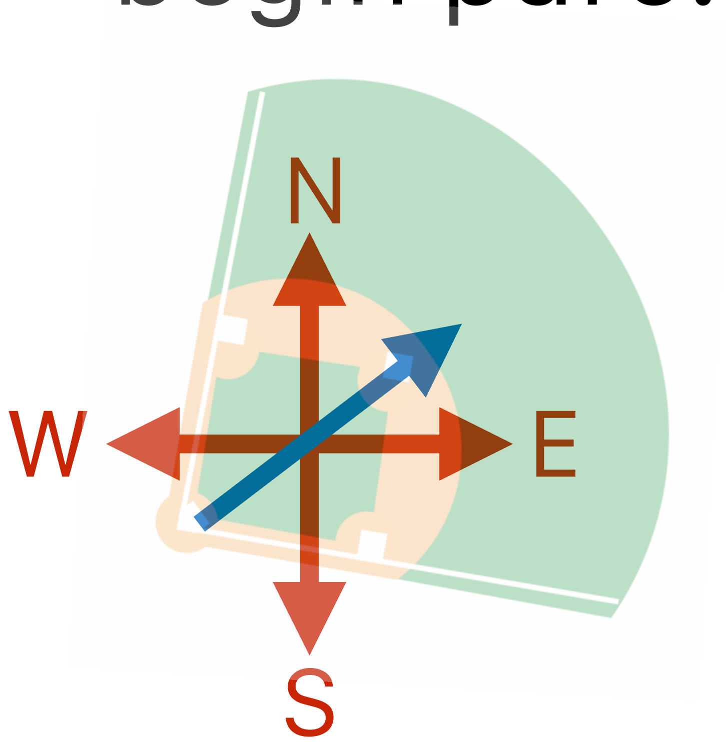
Hermann Minkowski

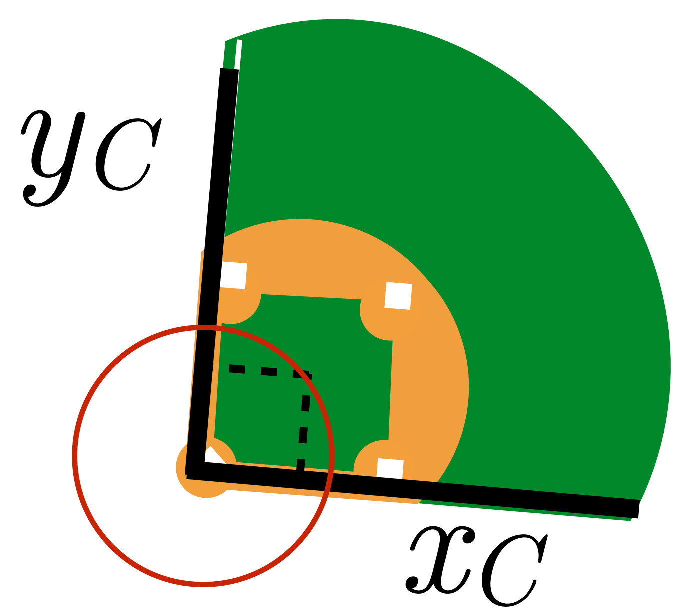
ISP220: Quarks, Spacetime, and the Big Bang

Space and time are mixed together
and together become a single entity

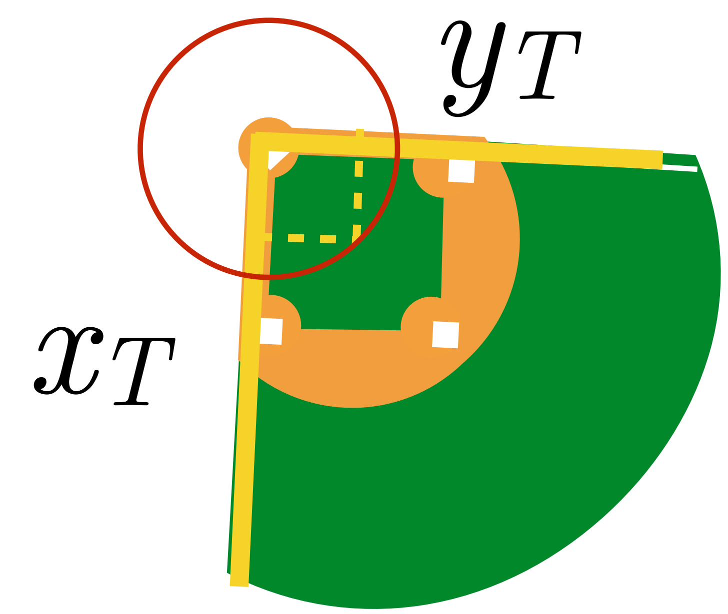
lay out a baseball field

begin pure: rule book says "East, north-East"





Wrigley Field: Cubs



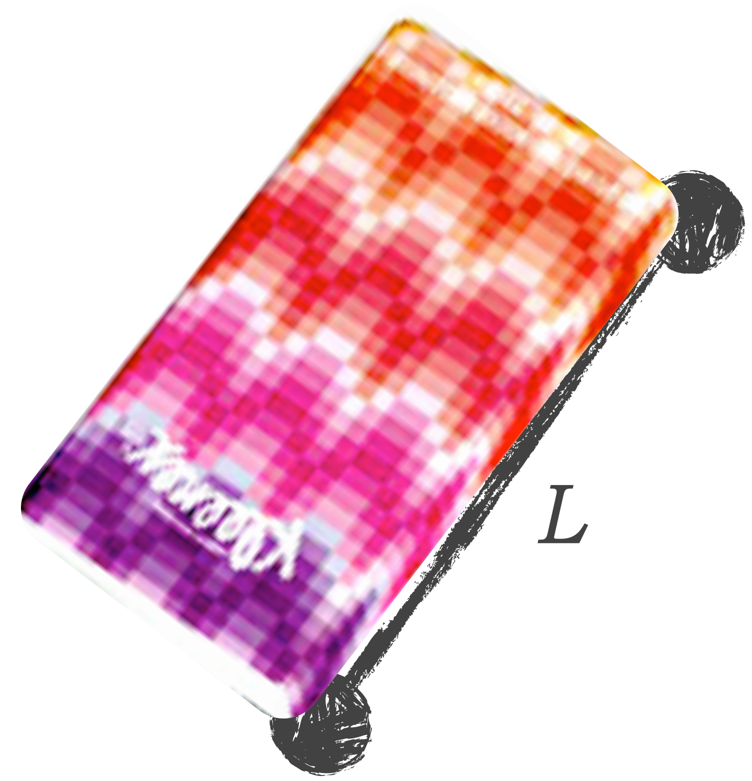
CoAmerica Park, Tigers

Distance to pitcher's rubber at Wrigley? 60 feet, 6 inches

Distance to pitcher's rubber at CoAmerica? 60 feet, 6 inches

$$L_P^2 = x_C^2 + y_C^2 = x_T^2 + y_T^2$$

take your board



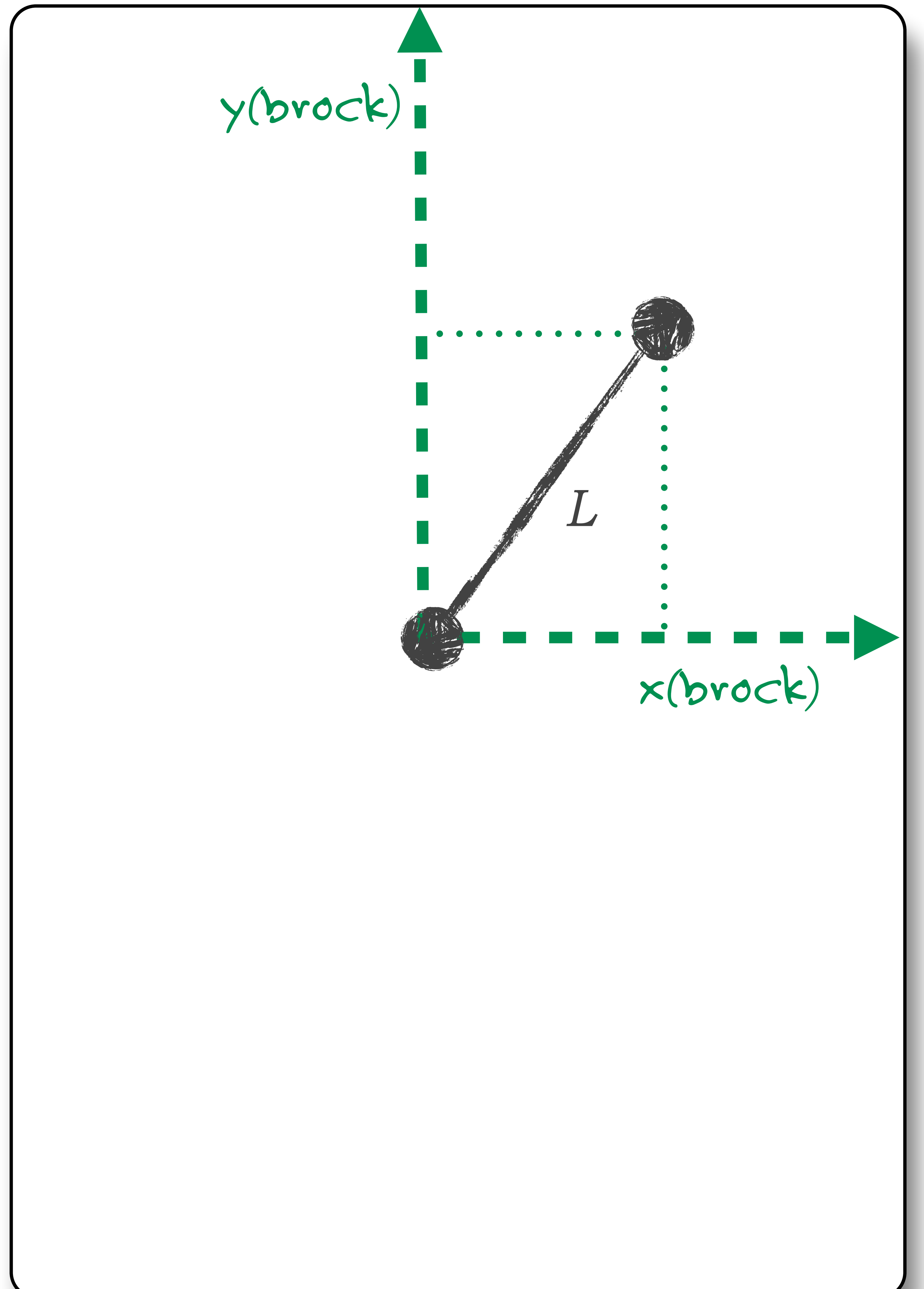
let's say L = 5 cm

take your board in your lap

about the lower dot:

Last name: A-M

draw an x-y set of axes with y pointing towards me

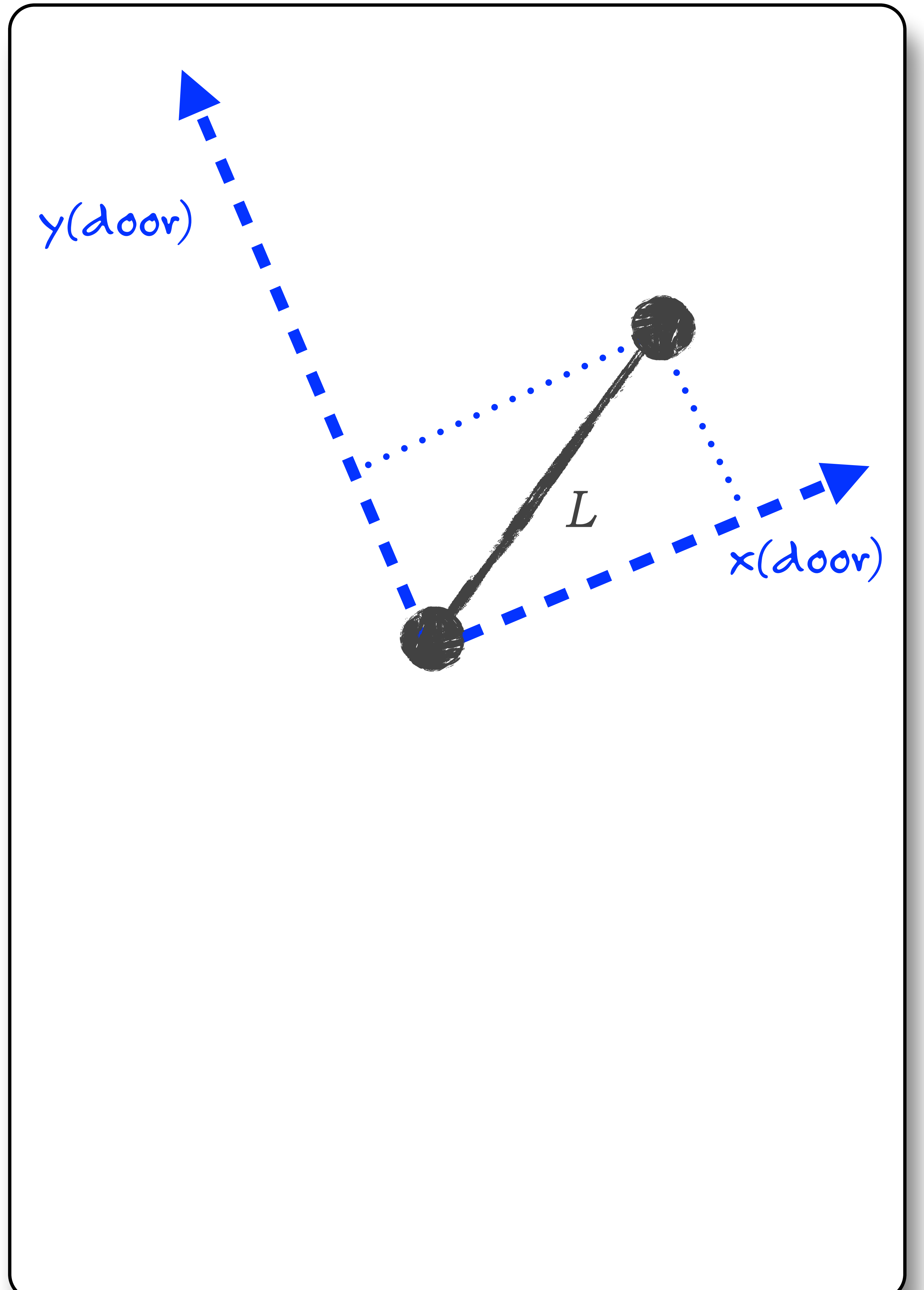


take your board in your lap

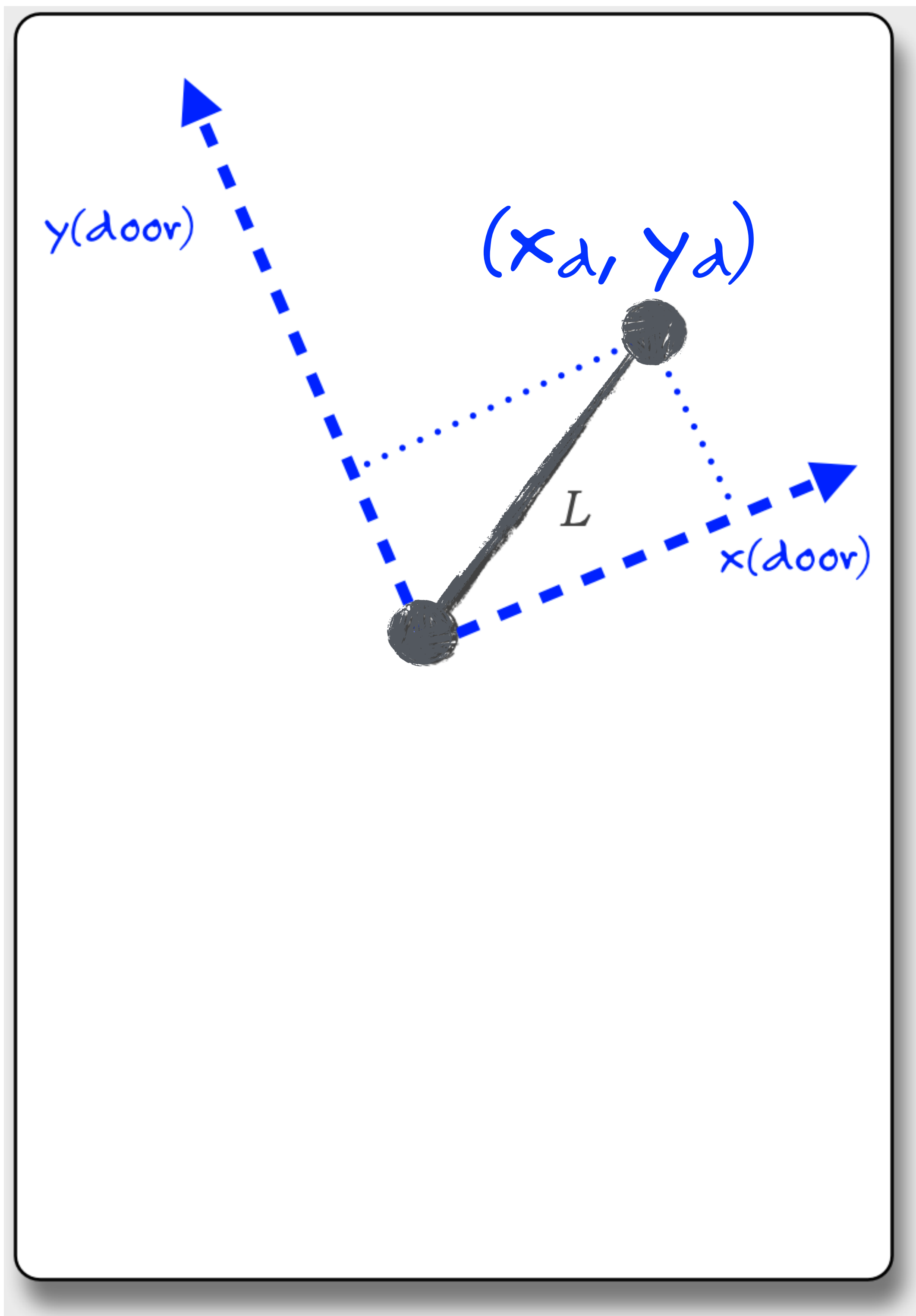
about the lower dot:

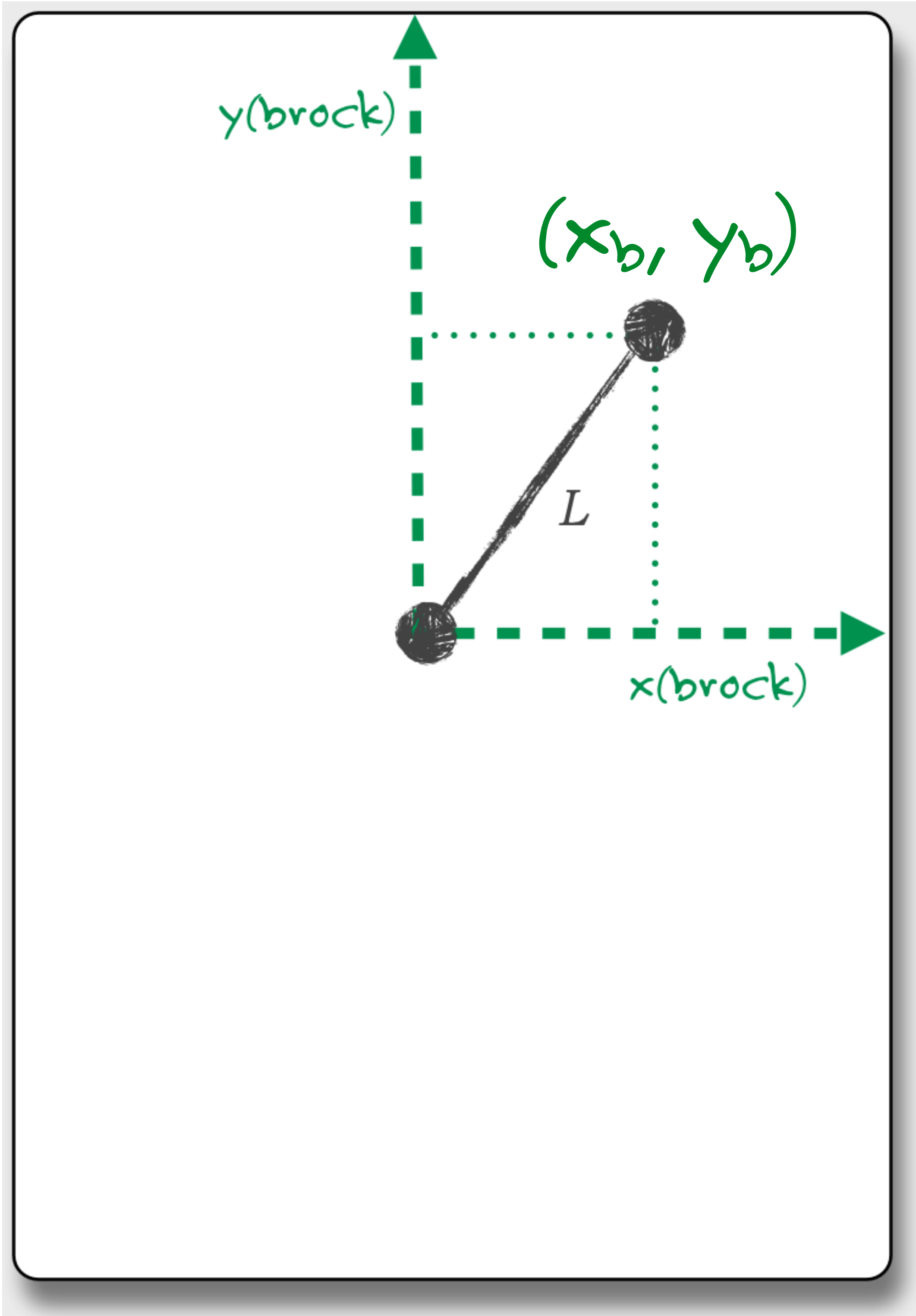
Last name: A-M
- draw an x-y set of axes with y pointing towards me

Last name: N-Z
- draw an x-y set of axes with y pointing vaguely towards the outside door



2 different coordinate systems...same line





Would the A-M people report the same x and y values as the N-Z people?

That is, is xb = xd and is yb = yd ??

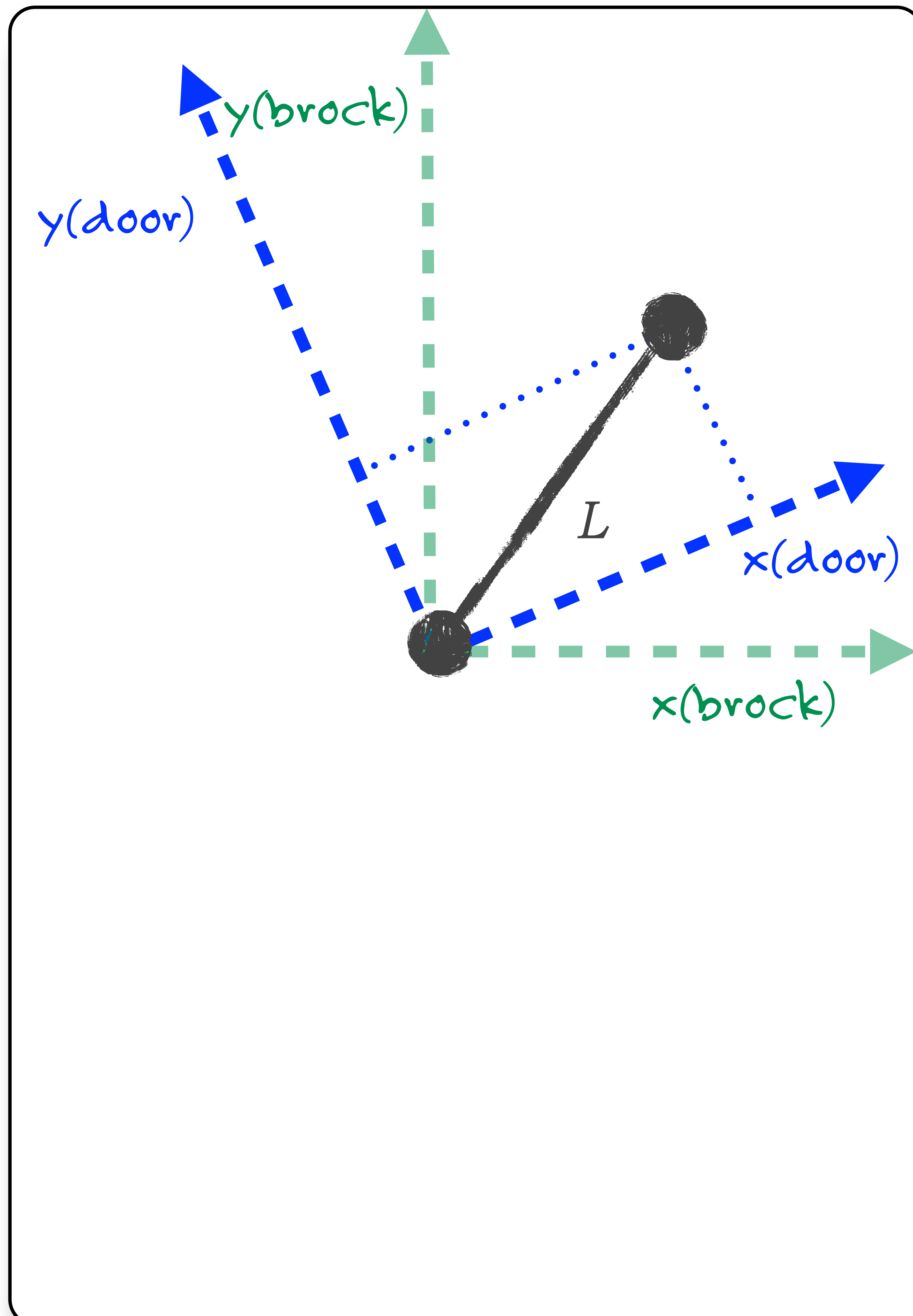
But both would report that the line, L, is 5 cm long.

take your board

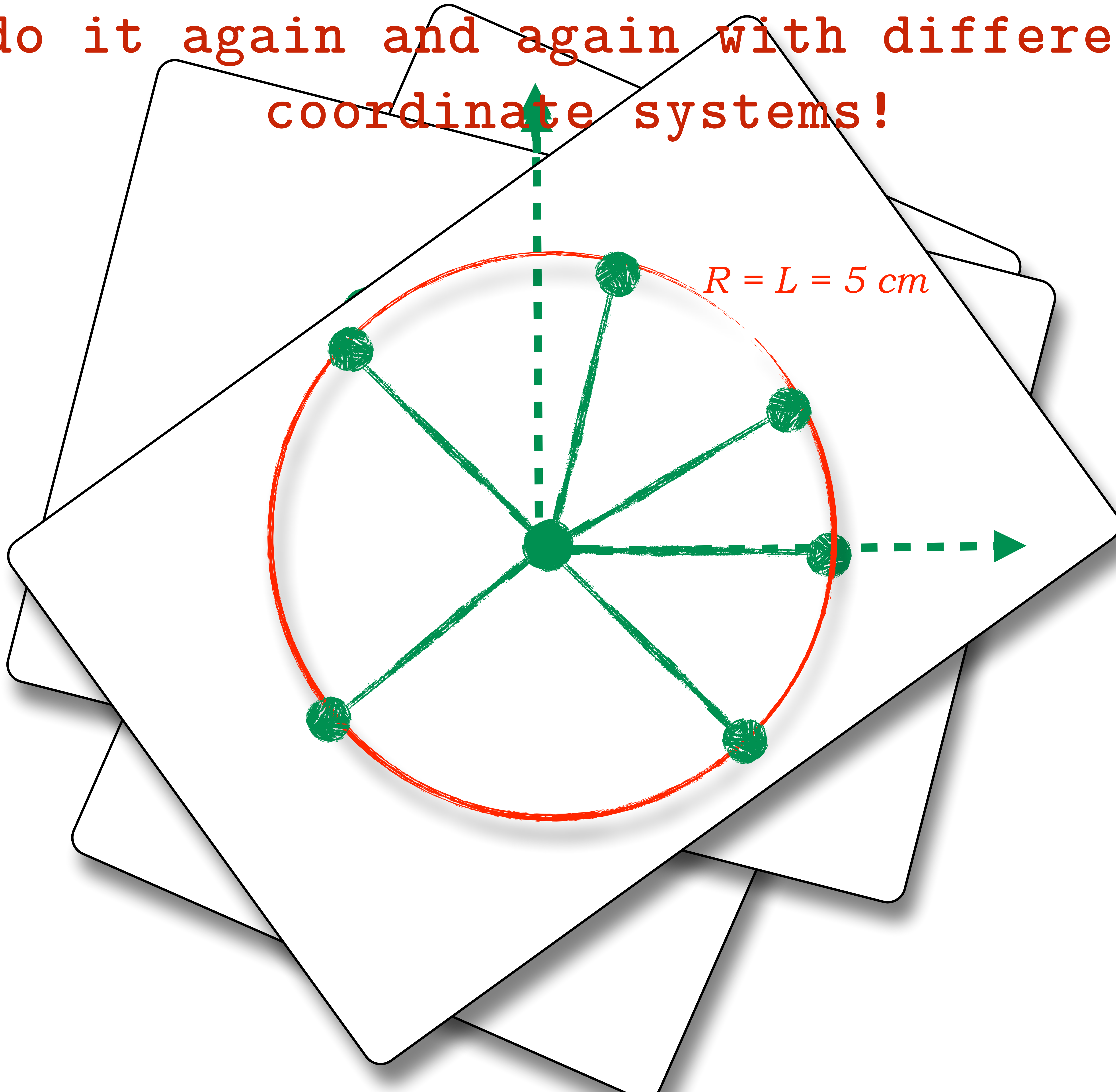
about the lower dot:

Last name: A-M
- draw an x-y set of axes with y pointing towards me

Last name: N-Z
- rotate your board so that your axes align with the A-M axes



do it again and again with different coordinate systems!



in ‘‘regular’’* geometry

Length is “invariant” with respect to coordinate systems

regular space distances preserve lengths

* “Euclidean Geometry”…of flat spaces

in SPACE: regardless of the coordinate system

in 2, 3, 4...any number dimensional space

LENGTH is "invariant" with respect to coordinate system

$$L^2 = x_1^2 + y_1^2 + z_1^2$$

coordinate system "1"

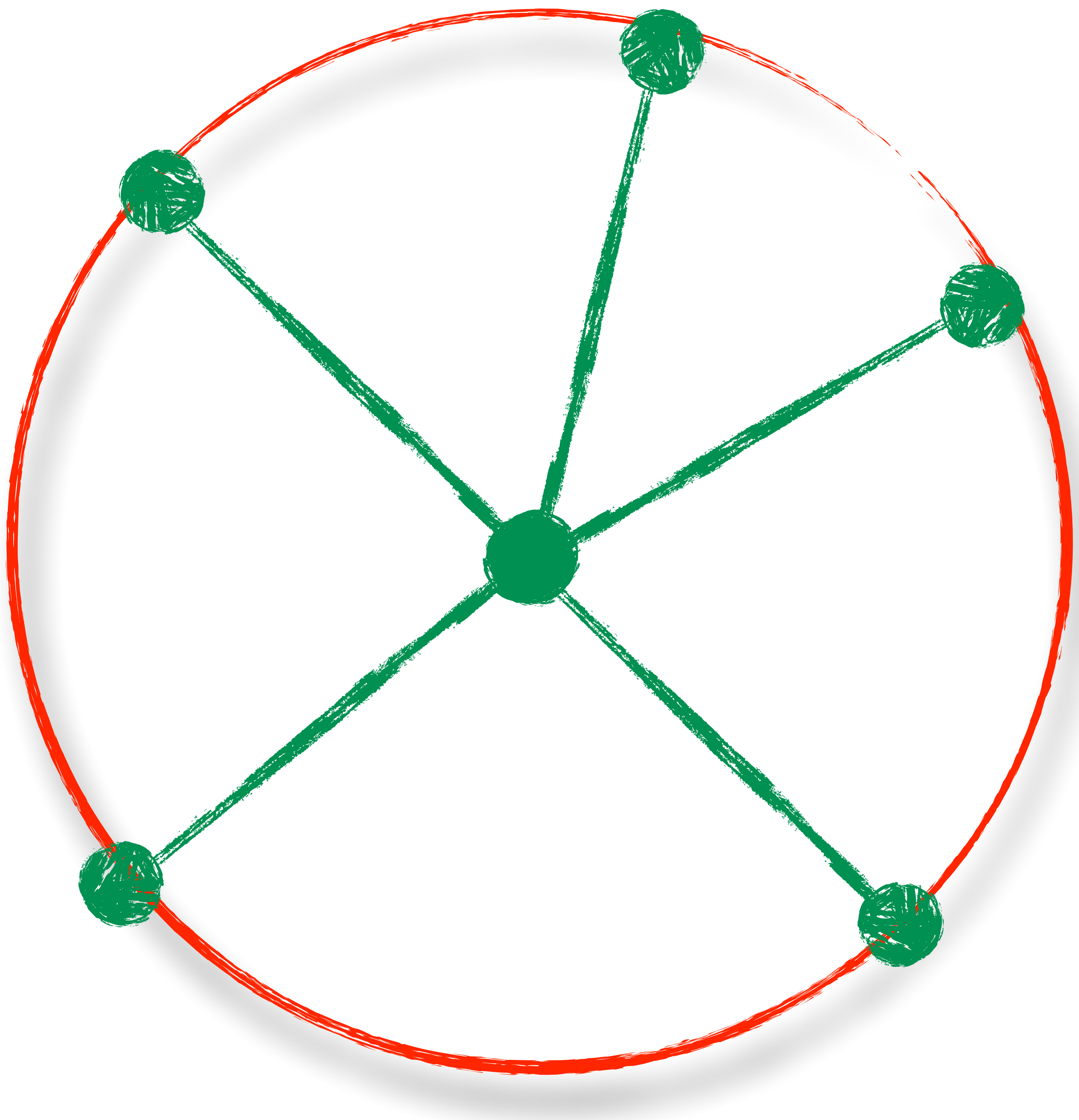
same length

This is a feature of "**Euclidean geometry**"

The "+" signs are the feature.

all coordinate systems in space

will place that point on the circle.



$$L^2 = x^2 + y^2 = x'^2 + y'^2 = x''^2 + y''^2 = x'''^2 + y'''^2$$

Let's call this: **the invariant curve**

What about SPACETIME?

what’s constant? What’s a Spacetime “Length”?

Try the same approach for spacetime as for space: Euclidean

construct the Invariant Curve for spacetime

‘‘coordinate systems’’

can mean “reference frames”

let's do some superfluous learnedness

"Minkowski Space"
SPACETIME
respects a different kind of "length" calculation

A quick aside:

Space and time are not exactly the same thing.

Space is measured in length units - meters.
Time is measured in time units - seconds.

Spacetime units need to be the same:
length is the convention.

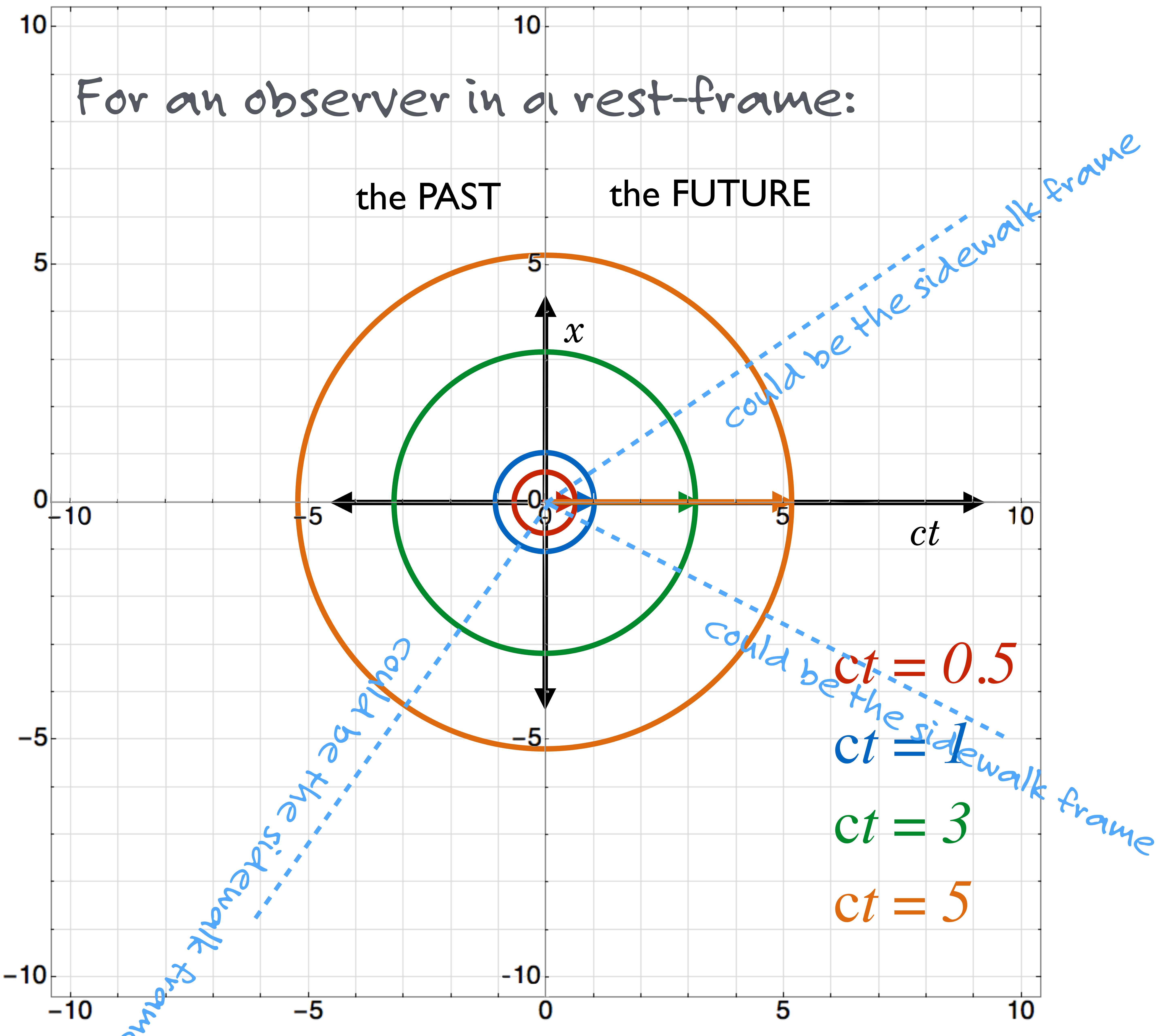
Spacetime - *time* is measured with a scale factor that gets the units right: multiply by the speed of light, *c*.

So, a spacetime "distance" of 2 seconds?
represent as 2•*c* meters, or

(2 seconds) • (3×10^8 m/s) = 6×10^8 m - "lightseconds"?

Rule of thumb: light-length can be visualized as about 1 foot per nanosecond.

For an observer in a rest-frame:



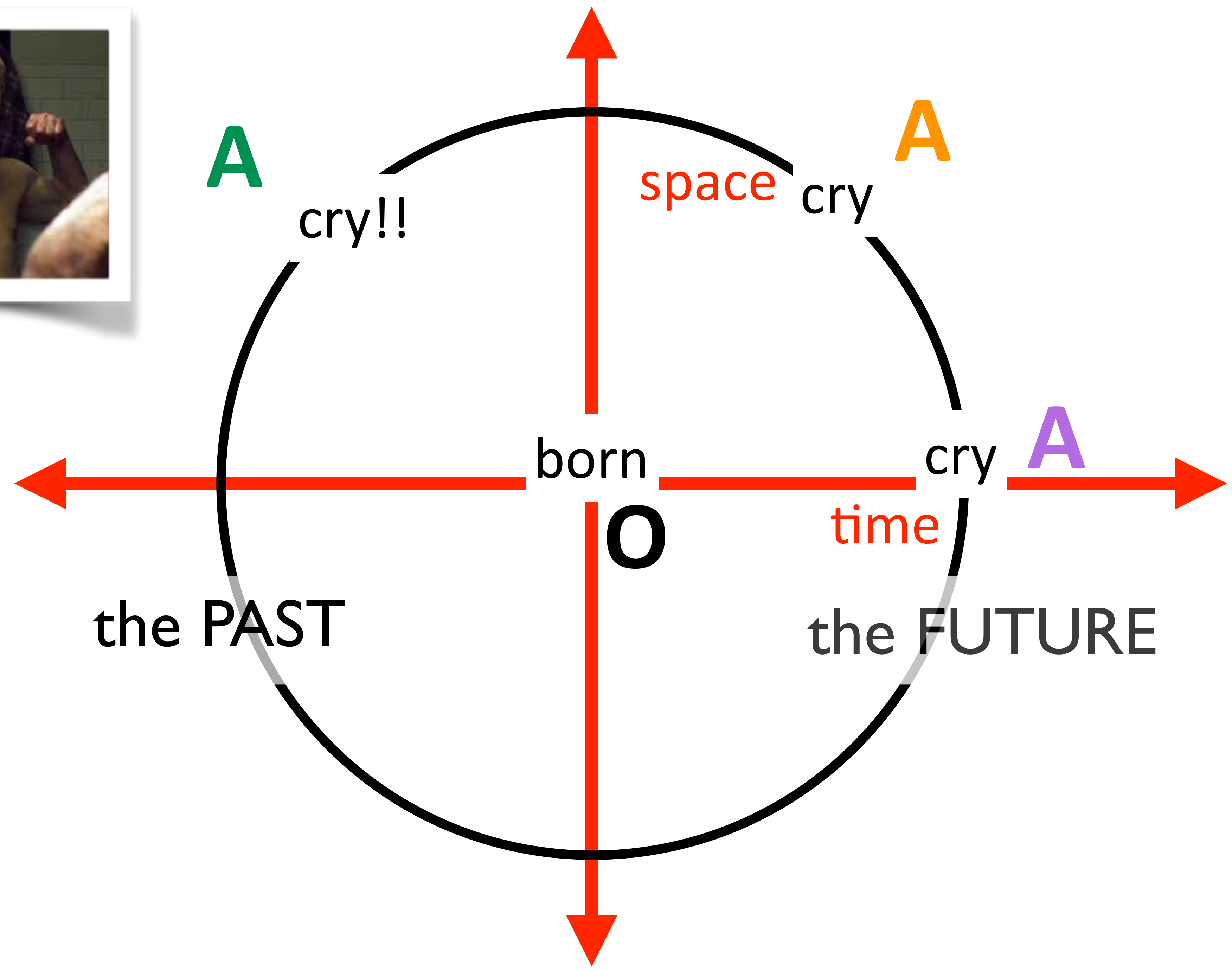
we're after

the invariant curve for spacetime

guess that it's a circle like "regular" geometry!

How about a hospital?

Now, OA are event intervals, not just space-lengths



<u>If</u> spacetime's invariant curve is a circle...then

- if A is the event in one frame,
- then A is another viewpoint from another frame

But...so is A okay in a third frame. Uh oh.

the invariant "length" in spacetime

"the interval," s

Remember:

The invariant curve for space is the equation of a circle:

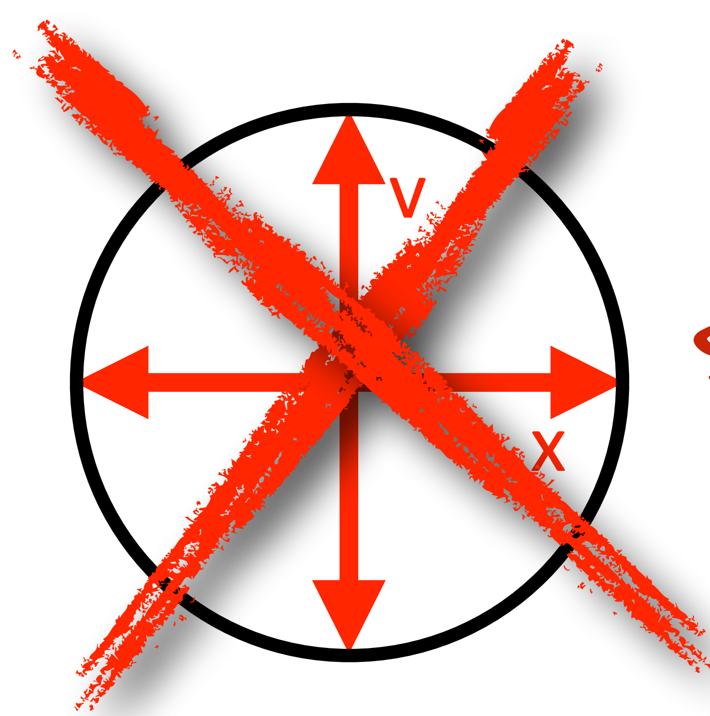
$$L^2 = x^2 + y^2 = x'^2 + y'^2 = x''^2 + y''^2 = x'''^2 + y'''^2$$

Minkowski's discovery was that the invariant curve for spacetime is

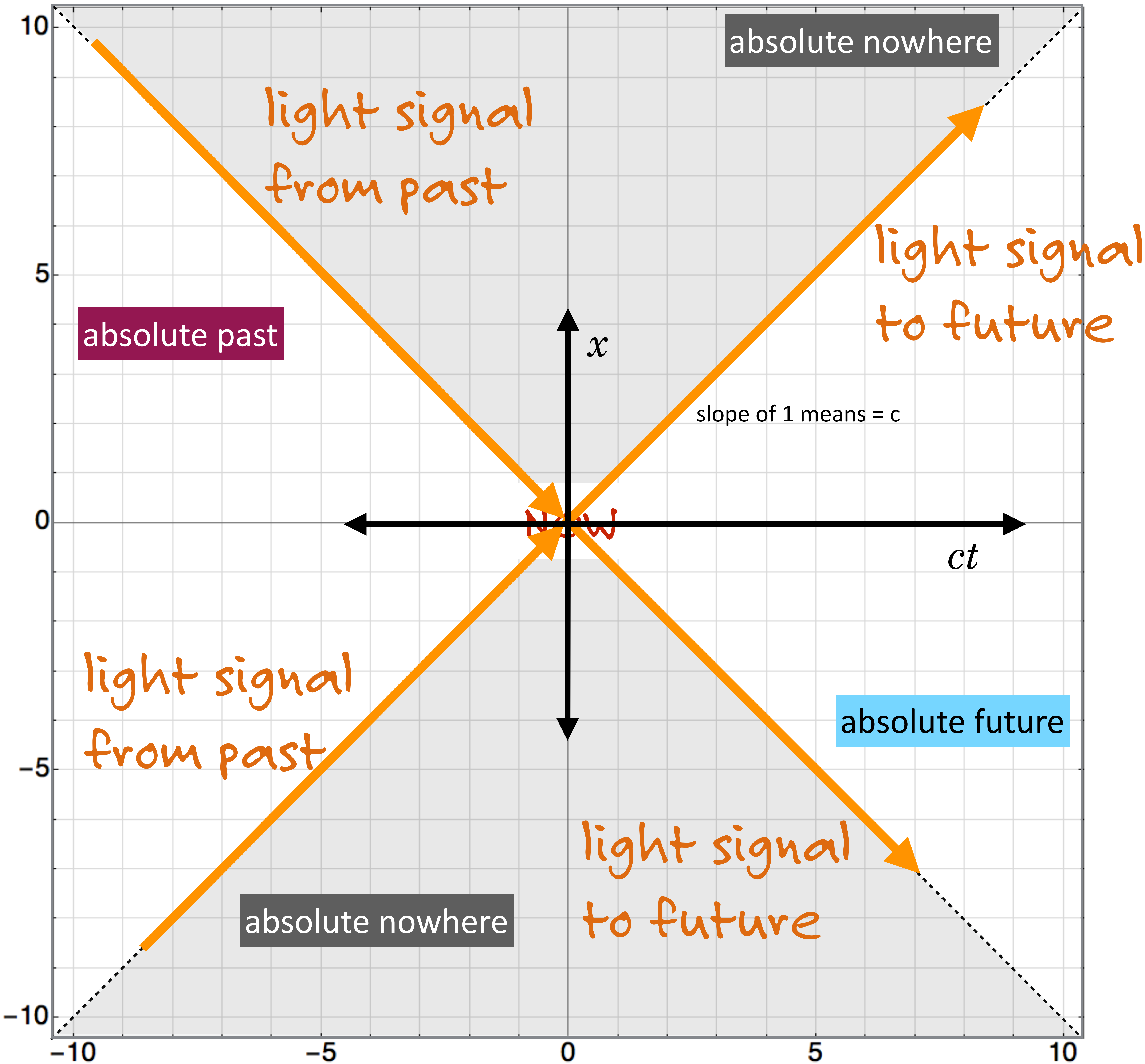
$$s^2 = c^2 t_H^2 - x_H^2$$

the equation of a hyperbola

This is the spacetime "length" that all inertial observers would agree on.



Spacetime is hyperbolic


absolute nowhere
light signal from past
light signal to future
absolute past
x
slope of 1 means = c
Now
ct
light signal from past
absolute future
light signal to future
absolute nowhere
10
5
0
−5
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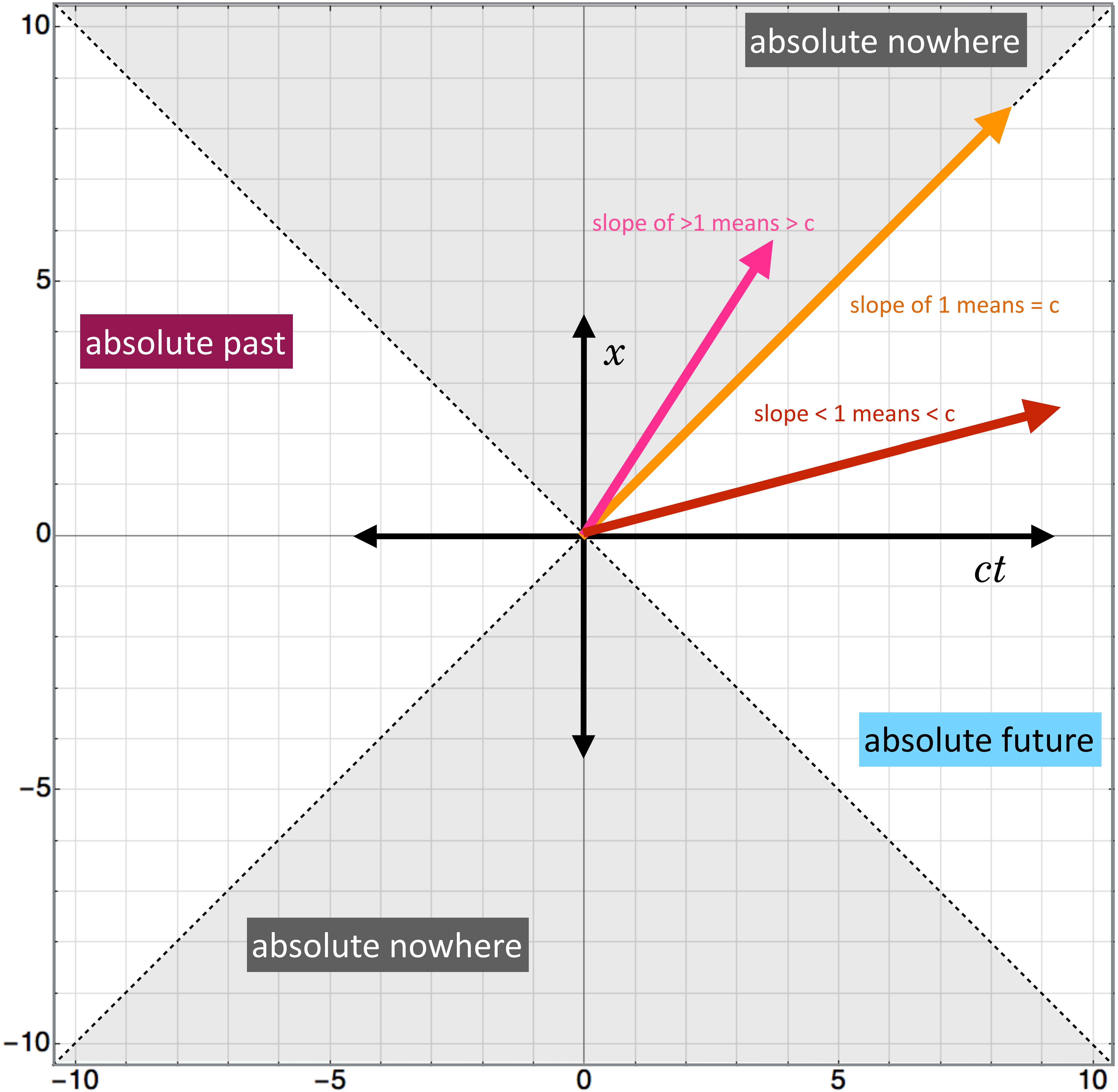
−5
0
5
10


absolute nowhere
slope of >1 means > c
slope of 1 means = c
absolute past
x
slope < 1 means < c
ct
absolute future
absolute nowhere
10
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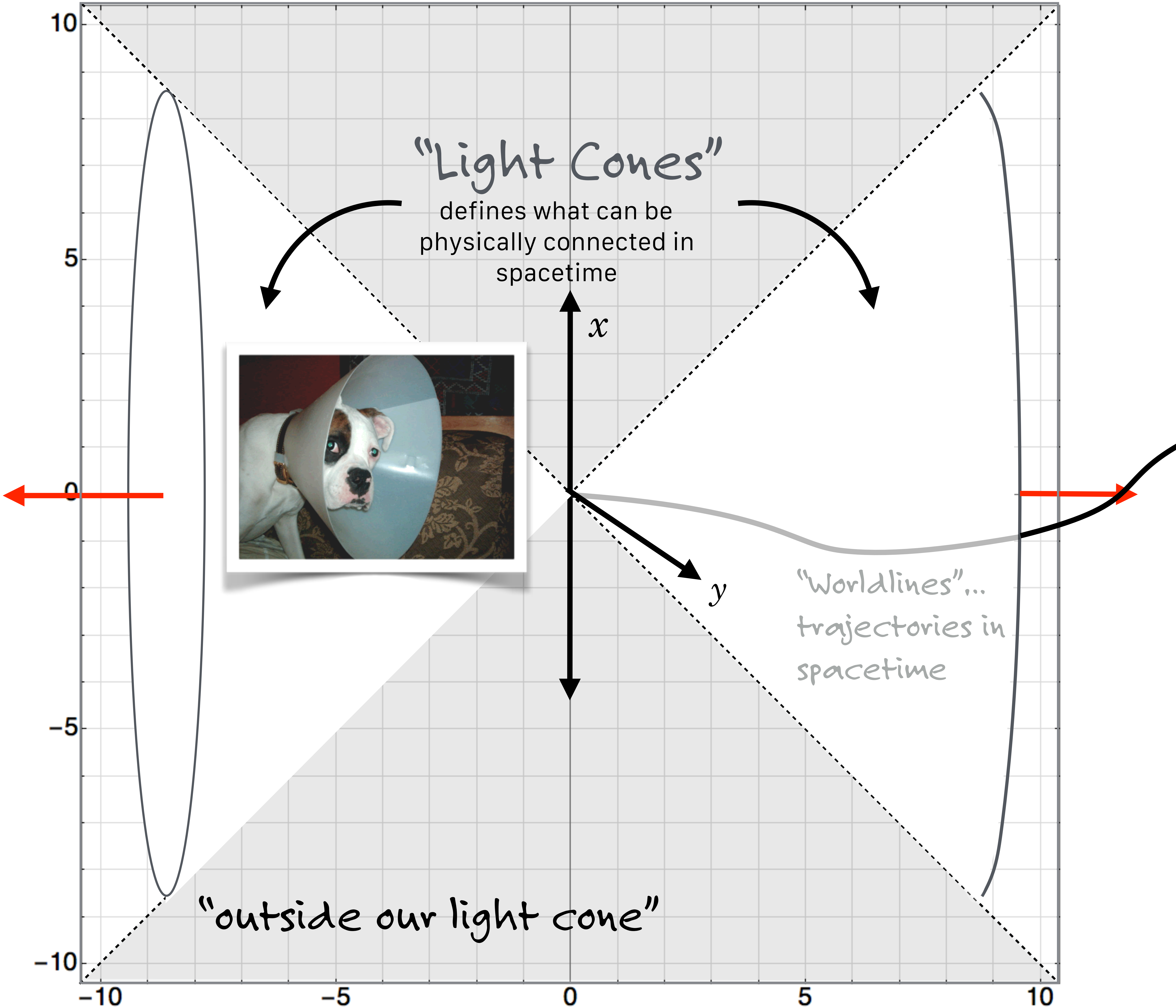
−10
−5
0
5
10


"Light Cones"
defines what can be physically connected in spacetime
x
y
"Worldlines"... trajectories in spacetime
"outside our light cone"
10
5
0
−5
−10
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5
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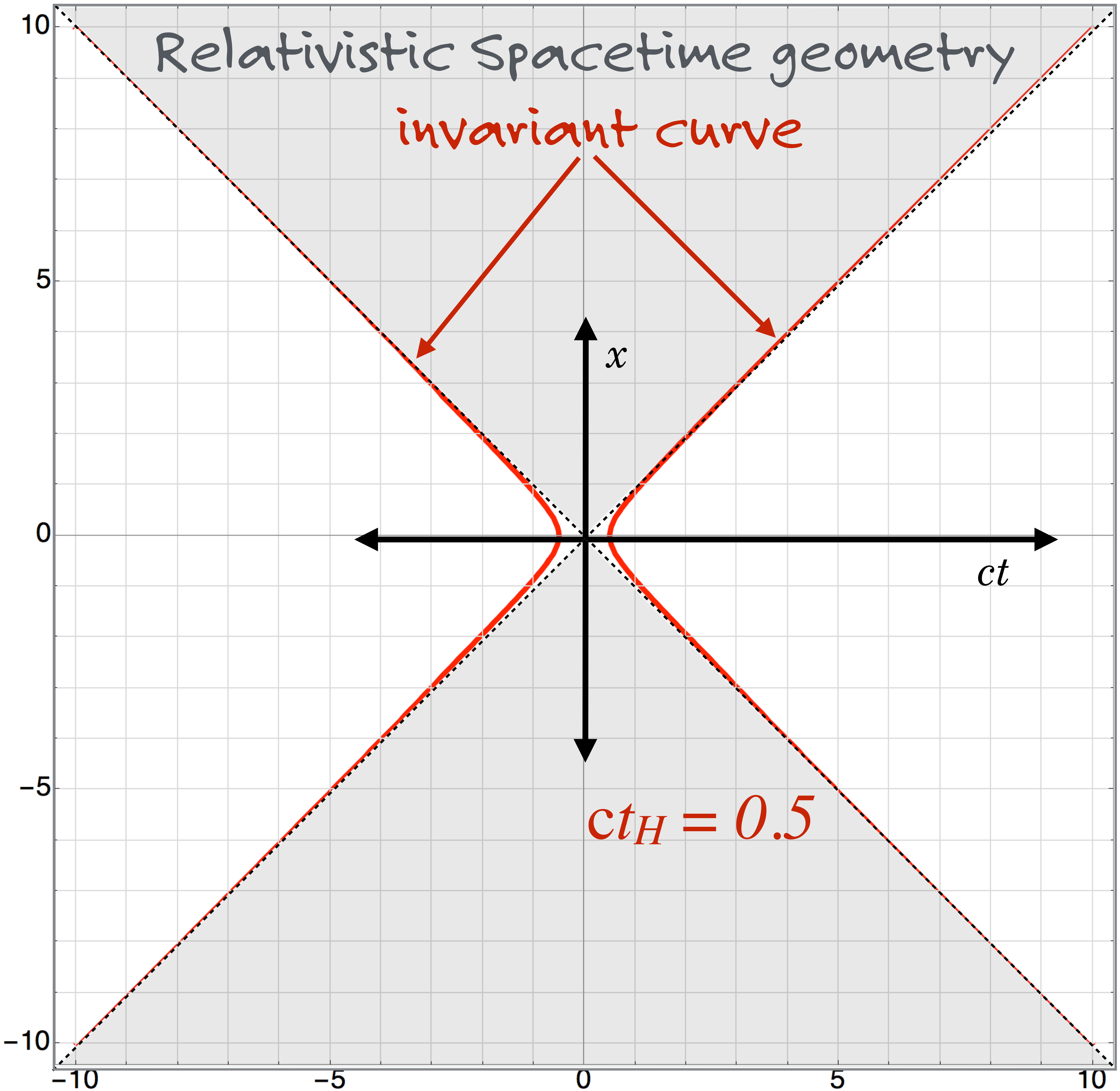
Relativistic Spacetime geometry
invariant curve
x
ct
$ct_H = 0.5$
10
5
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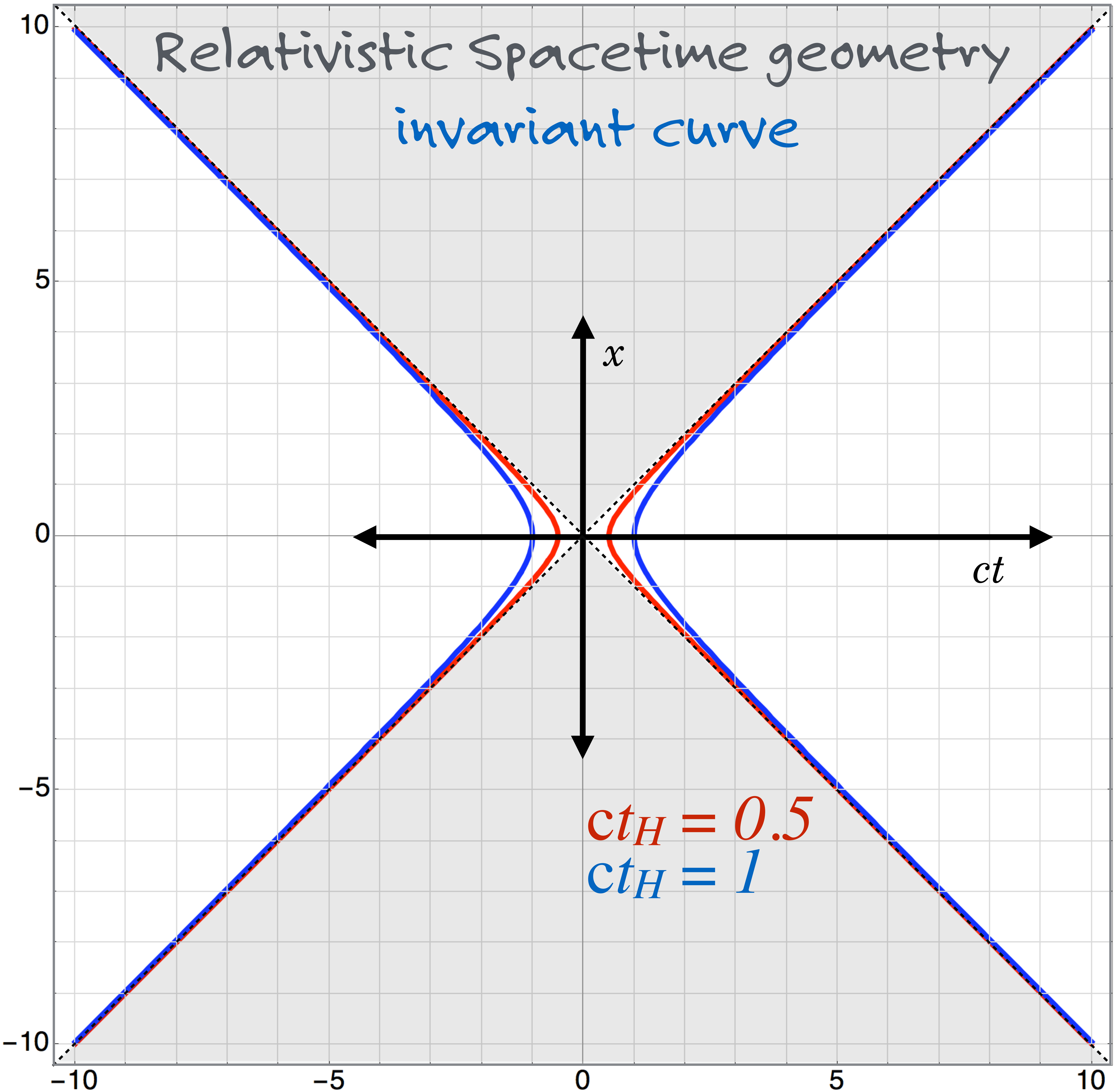
5
10


Relativistic Spacetime geometry
invariant curve
10
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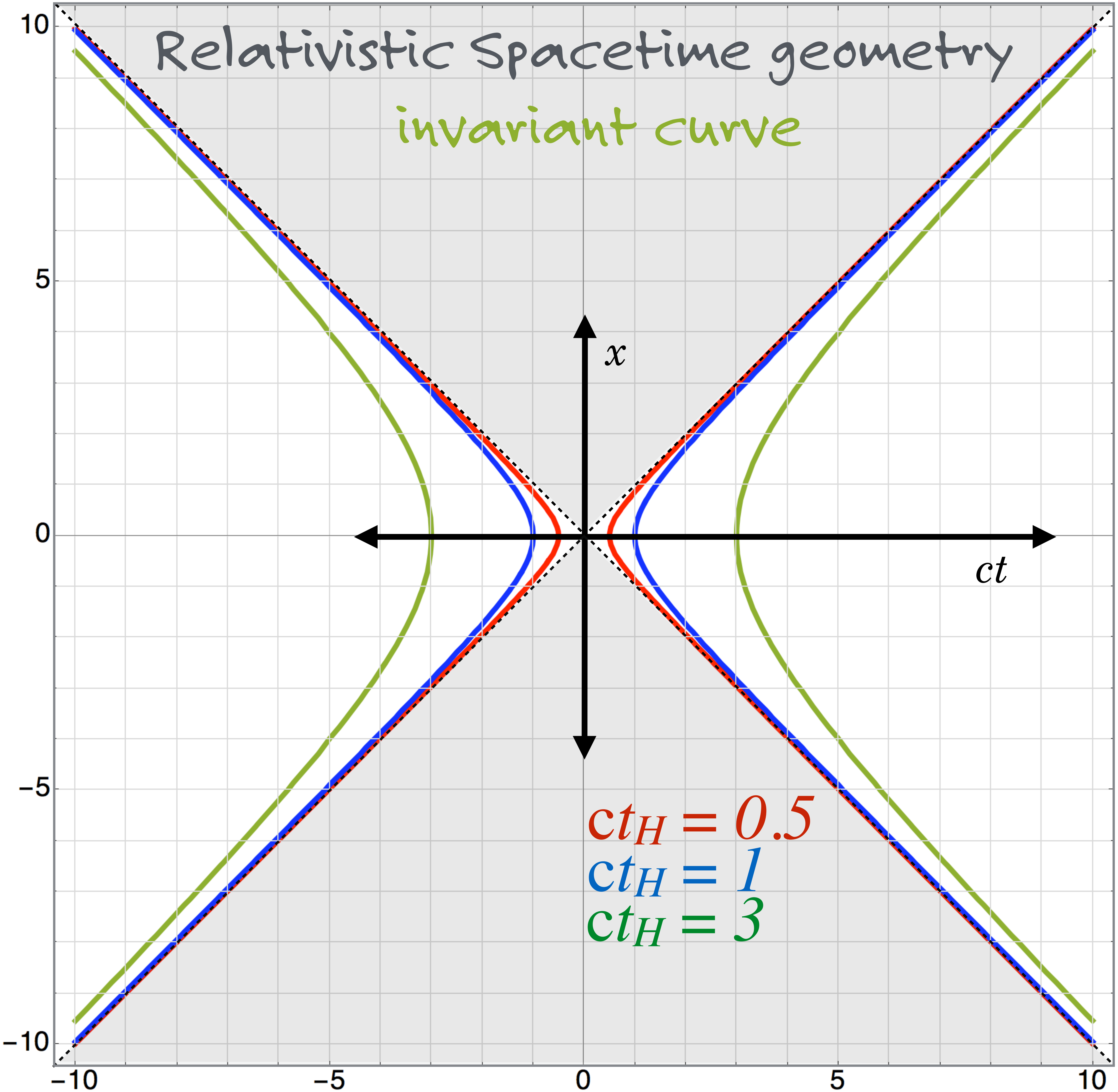
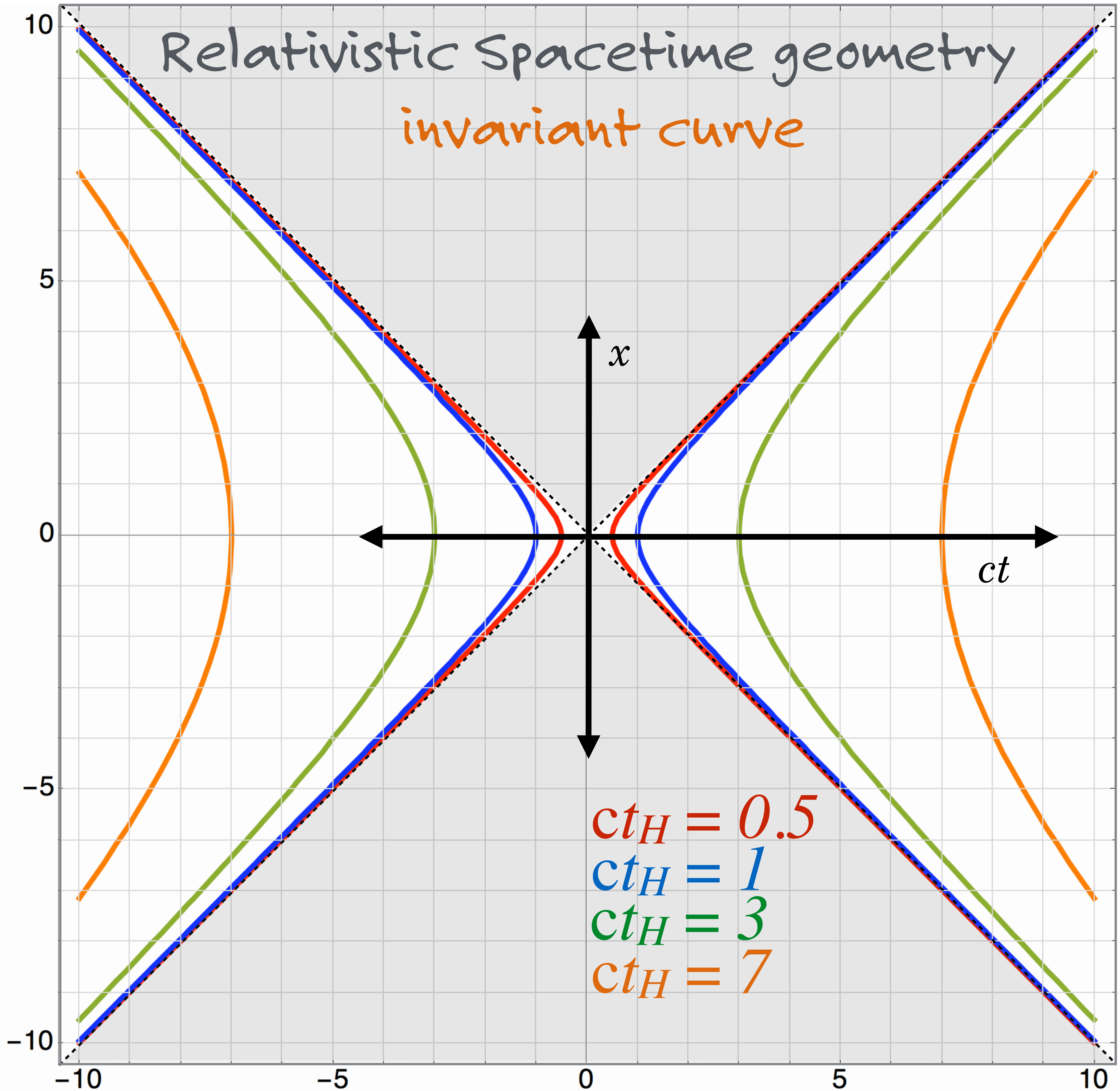
0
5
10
x
ct
$ct_H = 0.5$
$ct_H = 1$


Relativistic Spacetime geometry
invariant curve
x
ct
$ct_H = 0.5$
$ct_H = 1$
$ct_H = 3$
10
5
0
−5
−10
−10
−5
0
5
10


Relativistic Spacetime geometry
invariant curve
x
ct
ctH = 0.5
ctH = 1
ctH = 3
ctH = 7
10
5
0
−5
−10
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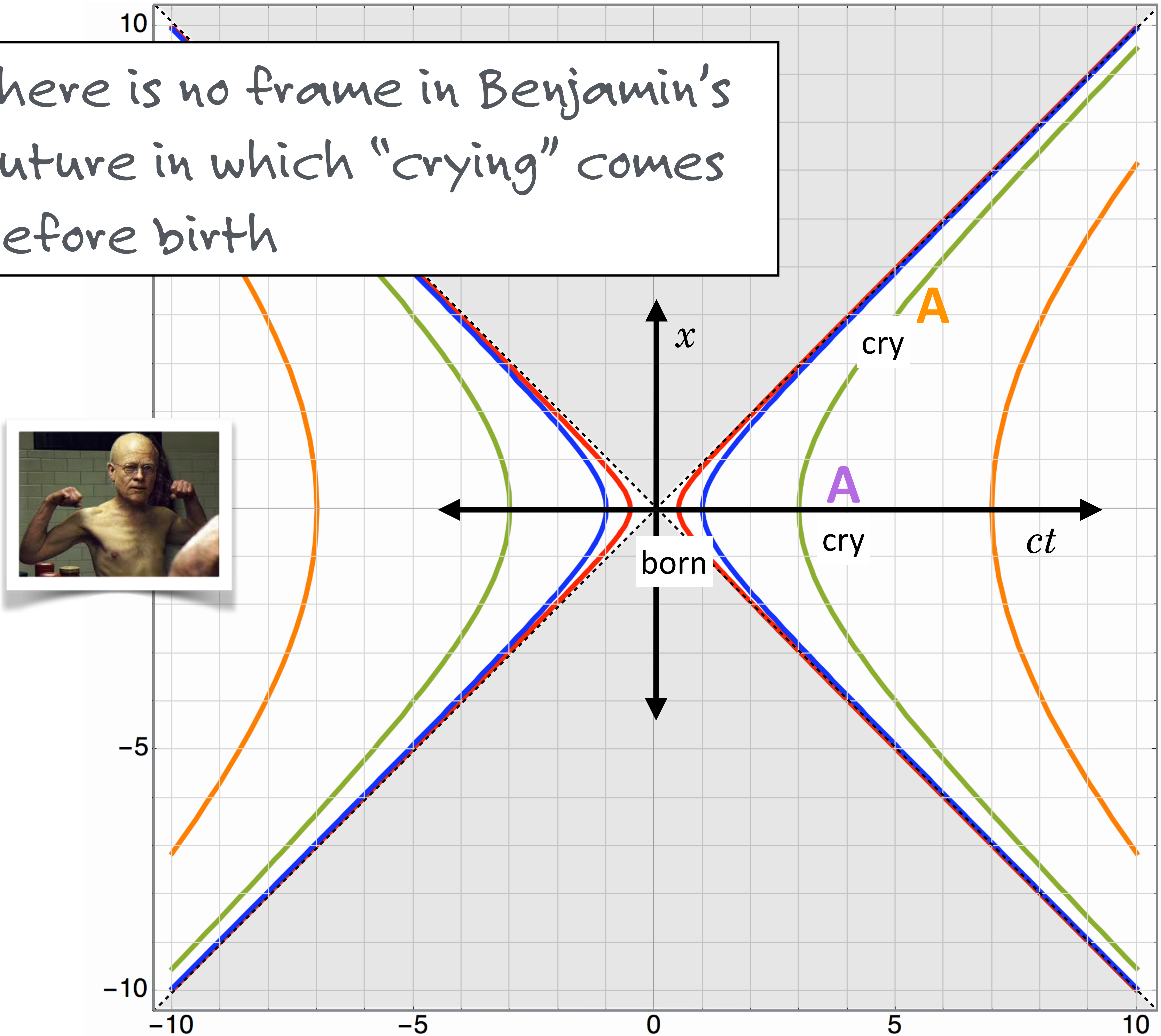
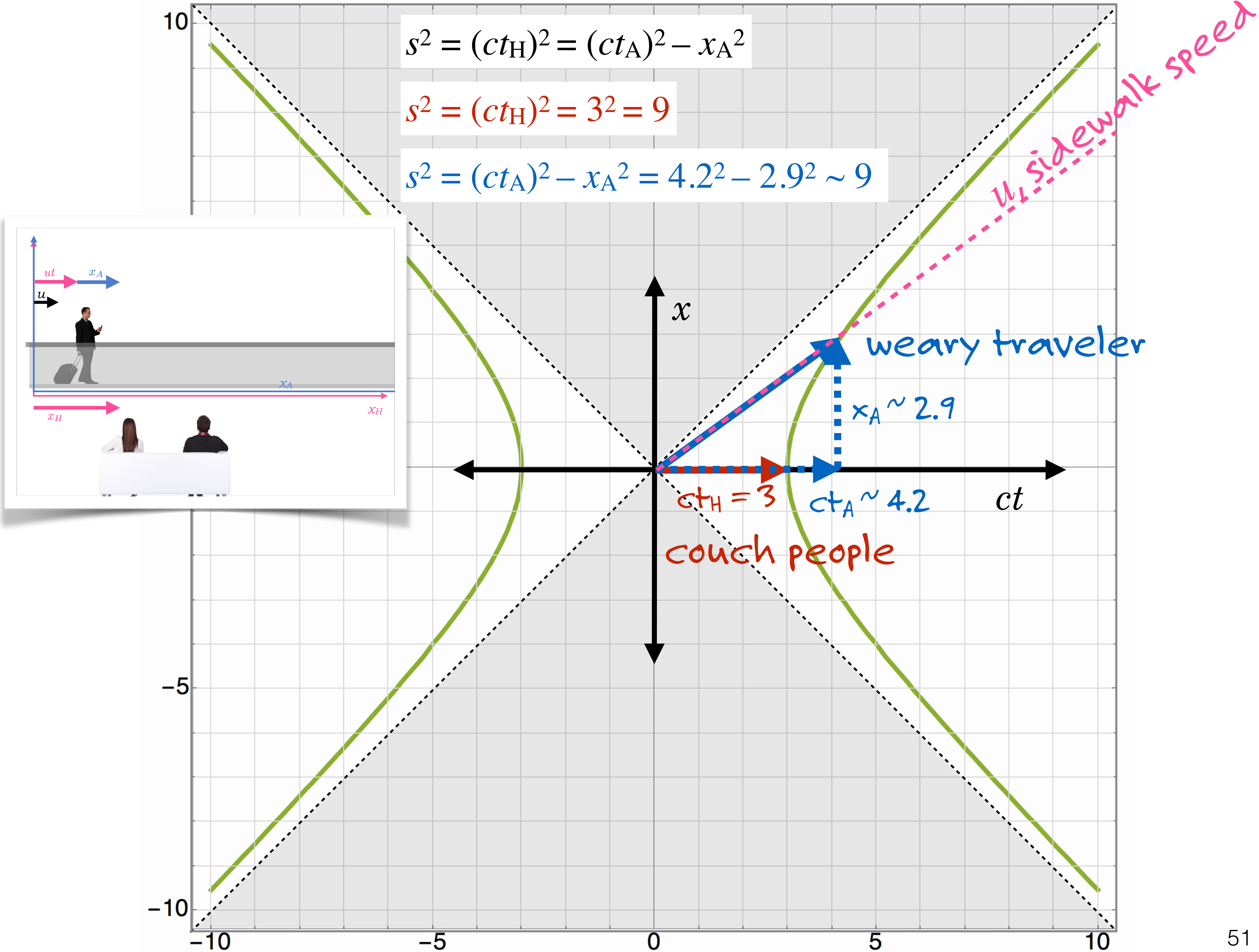
−5
0
5
10

there is no frame in Benjamin's future in which "crying" comes before birth




$s^2 = (ct_H)^2 = (ct_A)^2 - x_A{}^2$
$s^2 = (ct_H)^2 = 3^2 = 9$
$s^2 = (ct_A)^2 - x_A{}^2 = 4.2^2 - 2.9^2 \sim 9$
u, sidewalk speed
weary traveler
$x_A \sim 2.9$
$ct_H = 3$
$ct_A \sim 4.2$
couch people
x
ct
ut
x_A
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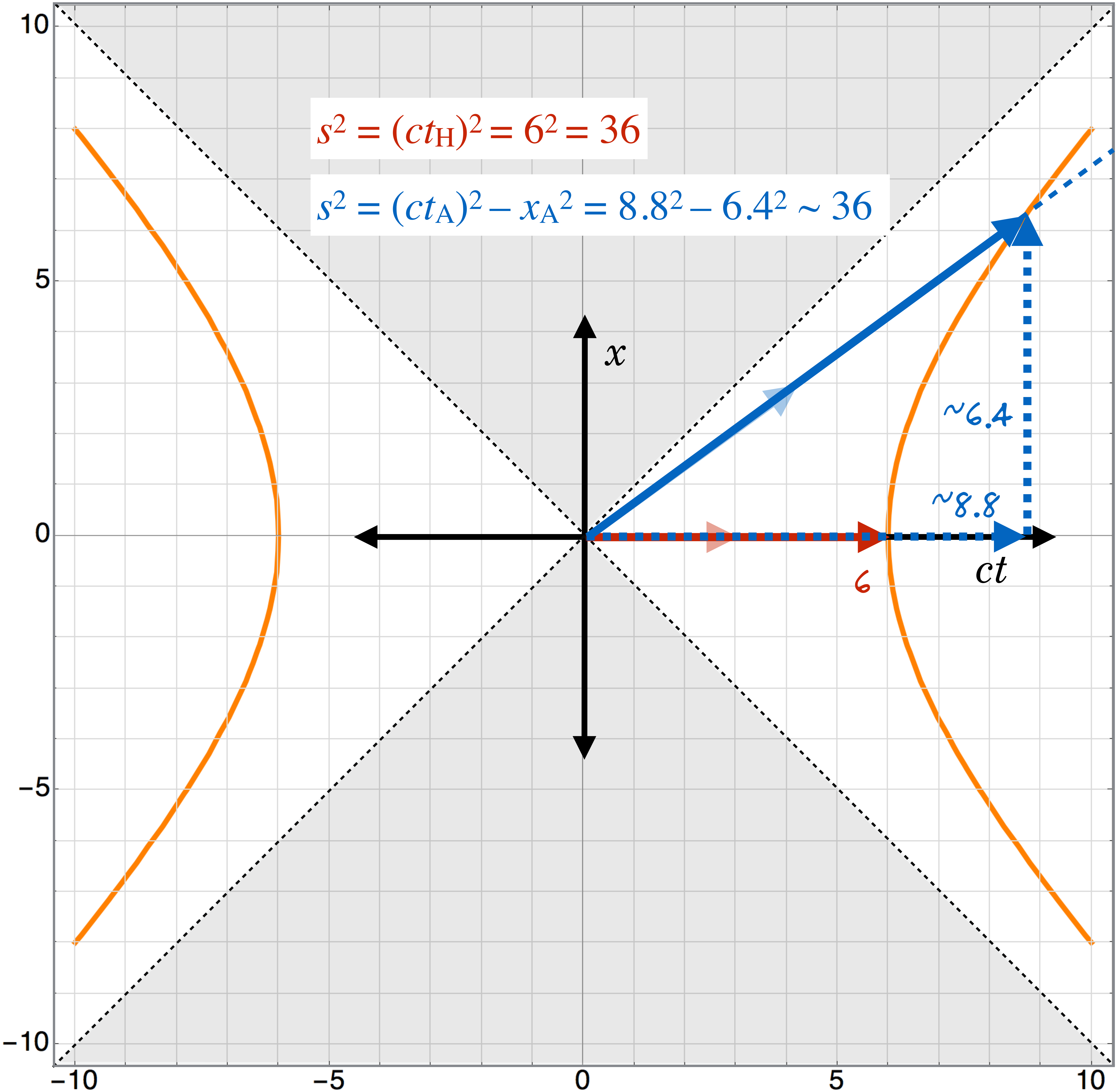
0
5
10



$s^2 = (ct_H)^2 = 6^2 = 36$
$s^2 = (ct_A)^2 - x_A^2 = 8.8^2 - 6.4^2 \sim 36$
x
ct
6
~6.4
~8.8
10
5
0
−5
−10
−10
−5
0
5
10

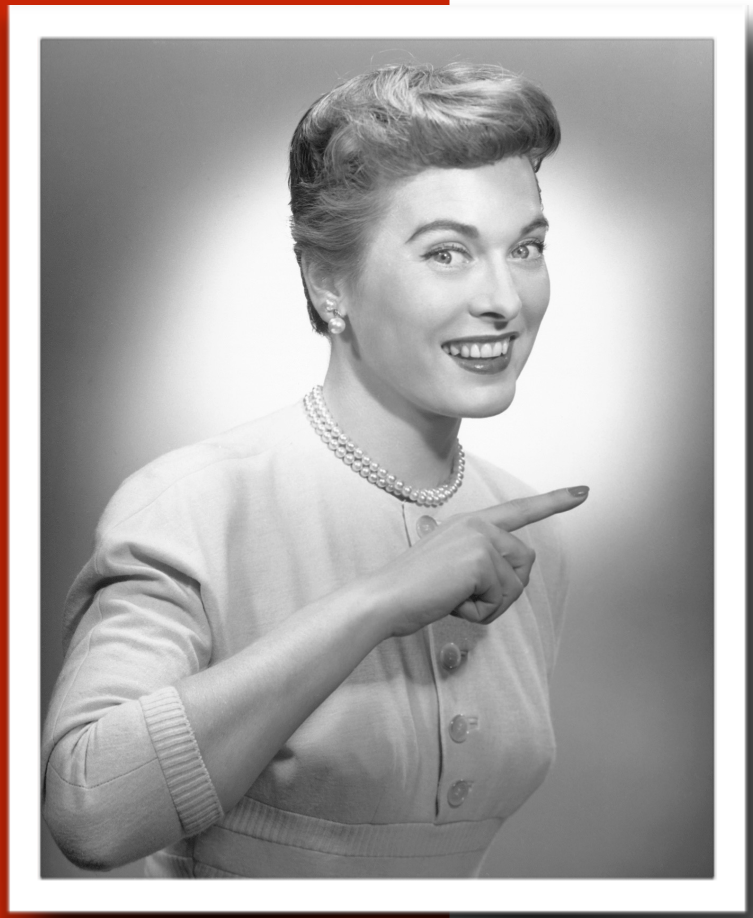
a useful invariant

and an important formal linkage

$$E_m = mc^2$$

$$E_T = m\gamma c^2$$

$$p = m\gamma v$$



fun fact...just with a little algebra...

$$m^2c^4 = E_m^2 = E_T^2 - p^2c^2$$

$$E_m^2 = E_T^2 - p^2c^2$$

another invariant...independent of the frame, just like:

$$s^2 = (ct)^2 - x^2$$

$$m^2c^4 = E_T^2 - p^2c^2$$

kinship:

t and E

x and p

three things are always, always constant

c

s

mc^2

Einstein preferred “Invariant Theory” to “Relativity”

Cousin Quantities!

- **Space and time** are not separate entities, but linked as spacetime
- **Electric and magnetic fields** are not separate entities, but linked as electromagnetism
- **Energy and momentum** are not separate entities, but linked as 4-momentum

so, how was this all received?

According to Einstein's sister,

…he anticipated a large reaction with much criticism

What he got at first was silence.

oh, a nice note from Max Planck asking for some clarification

then a seminar by Planck in Berlin which touched on Relativity…

- only then… a little professional attention, to "Prof. Einstein, University of Bern"

The first paper published on Relativity by not-Einstein:

also by Planck, who derived the relativistic momentum relation, $p = \gamma mv$

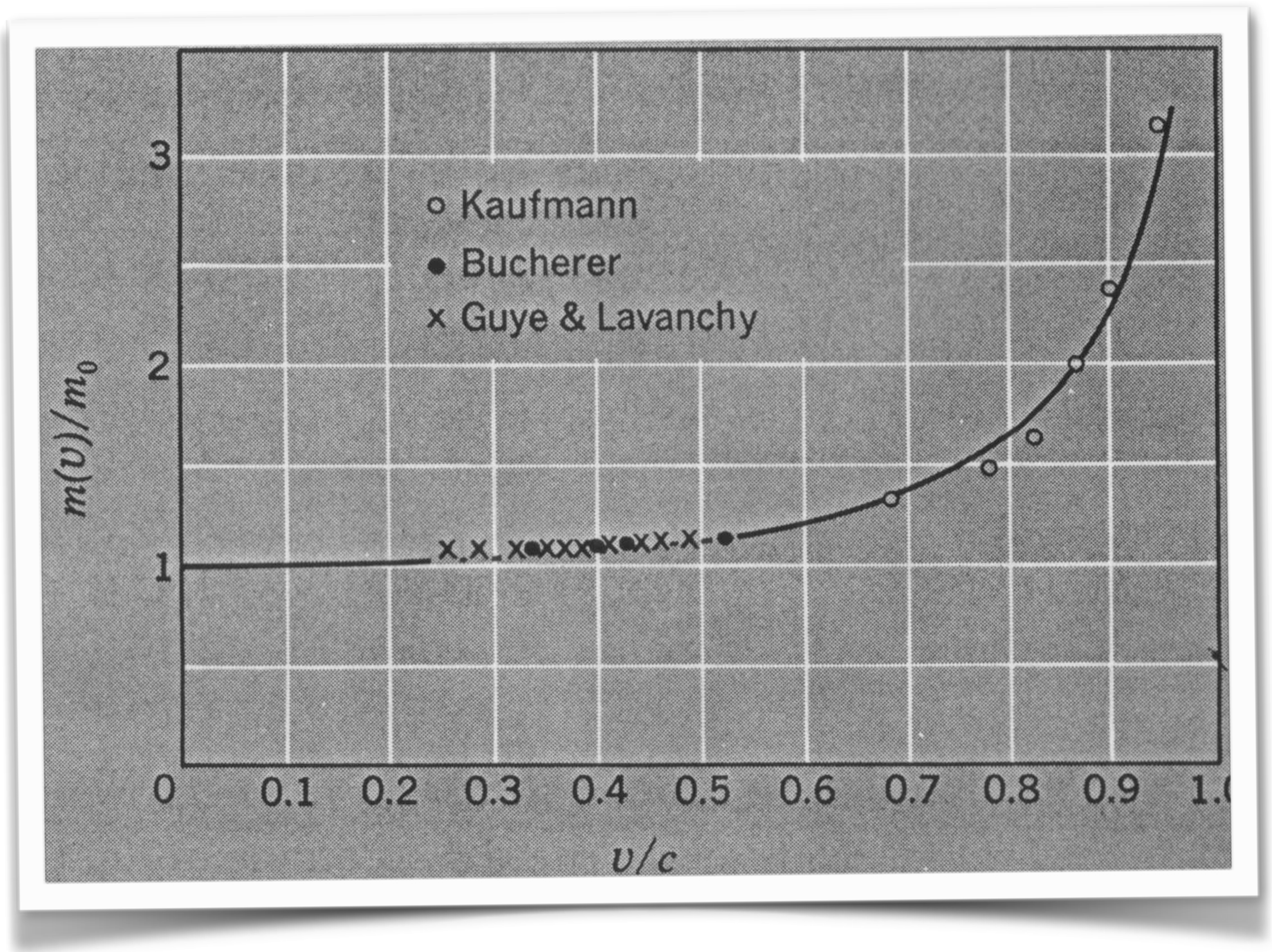
The 1908 Minkowski lecture, in which he worked out completely in modern form the mathematics of relativity and the spacetime view got people's attention

What about experiment?

the first experimental confirmation

New experiments were done,

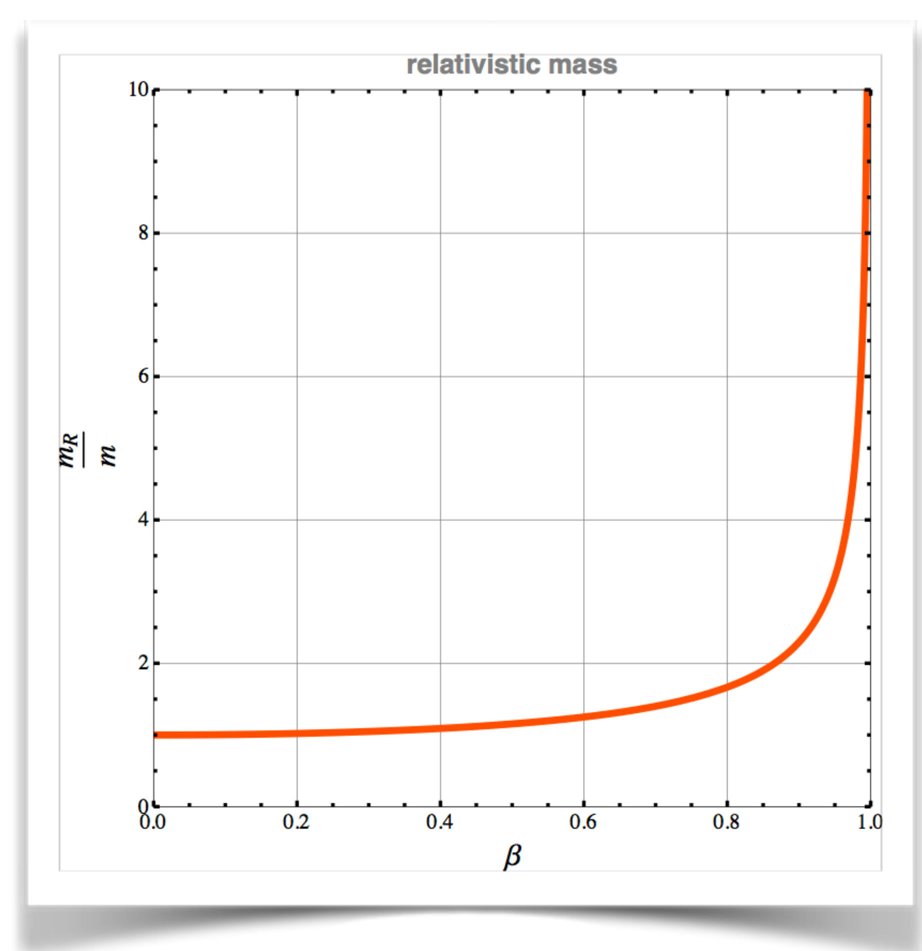
and by 1910, the results were:



These results are from 1910 for three experiments, and the curve is the special relativity prediction

From this point on relativity has become a part of everyday scientific and engineering life

Kaufmann lost again…Max Planck corrected his analysis



General Relativity

principle of equivalence

Special
Relativity

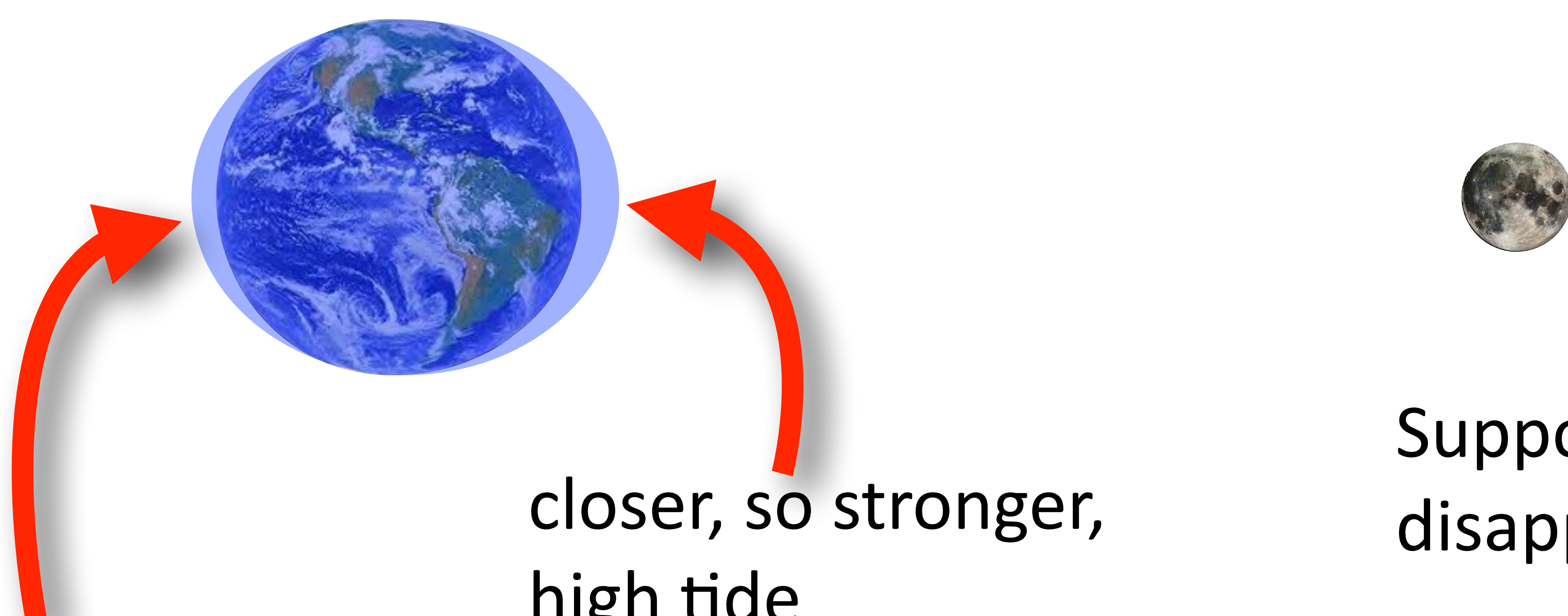
created a problem

What about most of the universe...?

Where gravitation is a fact of life?

In particular...the action-at-a-distance thing.

Think about the tides...caused by Moon



closer, so stronger,
high tide

further, so weaker,
low tide

Suppose Moon
disappeared?

Would the tides
flatten instantly?

That violates the rules of Special Relativity

what's worse

How do you deal with Newton's Universal Gravitational formula?

Masses appearing different from different frames?

$$F_{1,2} = G\frac{M_1 M_2}{R^2}$$

Start length-contracting the distance?

Worrying about Gravity led Einstein to

think hard about

SPACE and TIME

~~moving~~ coordinate systems

accelerating

BTW, his birthday was Pi-day

the general theory of relativity

What’s the “special” in “special” relativity?

the physics of inertial frames

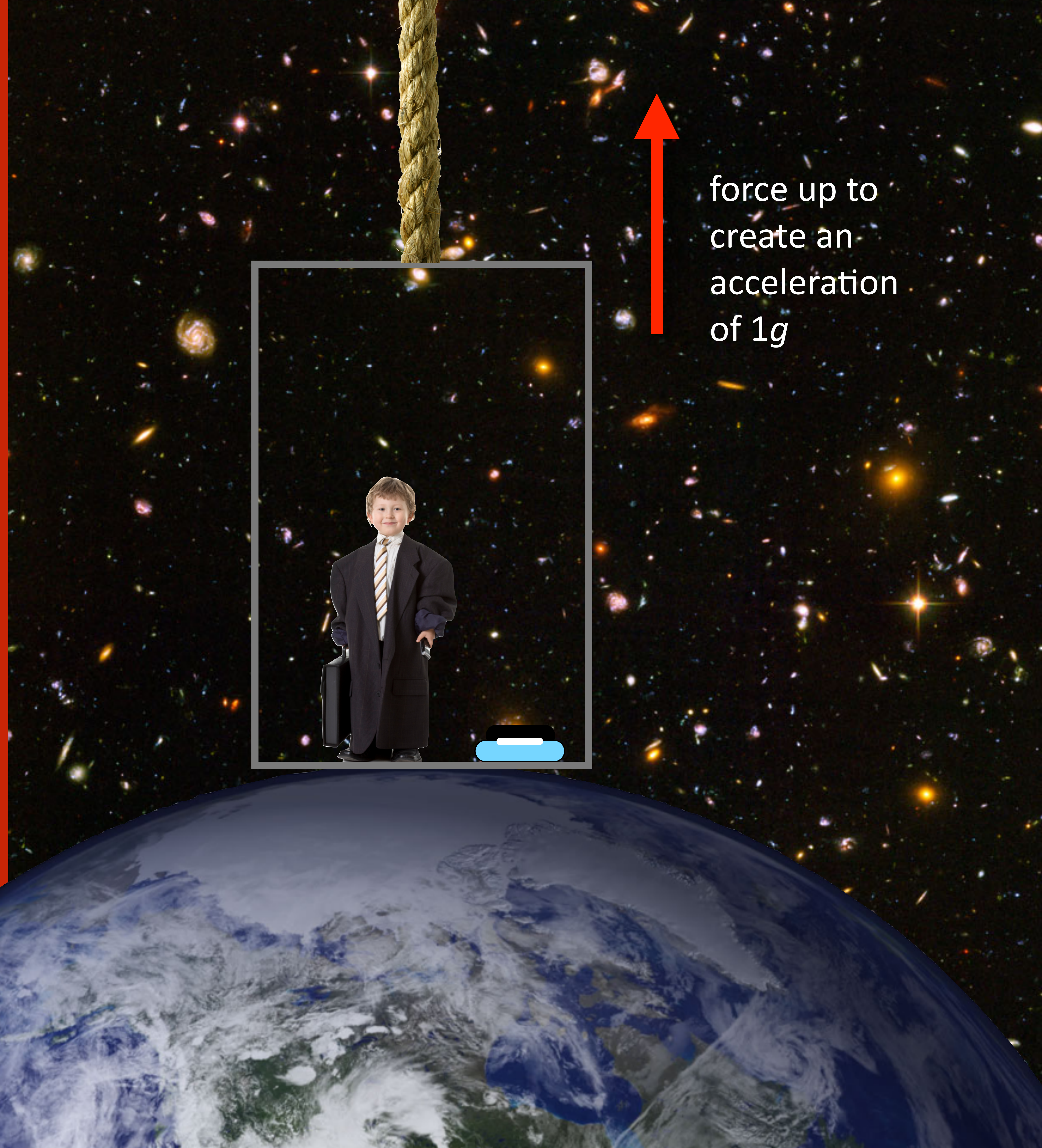
What about the most obvious accelerating condition?

stupid elevator trick, #1

gravitational attraction

stupid elevator trick, #2

gravitational attraction



Here comes a Relativity-like statement:

similar to Galileo’s ship-hold...

“you can not perform any mechanical experiment to tell you that a ship is moving at constant speed relative to the land.”

or Einstein’s...

“you can not perform any mechanical or electromagnetic experiment to tell you that a ship is moving at constant speed relative to the land.”

There is no mechanical or electromagnetic experiment he can perform

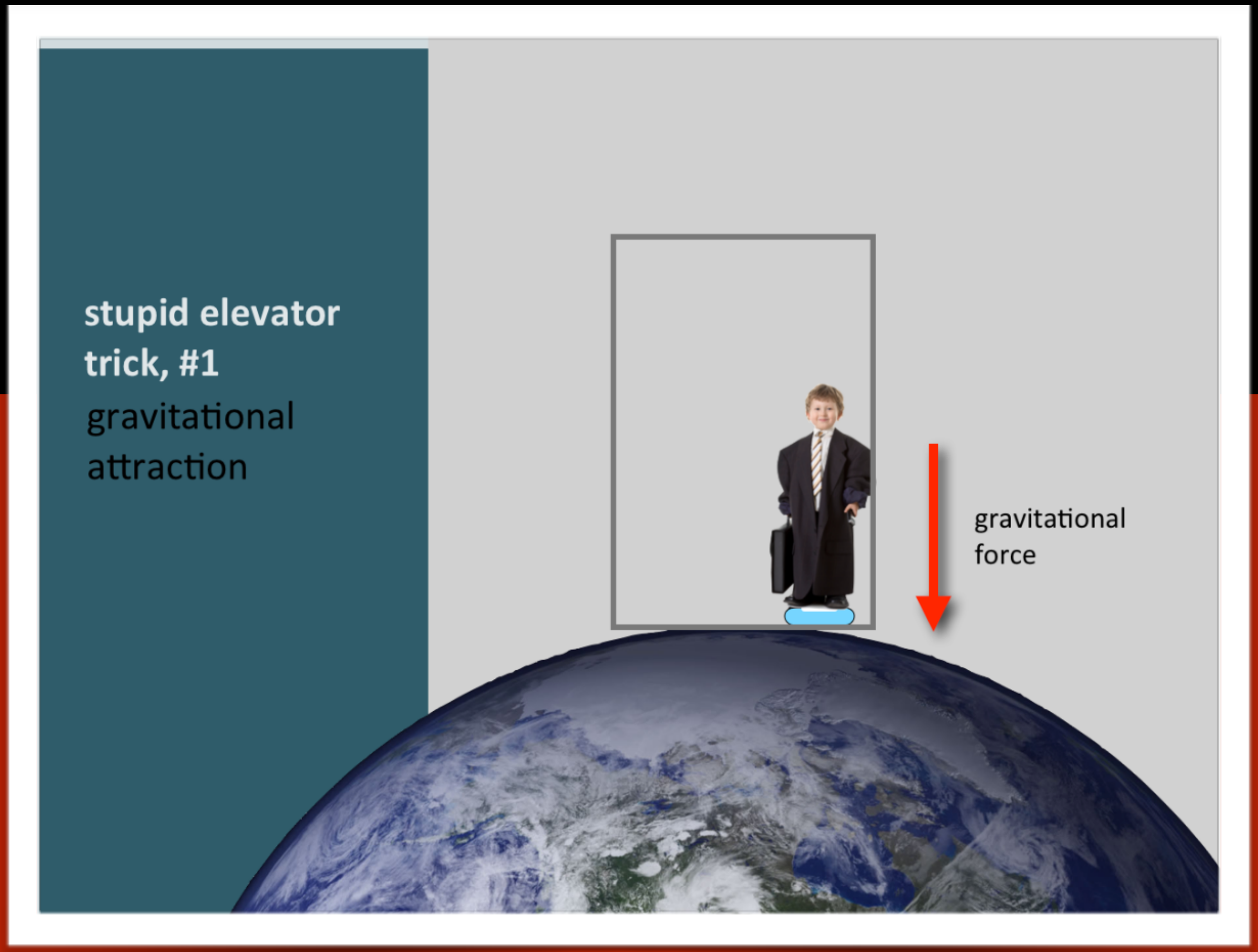
that would tell him that he was

1. being attracted by the Earth **due to gravity** or

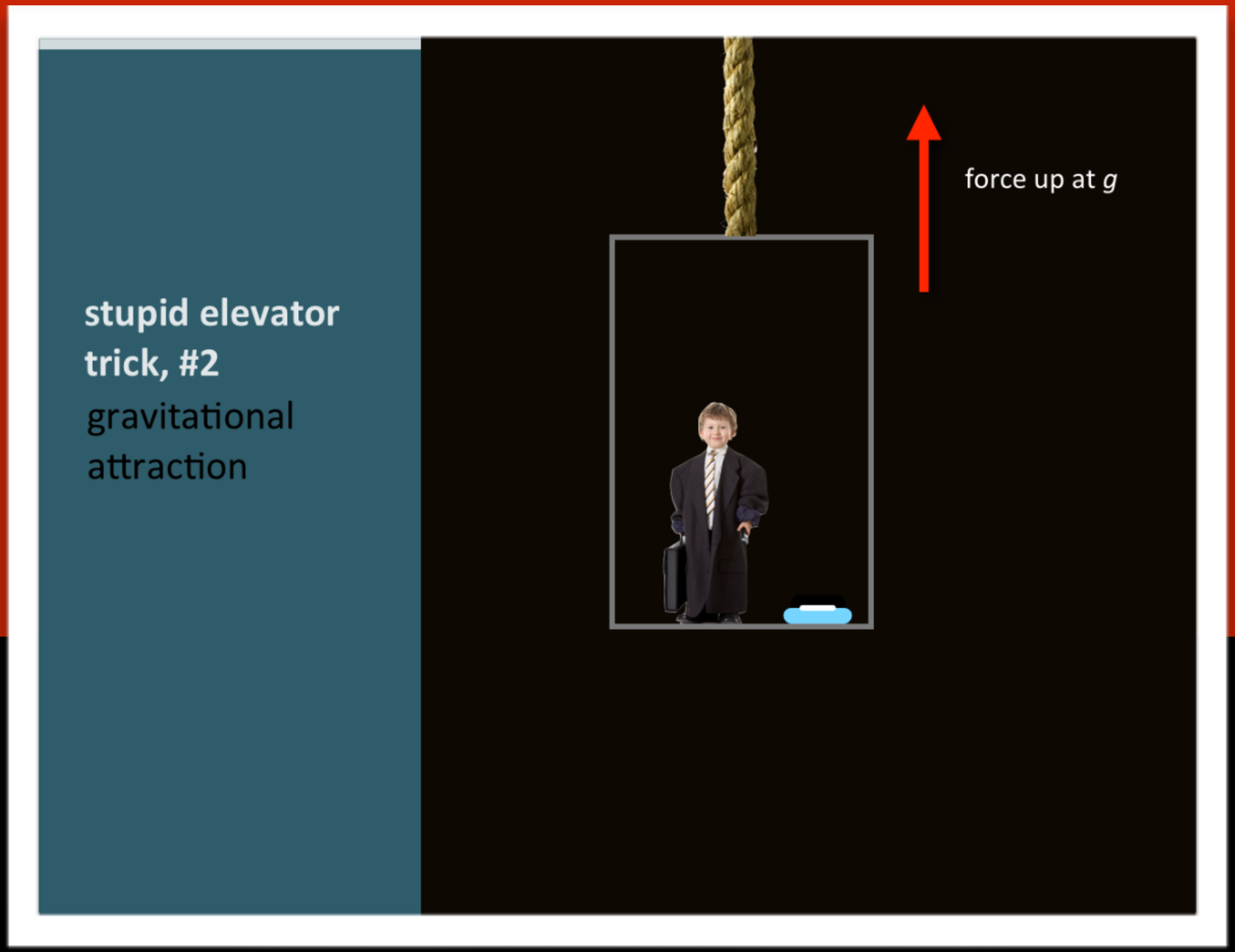
2. being pulled **and accelerated** g with no gravitational field anywhere

called sometimes

weak Equivalence Principle



identical



`gravitation`

is relative