# hi

Lecture 23, 04.04.2017

Quantum Mechanics 3

# housekeeping

Question about anything?

*I'll make a movie for you:*


Poster selection:

*April 13, outline due April 20…read the instructions.*

Homework:

*For month of April, I've shifted due dates to Saturdays.*

can't meet office hours today…sick

# Honors Project

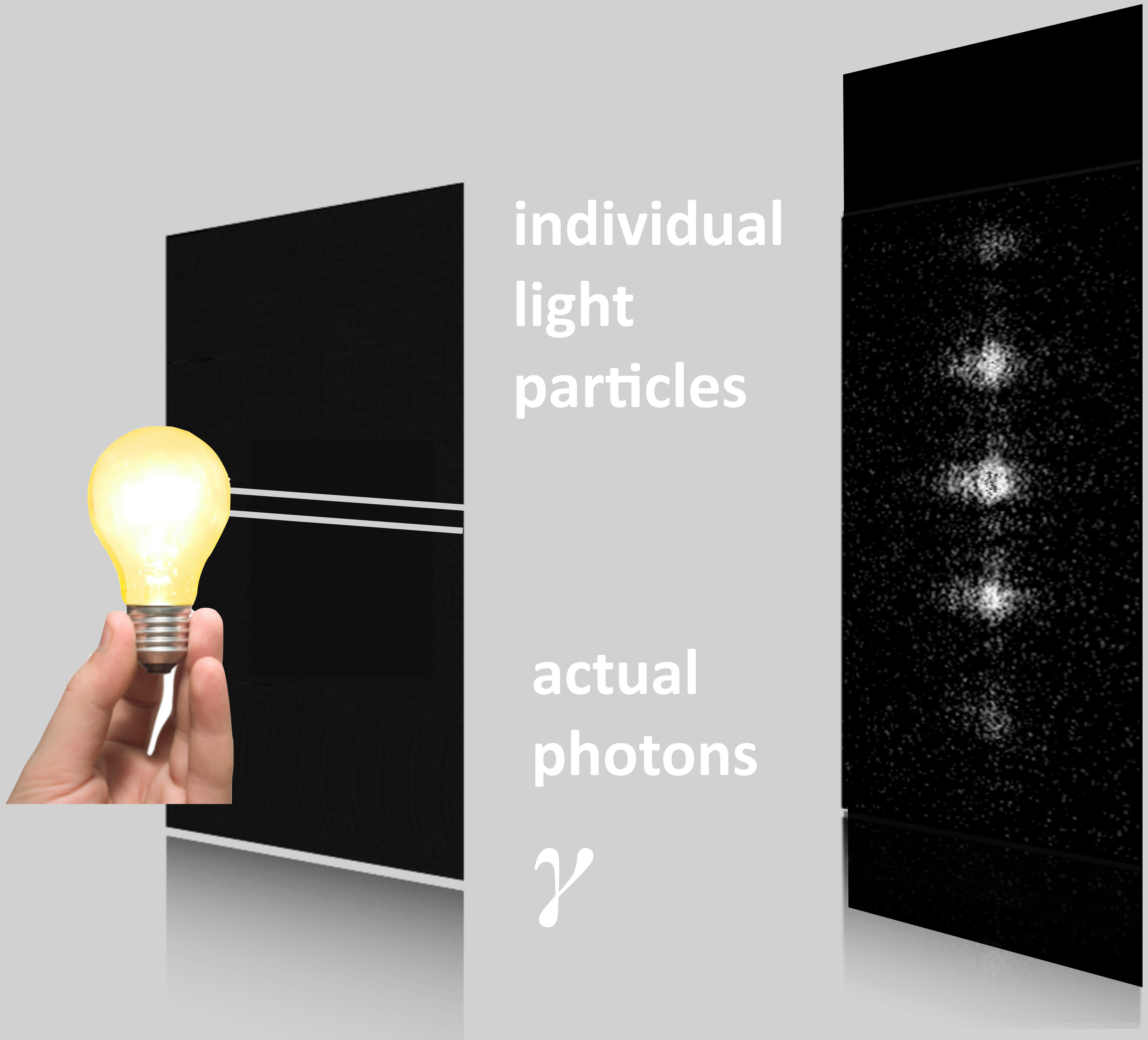
Data due April 22. Paper due on May 4 (final day).

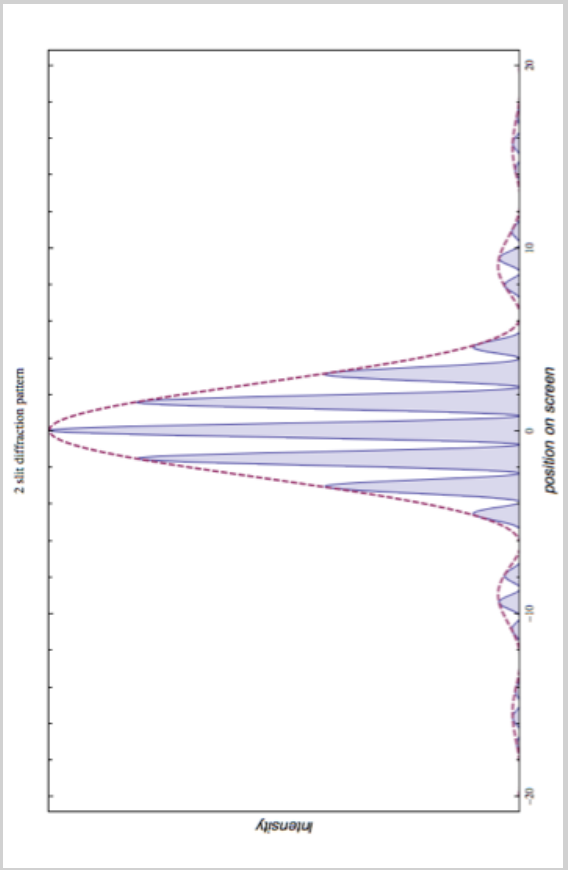
Read the Second of two sets of instructions:

`MinervaInstructions2_2017.pdf` in

www.pa.msu.edu/~brock/file_sharing/QSandBB/2017homework/honors_project_2017/

# here's how it works



let light go through a double slit

but sensitively count individual photons


David Dykstra, Steven Busch, Wouter Peeters, Martin vanExter, Leiden University, 2008

http://www.youtube.com/watch?v=MbLzh1Y9POQ

# 1899: he carefully isolated 2 components of radiation:

one stopped by thin aluminum

one highly penetrating

and one more

and figured out another found in 1903:

$\gamma$ neutral gamma rays

negatively charged, passes through matter relatively easily

$\beta$ beta rays

$\frac{q}{m}$ → electrons

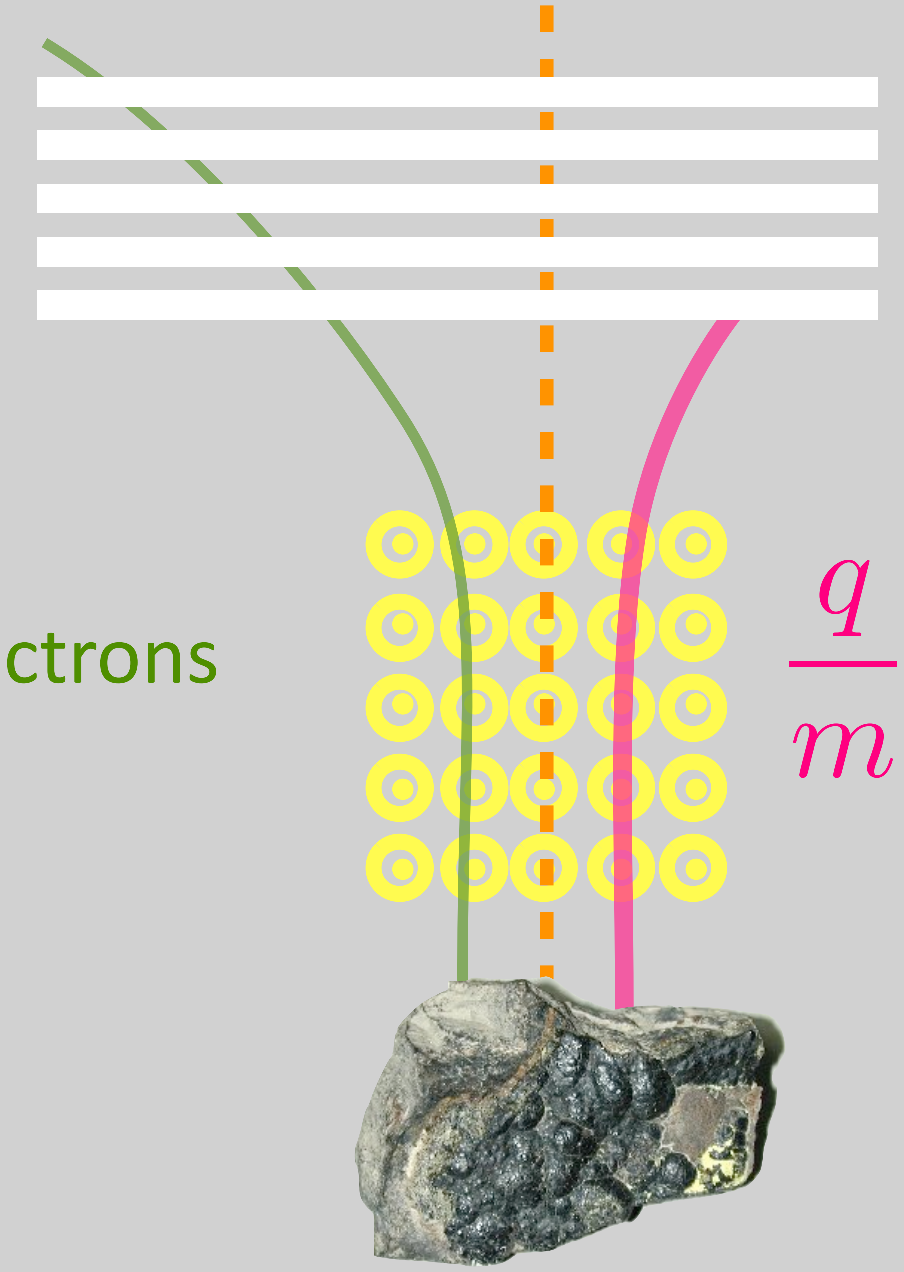
positively charged, easily stopped in matter

$\alpha$ alpha rays

$\frac{q}{m}$ → 2 x H atom

↓

**Helium nuclei**

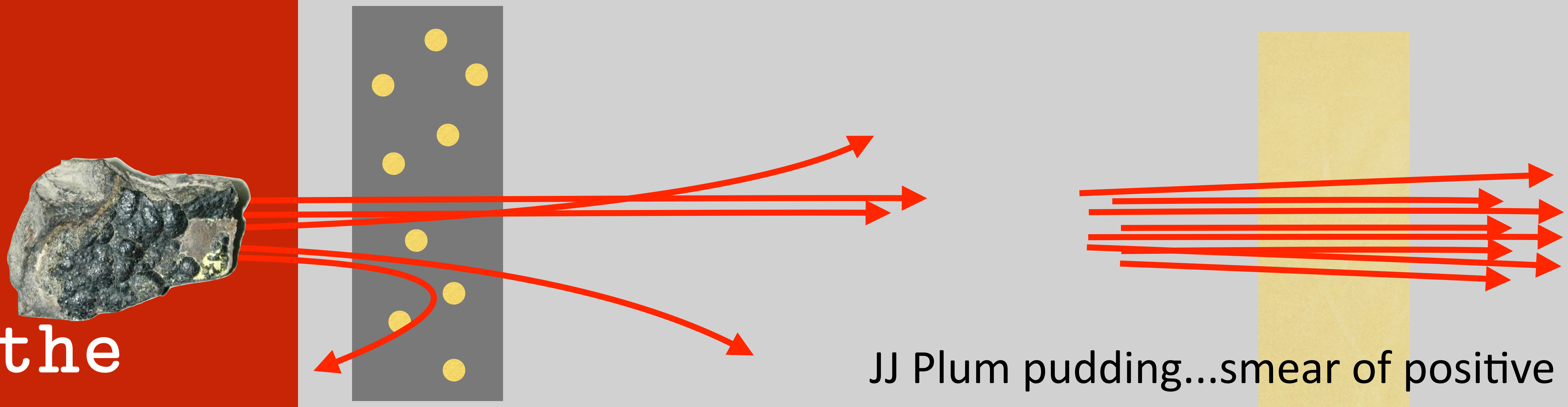


He had the solution after 2 years of work

he found:

1911: that the Atomic Number was +Ze

and made a model of the atom...



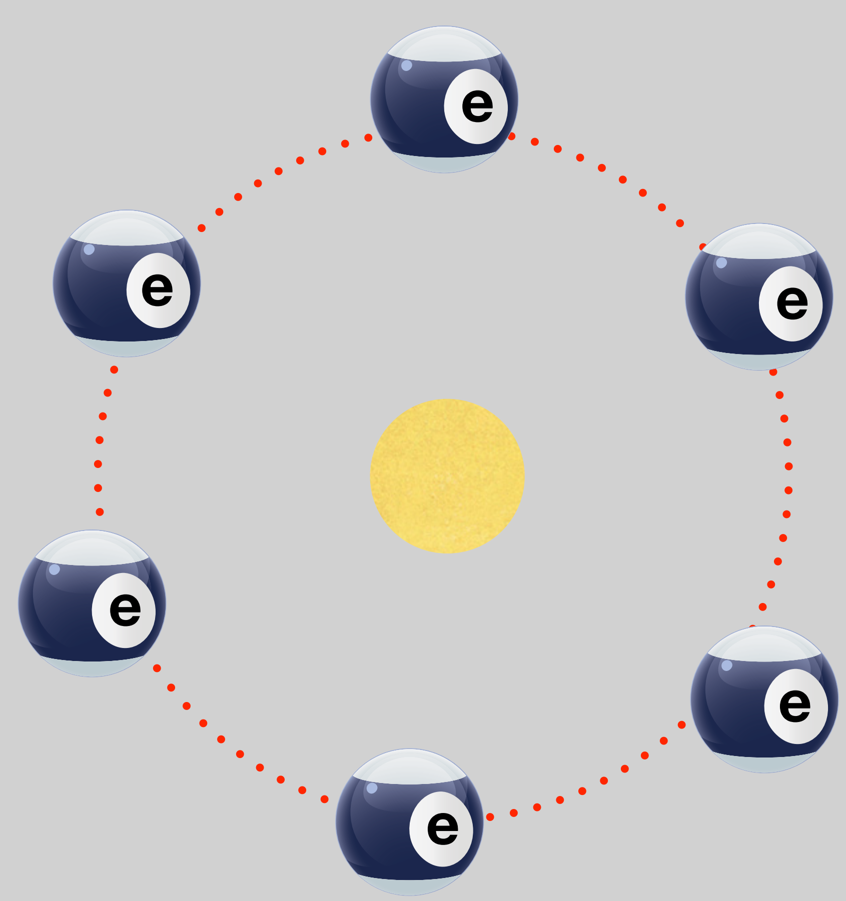
JJ Plum pudding...smear of positive charge - tiny individual deflections

**the Rutherford Model of the atom:**

**Matter** consists of **hard-cores of positive charge**.

The nucleus. This matched his alpha-scattering data.

The **electrons**? Somewhere around the outside?




That's problematic, the electrons would accelerate...and radiate.

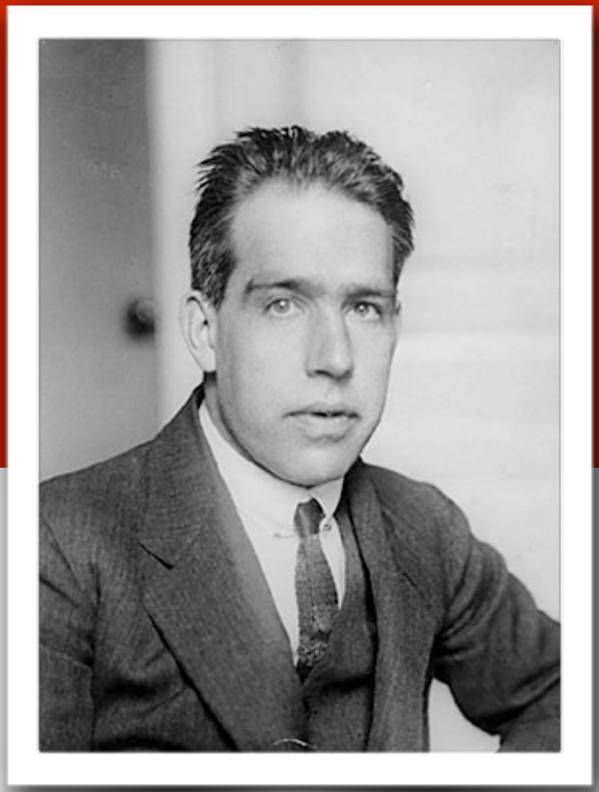
a spiral of death.

# In 1913 Bohr simply asserted

That at atomic distances…

there are electron orbits that simply don't radiate - "stationary states"

fixed "quantized" orbital radii and orbital velocities



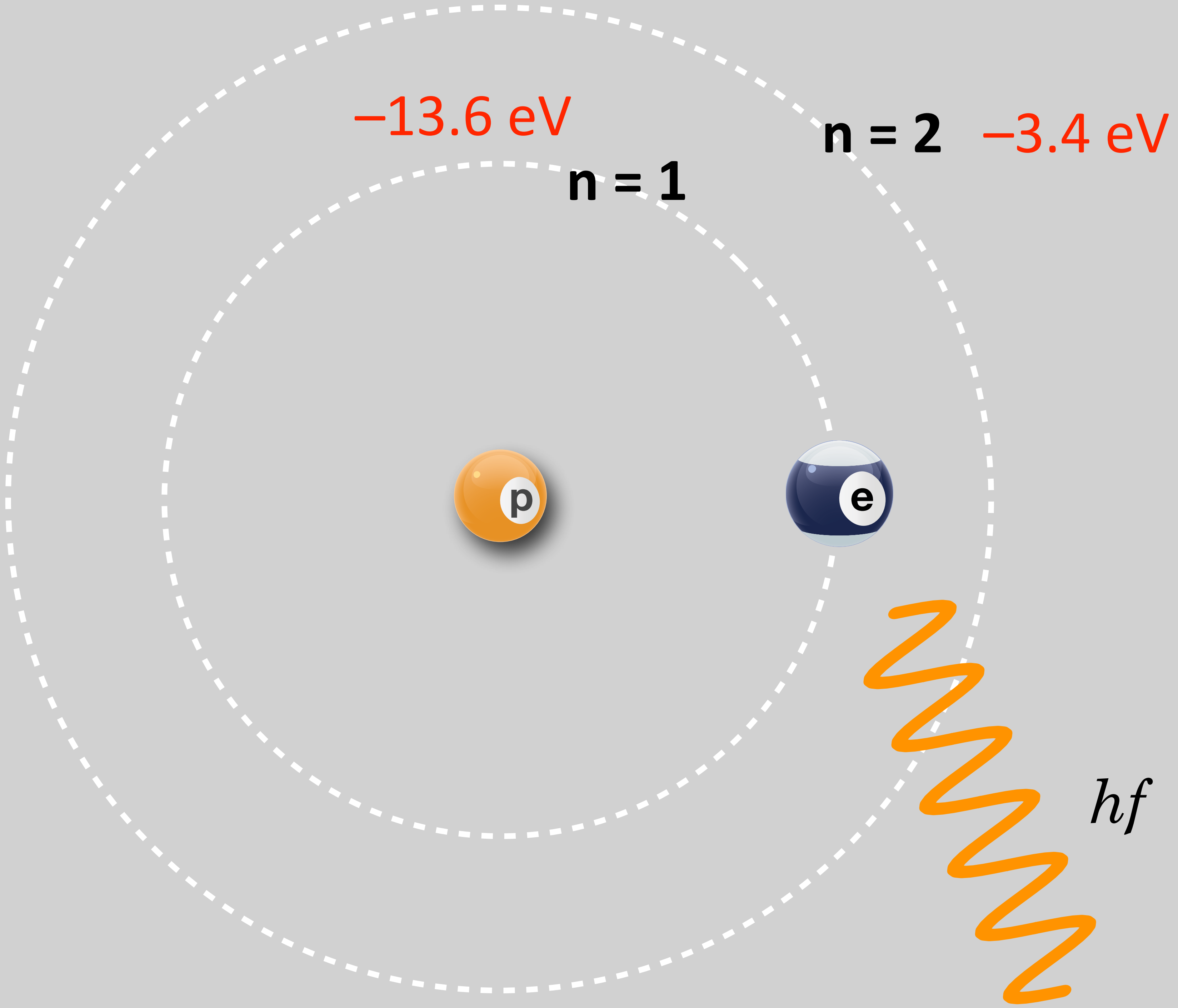
Niels Bohr

1885 – 1962

a talker.

# the magic of Bohr's model:

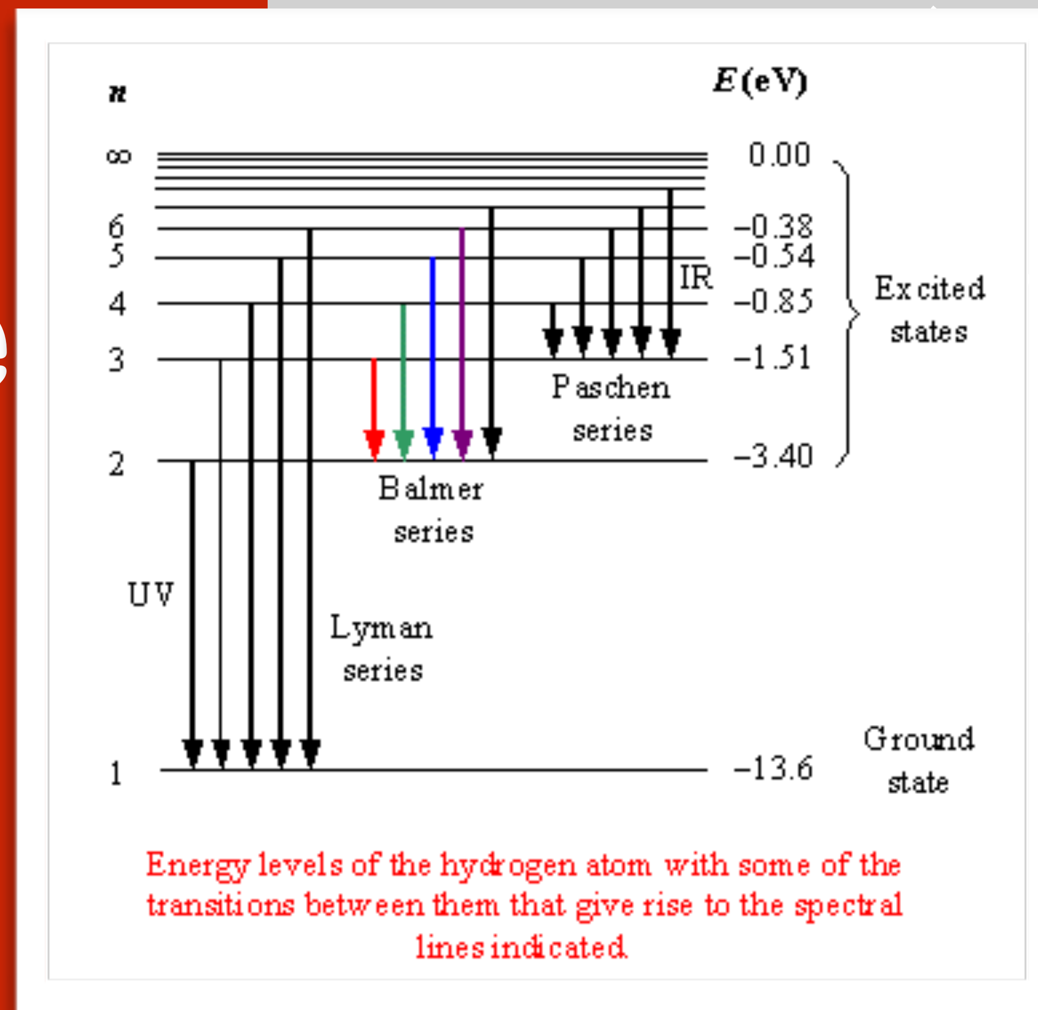
the idea of an atomic transition

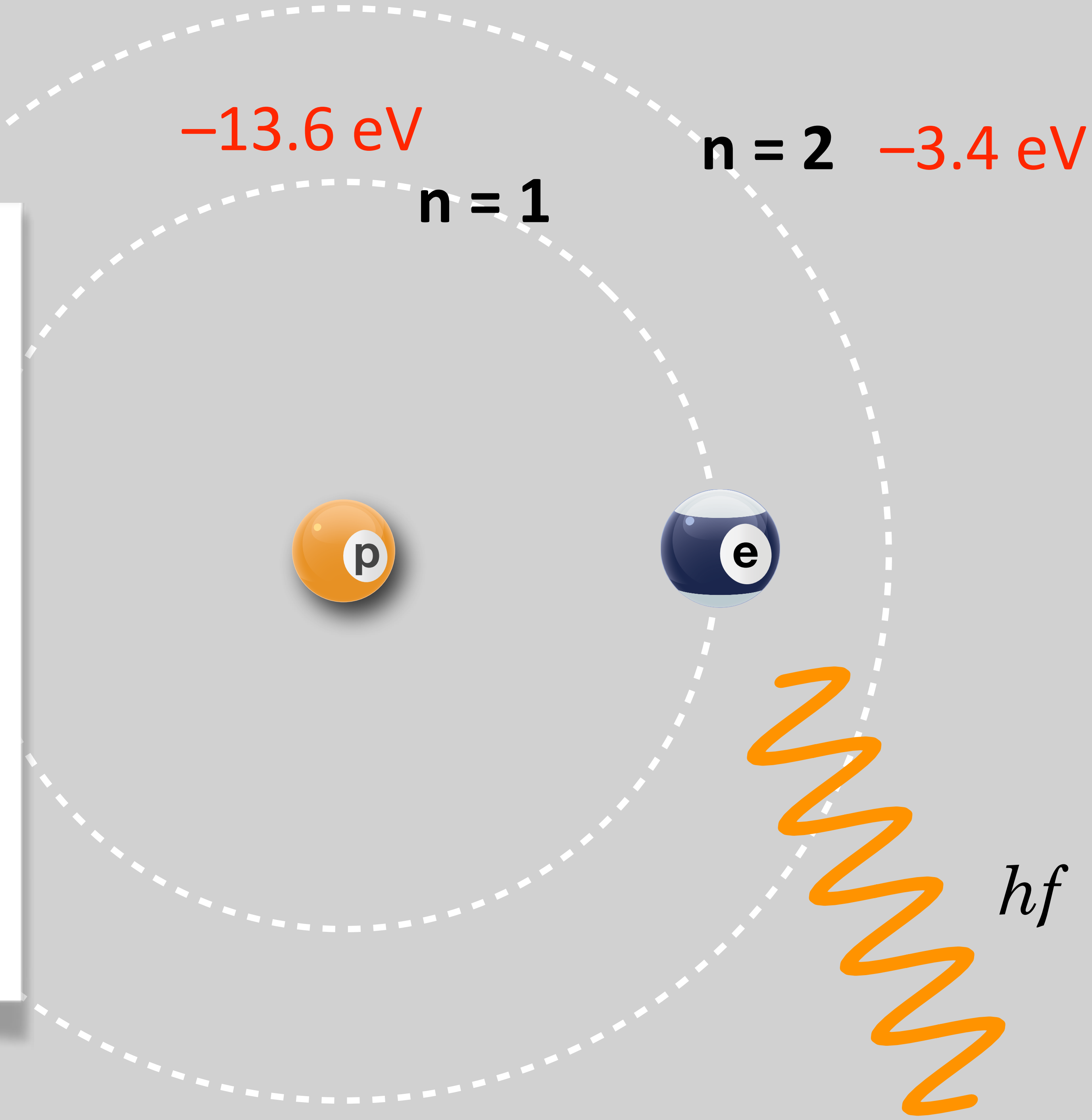


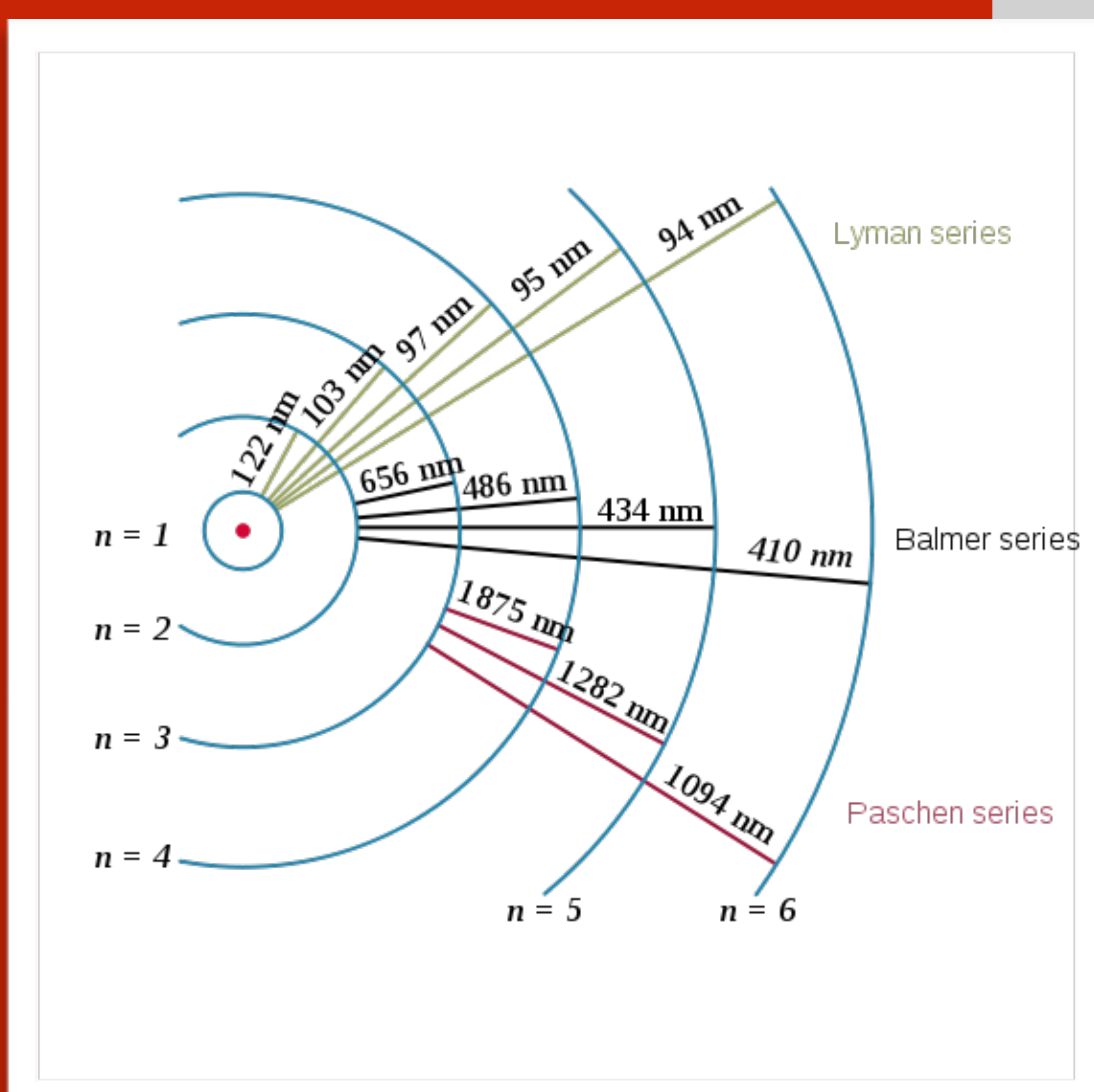

The idea: transition of electrons results in the released energy of a photon...of a particular energy

# imagine his surprise

1913: his way.

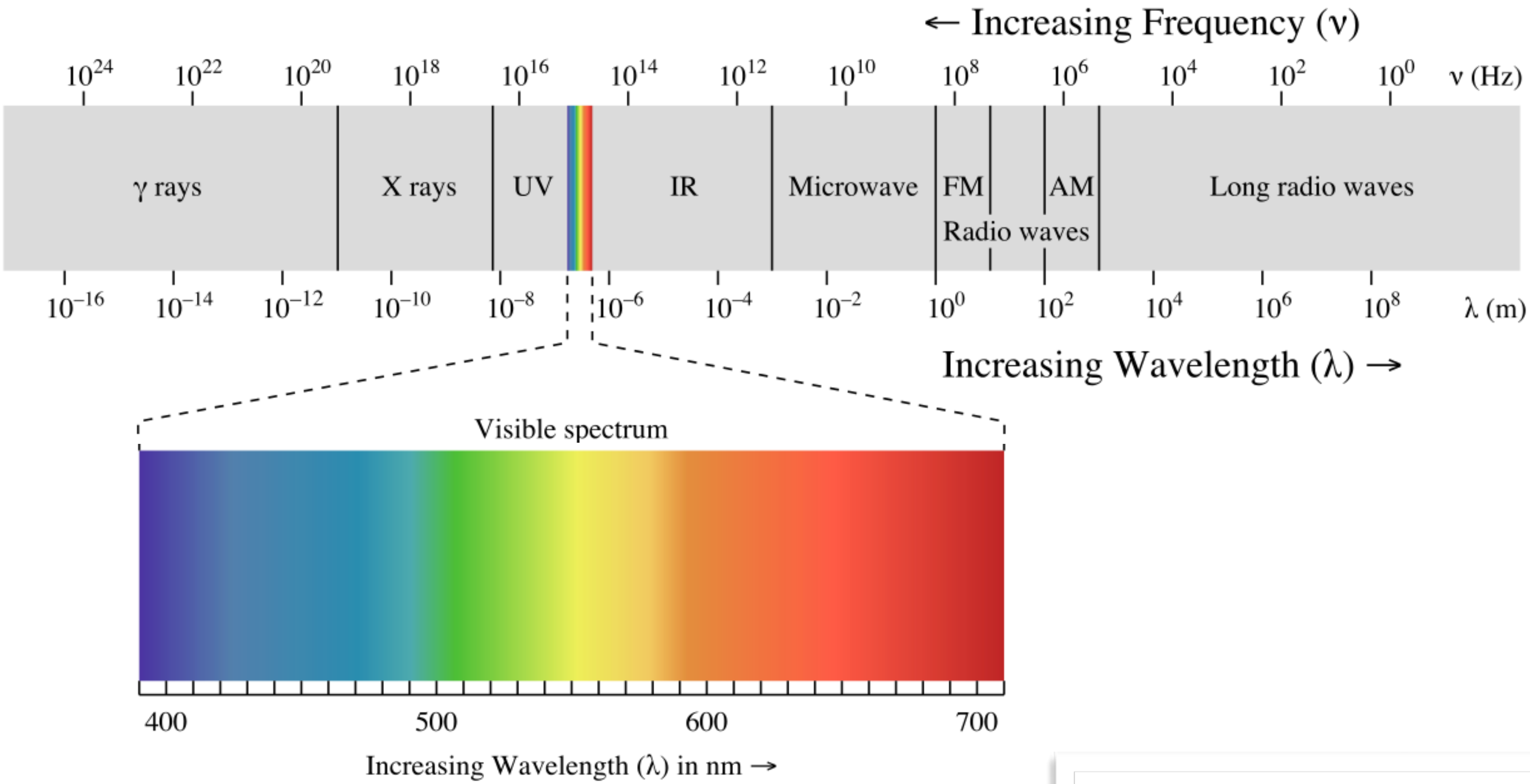



$$E_2 - E_1 = (13.6\ \text{eV})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = hf$$

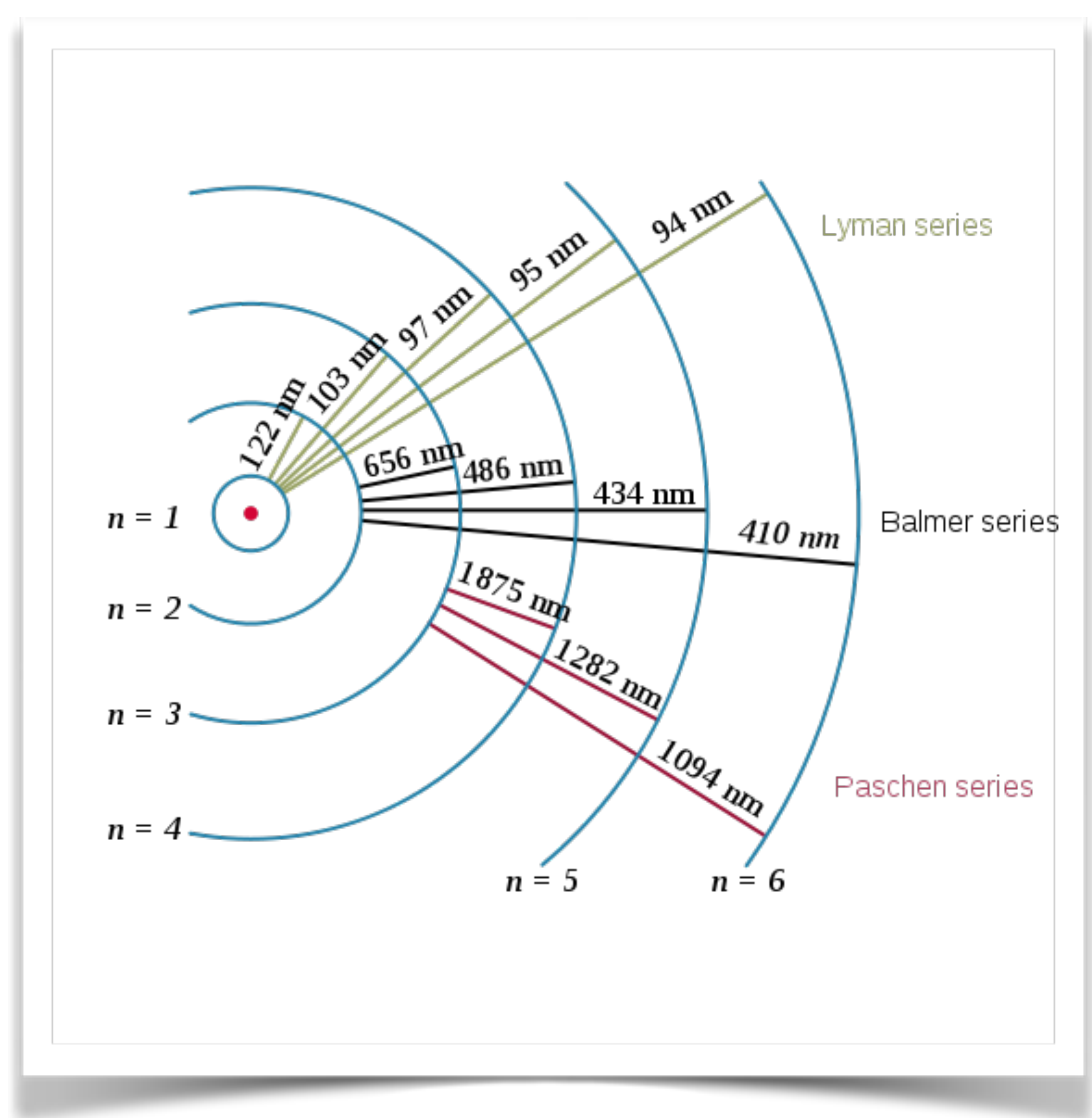
$$E_2 - E_1 = 10.1\ \text{eV} \longrightarrow \lambda = 122\ \text{nm}$$




410 434 486 656

nm

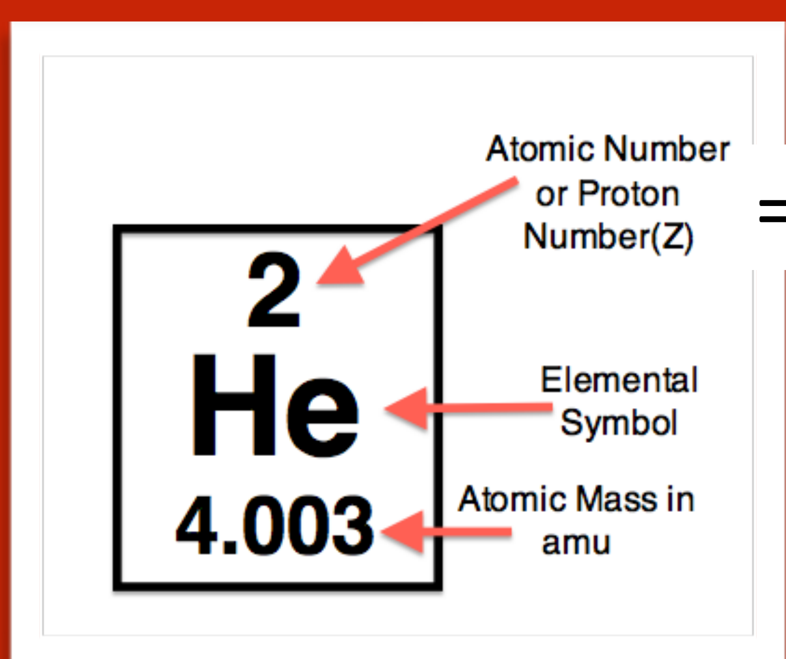
Hδ Hγ Hβ Hα



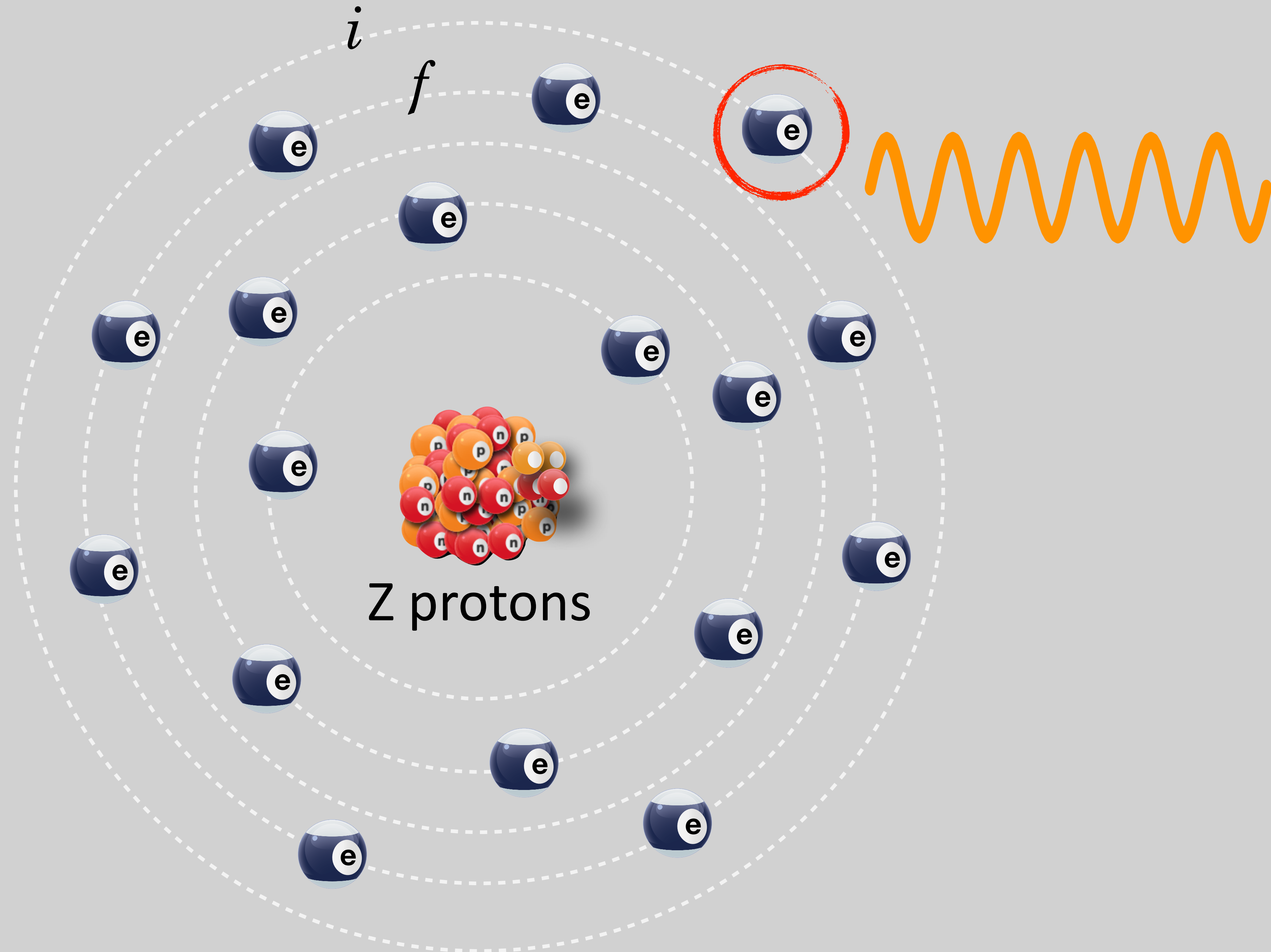
# hydrogen, fine

how about more complex elements?

Higher atomic number, Z?




= # of electrons also!



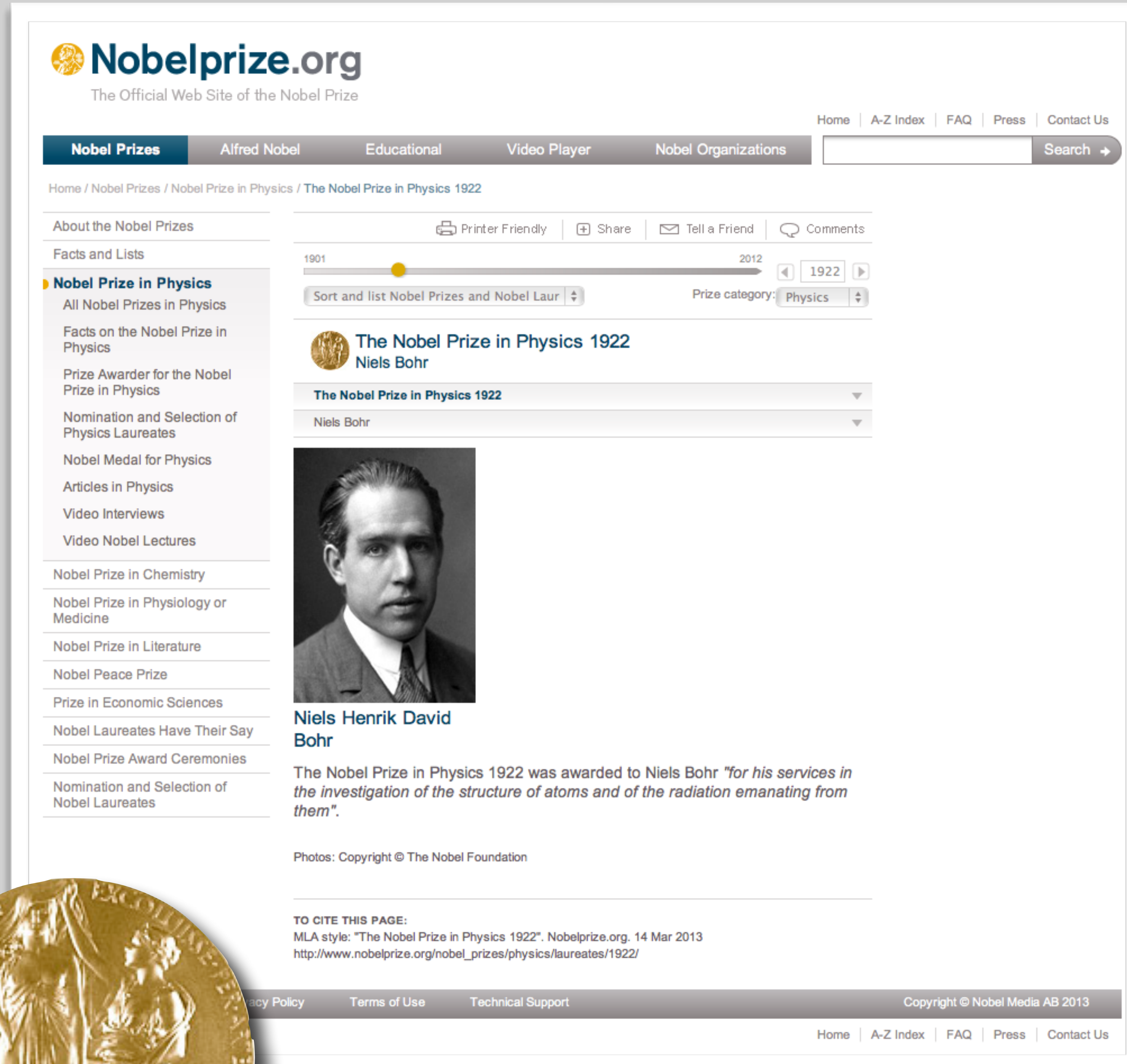

lots of electrons, but as long as there's one lone one..the Bohr Formula still works.

$$E_f - E_i = -\frac{1}{2}\frac{4\pi^2 k^2 Z^2 e^4}{h^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -hf$$

Go looking for new elements....

# yup, 1922

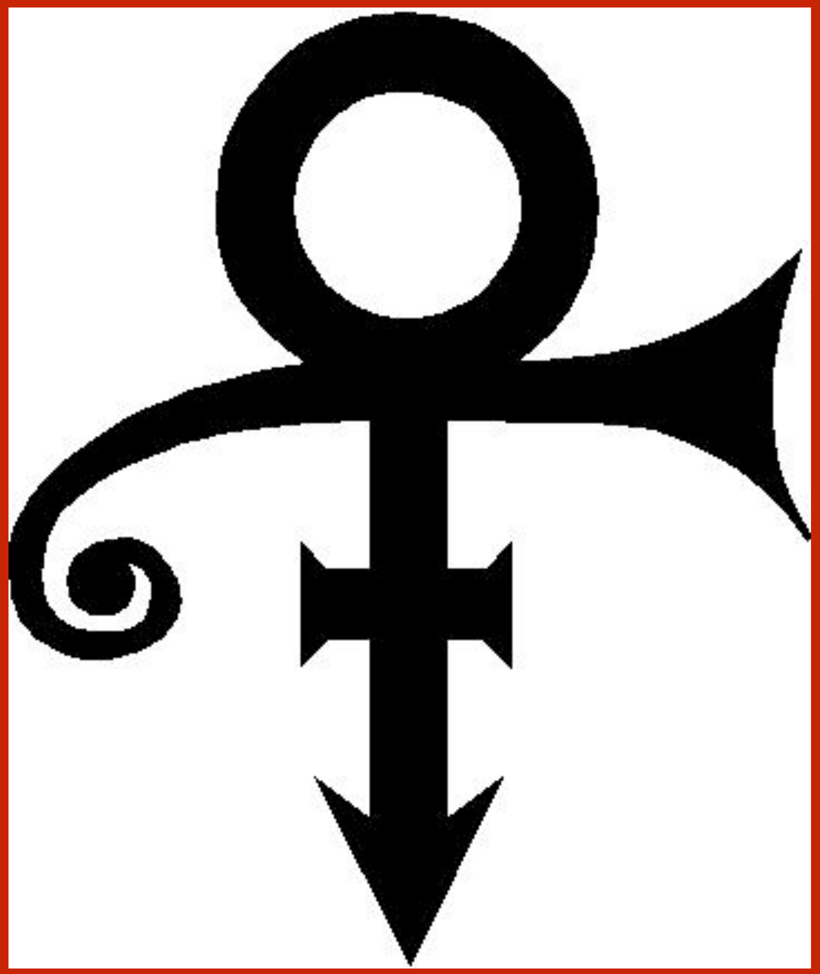
actually with Einstein's delayed prize




# `then`

it got strange

# quantum idea of electrons



# Prince Louis de Broglie

His 1922 PhD thesis:

"The French Comedy"

must have been disconcerting

The Prince looking self-satisfied

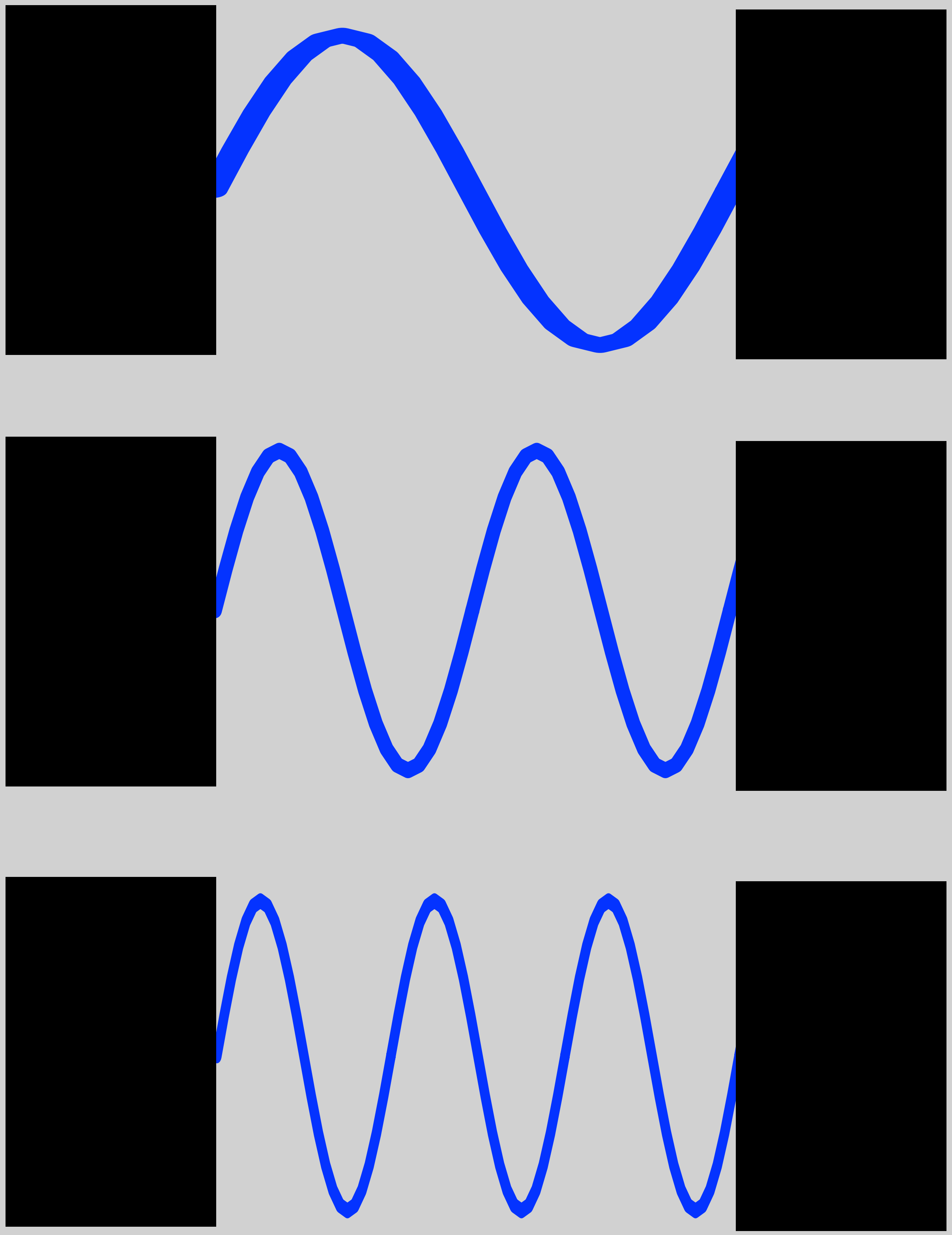
# the quantum idea:

made use of **integers**

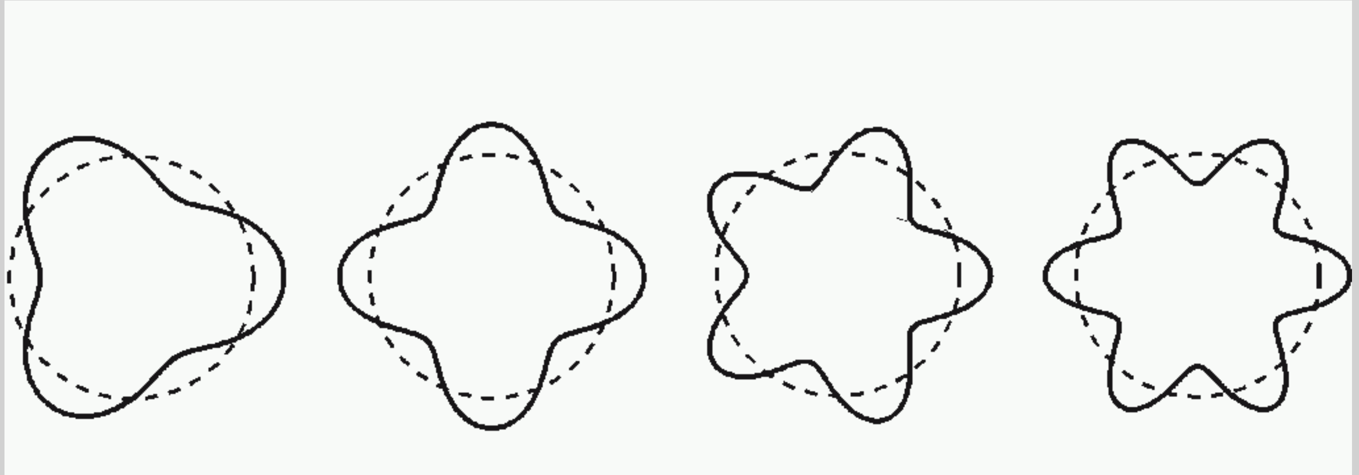
so do waves

# a standing wave

uses integers




Suppose the integer's in Bohr's formula...had to do with standing waves? Wrapped around a circle?



But...you sputter...I thought the orbits were electrons?

# Following Bohr:

photons

undeniably wave and particle-like

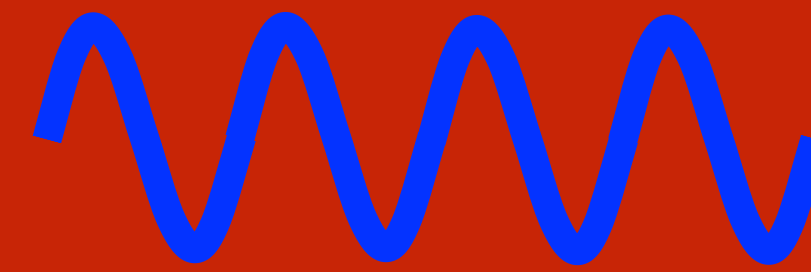
in atoms they involve integers directly.

hmmm, thought the Prince

One other thing involves integers

*standing waves*

A standing wave, wrapped around in a circle



## well

go from photons

to matter...!

**Remember the total energy relation?**

$$E_T^2 = (mc^2)^2 + (pc)^2$$

**In which objects with m = 0 have energy:**

$$E = pc$$

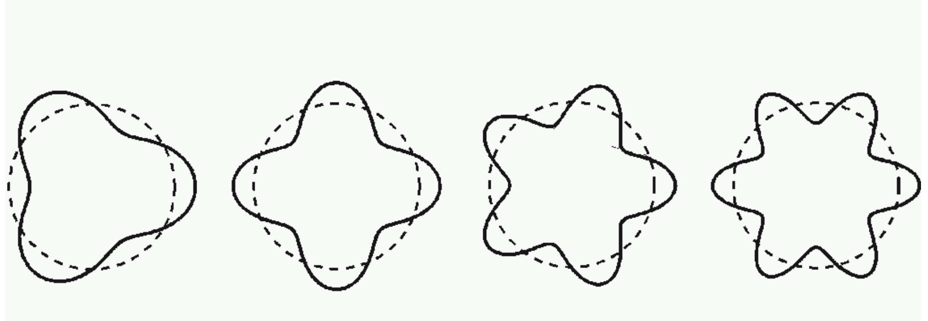
**rearrange...** $$p = \frac{E}{c}$$

**use the Planck relation for E:**

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

Pretend that this Photon-inspired, standing wave idea **works for electrons of momentum p.**

**Electrons with a wavelength!**



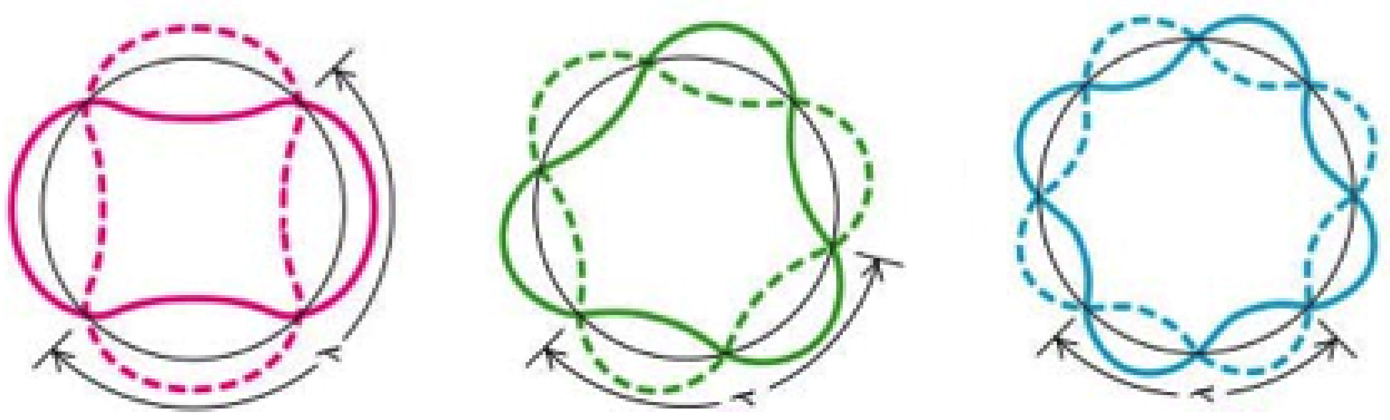
# the momentum of an electron

related to the wavelength of an electron

the wavelength of an electron??

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

now, a relation for an electron!



n = 2 3 4

deBroglie guessed that the Bohr quantum number was related to the number of standing waves of the electron around the nucleus

photons:

$$\lambda_\gamma = \frac{h}{p_\gamma}$$

electrons:

$$\lambda_e = \frac{h}{p_e}$$

$$\lambda_e = \frac{h}{m_e v}$$

# that was deBroglie's hypothesis

electrons are particles and waves

his PhD examination committee was so scandalized

they actually asked Einstein for advice

Who said: "sounds good to me."

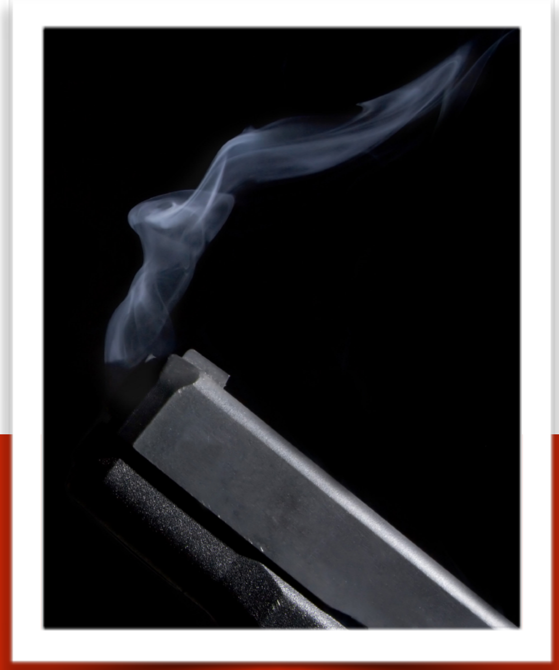
## this relation will be important

relating the wavelength of a quantum object

to its momentum

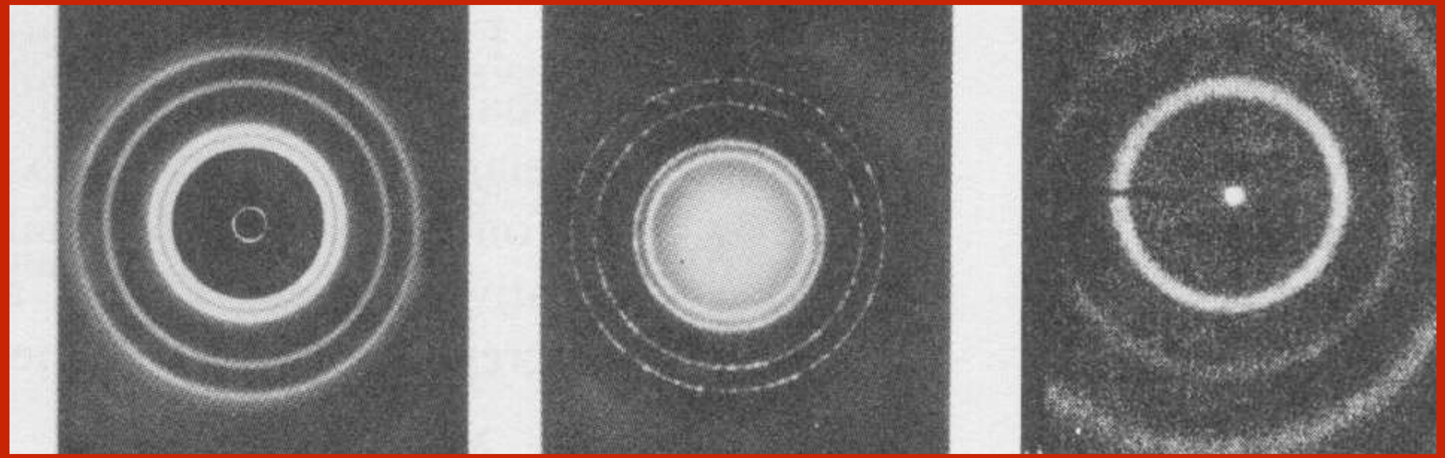
$$\lambda = \frac{h}{p}$$

# particles as waves?



deBroglie suggested how:

they should exhibit diffraction

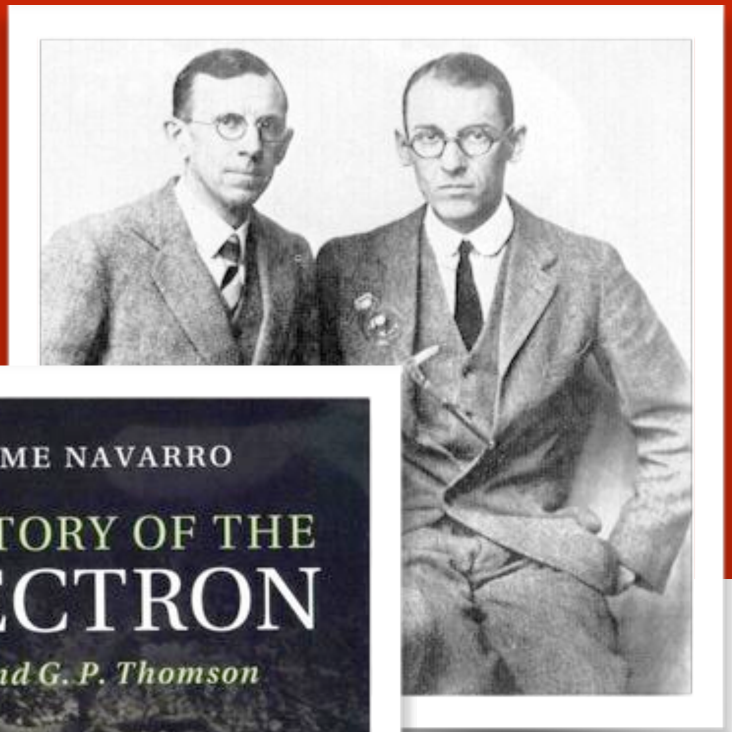


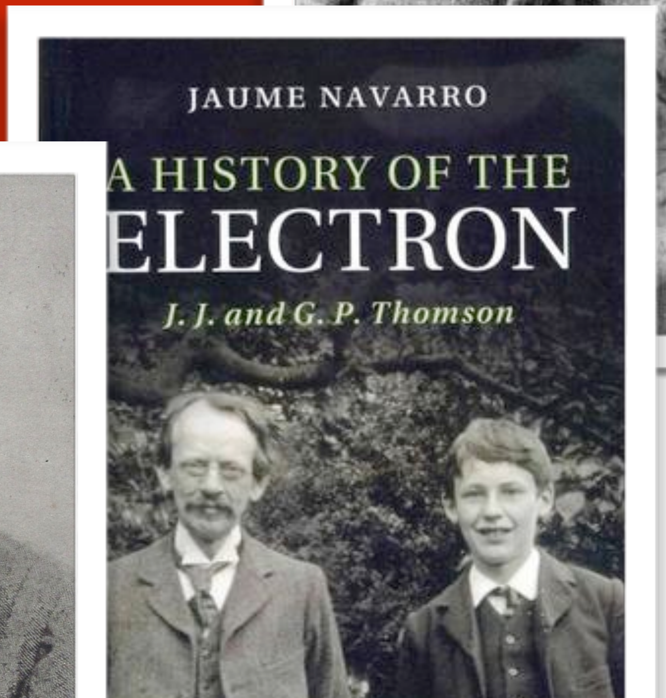
| 0.071nm X-ray diffraction on a polycrystal | 600 Ev electron diffraction on a polycrystal | 0.057 ev neutron diffraction on a polycrystal |
|---|---|---|

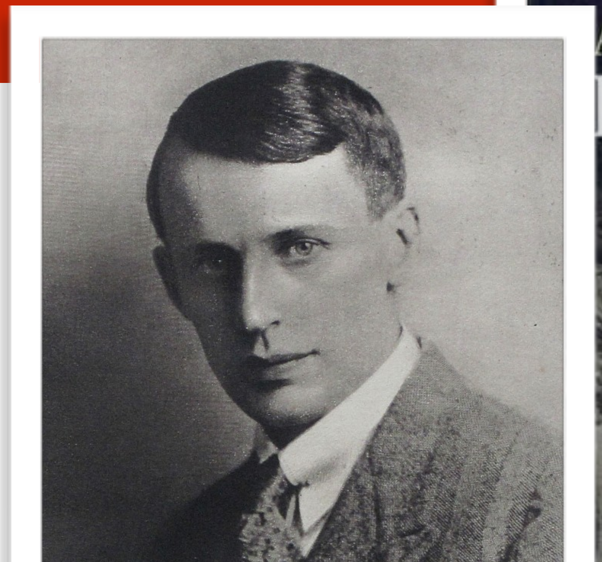
a "slit" appropriate for X-ray wavelengths

1927

*Davisson & Germer*





# JJ's son GP

JJ got the Nobel for showing that the electron exists and is a particle

GP got the Nobel for showing that the electron is a wave

Germer lost out

*Nobel rules: 3 people.*


# in one picture

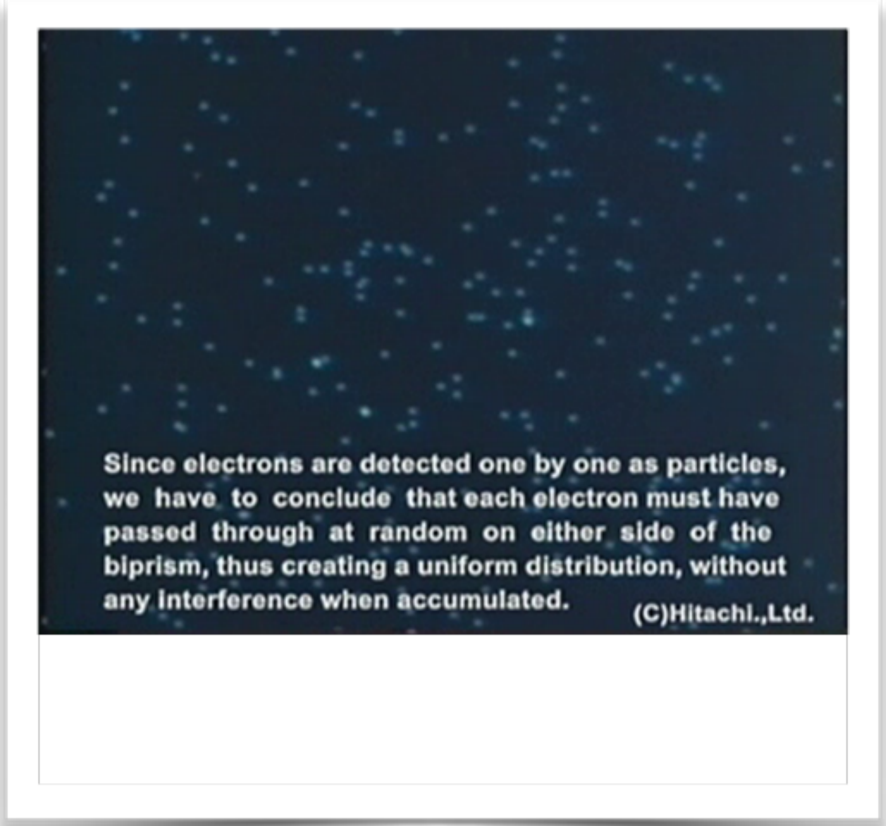
both the particle like features of electrons

the dots

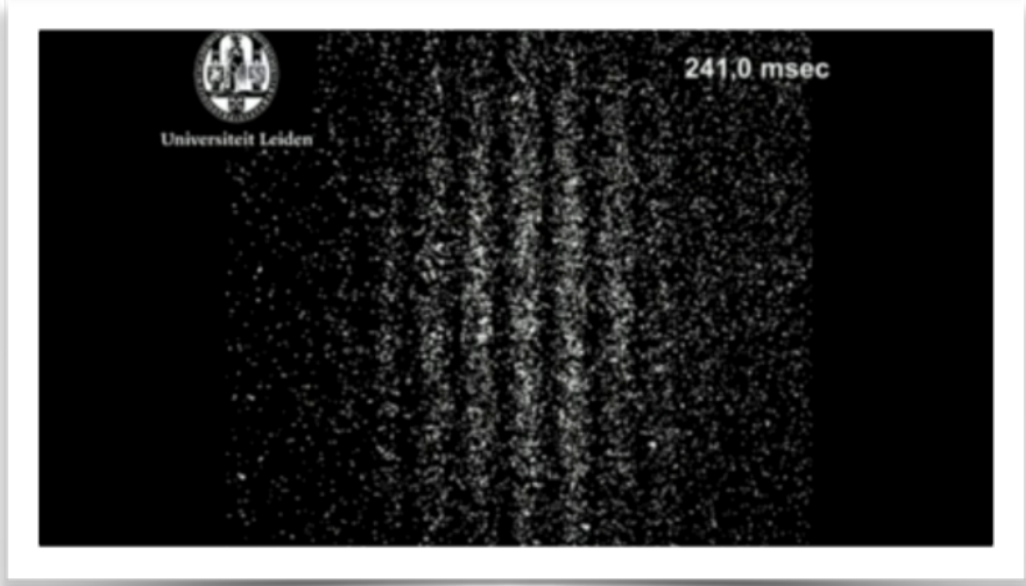
and the wavelike features of electrons

the diffraction pattern

http://www.hqrd.hitachi.co.jp/em/doubleslit.cfm



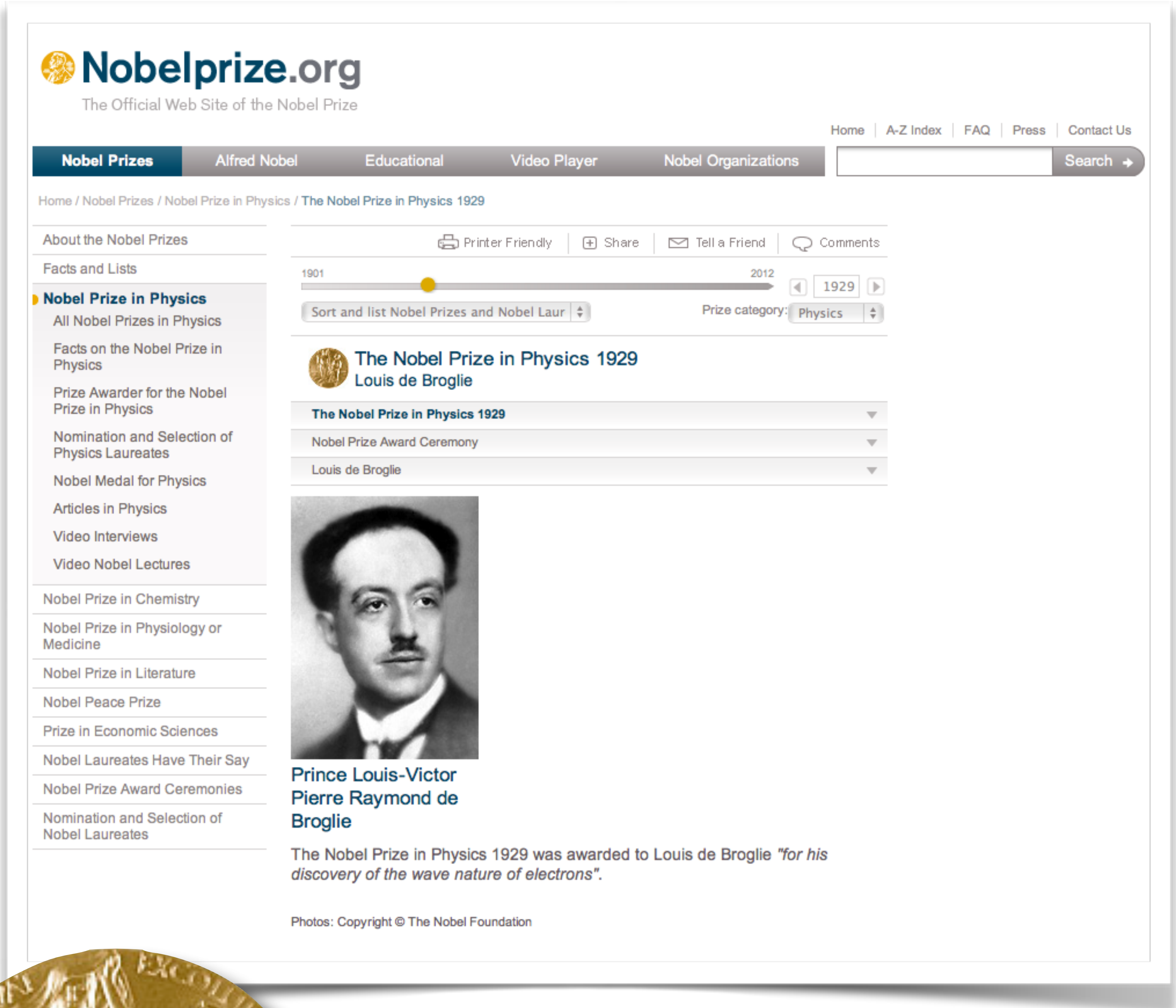

electrons!




photons!

# sole winner

1929




# get real

I weigh 200 lbs & I walk 5 mph

what's my wavelength?

$$p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv} = 3 \times 10^{-36} \text{ m}$$

Smaller than the nucleus...My waviness doesn't show.

**Why is it so small?**

*Two reasons:*

1. My momentum is huge, downstairs
2. Planck's Constant is tiny

# Quantum Mechanics born of some anxiety

the lack of radiation of Bohr's accelerating electrons was still a problem: Bohr knew it and figured there would be a more complete answer.

what in the world is an electron in deBroglie's scheme?

There was much that was ad hoc and not believable

*both in Bohr's approach and deBroglie's*

however, the experimental situation made it clear that the broad suppositions of both had to be a part of the truth.

Quantum Mechanics, proper was the child of 3+1 people:

*Werner Heisenberg - 1925; invention #1*

*Erwin Schrödinger - 1926; invention #2*

*Paul Dirac - 1925; showed #1 and #2 are equivalent*

*Max Born - 1926; gave the modern interpretation*

# the breakthrough

from an unlikely source

Erwin Schrödinger


Erwin Schroedinger 1887-1961

## where do you look for your keys in the dark?

Schroedinger was an expert

in the mathematics of waves

EM waves, material waves, fluids, elastic media, sound...

# the quantum idea:

made use of **integers**

so do *complicated* waves

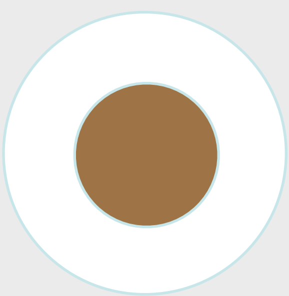
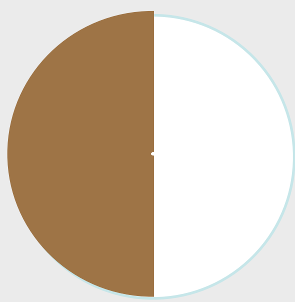
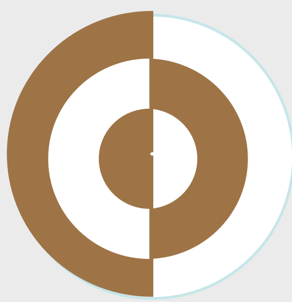
# integers again

$$u(r,t) = \sum_{m=0}^{m=\infty} \sum_{k=0}^{k=\infty} [(A_{mk} \cos \omega_{mk} t + B_{mk} \sin \omega_{mk} t) \cos \theta + (A_{mk} \cos \omega_{mk} t + B_{mk} \sin \omega_{mk} t) \sin \theta] J_m (\frac{\omega_{mk} r}{v})$$

Solutions for the vibrations of a drumhead, or a violin string, or that vibrating hoop...

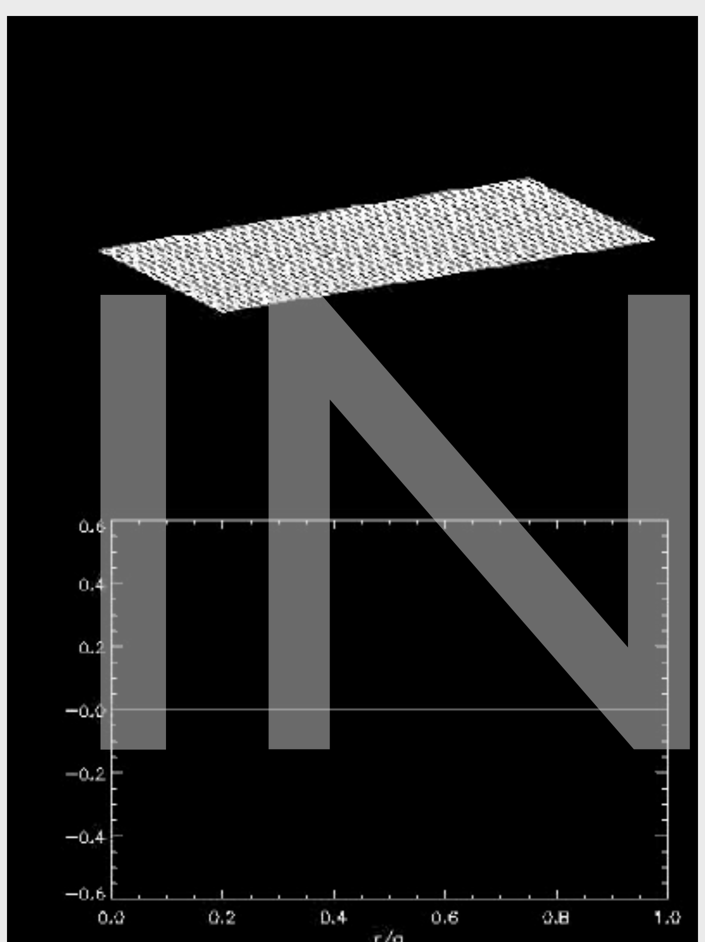
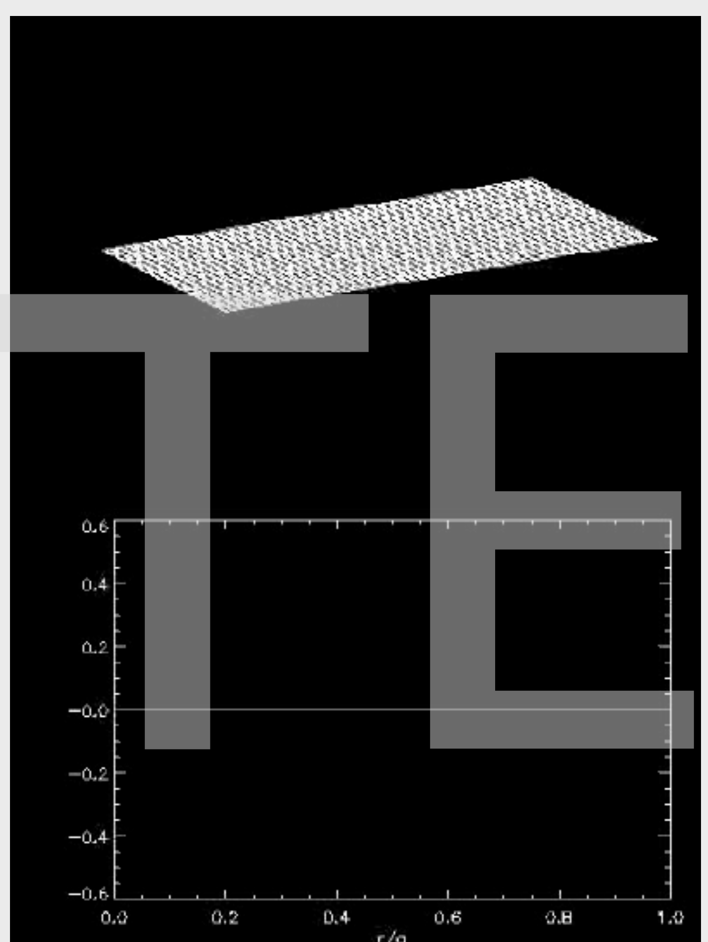
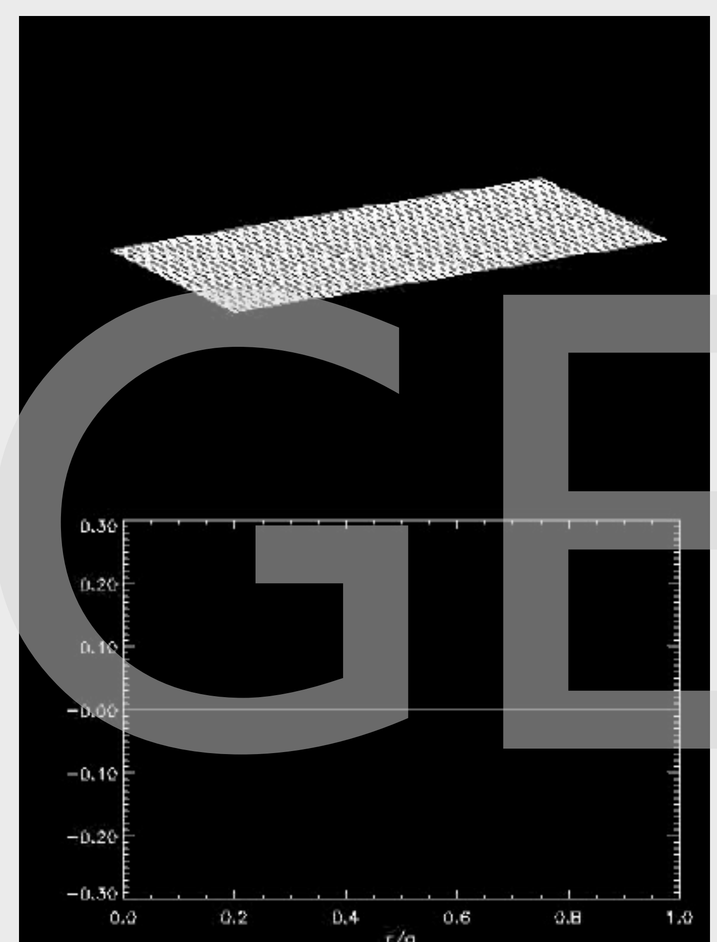
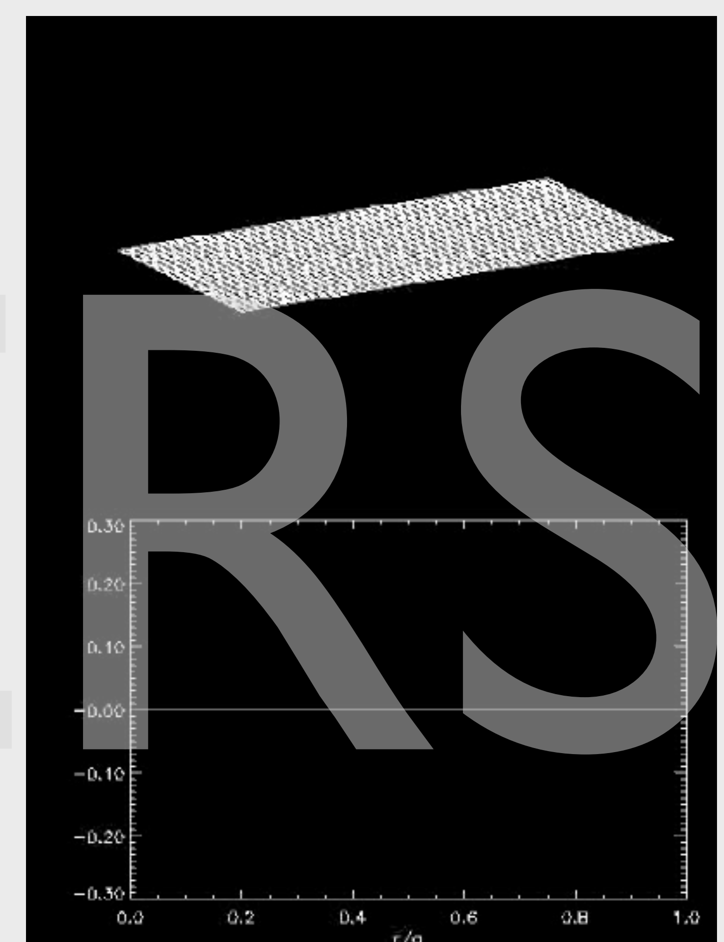
Forget the details...just notice the mixing of lots of waves...the m's and k's? **Integers**.

Here are some of these infinite modes of vibration as described by some of the functions (white and brown are moving in opposite directions (the drum is clamped down at the edges)

these are both m=0 modes

these are both m=1 modes

INTEGERS

I found these nice movies at: http://photon.phys.clemson.edu/brad/courses.dir/movies.dir/phys841-01.dir/movies.html

# terrific

what’s waving???

Schroedinger “solved” a drum-head-like equation for the hydrogen atom

Discrete, vibrational modes...of a something.

## However, he was in for a surprise -

Brave guy: worked in the alps over Christmas 1925 with his girlfriend while his wife stayed in Zurich.

The surprise, is that the mathematics required that the **state** of such a system had to be

imaginary!!

Solutions: the Bohr atom bang-on.
*but with a twist.*

## the ‘‘quantum field’’

“psi”…also called the “wavefunction”

$$\psi(x,t)$$

the “state” of something.

The “Schroedinger Equation”

predicts its behavior in space and time

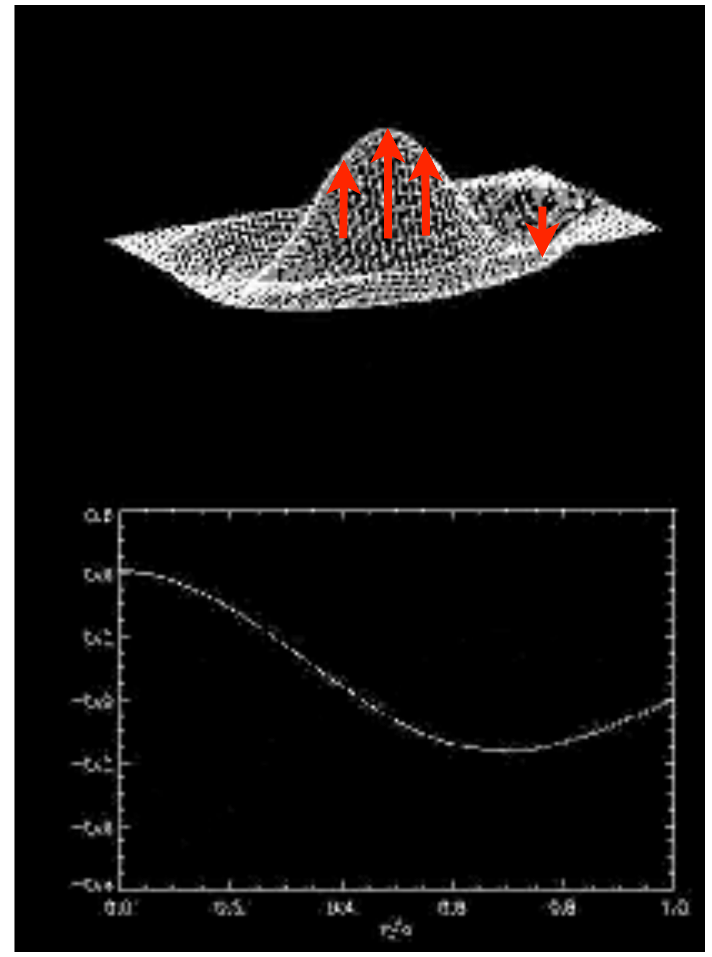
# what is the "state" of a system

a function:

you give me a time and a position in space

I'll give you the "state" of the system

# There can be classical states:

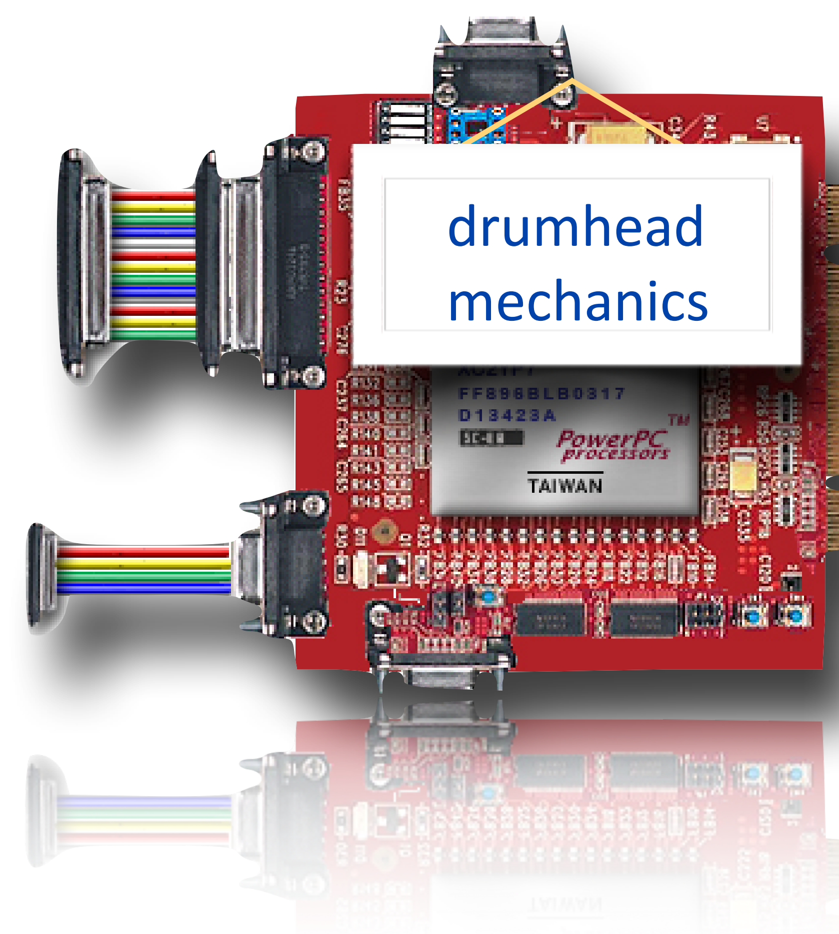


Let's call the "state" of the drumhead, $S$...which is a function of time and space.

The *value* of $S$ is the height above the plane.

forces

initial state at $x_0$, $y_0$, $t_0$




& energy

at any time, all over the surface

# what is the "state" of a system

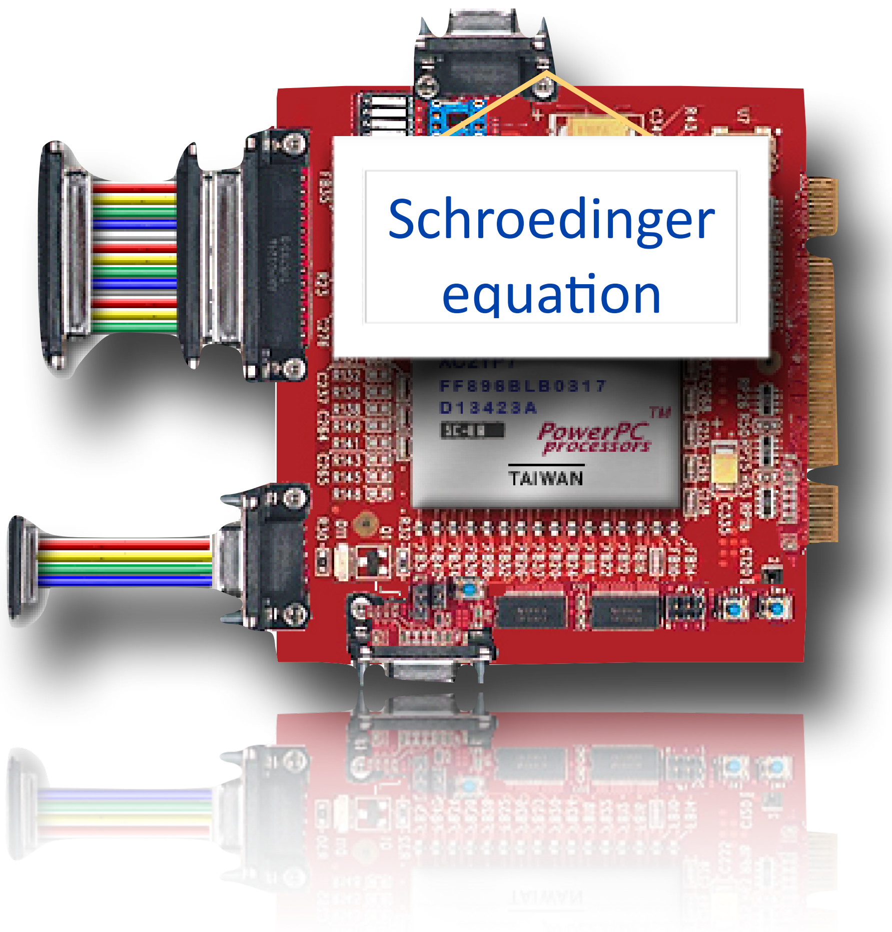
but for quantum systems?

Schroedinger didn't know what it was

but he could solve the equation

forces

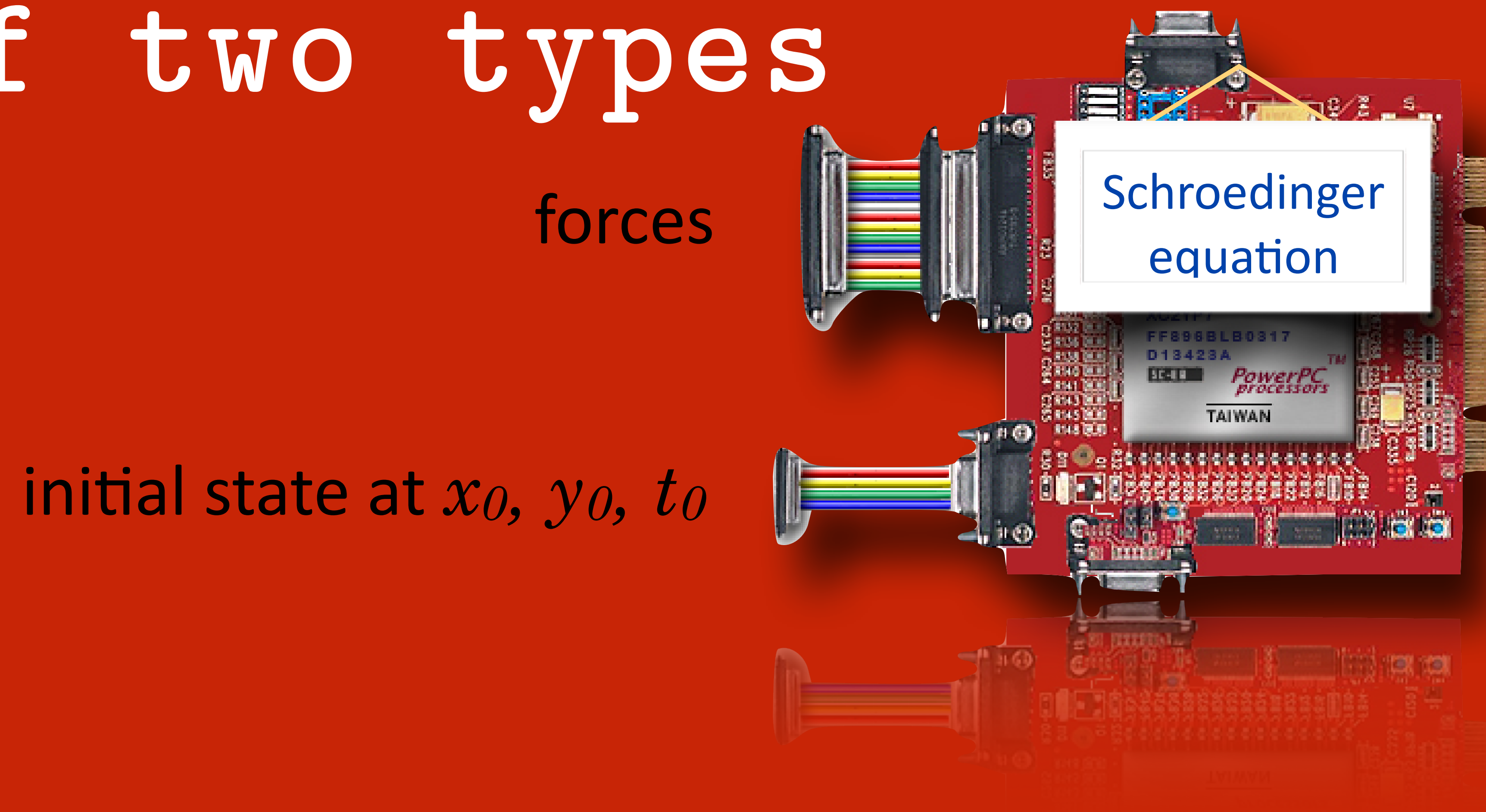
initial state at $x_0, y_0, t_0$




& energy

at any time, all over the volume

# forces can be of two types




attractive, bound forces…

with negative energies - atoms

free forces

with positive energies…often the forces of other particles - particle physics

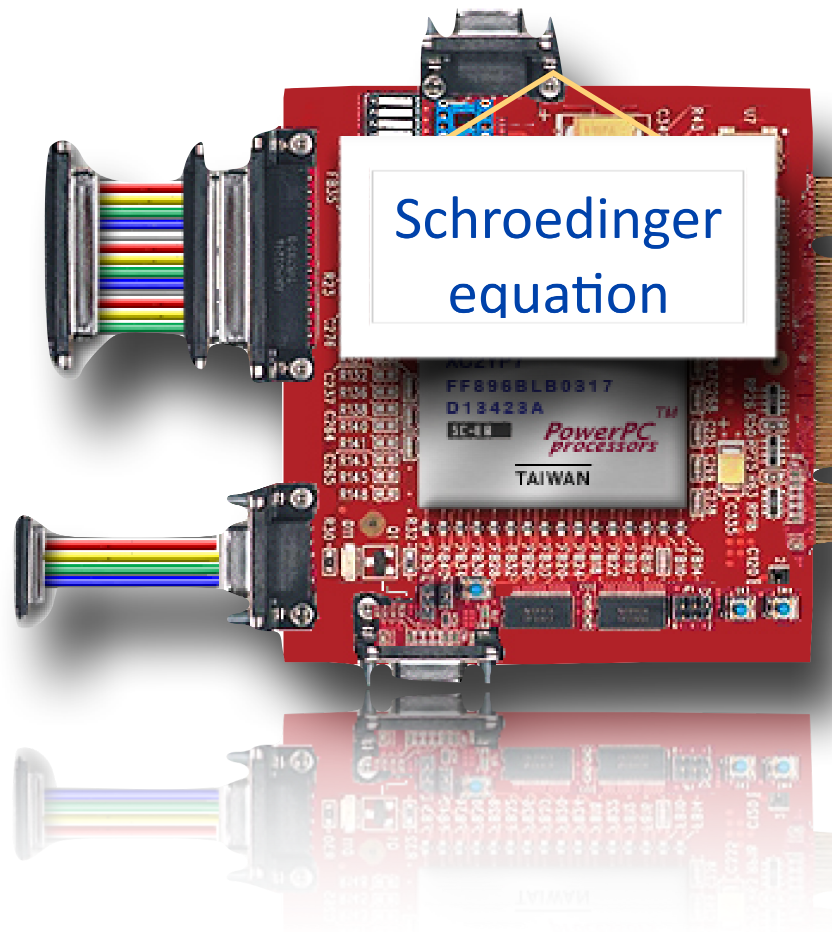
# what is the “state” of a system

for an electron and proton

coupled by the Coulomb’s Force?

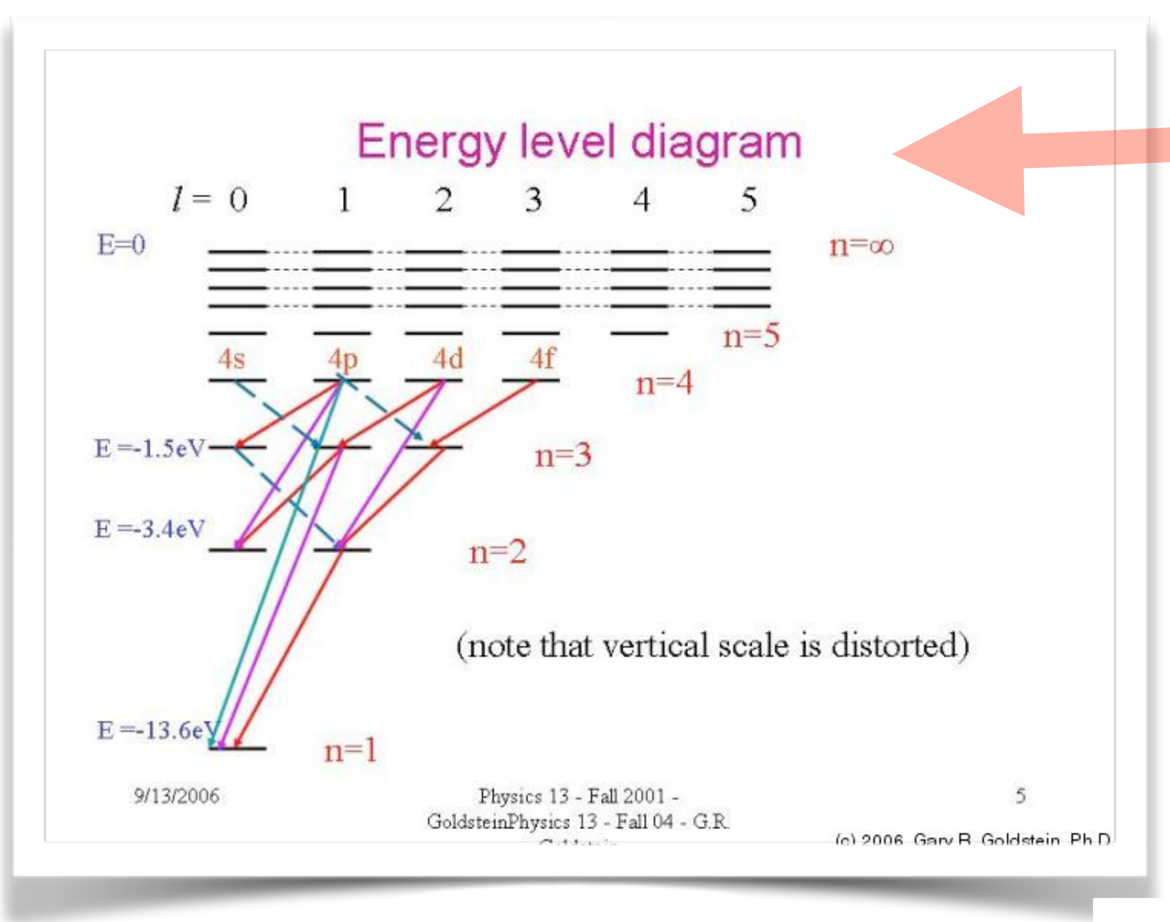
Coulomb’s Law

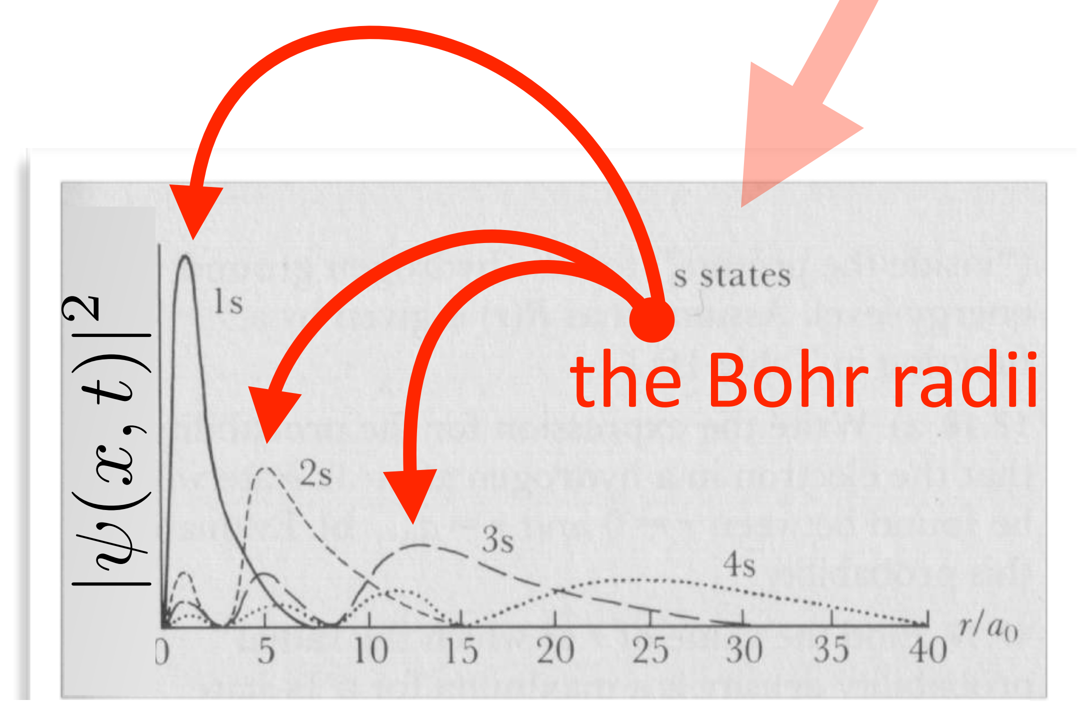
initial state at $x_0, y_0, t_0$




& $E_{n,l}$

at any time, all over the volume




interpreted as the charge distribution

# The prize

with Paul Dirac

about whom I will swoon soon!


# WHAT'S WAVING??

"wavefunctions"

...but they're imaginary!

$$\psi(x,t)$$

# Schroedinger had to work with real quantities

built from the imaginary quantum field function

**With only a half-baked clue of what he was doing.**

Remember what imaginary quantities are?

$$i = \sqrt{-1} \longrightarrow A = a + ib$$

has both real and imaginary parts

Nature... does "Real."

So, Schroedinger created a real number out of $\psi$

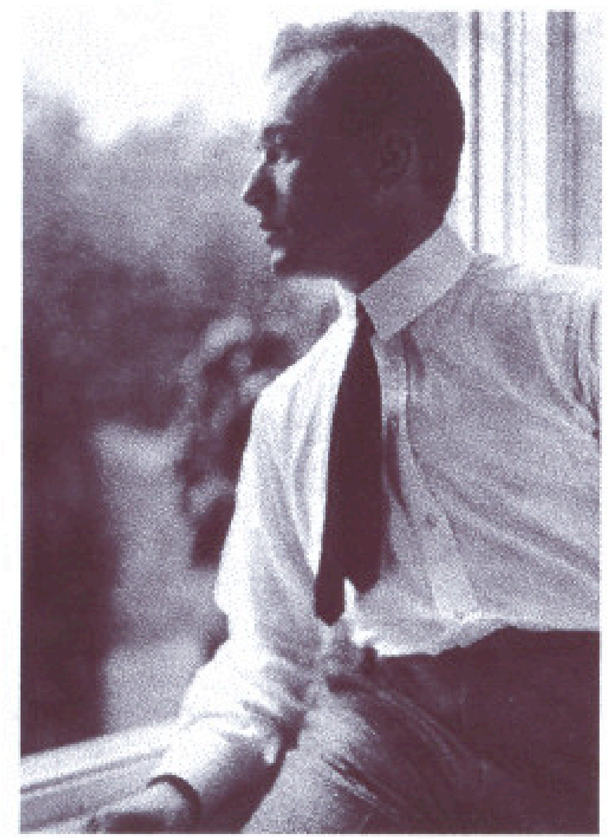
The "complex conjugate" of $A$ is: $A^* = a - ib$

And a real combination of them is the "norm" $|A|^2$

$$AA^* = (a + ib)(a - ib)$$

Schroedinger thought that $|\psi|^2$ might refer to the distribution of electrons' electrical charge.

Sandy: Oh Danny, is this the end?
Danny: No Sandy. It's only the beginning.



"I am now convinced that theoretical physics is actually philosophy."

# probably, it's probability

Where Max Born (Olivia Newton-John's grand-dad) comes in

$|\psi|^2$ *is the **probability of finding the electron***

a measure of the likelihood that an electron will be at a given place at a given time…that's all we can know

*then: no radiation problem…since the electron is not actually orbiting*

We calculate the **shape of its probability density**

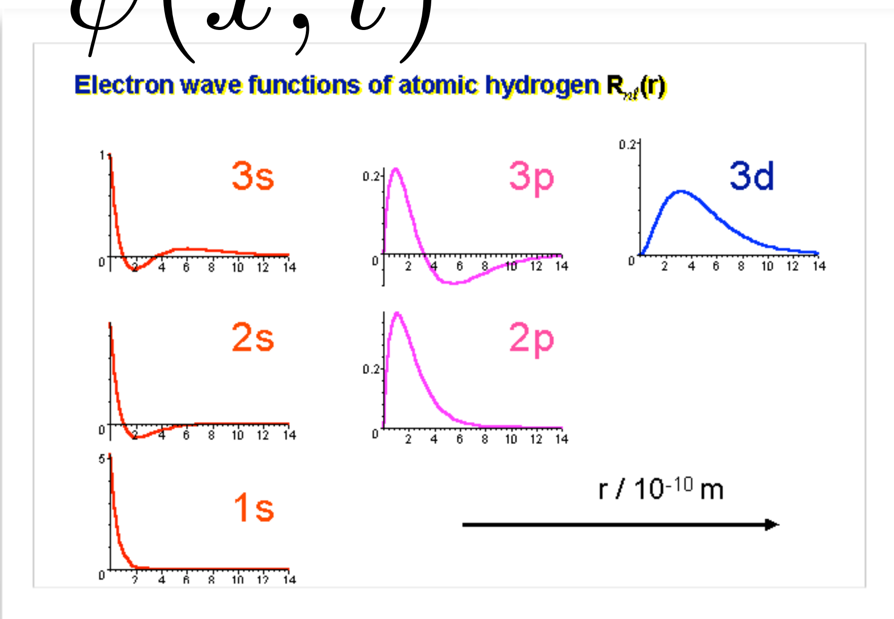
*a **probability***

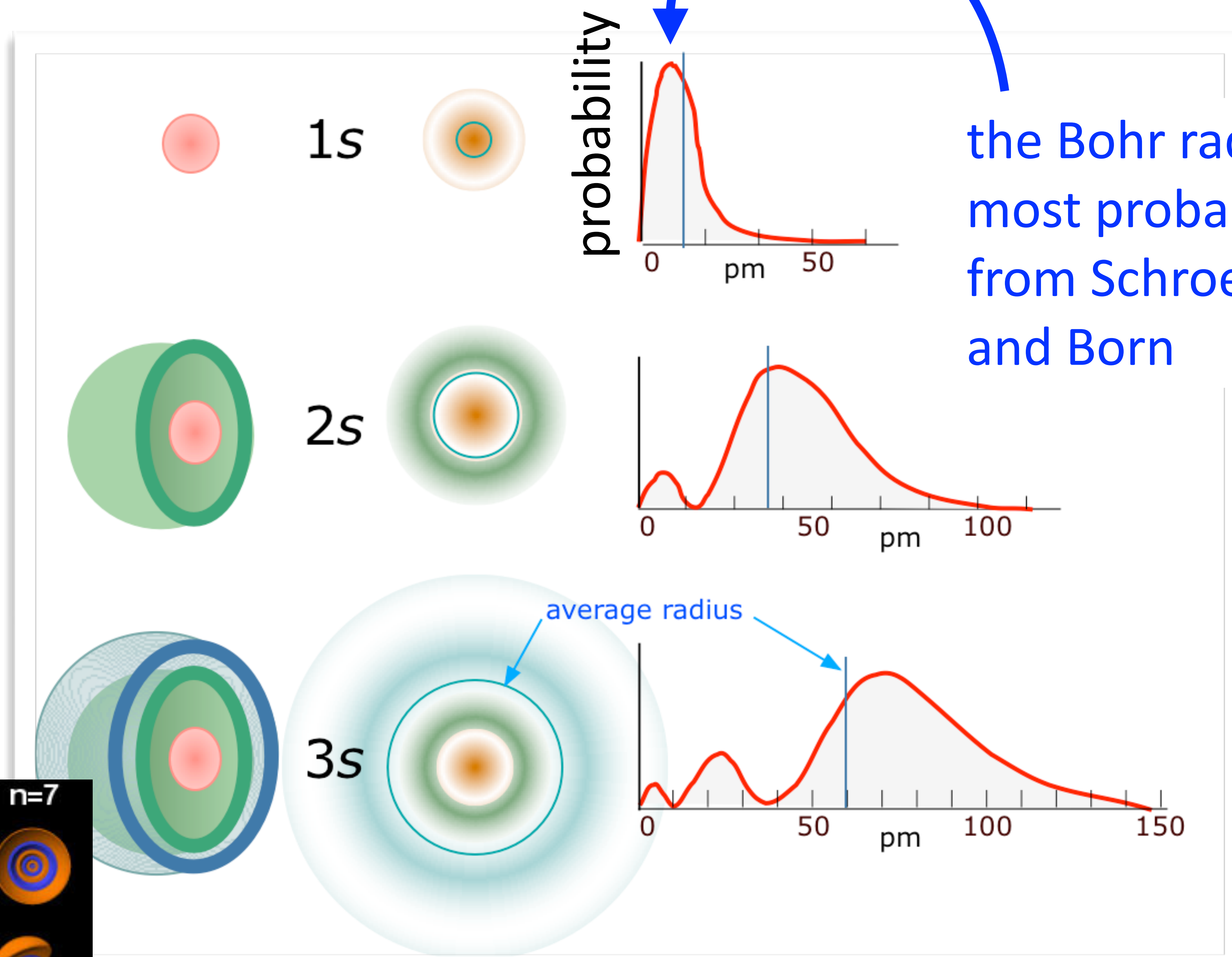
**The concept of normal matter disappears, never to return**

# slice through the probability density of Hydrogen

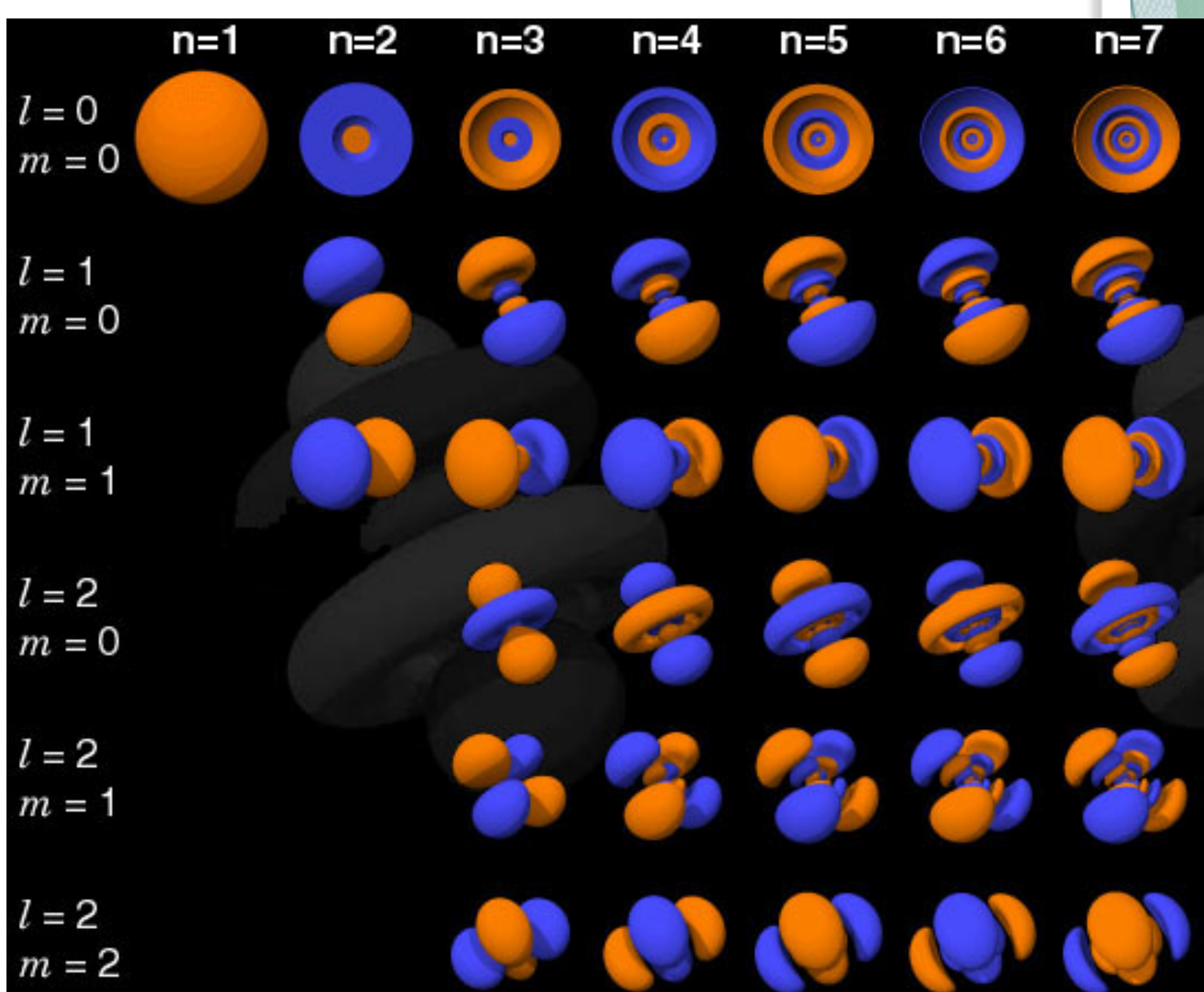
Square these:

$$\psi(x,t)$$



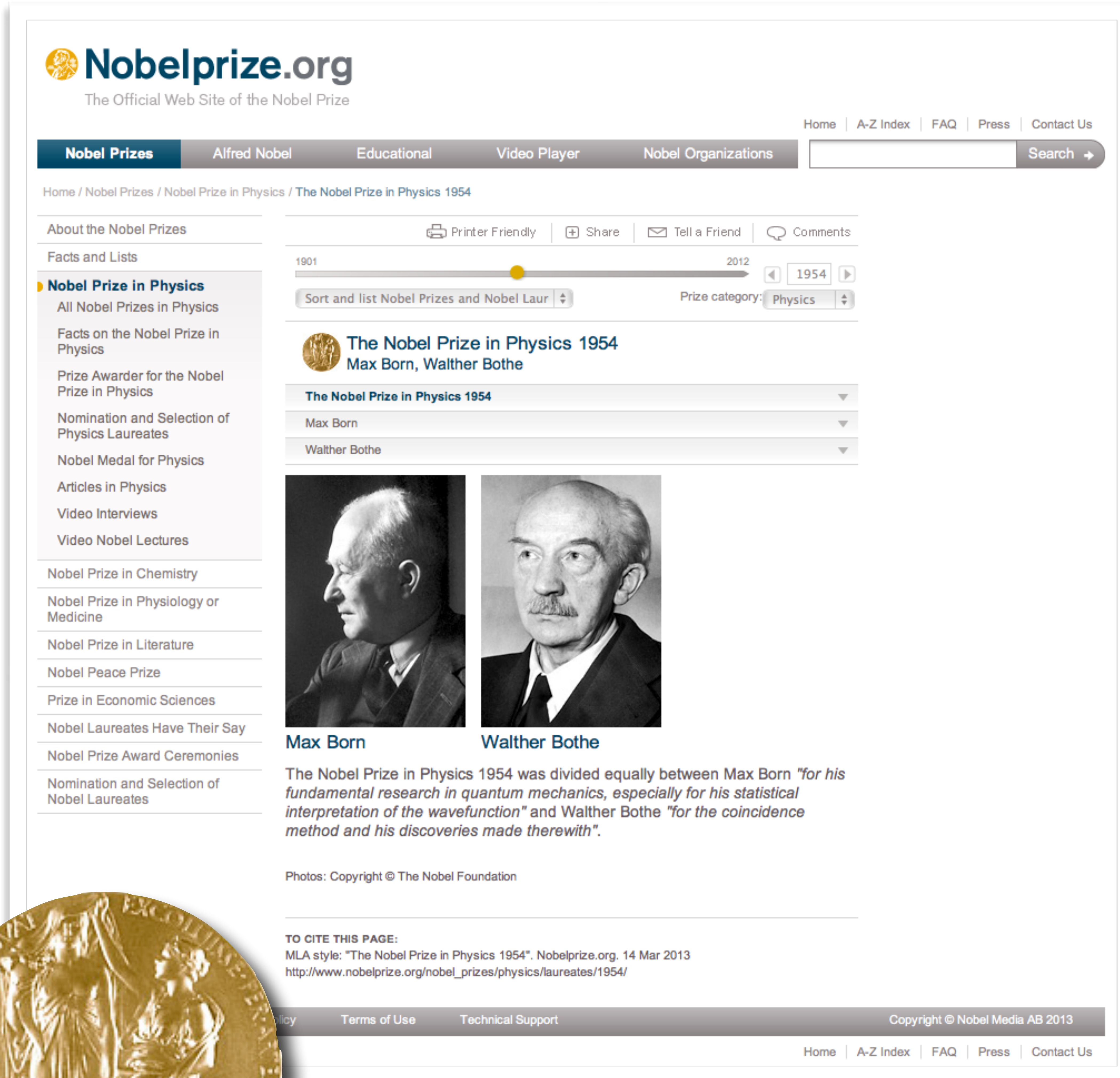




the Bohr radius - the most probable radius from Schroedinger and Born



# finally

in 1954




## **I'm now uncertain.**

This probabilistic interpretation stresses your intuition

intensely pursued by Heisenberg, who in the best Einsteinian tradition, asked a simple question:

what's involved in measuring something…?

**`this relation will be important`**

relating the wavelength of a quantum object

to its momentum

$$\lambda = \frac{h}{p}$$

# it was hard enough

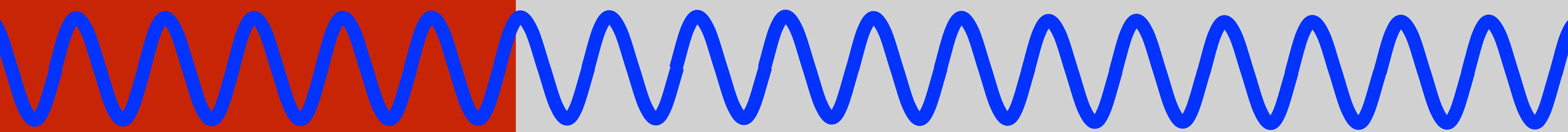
for photons

but for an electron?

A particle is HERE: 

$$p = mv$$

A wave is EVERYWHERE:



The deBroglie hypothesis:

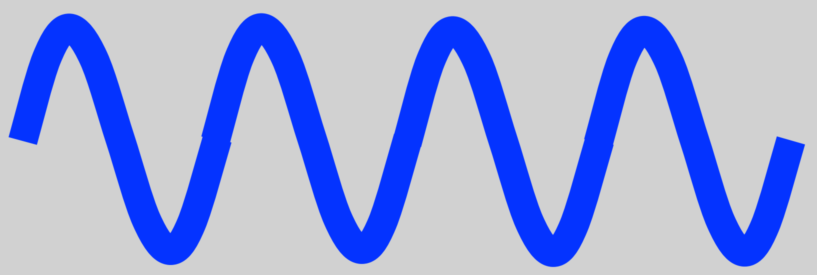
***of given momentum***

<u>also</u> has

***a single wavelength***

$$p = \frac{h}{\lambda}$$

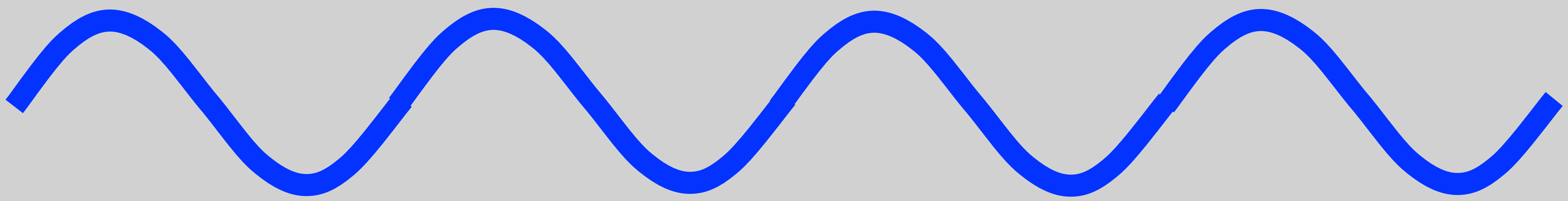
# immediate implications



$$p_1 = \frac{h}{\lambda_1}$$

wavelength and momentum are inversely linked

# immediate implications

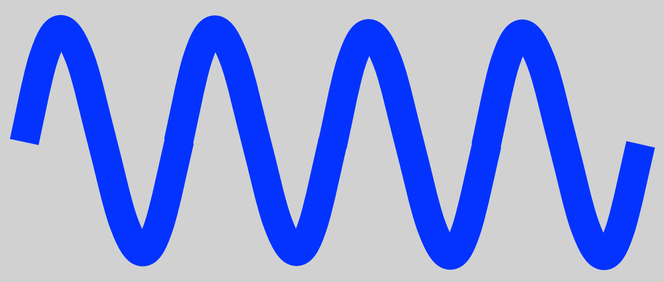


$$p_2 = \frac{h}{\lambda_2}$$

$$p_2 < p_1$$

long wavelength: low momentum

# immediate implications



$$p_3 = \frac{h}{\lambda_3}$$

$$p_3 > p_1$$

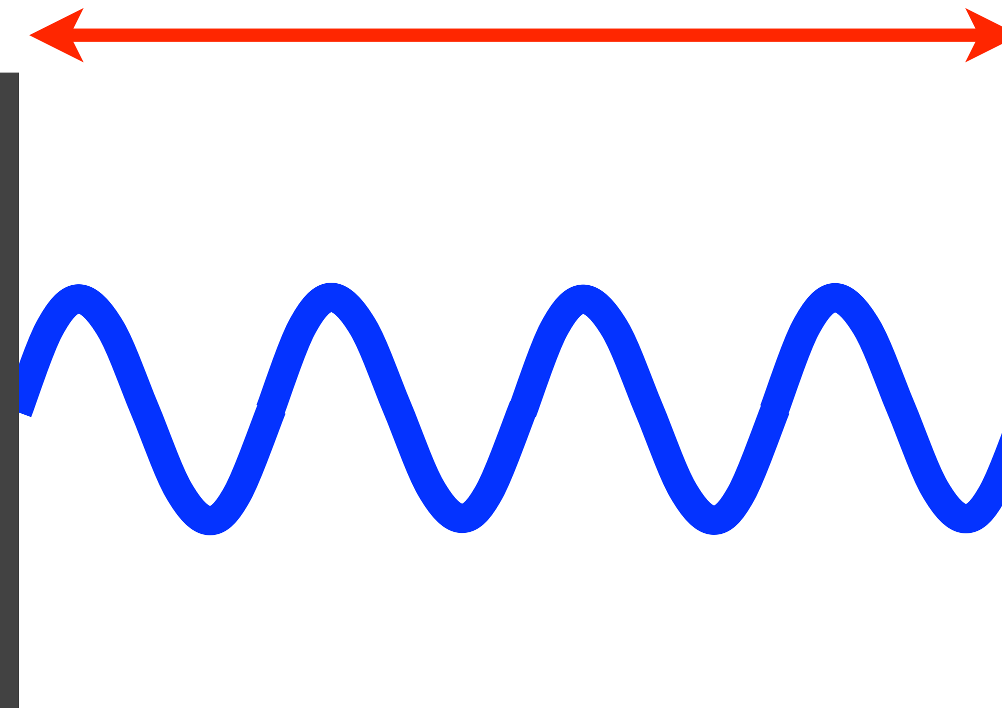
short wavelength: high momentum

# suppose we trap

an electron

Where's the electron?

somewhere here:



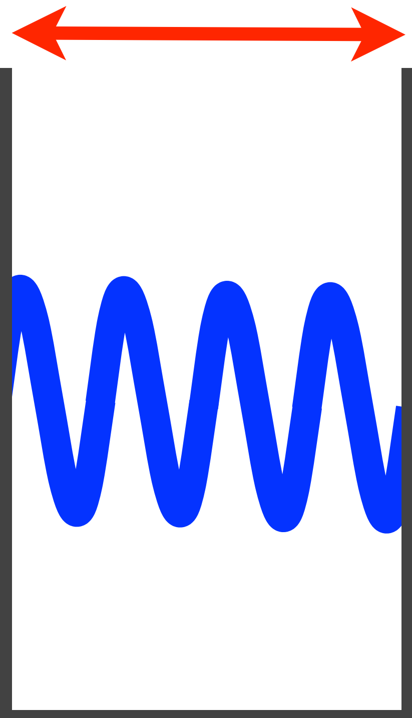
how to locate it better?

# suppose we trap

an electron

Where's the electron?

somewhere here:



make the trap smaller

$$p = \frac{h}{\lambda}$$

The wavelength is shorter...
So the momentum is higher!

## an inevitable trade-off

in order to make the location more precise

you pay the price that its **speed is higher**

# Heisenberg Uncertainty Principle

# the Heisenberg Uncertainty Principle

was from 26 year old Werner Heisenberg

an enigma

inventor of many important concepts

did he save the west from a German nuclear bomb?

or the opposite?


Werner Heisenberg 1901-1976

# measuring something...

you have to "look" at it

by eye or some external, intermediate probe

remember for waves what determines the scale?

**wavelength**

What if the object is atomic sized or smaller? ... what is it to "look"??

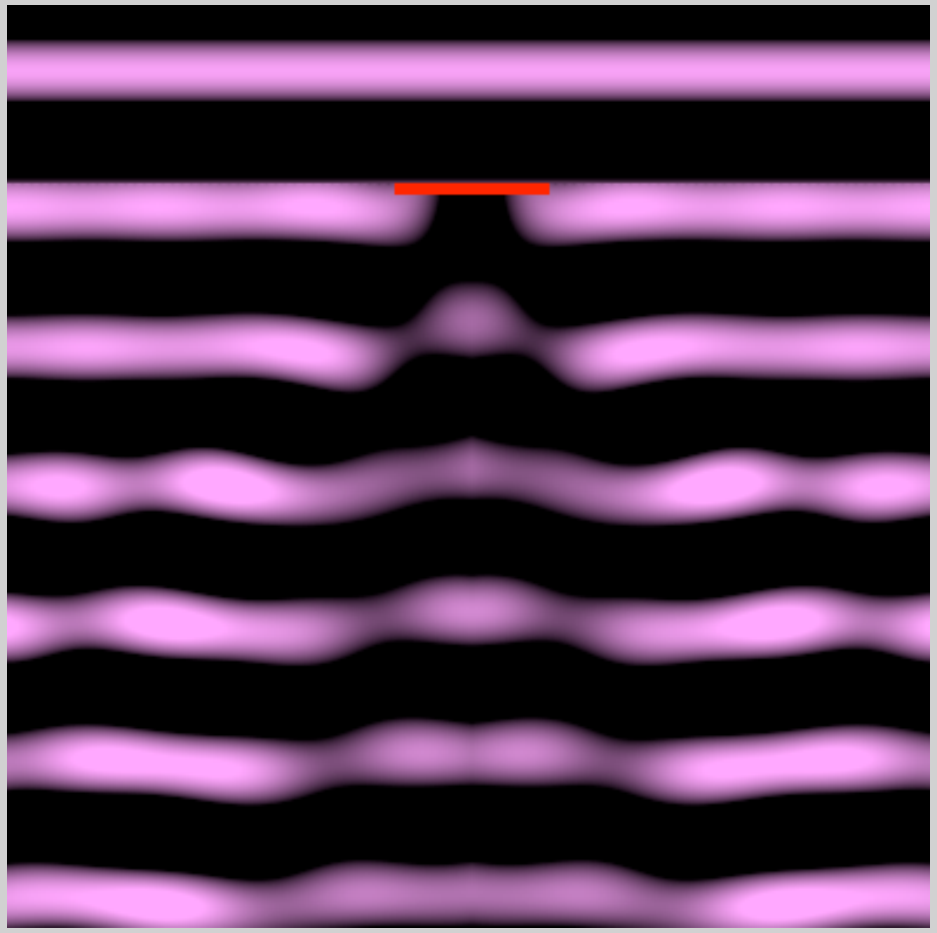
# Heisenberg Uncertainty ... really!

how do you measure the trajectory of an object?

look at it in Time

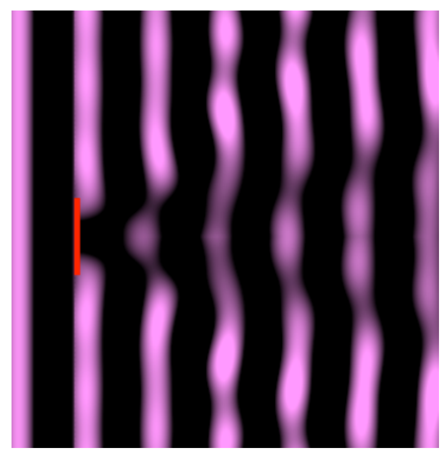
→ bounce light off it

Sweet spot for identifying an object:
need $\lambda$ ~ size of the object



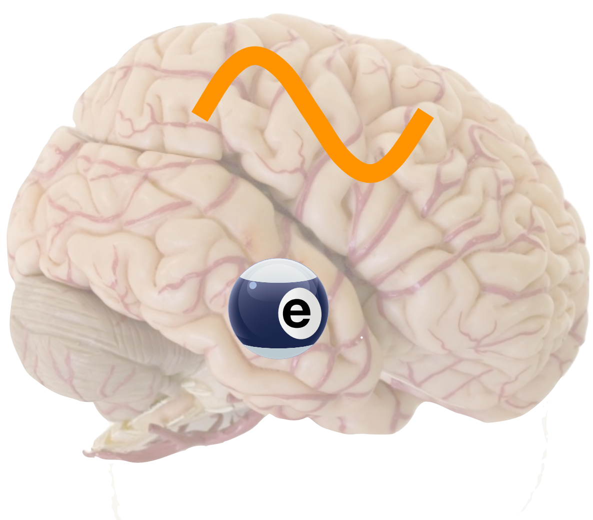
# uncertainty - sometimes called "indeterminancy"

Try to "see" and electron.
Electrons are small.
So...need light wavelength small.



$\} \Delta x \sim \lambda$

So, make $\lambda$ small to reduce $\Delta x$



Gedankenexperiment

Photon diffracts by the electron "barrier" and blurs the electron position by about the amount of the photon wavelength

But, $p = \frac{h}{\lambda}$ makes $p$ large!

$$\Delta p \sim \frac{h}{\Delta x}$$

so now knowledge of the momentum is blurred

$$\Delta p \Delta x \sim h$$

**there is**

NO WAY to beat it in any of these measurement scenarios

the inverse relation between p and λ messes with you every time

$$p = \frac{h}{\lambda}$$

## but here's the hard part

what does it mean?

the inability to determine position or momentum to arbitrary precision

is not about poor instruments

It. Is. About. Nature.

relation alert:

# Heisenberg Uncertainty Relation

refers to: $\Delta x \Delta p \geq h$ & $\Delta t \Delta E \geq h$

an inherent property of Nature

example: objects to not possess precise position and precise velocity at the same time.

# 1932 Nobel

31 years old

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## The Nobel Prize in Physics 1932
Werner Heisenberg

The Nobel Prize in Physics 1932

Werner Heisenberg

Werner Karl Heisenberg

The Nobel Prize in Physics 1932 was awarded to Werner Heisenberg *"for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"*.

Werner Heisenberg received his Nobel Prize one year later, in 1933. During the selection process in 1932, the Nobel Committee for Physics decided that none of the year's nominations met the criteria as outlined in the will of Alfred Nobel. According to the Nobel Foundation's statutes, the Nobel Prize can in such a case be reserved until the following year, and this statute was then applied. Werner Heisenberg therefore received his Nobel Prize for 1932 one year later, in 1933.

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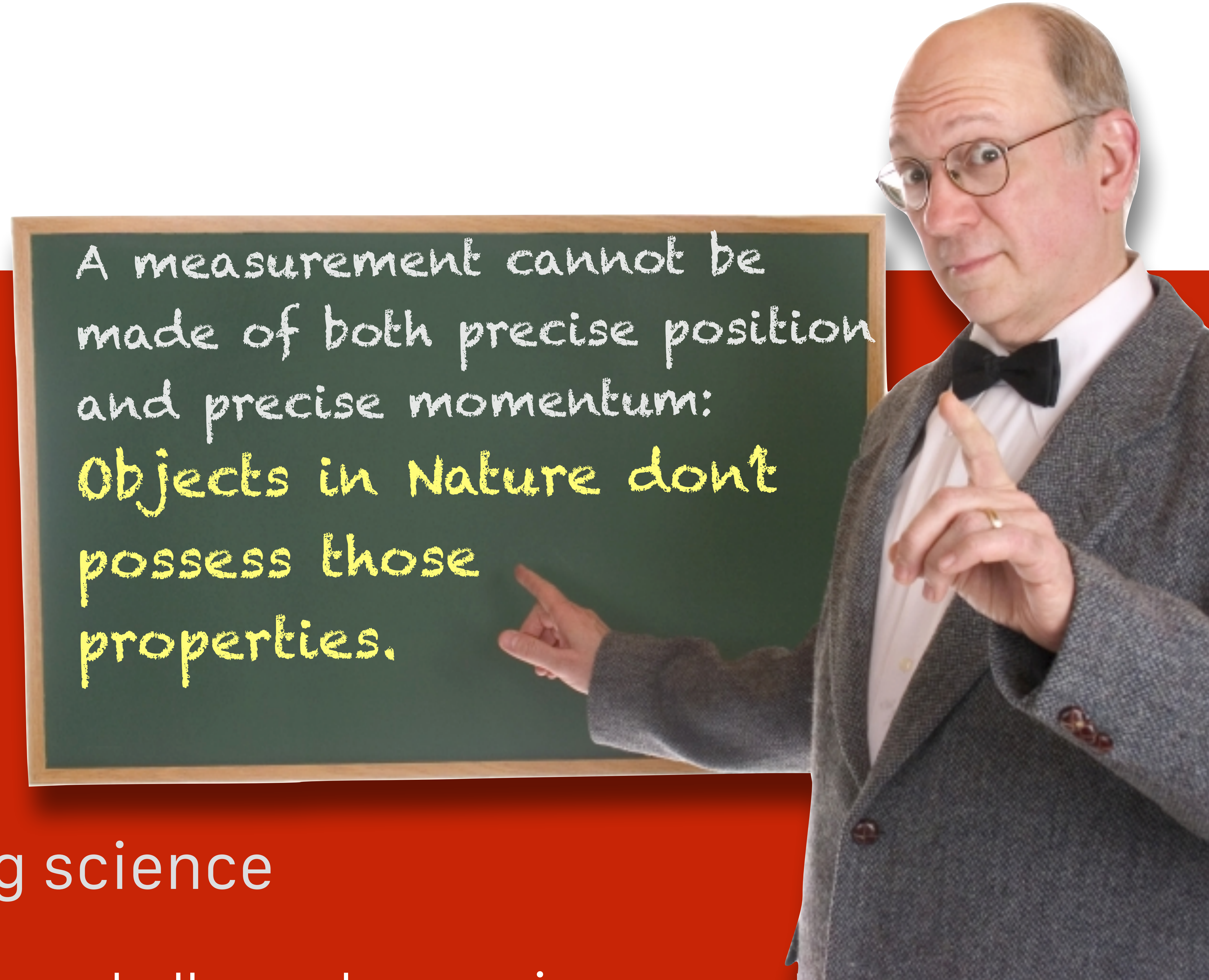



## a new way

Of thinking and doing science

we lose another classical, unchallenged scenario

**there is no such thing as a precise trajectory**

and a measurement is not isolated from the thing being measured

which is where new-age-y analyses of physics go off the rails

## get real

I got pulled over for doing 105 mph*

The state police use radar

~20 GHz, $\lambda \sim$ 14cm

How uncertain was my position?

$$\Delta p \Delta x \sim h$$

about $6 \times 10^{-33}$ m

* it was a different black Bimmer that had passed me a while back.

## instead of midlife-crisis sports cars

how about:

a proton at 0.9c

what's its position uncertainty?

$$\Delta p \Delta x \sim h$$

$$\Delta x = \frac{h}{p} \sim \frac{h}{m\gamma v} \sim 10^{-15} \text{ m}$$

about 1/3 the size of a nucleus

## the whole story

for technical reasons, we use: $$\Delta x \Delta p \geq \frac{h}{4\pi}$$

plus

the other form: $$\Delta t \Delta E \geq \frac{h}{4\pi}$$

# one more...

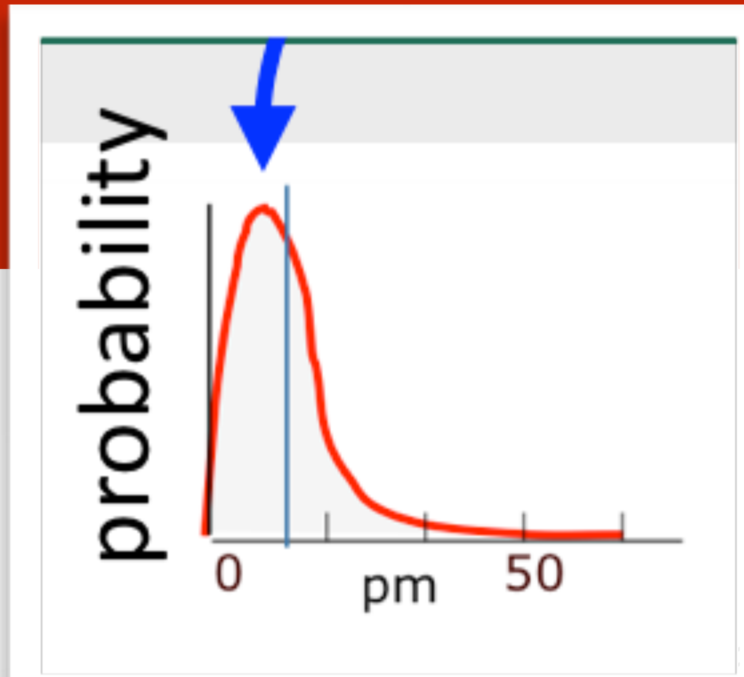
from the Bohr model, the speed of the electron is

~ 2 x $10^6$ m/s – let's use non-relativistic momentum:

for $\Delta p$ for an electron: $\Delta x \Delta p \geq \frac{h}{4\pi}$ $\quad \Delta x \sim \frac{h}{4\pi \Delta p} \sim 3 \times 10^{-11} m$

just about the Bohr radius!

**So, the size of the atom is consistent with the electron being smeared all over the "fixed" Bohr radius.**

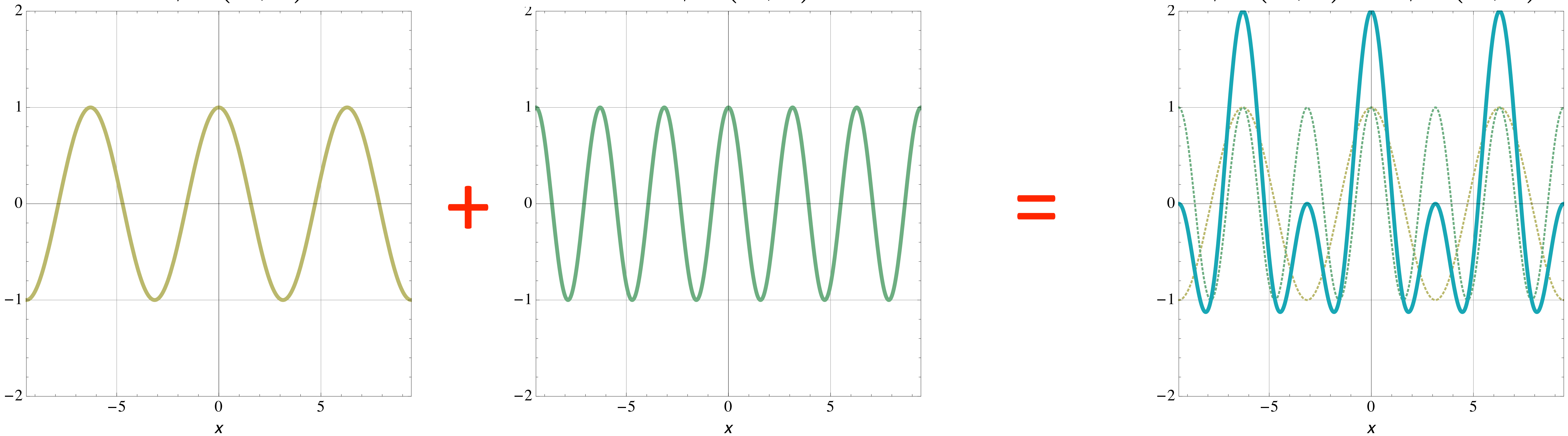


# I’m dancing around a tough question

But, if particles are waves and if waves are “everywhere”

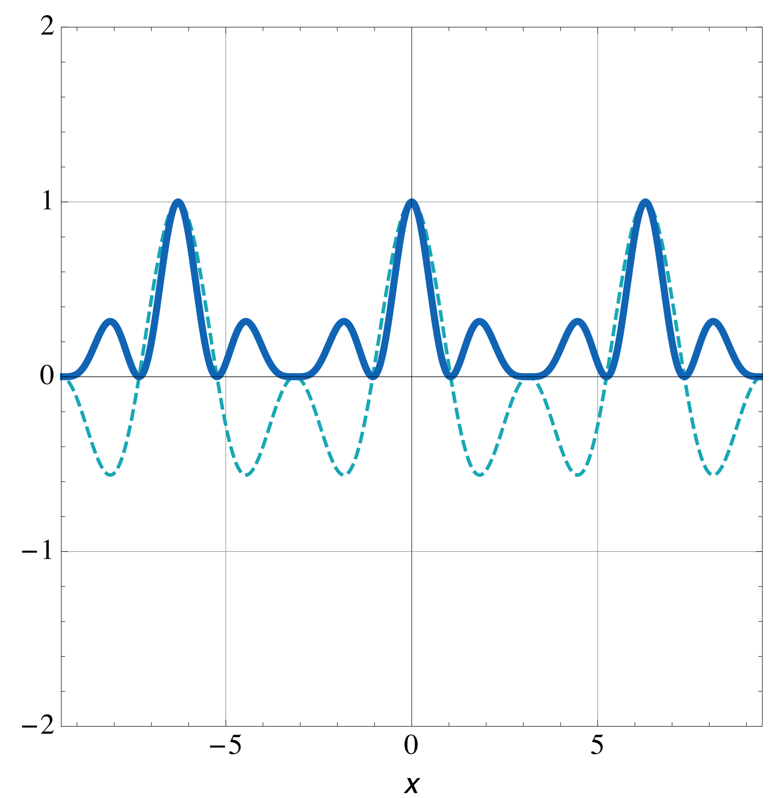
...what’s the “particle” in Particle Physics?

$\psi_1(x,t)$ + $\psi_2(x,t)$ = $\psi_1(x,t) + \psi_2(x,t)$



But, remember that what's real about the quantum fields is the square: $|\psi(x,t)|^2$

$|\psi_1(x,t) + \psi_2(x,t)|^2$



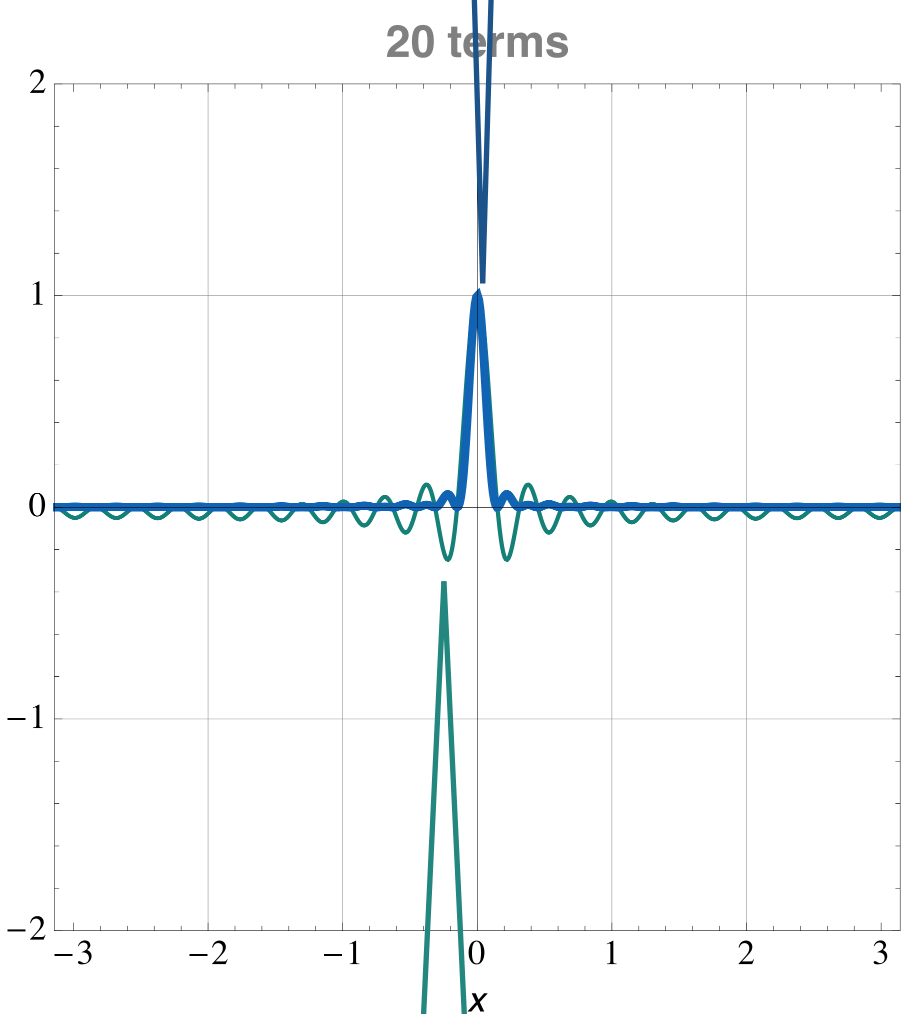
notice the peaking

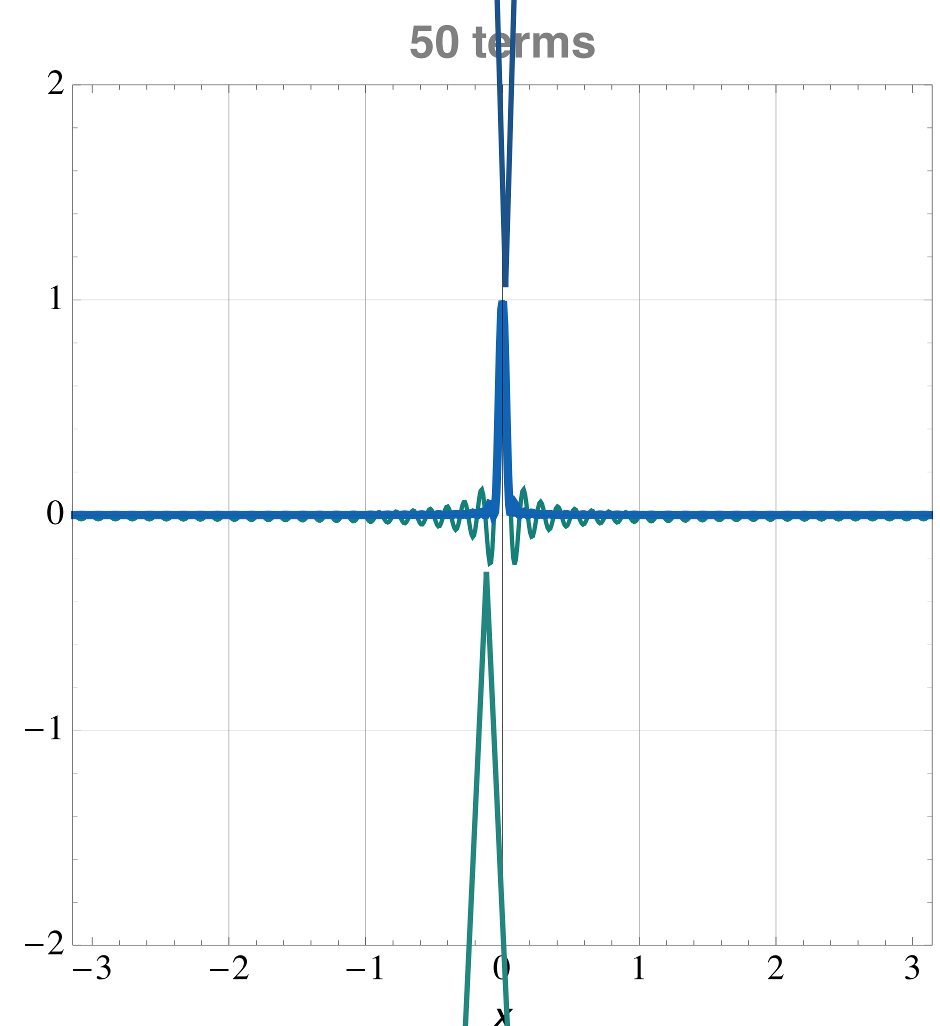
(I've changed the heights)

# add quantum field functions - more terms

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)|^2$$

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)|^2$$




$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)$$

$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)$$

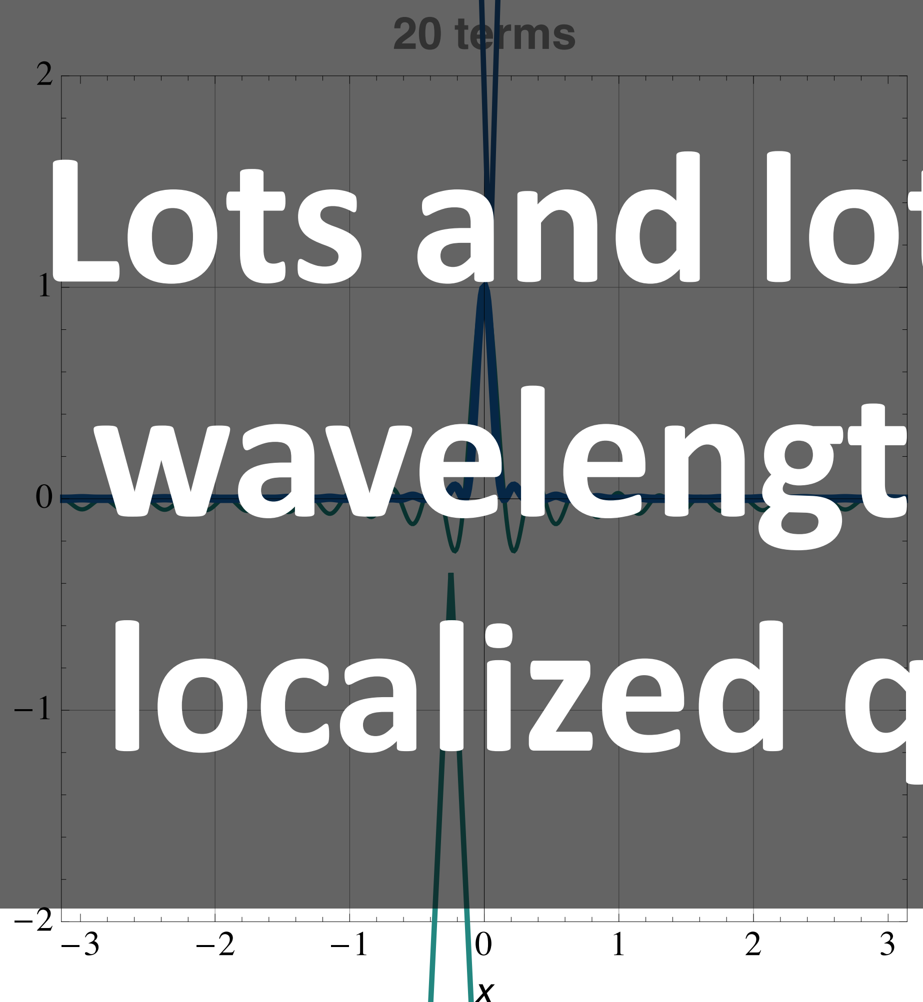
peaking is even more pronounced

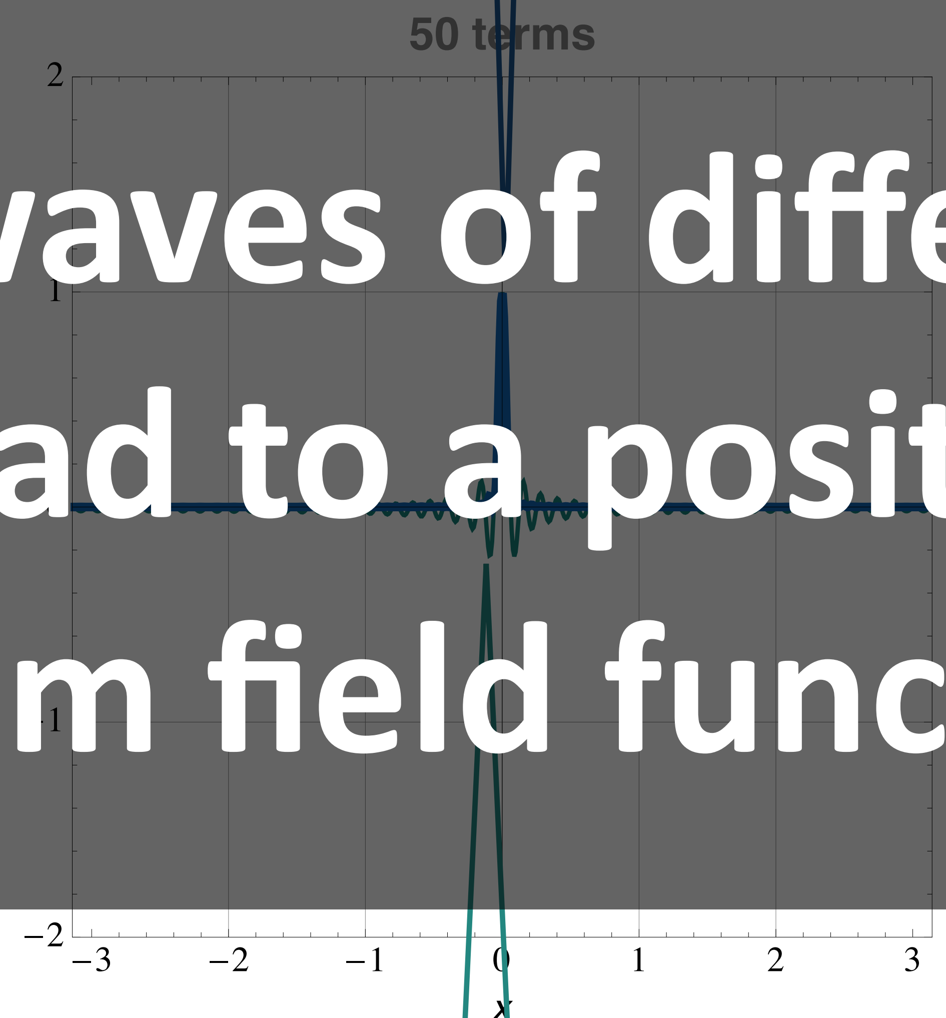
(I've changed the heights)

# add quantum field functions - more terms

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)|^2$$

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)|^2$$




**Lots and lots of waves of different wavelengths...lead to a position-localized quantum field function**

$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)$$

$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)$$

peaking is even more pronounced

(I've changed the heights)

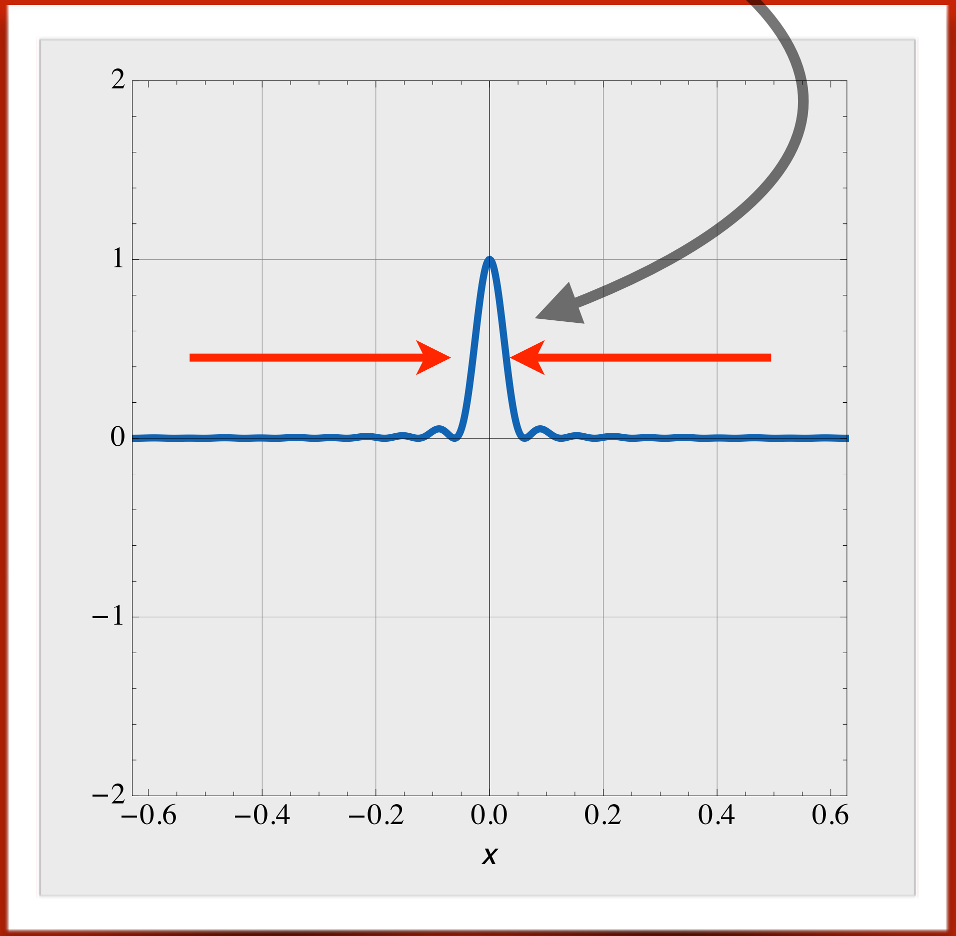
a classical particle (dot) and its wavefunction

the wave combinations localize the state...with some spread in $x$

**waves of different wavelengths?**

$$p = \frac{h}{\lambda}$$

different momenta




Heisenberg Uncertainty Relation at work again

called "wavepackets"

all of the wave combinations means all of the momenta contribute: an spread in $p$.

**the larger the momentum spread**

the smaller the localization

“particles” are more particle-like at large momentum the larger the momentum spread

the smaller the localization

“particles” are more particle-like at large momentum

# THAT’S WHY WE CALL IT “PARTICLE PHYSICS” and not “wave physics”