

hi

Lecture 25, 04.11.2017

Quantum Mechanics 5

housekeeping



Question about anything?

I'll make a movie for you:

Poster selection:

April 13, outline due April 20...read the instructions.

Homework:

For month of April, I've shifted due dates to Saturdays.

Readings:

*There is a manuscript chapter on antimatter and Dirac...Antimatter,
Paul Dirac's Second Big Score*

Honors Project

Data due April 22. Paper due on May 4 (final day).

the dropbox instructions? Forget them. We'll be uploading files to a site in Norway. I'll let you know.

Read the Second of two sets of instructions:

MinervaInstructions2_2017.pdf in

www.pa.msu.edu/~brock/file_sharing/QSandBB/

Quantum Mechanics, so far:

Light has both wave and particle-like properties

Bohr Model:

- electrons are in atomic orbits

- fixed in radius and energy

- electrons make transitions - spectra

Electrons have both wave and particle-like properties

- for both light and electrons, $p = \frac{h}{\lambda}$

- standing wave patterns at Bohr radii worked

Electrons are represented by imaginary wavefunctions, ψ

- the square of the wavefunctions represent the probability of finding an electron at a point at a time

Heisenberg Uncertainty Principle:

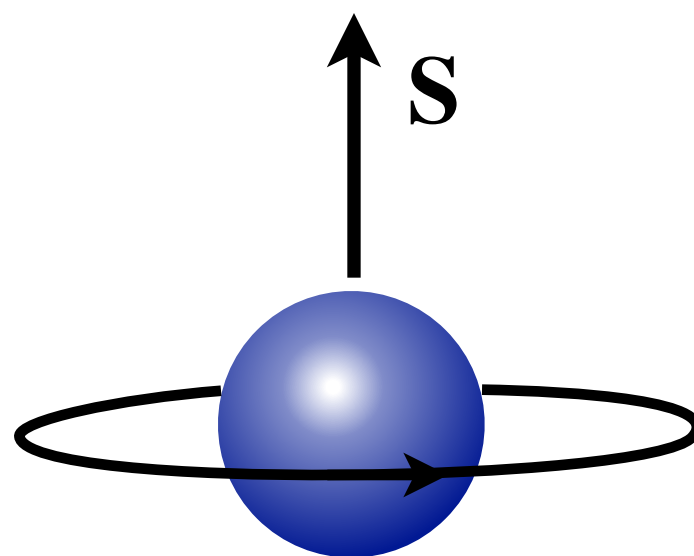
- measuring a precision location of a quantum makes momentum imprecise

- measuring a precision time interval of a quantum makes energy imprecise

electrons are little magnets

They behave in a
magnetic field as
if they are little
spinning current
spheres

The electron **itself** is *like* a spinning charge...



Electrons have an **intrinsic** angular momentum, "S": "spin"

$$S_z = m_s \frac{h}{2\pi}$$

But, the "spin" can only take on two values:

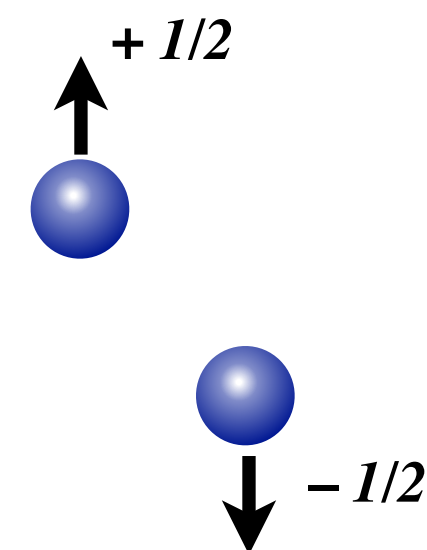
$$m_s = +\frac{1}{2} \quad \text{or} \quad m_s = -\frac{1}{2}$$

We say

"spin, plus 1/2" or "spin up"

and

"spin, minus 1/2" or "spin down"



Pauli Exclusion Principle:

No two electrons can be in the same quantum state

that is, have identical "quantum numbers"

...integers that characterize the atom

Relativistic Quantum Mechanics, so far:

Making Quantum mechanics relativistic was problematic:

negative energies were inevitable $E_T = \pm\sqrt{(m_0c^2)^2 + (pc)^2}$

negative probabilities seemed inevitable

Paul Dirac creatively changed Schrodinger's approach:

Schrodinger Equation: one wavefunction...the electron, non relativistic

Dirac Equation: 4 wavefunctions, completely relativistic

two wavefunctions with positive energies, one spin up and one spin down

two wavefunctions with negative energies, one spin up and one spin down

"spin" came naturally out of Dirac's equation...a purely relativistic property with no classical analogue

Vacuum and antimatter

Dirac embraced the negative energies and modeled the vacuum as full of negative energy electrons

But he envisioned a multi-particle model in which a negative energy electron can be promoted to a positive energy electron and a negative energy, "anti-electron"

The process was: photon \rightarrow (-e, +E electron) + (+e, -E electron) = electron + anti-electron

Positron

The anti-electron, aka Positron, was discovered soon after in cosmic rays

jargon alert:

fermion

refers to:

any particle with half-integer spin

entomology:

from Fermi's theoretical work on the behavior of large numbers of Fermions

example:

electron, proton, neutron

jargon alert:

boson

refers to:

any quantum object with integer spin

etymology:

from Satyendra Nath Bose, who worked on the effects of multiple boson aggregates

example:

photon, pion, Higgs Boson

spin is a defining quality of an electron

electron

symbol:

e

charge:

$-1e$

mass:

$m_e = 9.0 \times 10^{-31} \text{ kg} \sim 0.0005 \text{ p}$

spin:

$1/2$

category:

fermion, lepton

the bar over the top will mean
“antiparticle”

anti-electron, aka “positron”

symbol:

\bar{e} or e^+

charge:

$+1e$

mass:

$m_e = 9.0 \times 10^{-31} \text{ kg} \sim 0.0005 \text{ p}$

spin:

$1/2$

category:

anti-fermion, anti-lepton

particle:

proton

symbol:

p

charge:

$+1e$

mass:

$m_p = 1.6726 \times 10^{-27} \text{ kg} = 1 \text{ p}$

spin:

$1/2$

category:

fermion, hadron

again, an inherent angular momentum and a defining property of photons

particle:

photon

symbol:

γ

charge:

0

mass:

$m_{\gamma} = 0$

spin:

1

category:

boson, aka Intermediate Vector Boson

Dirac's result

required: 4 quantum fields, rather than 1 $\psi_{up}(E, \vec{p}) \psi_{down}(+E)$

2 have positive energy, 2 have negative energy $\psi_{up}(E, \vec{p}) \psi_{down}(-E)$

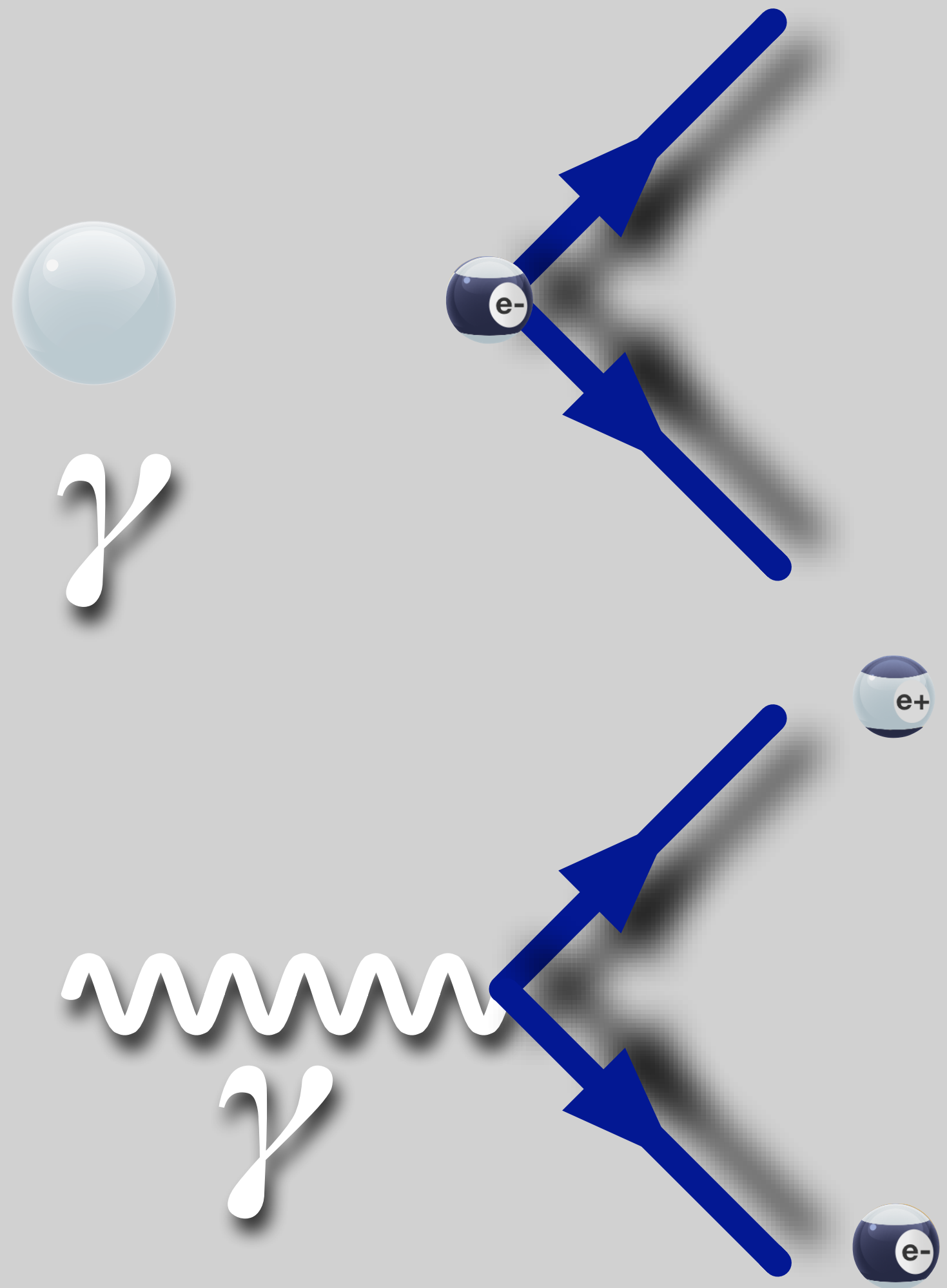
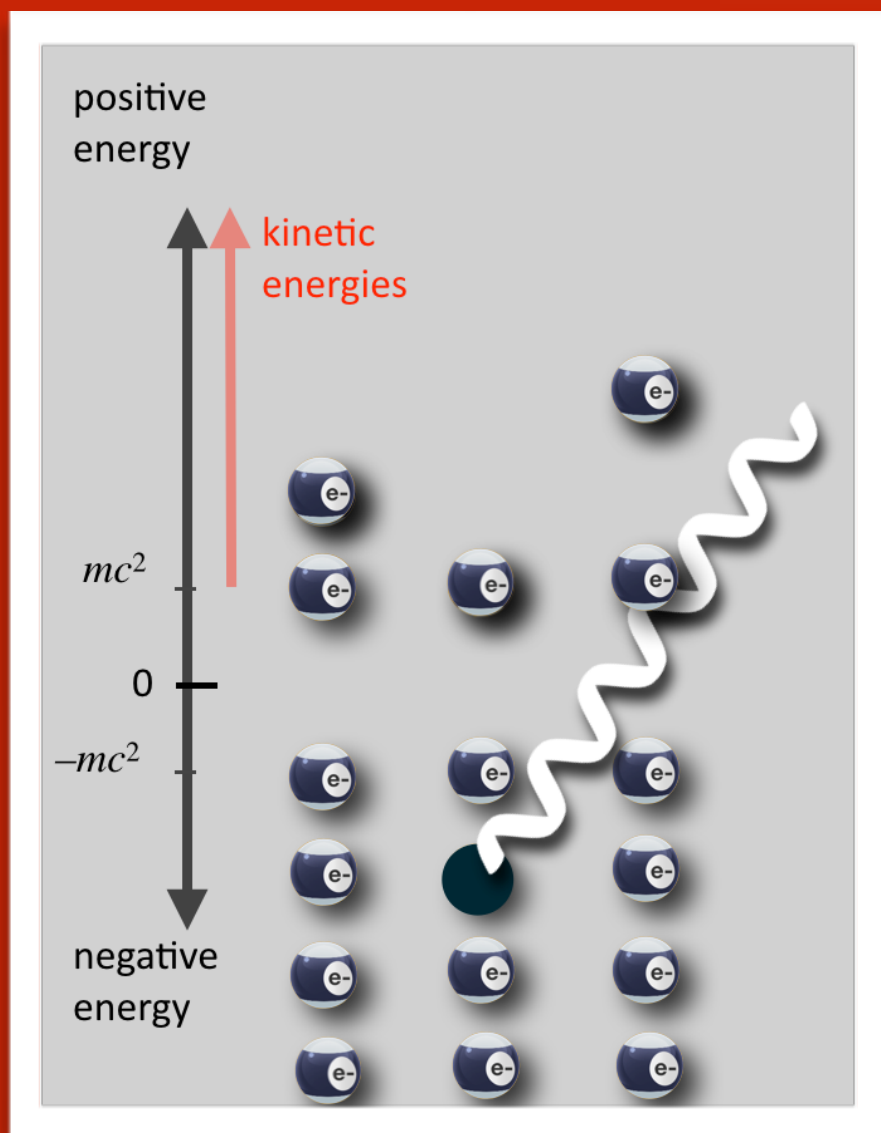
each pair is related precisely to spin

**Dirac showed that spin is a wholly relativistic effect
...it just popped out of his equation.**

modern
interpretat



a photon
poof-disappears



antimatter

is a fact of life

every particle has it's anti-particle partner

same mass, different electrical charge

Dirac Nobel

at the age of 31



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1901 2012 1933
Sort and list Nobel Prizes and Nobel Laur Prize category: Physics

The Nobel Prize in Physics 1933
Erwin Schrödinger, Paul A.M. Dirac

The Nobel Prize in Physics 1933
Erwin Schrödinger
Paul A.M. Dirac

 
Erwin Schrödinger **Paul Adrien Maurice Dirac**

The Nobel Prize in Physics 1933 was awarded jointly to Erwin Schrödinger and Paul Adrien Maurice Dirac *"for the discovery of new productive forms of atomic theory"*

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http://www.nobelprize.org/nobel_prizes/physics/laureates/1933/

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Carl Anderson and Victor Hess

Anderson was 31

The screenshot shows the Nobelprize.org website. At the top, the logo and tagline "The Official Web Site of the Nobel Prize" are visible. A navigation menu includes "Nobel Prizes", "Alfred Nobel", "Educational", "Video Player", and "Nobel Organizations". The breadcrumb trail reads "Home / Nobel Prizes / Nobel Prize in Physics / The Nobel Prize in Physics 1936".

On the left, a sidebar menu lists various categories, with "Nobel Prize in Physics" selected. Below it, there are links for "All Nobel Prizes in Physics", "Facts on the Nobel Prize in Physics", "Prize Awarder for the Nobel Prize in Physics", "Nomination and Selection of Physics Laureates", "Nobel Medal for Physics", "Articles in Physics", "Video Interviews", and "Video Nobel Lectures".

The main content area features a timeline from 1901 to 2012, with 1936 highlighted. Below the timeline, there are options for "Sort and list Nobel Prizes and Nobel Laur" and "Prize category: Physics".

The central focus is the "The Nobel Prize in Physics 1936" section, which lists the laureates: Victor F. Hess and Carl D. Anderson. Below their names are two black and white portrait photographs. Underneath the photos, the text reads: "The Nobel Prize in Physics 1936 was divided equally between Victor Franz Hess 'for his discovery of cosmic radiation' and Carl David Anderson 'for his discovery of the positron'".

At the bottom of the page, the date "20 Mar 2013" and the URL "nobelprize.org/laureates/1936/" are visible.



this is where it gets interesting

we need to establish a language for Dirac-like reactions

**“Relativistic Quantum Field Theory”
essentially invented by Paul Dirac**

notice a couple of things about what appears in Dirac's equation

1. it's about more than one thing: two electrons and a photon

"regular" Quantum Mechanics is about single objects only

2. stuff appears and stuff disappears

what's

nothing.

what's nothing

you'd maybe say:

no objects (particles...quanta)

zero energy

the Heisenberg Uncertainty Principle:

there's no state of Nature that can possess any precise value of, say, energy

and that includes Zero.

the Heisenberg Uncertainty Principle

will not allow a **void**.

but we *still* have a notion of the **vacuum**

it's the lowest energy state in Nature

where there are no real particles

understanding what's going on here

requires some mental fortitude

remember

trying to trap an electron?

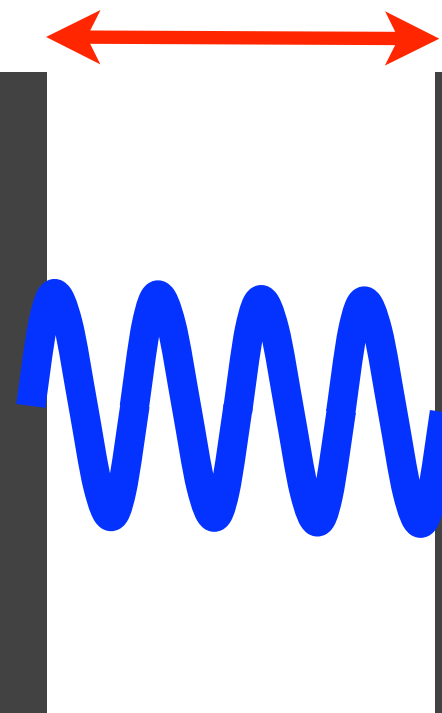
let's make it all about nothing.



What's in Nothing?



nothing...somewhere here:



make the trap smaller

remember

trying to trap an electron?

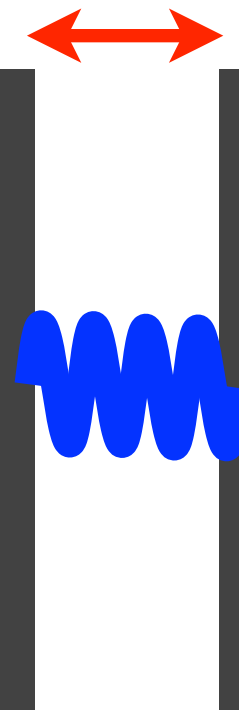
do nothing tighter



What's in Nothing?



or nothing, somewhere here:



make the trap smaller to this value:

$$\Delta x \sim \frac{h}{m_e c}$$

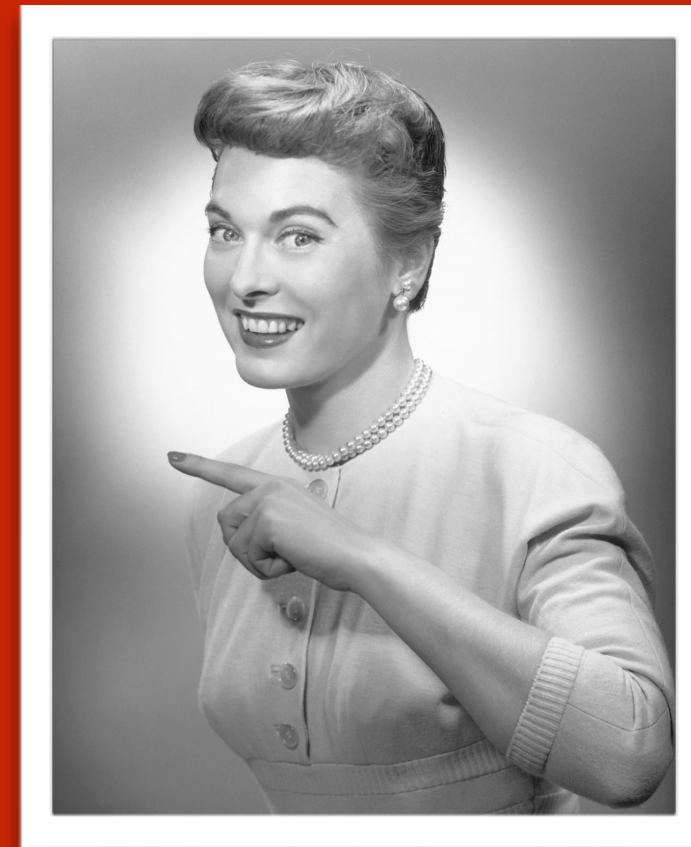
$$\sim 2.2 \times 10^{-12} \text{ m}$$

The size of a Hydrogen atom... $5 \times 10^{-11} \text{ m}$

The size of a proton... $\sim 10^{-15} \text{ m}$

an important

but simple calculation about
nothing



a very
important
“length”

Compton
Wavelength

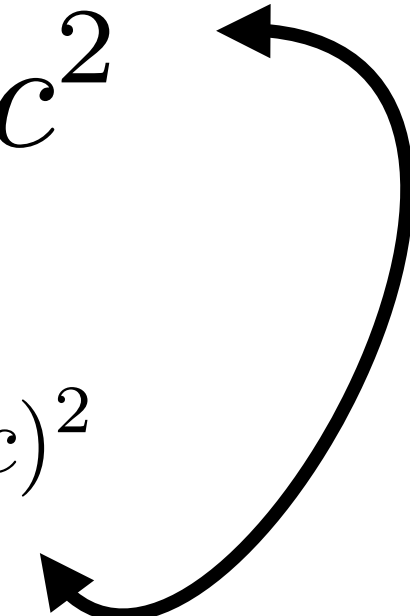
we consider this to be
“the size of a
particle”

$$\Delta x \sim \frac{h}{m_e c} = \lambda_{\text{Compton}} = \lambda_C$$

$$\Delta x \Delta p \sim h$$

$$\frac{h}{m_e c} \Delta p \sim h$$

$$\frac{1}{m_e c^2} \Delta p c \sim 1$$

$$\Delta p c \sim m_e c^2$$


Remember: $E_T^2 = (m c^2)^2 + (p c)^2$

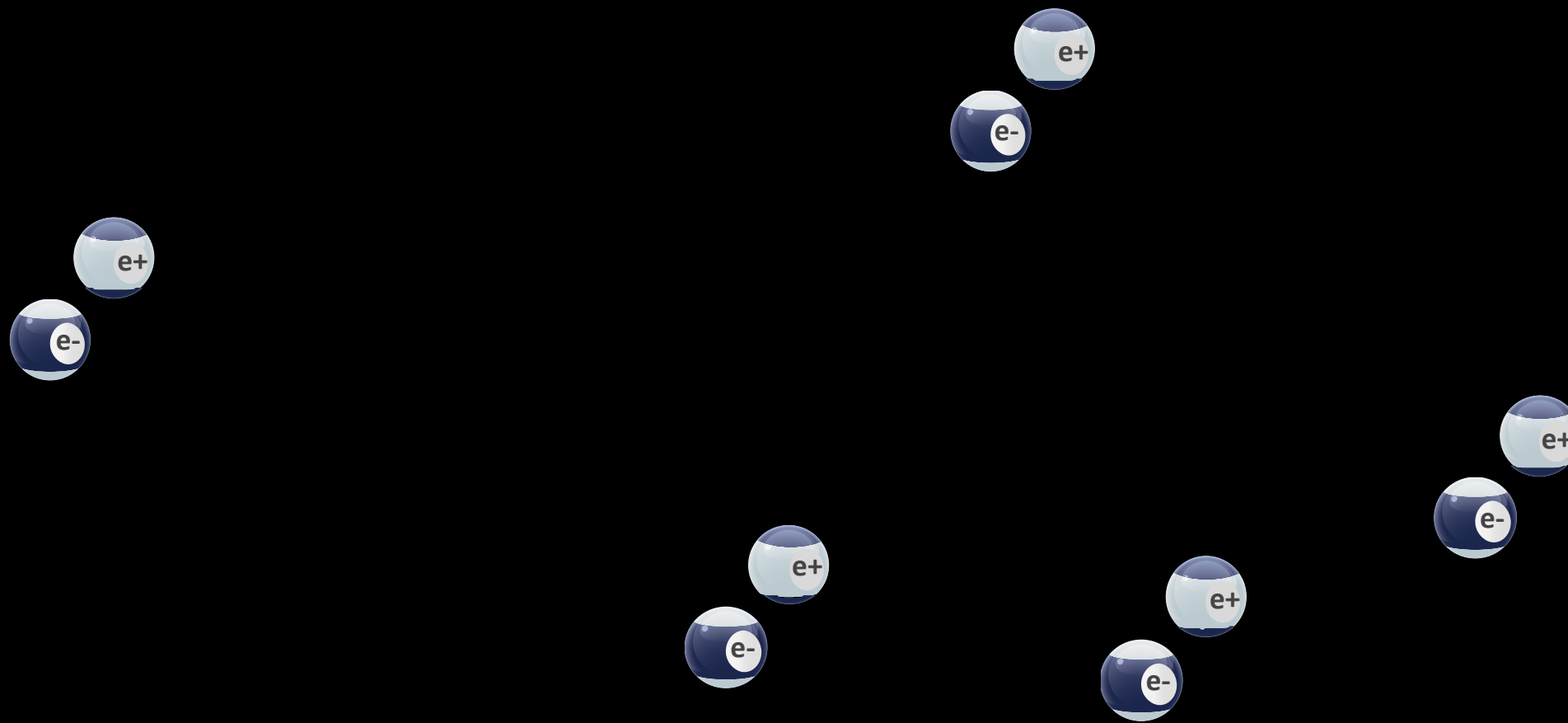
An energy equivalent to the mass energy...all by looking
closely at nothing.

pop

the Uncertainty Principle requires

that particle-antiparticle pairs pop into and out of existence

all the time



uncertainty principle

+ the particular length of:

$$\lambda_C = \frac{h}{mc}$$

makes the vacuum very active.

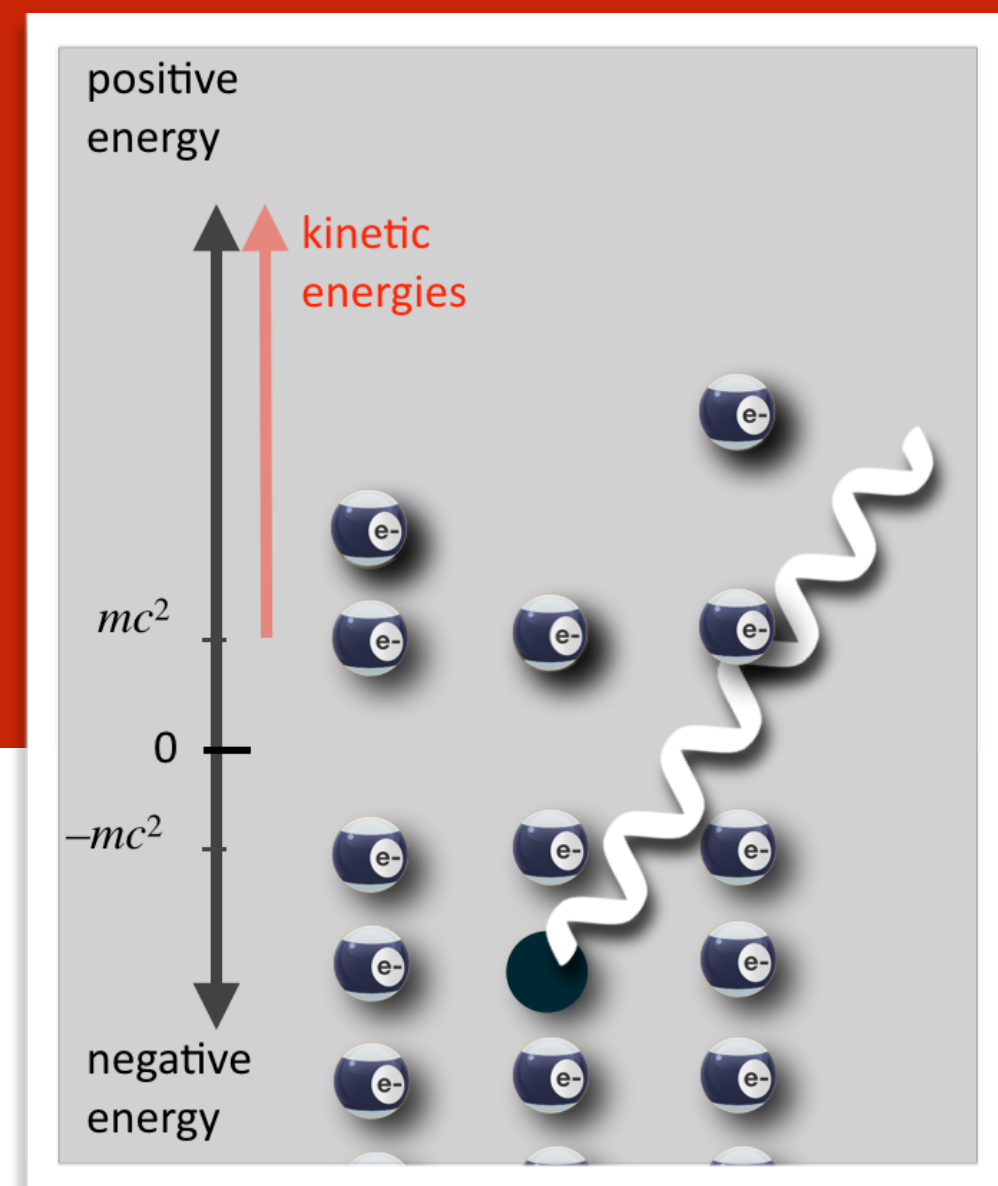


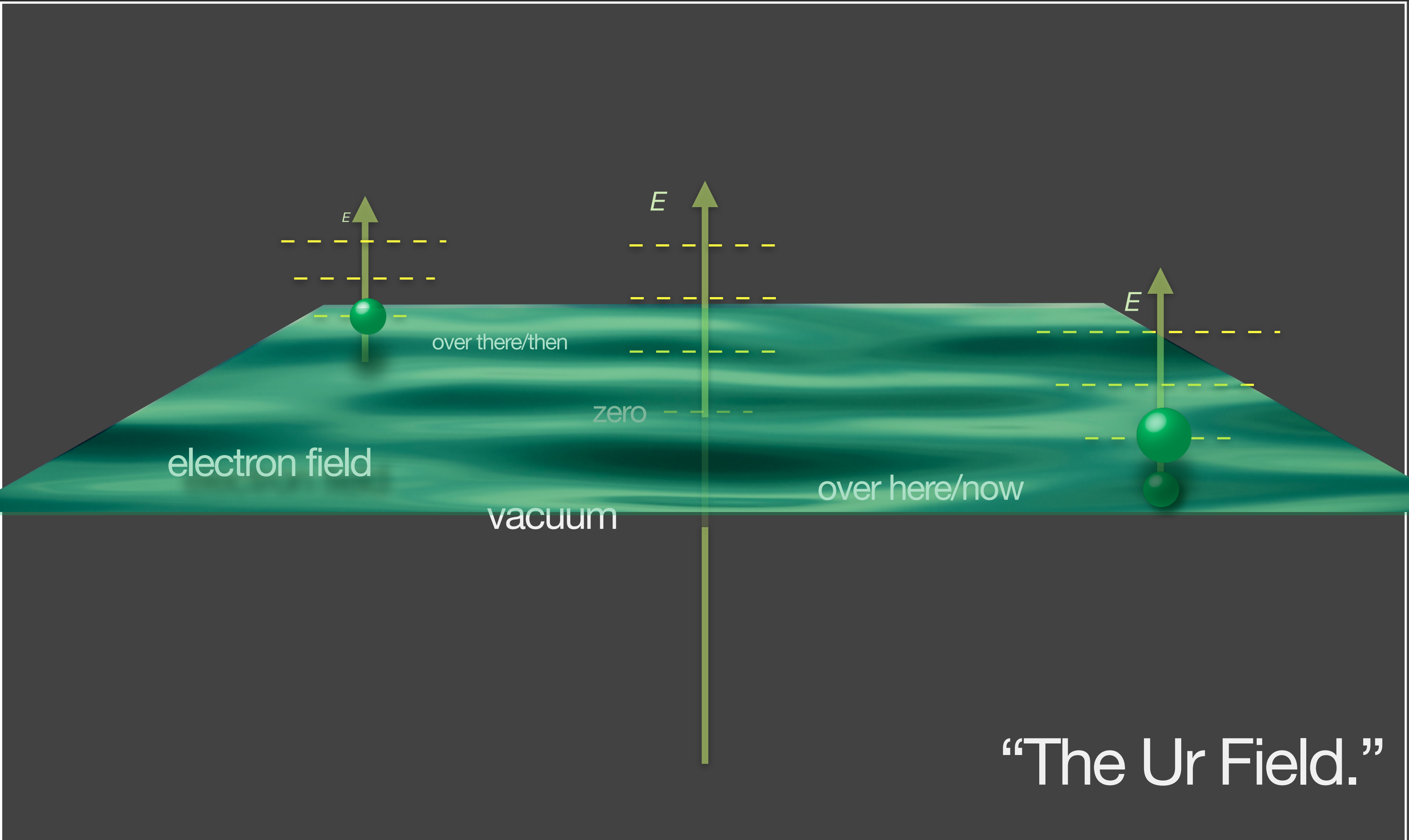
they are all popped out of the same stuff

the vacuum Field of the electron

electrons appear because they're coerced out of the vacuum

like by a photon

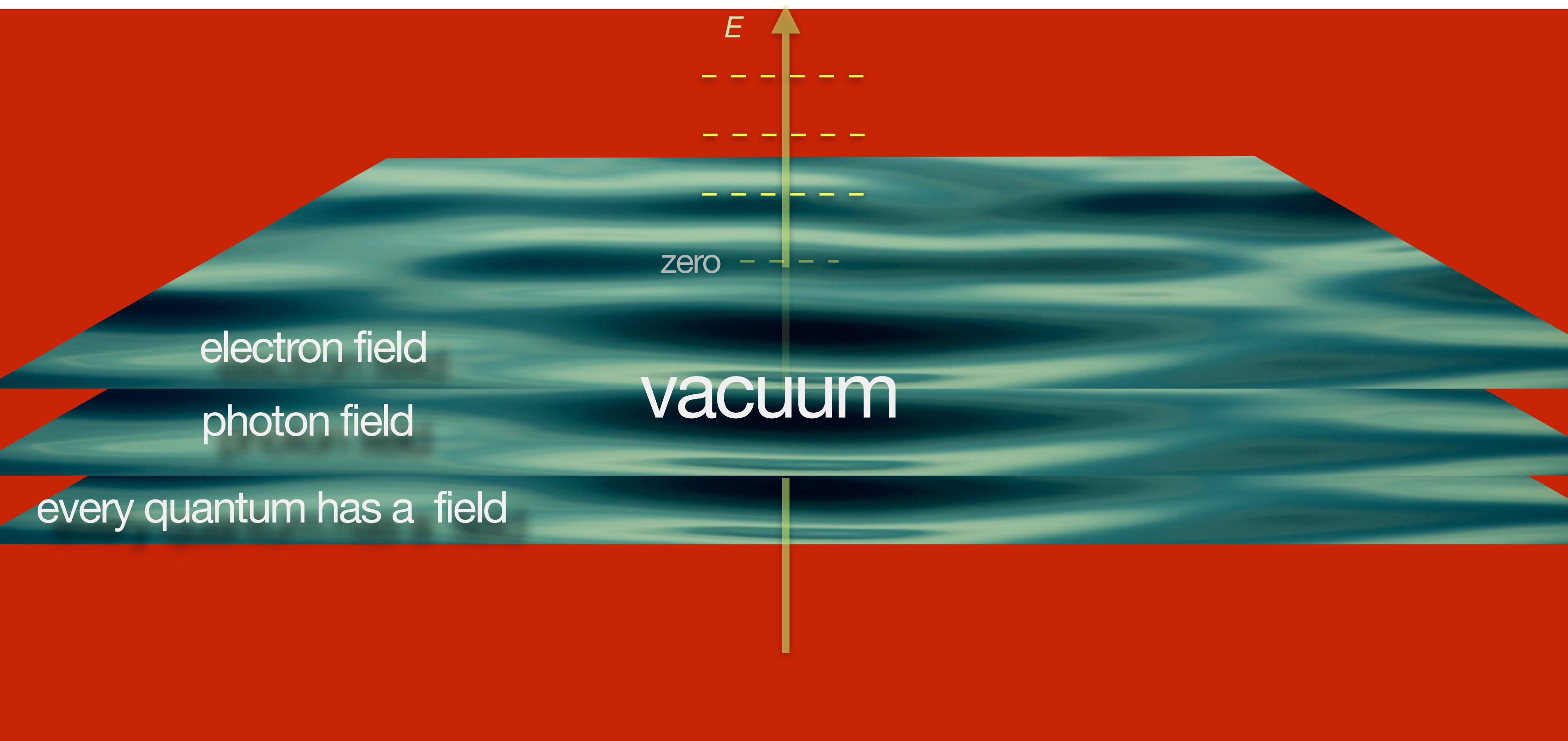




ur- |ʊ(ə)r| |ʊə| combining form primitive;
 original; earliest : *urtext*. ORIGIN from German.

but wait, there's more

an Ur Field for every quantum



E

zero

vacuum

electron field

photon field

every quantum has a field

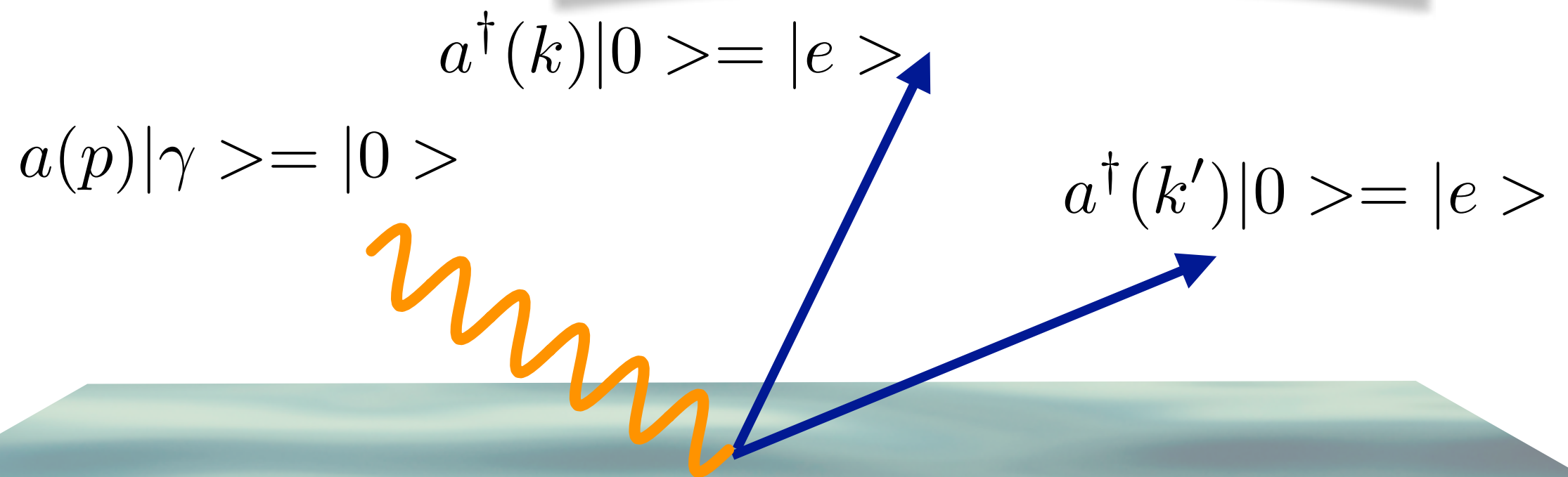
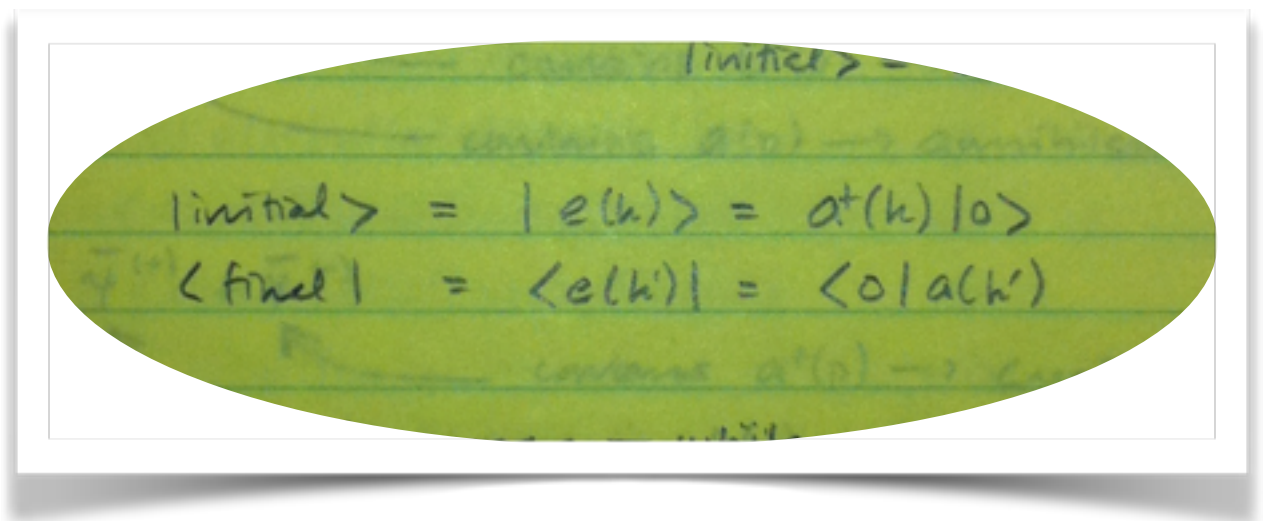
this was first understood for the EM field

Dirac put all particles on the same footing

a Field \leftrightarrow quanta of that Field

the vacuum
is a
complicated
place

what the
mathematics tells
us



it's not like the photon is now "in" the electron

the photon pops the electron- positron pair out of the Ur
electron field

and itself disappears back into the Ur photon field.

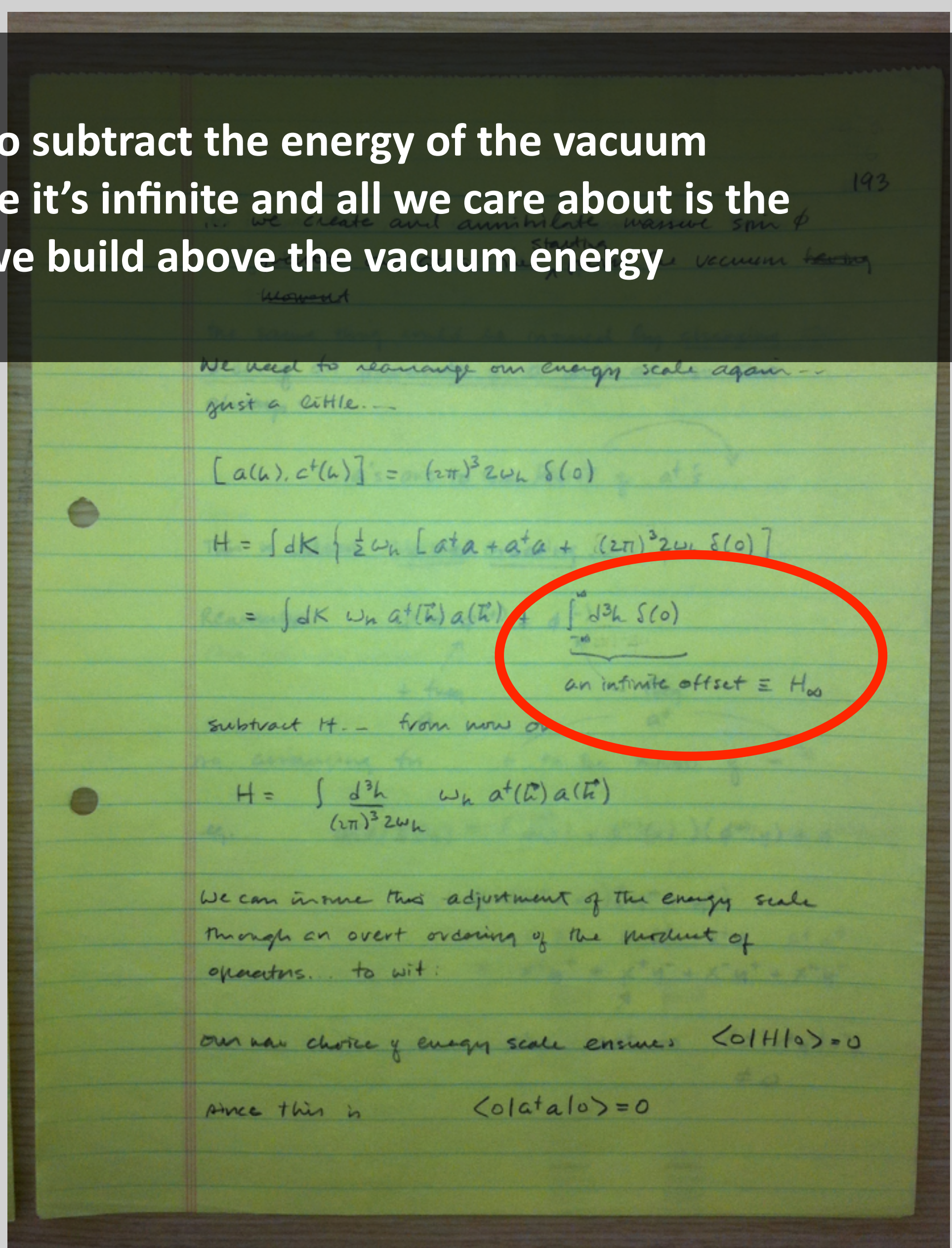
but what
does that
mean?

we have to subtract the energy of the vacuum
away... because it's infinite and all we care about is the
states we build above the vacuum energy

it means that the
vacuum is full of
energy

like a reservoir

particles are
created out of the
vacuum



Okay. So we don't like infinity

we subtract it away and worry about the difference
between

infinity and finite energies of real particles

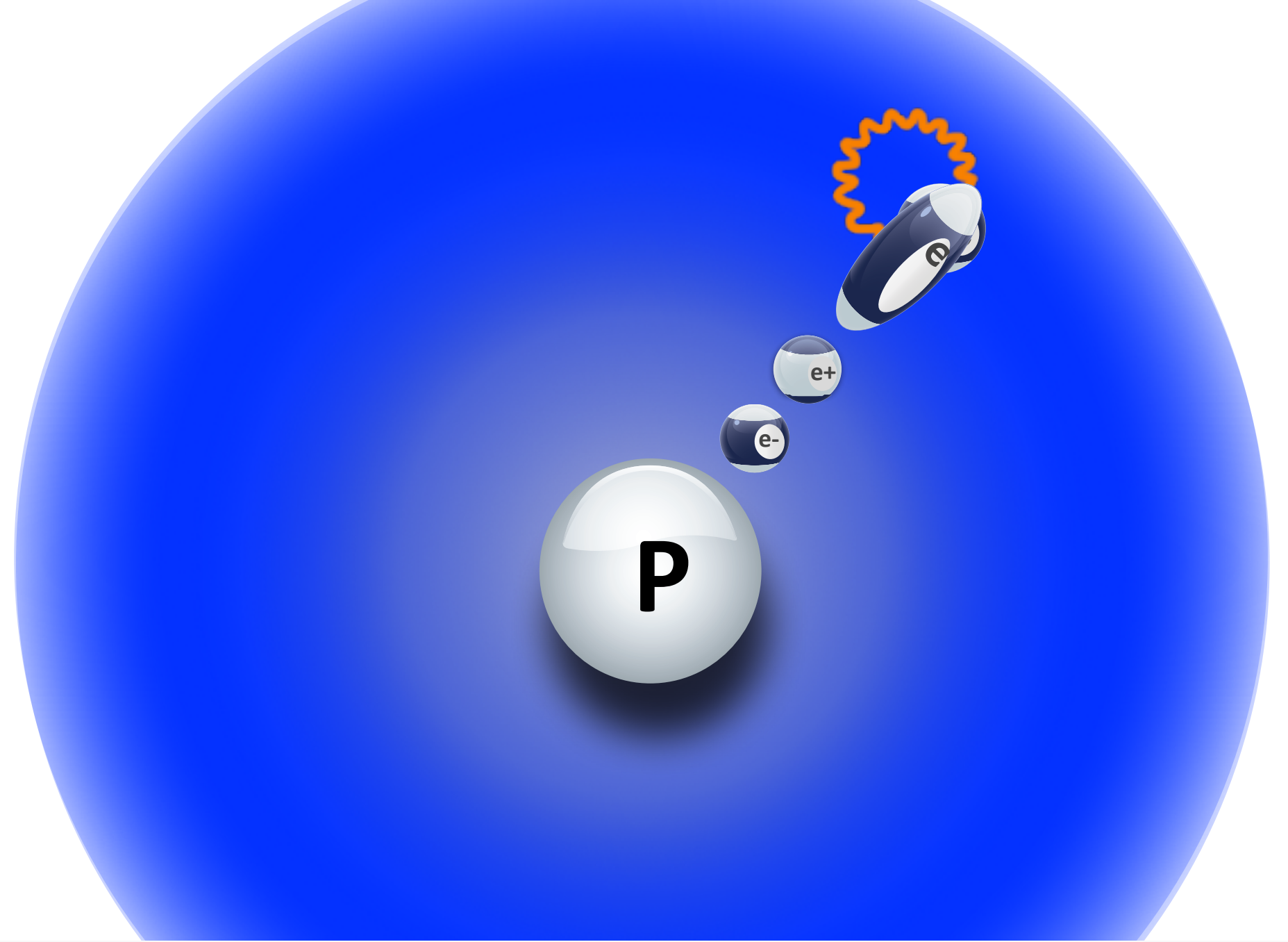
Seriously?

That seems to be the case.

This picture works with exquisite precision and accuracy.

but the
vacuum

is roiling with
particle-
antiparticle
“virtual pairs”
popping into and
out of existence
multiple ways we
know this.



A “regular” model of the hydrogen atom...needs
modification to take into account the effects of the vacuum

The electron cloud is spread out by the virtual
photon and the positron’s effects...and that changes
the emission spectrum of hydrogen:
The “**Lamb Shift**”...measured after WWII with
microwave technologies

1955 Nobel Prize

Willis E. Lamb

died just a few years ago at the University of Arizona



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Nobel Laureates Honored

1901 2012
1955
Sort and list Nobel Prizes and Nob... Prize category: Physics

The Nobel Prize in Physics 1955
Willis E. Lamb, Polykarp Kusch

The Nobel Prize in Physics 1955
Nobel Prize Award Ceremony
Willis E. Lamb
Polykarp Kusch



Willis Eugene Lamb Polykarp Kusch

The Nobel Prize in Physics 1955 was divided equally between Willis Eugene Lamb "for his discoveries concerning the fine structure of the hydrogen spectrum" and Polykarp Kusch "for his precision determination of the magnetic moment of the electron".

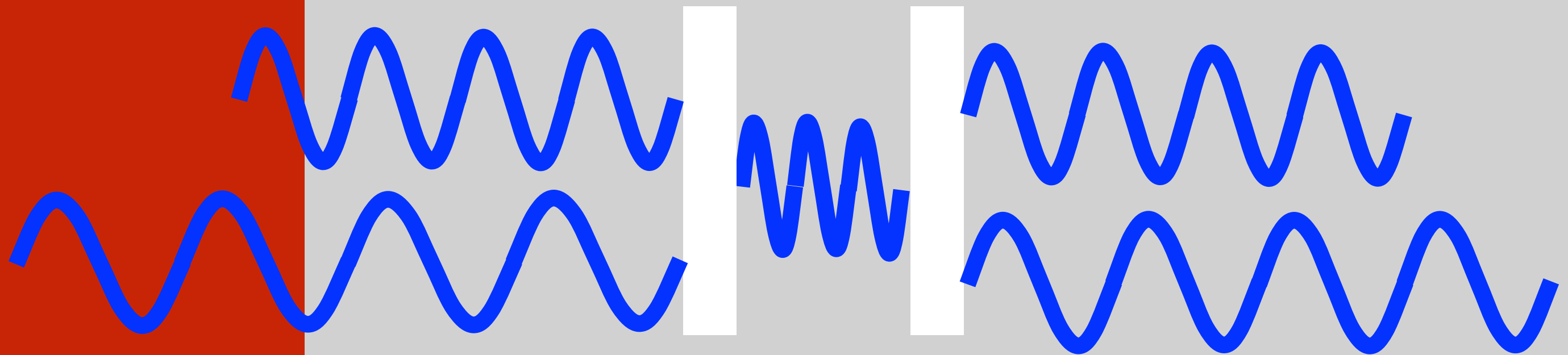
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the “Casimir Effect”

**two highly polished mirrors isolated
from all external effects**

The vacuum has all wavelengths of virtual waves from
particles and fields...but fewer can fit between the walls

...and the pressure from the outside, moves them closer
together



The amount is precisely predicted...and a few years ago the
experiments confirmed it convincingly in 2001

the vacuum

is a very complicated thing

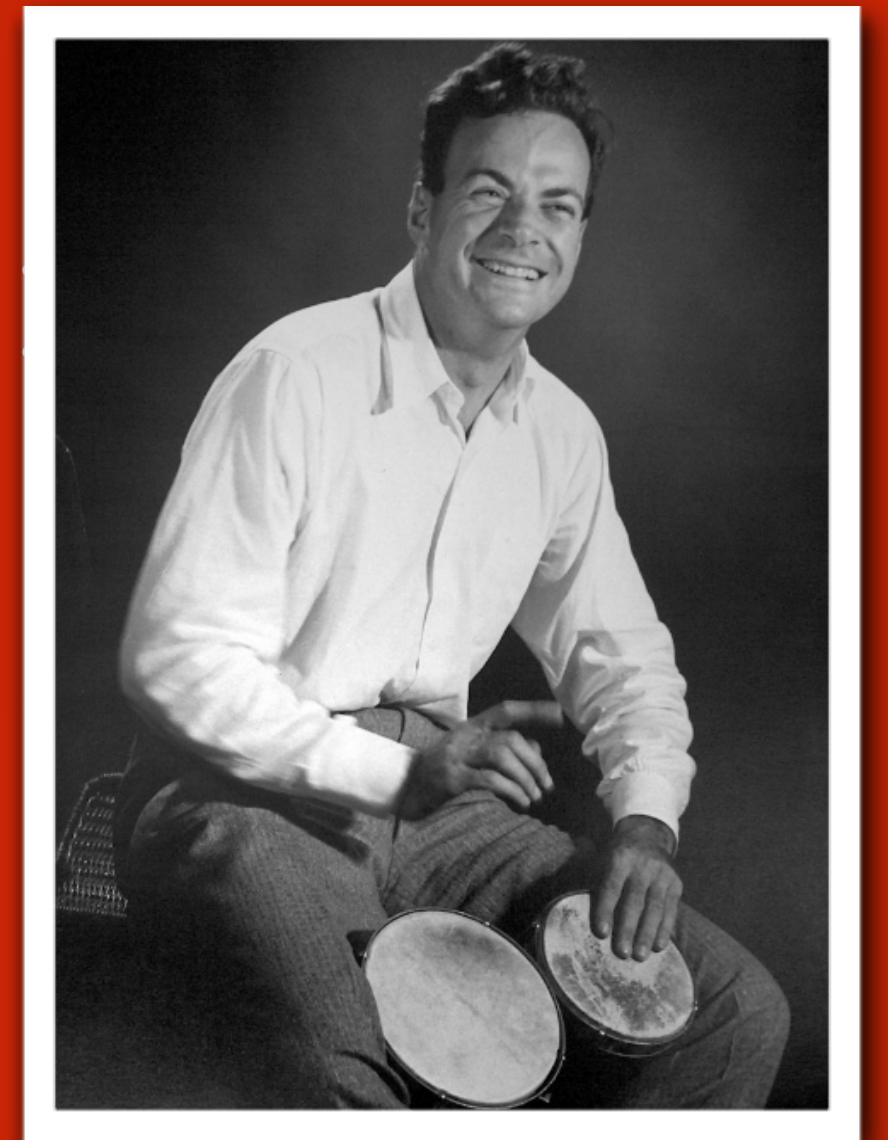
as we'll see when we get back to cosmology



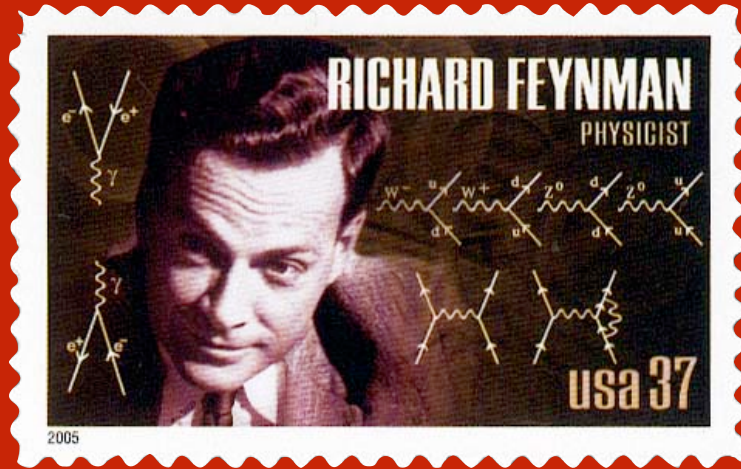
Feynman Diagrams

now for real.

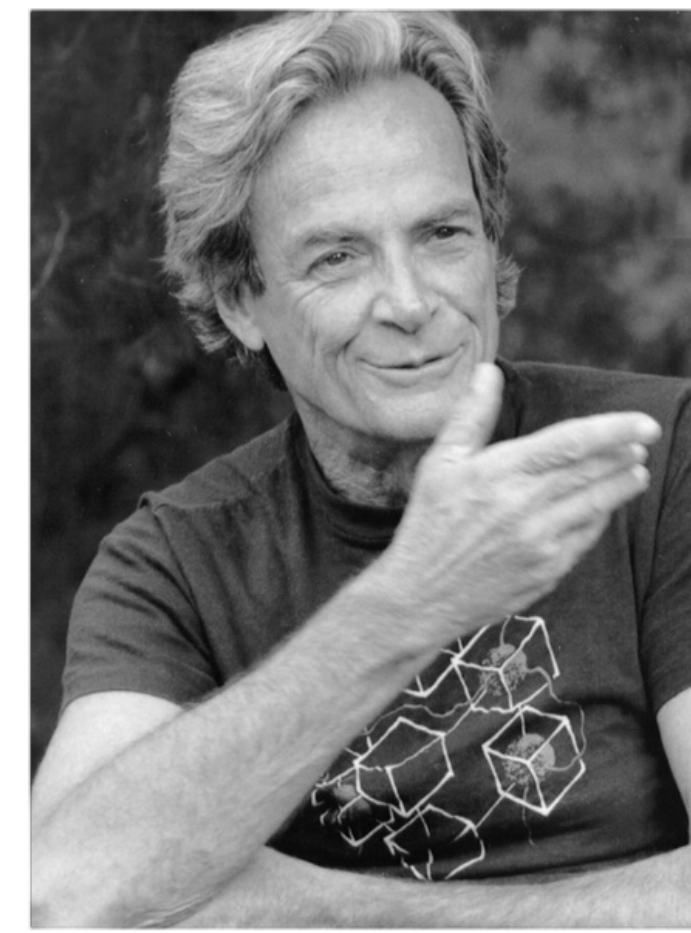
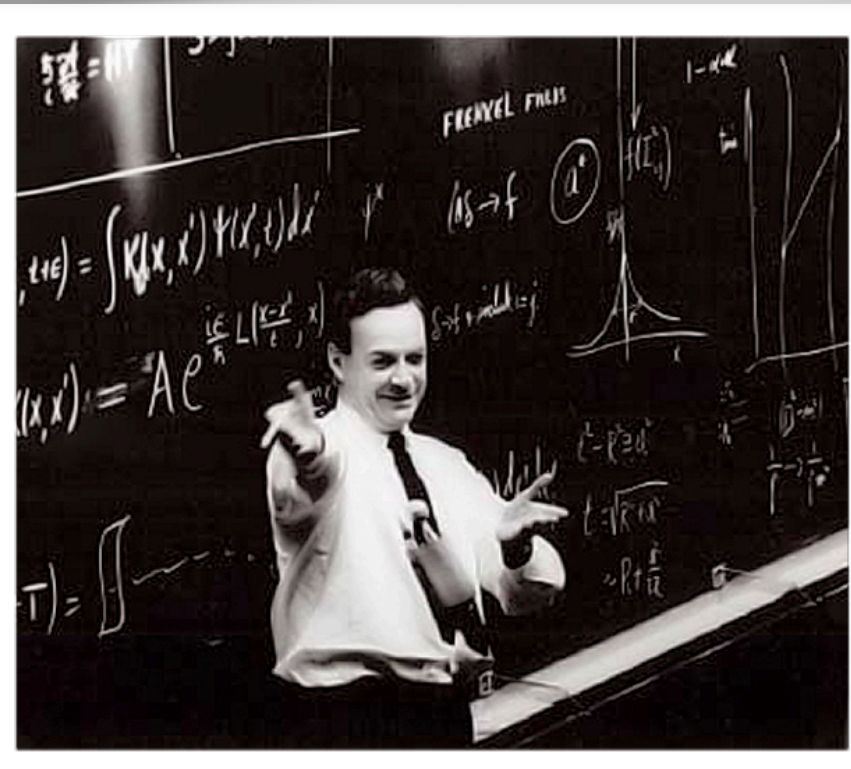
creation and annihilation of
can be embodied in Feynman Diagrams



hero worship



about as close as
we come



Richard Feynman, Sin-Itiro Tomonaga, Julian Schwinger

1965 Nobel



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1901 2012
1965
Sort and list Nobel Prizes and Nobel Laur | Prize category: Physics

The Nobel Prize in Physics 1965
Sin-Itiro Tomonaga, Julian Schwinger, Richard P. Feynman

The Nobel Prize in Physics 1965
Sin-Itiro Tomonaga
Julian Schwinger
Richard P. Feynman

Sin-Itiro Tomonaga **Julian Schwinger** **Richard P. Feynman**

The Nobel Prize in Physics 1965 was awarded jointly to Sin-Itiro Tomonaga, Julian Schwinger and Richard P. Feynman *"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"*.

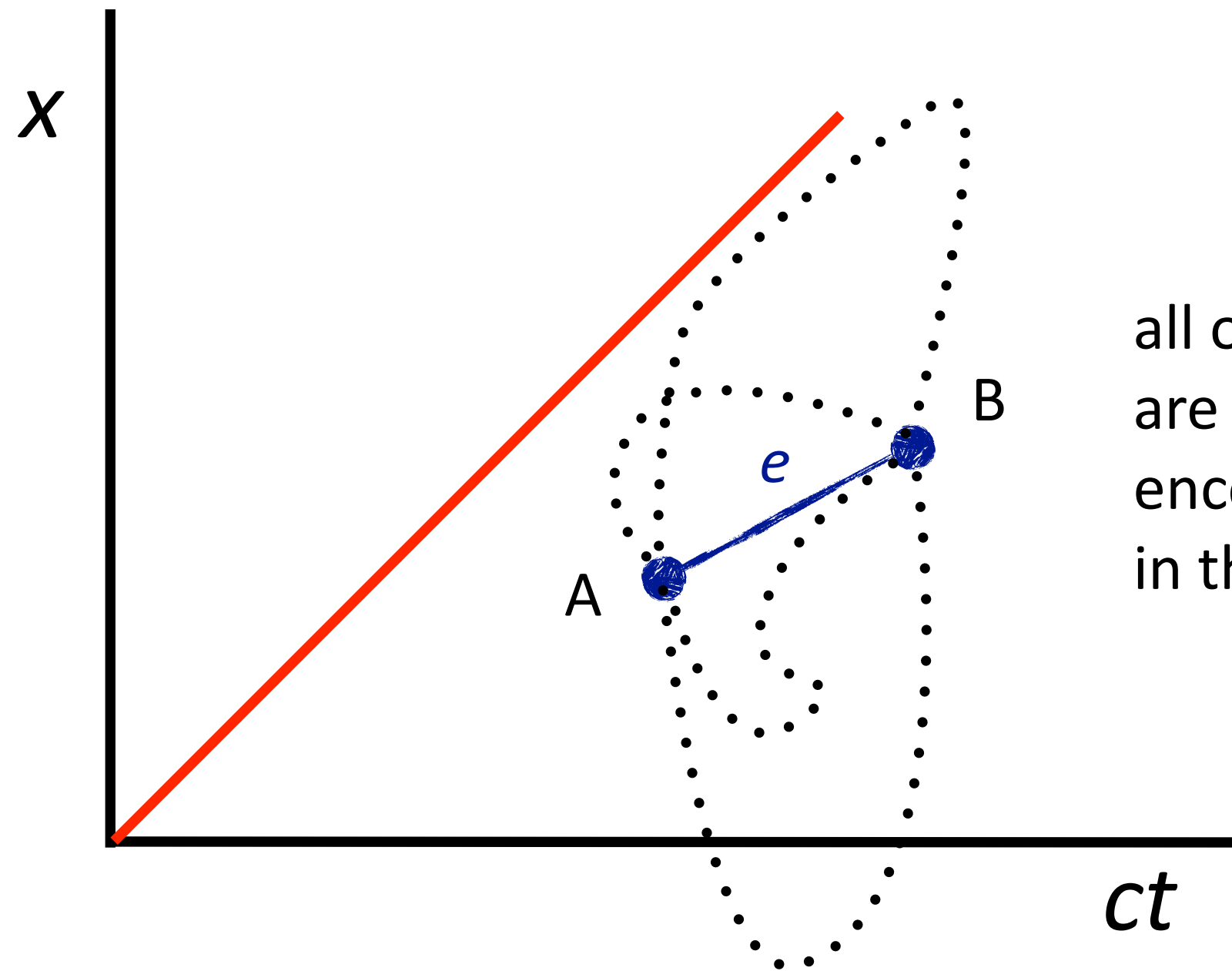
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http://www.nobelprize.org/nobel_prizes/physics/laureates/1965/

the symbols of Feynman Diagrams

each line represents an entire "history" of trajectories

to go from A to B, represent all histories with a single line.



Feynman's lines include rules on how to calculate the possibilities in a relativistically consistent way.

very
efficient

avoids lots of
technicalities.

When I teach these techniques to second year graduate students, I first do the calculation of Compton Scattering and do it without Feynman's tools.

... spread. (ask look at 204)

$$\langle 0 | T [\psi(x) \bar{\psi}(y)] | 0 \rangle = \bar{\psi}(x) A(x) \psi(y) A(y) A(x) A(y)$$

$$\begin{aligned} (\bar{\psi}^+ + \bar{\psi}^-)(\psi^+ + \psi^-) \\ \bar{\psi}^+ \psi^+ + \bar{\psi}^+ \psi^- + \bar{\psi}^- \psi^+ + \bar{\psi}^- \psi^- \\ \bar{\psi}^+ \psi^+ + \bar{\psi}^- \psi^- = 1 \cdot 0 \cdot 0 + 1 \cdot 0 \cdot 0 \\ \bar{\psi}^+ \psi^- + \bar{\psi}^- \psi^+ = 0 \cdot 0 + 0 \cdot 0 = 0 \end{aligned}$$

... depends on the actual process.

... considering classical Compton scattering. $\epsilon \rightarrow \epsilon'$

$$\begin{aligned} |initial\rangle &= |e, k\rangle \\ |final\rangle &= |e', k'\rangle \end{aligned}$$

... and $k^0 = -|k^0|$. An essential feature.

$$\frac{1}{k^2 - m^2} \rightarrow \frac{-i \pi \delta(k^2 - m^2)}{2m}$$

... and exponential decays.

$$\int_C \frac{d^4 k}{(2\pi)^4} = -i \pi^2 \int_{-\infty}^{\infty} \frac{d^3 k}{(2\pi)^3} \delta(k^2 - m^2)$$

$$-i \pi^2 \int_{-\infty}^{\infty} \frac{d^3 k}{(2\pi)^3} = i \pi^2$$

The pole is simple, so the residue is

$$\frac{(k+m) \epsilon}{(k^2 - m^2)(k^0 + i\epsilon)} \Big|_{k^0 = \sqrt{k^2 + m^2}}$$

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = \bar{\psi}(x) A(x) \psi(x) A(x)$$

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = \bar{\psi}(x) \psi(x) \theta(x_0 - x'_0)$$

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = \bar{\psi}(x) \psi(x) \theta(x_0 - x'_0)$$

$$\frac{1}{k^2 - m^2} = \frac{1}{(k^0 - \sqrt{k^2 + m^2})(k^0 + \sqrt{k^2 + m^2})}$$

$$\frac{1}{k^2 - m^2} = \frac{1}{(k^0 - \sqrt{k^2 + m^2})(k^0 + \sqrt{k^2 + m^2})}$$

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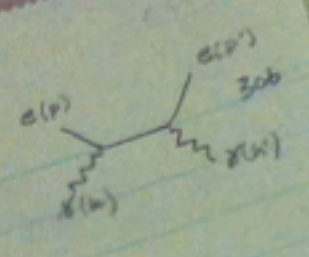
$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = \bar{\psi}(x) \psi(x) \theta(x_0 - x'_0)$$

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = \bar{\psi}(x) \psi(x) \theta(x_0 - x'_0)$$

Compton scattering

$$J^{(1)}(e^+e^-) = \frac{-ie}{2} \int d^4x_1 \int d^4x_2 \langle e^+ | \delta_0^+ \delta_{im}^- | e^- \rangle$$

$$= \int d^4p_1 \int d^4p_2 \int d^4k_1 \int d^4k_2 \langle 0 | a^{(1)}(p_1) a^{(1)\dagger}(p_2) a^{(1)}(k_1) a^{(1)\dagger}(k_2) | 0 \rangle$$



Do the standard $\{a(n), a^\dagger(m)\}$ - twice

$$\int d^4x_1 \int d^4x_2 \delta(\vec{p}-\vec{p}_1) \delta_{\mu\nu} \delta_{\alpha\beta}$$

first

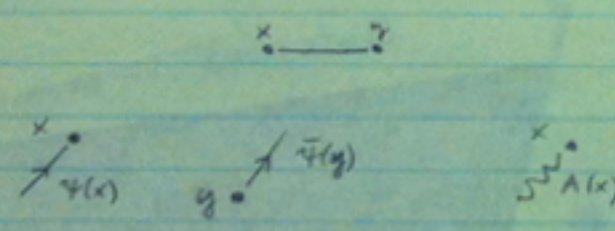
Put all the stuff, into the momentum, separate out the four space terms for do photons.

$$J^{(1)}(e^+e^-) = \frac{-ie}{2} \int d^4x_1 \int d^4x_2 \langle 0 | A_\mu(x_1) A_\nu(x_2) | 0 \rangle$$

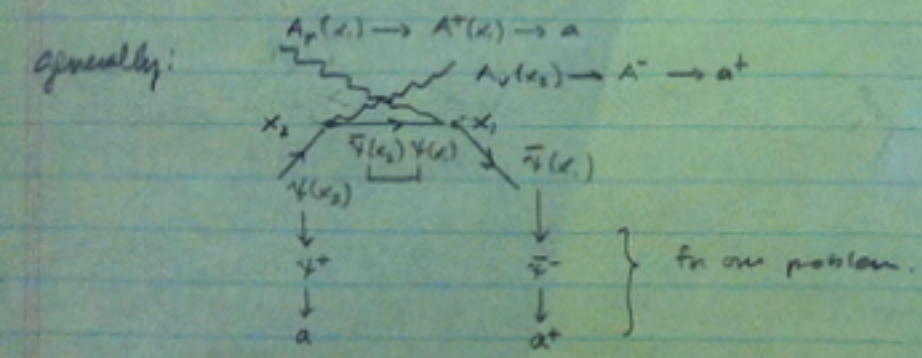
$$= \int d^4p_1 \int d^4p_2 \int d^4k_1 \int d^4k_2 \langle 0 | a^{(1)}(p_1) a^{(1)\dagger}(p_2) a^{(1)}(k_1) a^{(1)\dagger}(k_2) | 0 \rangle$$

Earlier, I attached a graphical meaning to the Wick expansion terms, let's recap that according to what we've calculated.

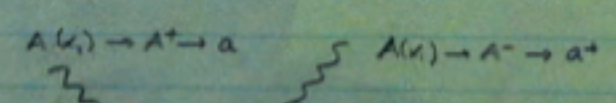
$$\bar{\psi}(x)\psi(y) = \langle 0 | T[\bar{\psi}(x)\psi(y)] | 0 \rangle$$



So, the first (1) or (3) graph would be



and (2) or (4)



twice (which cancels the funny minus signs). So we get

$$A_\mu^+(x_1) | \gamma \rangle = \int d^4k_1 \sum_\lambda \epsilon_{\mu\lambda}(k_1) e^{-ik_1 \cdot x_1} (\pi)^3 2\omega_1 \delta_{\lambda\lambda} \delta(\vec{k}-\vec{k}_1) | 0 \rangle$$

and we can do the momentum integrals giving

$$\langle \gamma(x_1) | : A_\mu(x_1) A_\nu(x_2) : | \gamma(x_2) \rangle = \epsilon_{\mu\lambda}(k_1) \epsilon_{\nu\lambda}(k_2) e^{ik_1 \cdot x_1 - ik_2 \cdot x_2} + \epsilon_{\nu\lambda}(k_2) \epsilon_{\mu\lambda}(k_1) e^{ik_2 \cdot x_2 - ik_1 \cdot x_1}$$

therefore, 4 terms overall.

$$i \int d^4x_1 \int d^4x_2$$

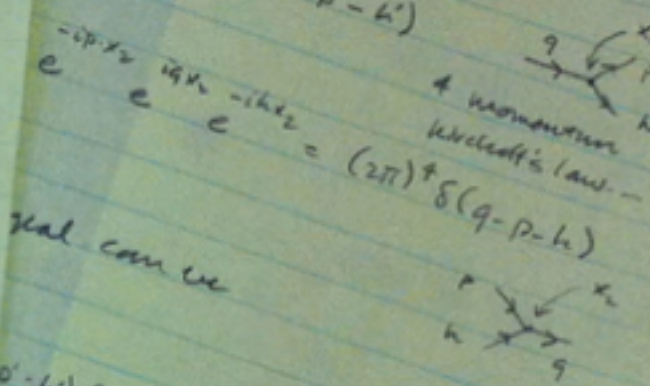
$$u^{(s)}(p) \bar{u}^{(s')}(p) e^{ip \cdot x_1} e^{-ip \cdot x_2}$$

$$\int d^4x_1 \int d^4x_2 e^{ip \cdot x_1 - iq \cdot x_2 + ik \cdot x_1} e^{-ik \cdot x_2}$$

$$= (2\pi)^4 \delta(p-q+k)$$

$$= (2\pi)^4 \delta(q-p-k)$$

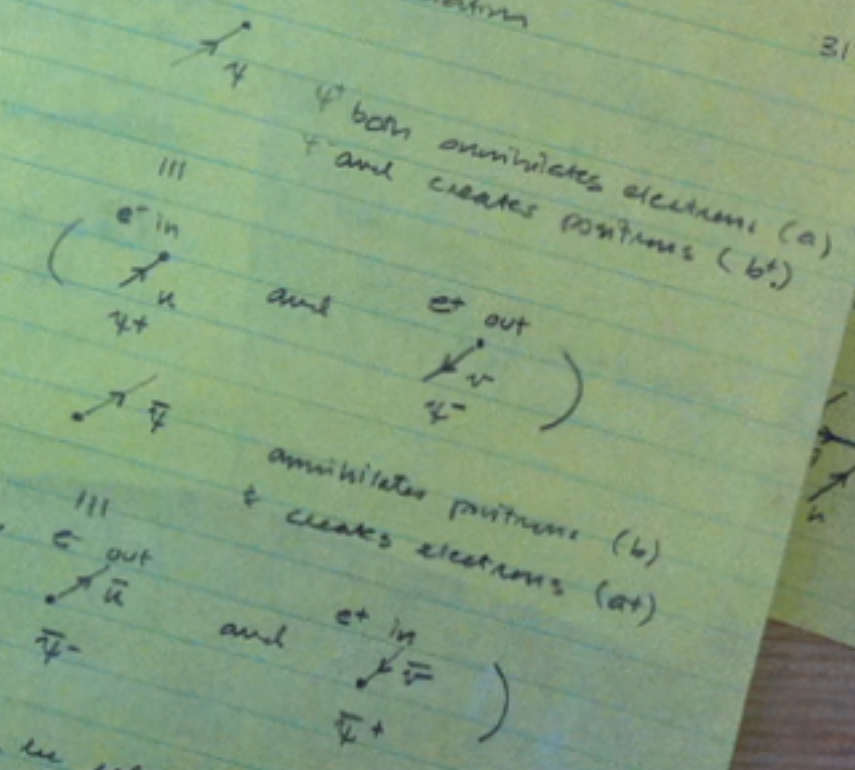
in (1)



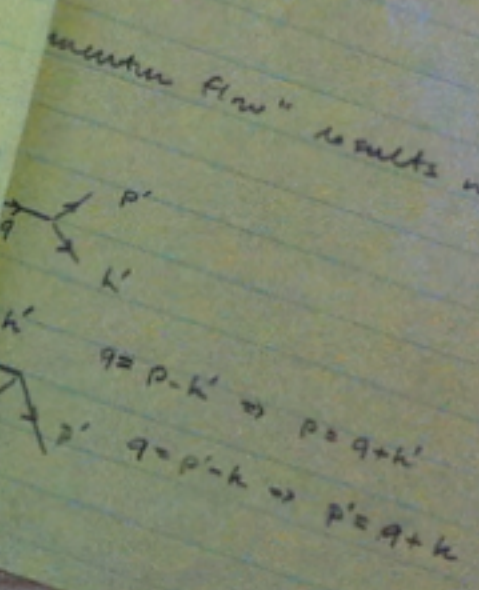
$$u^{(s)}(p) \bar{u}^{(s')}(p) e^{ip \cdot x_1} e^{-ip \cdot x_2}$$

Remember the Feynman interpretation

↑ time



$$\bar{u}^{(s')}(p') \left[\epsilon_{\mu\nu}(k) \frac{1}{\not{p} + \not{k} - m} \right] u^{(s)}(p)$$



can be related with the frequency indicated.

Feynman rules eliminate all γ and γ directly to

his rules
eliminate
all of
that

and I can just
write down the
"answer"

appropriately labeled, each line tells us what to put into a long
equation for further solving

23 Compton - finish! 325

Now let's do the Compton calculation as if we know
the rules all along. I want the cross section for
 $\gamma e \rightarrow \gamma e$ for an initial electron at rest.

$\gamma(k) + e(p) \rightarrow \gamma(k') + e(p')$ to 2nd order.

$(-i) \bar{u}^{(s)}(p') \epsilon_{\nu\alpha}(-k') \frac{i}{p+k-m} (-ie\gamma^\alpha) u^{(s)}(p)$

$(-ie\gamma^\nu) \epsilon_{\nu\alpha}(k) u^{(s)}(p)$

$(-i) \bar{u}^{(s)}(p') (-ie\gamma^\nu) \epsilon_{\nu\alpha}(k) \frac{i}{p-k-m} (-ie\gamma^\alpha) u^{(s)}(p)$

$(-ie\gamma^\nu) \epsilon_{\nu\alpha}(k') u^{(s)}(p)$

so,

$T_{fi} = -e^2 \bar{u}^{(s)}(p') \not{\epsilon}' \frac{1}{p+k-m} \not{\epsilon} u^{(s)}(p)$

$- e^2 \bar{u}^{(s)}(p') \not{\epsilon} \frac{1}{p-k-m} \not{\epsilon}' u^{(s)}(p)$

simplify:

$$\frac{1}{p+k-m} = \frac{p+k+m}{(p+k)^2 - m^2} = \frac{p+k+m}{p^2 + k^2 + 2p \cdot k - m^2}$$

$$= \frac{p+k+m}{2p \cdot k}$$

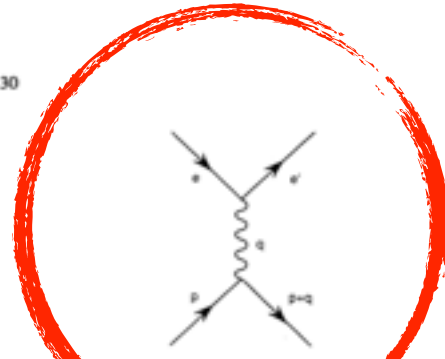
$$\frac{1}{p-k-m} = \frac{p-k+m}{p^2 - k^2 - 2p \cdot k - m^2}$$

24
24

our papers are full of diagrams

experimental ones:

30



A. F. SILL *et al.* 48

$G_E^p(Q^2) = G_M^p(Q^2)/\mu_p$. In terms of F_1 and F_2 , the form factor scaling relation becomes

$$F_2^p = F_1^p / (1 + \mu_p \tau) \quad (4)$$

The electric form factor G_E^p has not yet been measured with good precision above $Q^2 \approx 7$ (GeV/c)². If form factor scaling continues, the contribution of G_M^p to the cross section dominates over that of G_E^p at high Q^2 . The contribution of G_E^p to the cross section under this assumption is typically a few percent above $Q^2 = 5$ (GeV/c)², and so moderate deviations from form factor scaling would have little effect on the extracted value of G_M^p for most of the data in this experiment. Extraction of the Dirac form factor depends more heavily on the assumption of form factor scaling (see Sec. IV).

It is customary in these cases to refer to the momentum transfer squared as $Q^2 = -q^2$, which is a positive quantity.

1. Representation of elastic cross section in terms of form factors

The elastic cross section can be represented in terms of the proton magnetic and electric form factors $G_M^p(Q^2)$ and $G_E^p(Q^2)$ as

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega} \right]_{NS} \left[\frac{G_E^p(Q^2) + \tau G_M^p(Q^2)}{1 + \tau} + 2\tau G_M^p(Q^2) \tan^2(\theta/2) \right] \quad (1)$$

or in terms of the proton Dirac and Pauli form factors $F_1^p(Q^2)$ and $F_2^p(Q^2)$ as

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega} \right]_{NS} \left[F_1^p(Q^2) + \tau \kappa_p^2 F_2^p(Q^2) + 2\tau [F_1^p(Q^2) + \kappa_p F_2^p(Q^2)]^2 \tan^2(\theta/2) \right] \quad (2)$$

where $\tau = Q^2/4M_p^2$, $\kappa_p = \mu_p - 1$ is the anomalous magnetic moment of the proton (1.7928...), and $[d\sigma/d\Omega]_{NS}$ is the no-spin, "nonstructure" cross section:

$$\left[\frac{d\sigma}{d\Omega} \right]_{NS} = \frac{(\alpha \hbar c)^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2) [1 + 2(E/M_p) \sin^2(\theta/2)]} = \left[\frac{d\sigma}{d\Omega} \right]_{Mott} \frac{E'}{E} \quad (3)$$

The Sachs form factors G_E and G_M are related to the Dirac and Pauli form factors F_1 and F_2 by the expressions $G_M = F_1 + \kappa F_2$ and $G_E = F_1 - \tau \kappa F_2$. Here F_1 corresponds to the helicity-conserving part of the cross section, and F_2 corresponds to the helicity-flip part. For a pointlike, spin-1/2 particle with electric charge and no anomalous magnetic moment, $F_1 = 1$ and $F_2 = 0$.

2. Form factor scaling

At low-momentum transfers [$Q^2 \leq 7$ (GeV/c)²], G_E^p has been found to scale [1,2] with G_M^p such that

PHYSICAL REVIEW D VOLUME 48, NUMBER 1 1 JULY 1993

Measurements of elastic electron-proton scattering at large momentum transfer

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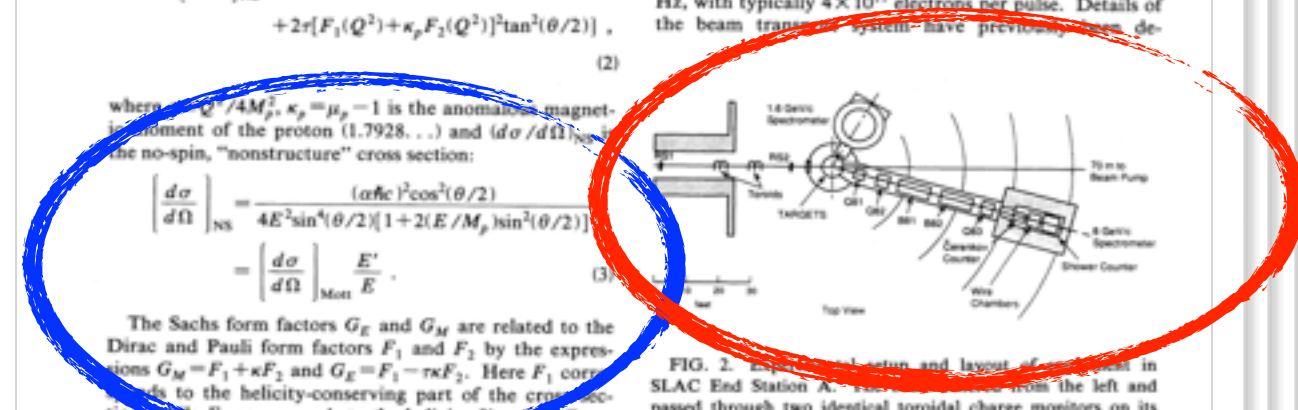
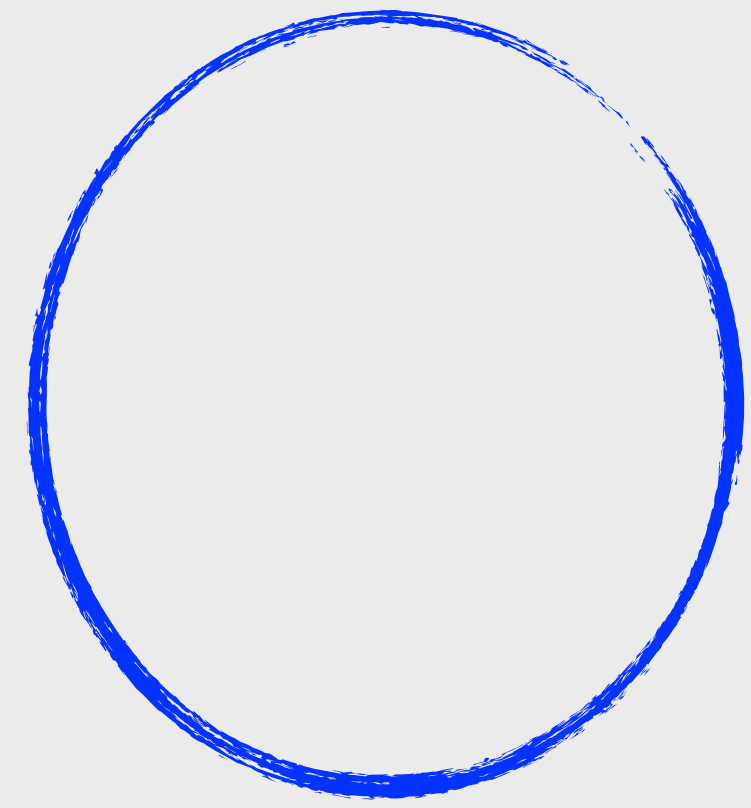


FIG. 2. Experimental setup and layout of the spectrometers in SLAC End Station A. The electron beam enters from the left and passed through two identical toroidal charge monitors on its way to the targets. The beam position, size, and angle were checked manually by inserting fluorescent screens (RS1 and RS2) every few hours. The beam position and profile were measured continuously with wire arrays, which were part of a computerized beam steering system described in the text. Scattered electrons were detected using the SLAC 8-GeV/c spectrometer. The SLAC 1.6-GeV/c spectrometer was used to monitor the density of the liquid-hydrogen targets.



theoretical papers

each diagram is a complicated calculation

m_{π_T} smaller than 260 GeV. The parameter ϵ refers to the proportion of the top quark mass generated by the extended technicolor which is taken in the range of $\epsilon \sim (0.01, 0.1)$.

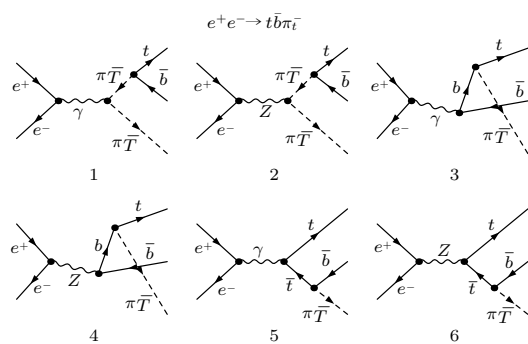


Fig. 1. Diagrams for $e^+e^- \rightarrow t\bar{b}\pi^-$.

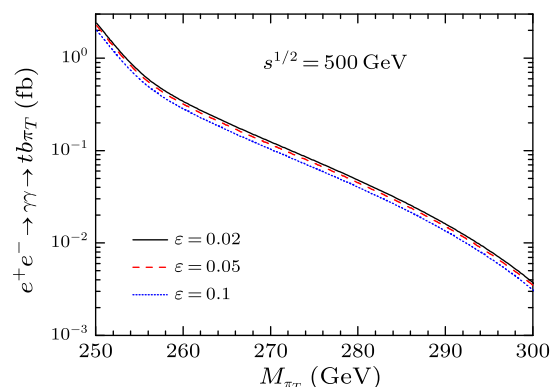


Fig. 2. Dependence of the cross section for $e^+e^- \rightarrow t\bar{b}\pi^-$ ($t\bar{b}\pi^-$) on the top-pion mass m_{π_T} at the ILC with energy of 500 GeV.

By using the laser back-scattering technique on electron beam, an e^+e^- LC which has the c.m. energy of hundreds of GeV to several TeV can be transformed to be a photon collider.^[19–21] By integrating over the photon luminosity in an e^+e^- linear collider, the total cross section for the process $e^+e^- \rightarrow t\bar{b}\pi^-$ can be obtained in the form

$$\sigma(s) = \int_{\frac{E_0}{\sqrt{s}}}^{x_{\max}} dz \frac{d\mathcal{L}_{\gamma\gamma}}{dz} \hat{\sigma}(\gamma\gamma \rightarrow t\bar{b}\pi^-, \text{ at } \hat{s} = z^2s), \quad (12)$$

where $E_0 = m_t + m_b + m_{\pi_T}$, $\sqrt{s}(\sqrt{\hat{s}})$ is the e^+e^- ($\gamma\gamma$) c.m. energy, and $\frac{d\mathcal{L}_{\gamma\gamma}}{dz}$ is the distribution function of photon luminosity, which is defined as

$$\frac{d\mathcal{L}_{\gamma\gamma}}{dz} = 2z \int_{z^2/x_{\max}}^{x_{\max}} \frac{dx}{x} F_{\gamma/e}(x) F_{\gamma/e}\left(\frac{z^2}{x}\right). \quad (13)$$

For the initial unpolarized electrons and laser photon beams, the energy spectrum of the back scattered

photon is given by^[22]

$$F_{\gamma} = \frac{1}{D(\xi)} \left[1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2} \right], \quad (14)$$

where $x = 2\omega/\sqrt{s}$ is the fraction of the energy of the incident electron carried by the back-scattered photon, the maximum fraction of energy carried by the back-scattered photon is $x_{\max} = 2\omega_{\max}/\sqrt{s} = \xi/(1+\xi)$, and

$$D(\xi) = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2}\right) \ln(1+\xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1+\xi)^2}, \quad (15)$$

$$\xi = \frac{2\sqrt{s}\omega_0}{m_e^2}, \quad (16)$$

where m_e and $\sqrt{s}/2$ are the mass and energy of the electron, ω_0 is the laser photon energy. In our evaluation, we choose ω_0 such that it maximizes the backscattered photon energy without spoiling the luminosity through e^+e^- pair creation. Then we have $\xi = 2(1+\sqrt{2})$, $x_{\max} \simeq 0.83$, and $D(\xi) \simeq 1.84$, as used in Ref. [23].

The processes $\gamma\gamma \rightarrow t\bar{b}\pi^-$ ($t\bar{b}\pi^-$) occurs through the u- and t-channel involving charged top-pion bremsstrahlungs originated from different positions on quark lines. The Feynman diagrams are drawn in Fig. 3, but the corresponding diagrams with interchange of the two incoming photons are not shown.

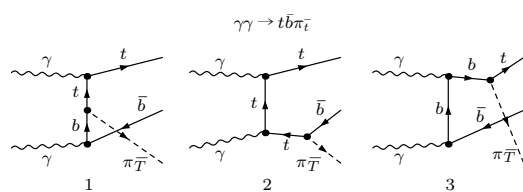


Fig. 3. Diagrams for $\gamma\gamma \rightarrow t\bar{b}\pi^-$.

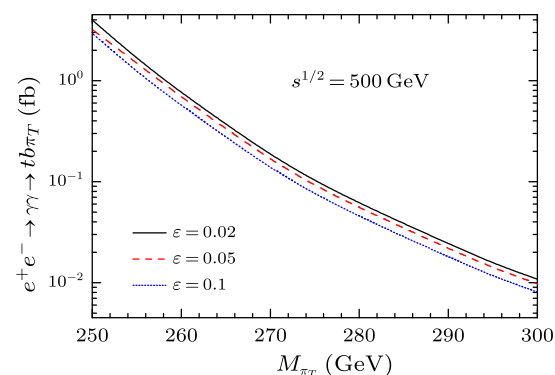


Fig. 4. Dependence of the cross section for $e^+e^- \rightarrow \gamma\gamma \rightarrow t\bar{b}\pi^-$ ($t\bar{b}\pi^-$) on the top-pion mass m_{π_T} at the ILC with energy of 500 GeV.

We show the cross section for $e^+e^- \rightarrow \gamma\gamma \rightarrow t\bar{b}\pi^-$ ($t\bar{b}\pi^-$) at the ILC with energy of 500 GeV as a

2.1.1 Lepton and heavy quark pair decays of the SM Higgs particle

In lowest order the leptonic decay width of the SM Higgs boson is given by [10, 37]

$$\Gamma[H \rightarrow l^+l^-] = \frac{G_F M_H}{4\sqrt{2}\pi} m_l^2 \beta^3 \quad (6)$$

with $\beta = (1 - 4m_l^2/M_H^2)^{1/2}$ being the velocity of the leptons. The branching ratio of decays into τ leptons amounts to about 10% in the intermediate mass range. Muonic decays can reach a level of a few 10^{-4} , and all other leptonic decay modes are phenomenologically unimportant.

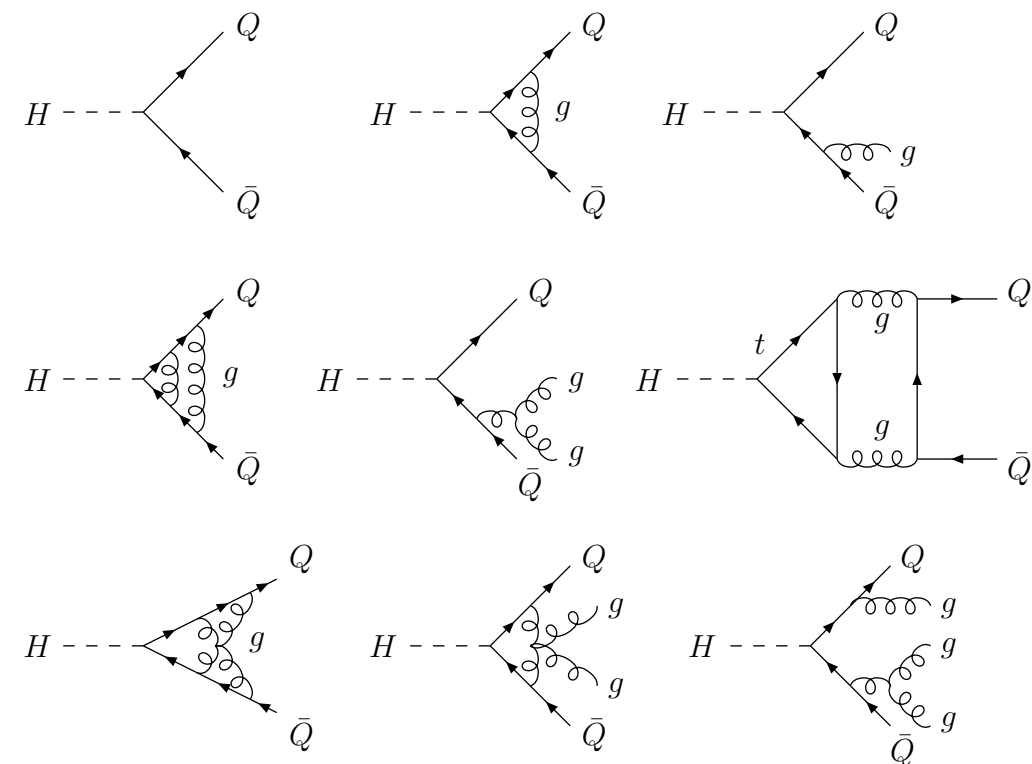
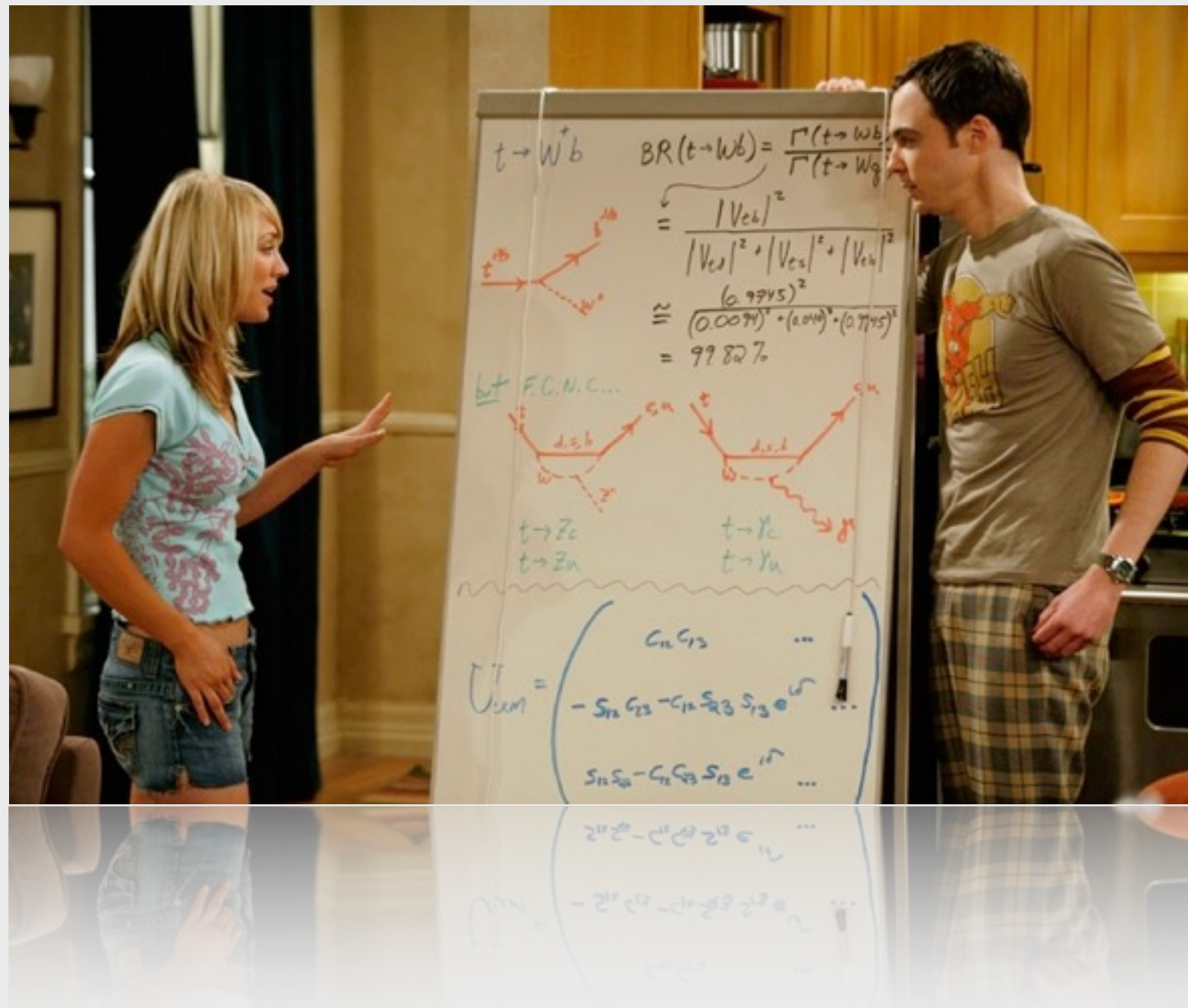


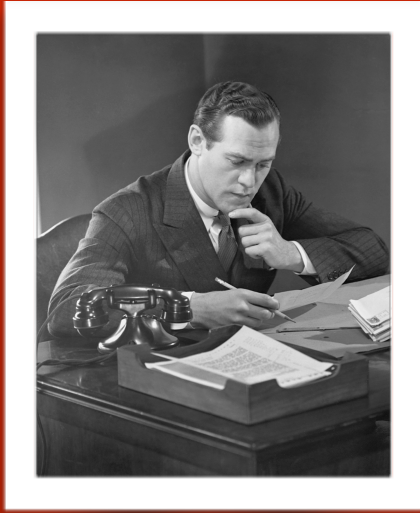
Figure 3: Typical diagrams contributing to $H \rightarrow Q\bar{Q}$ at lowest order and one-, two- and three-loop QCD.

For large Higgs masses the particle width for decays to b, c quarks [directly coupling to the SM Higgs particle] is given up to three-loop QCD corrections [typical diagrams are depicted in Fig. 3] by the well-known expression [38–40]

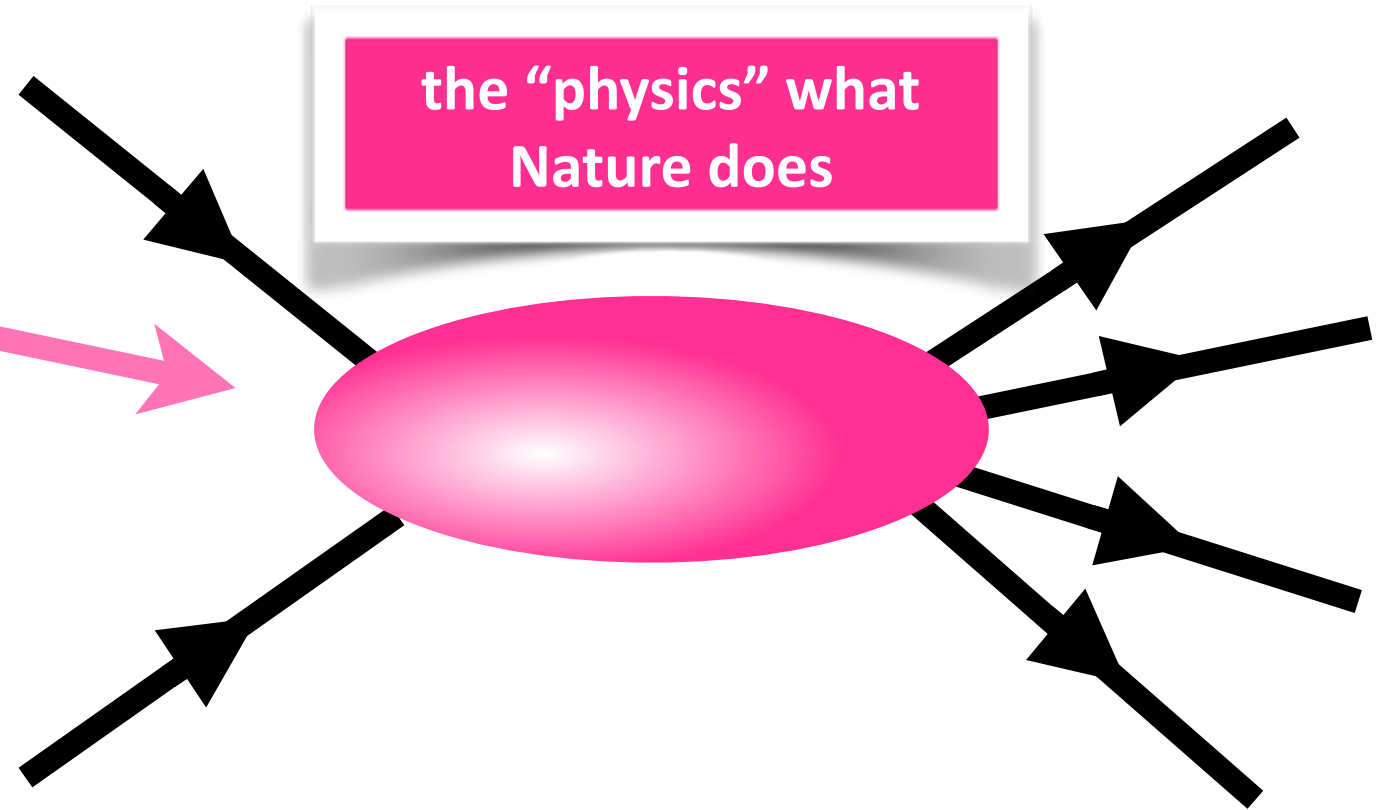
$$\Gamma[H \rightarrow Q\bar{Q}] = \frac{3G_F M_H}{4\sqrt{2}\pi} \bar{m}_Q^2(M_H) [\Delta_{\text{QCD}} + \Delta_t] \quad (7)$$

we really do use Feynman Diagrams





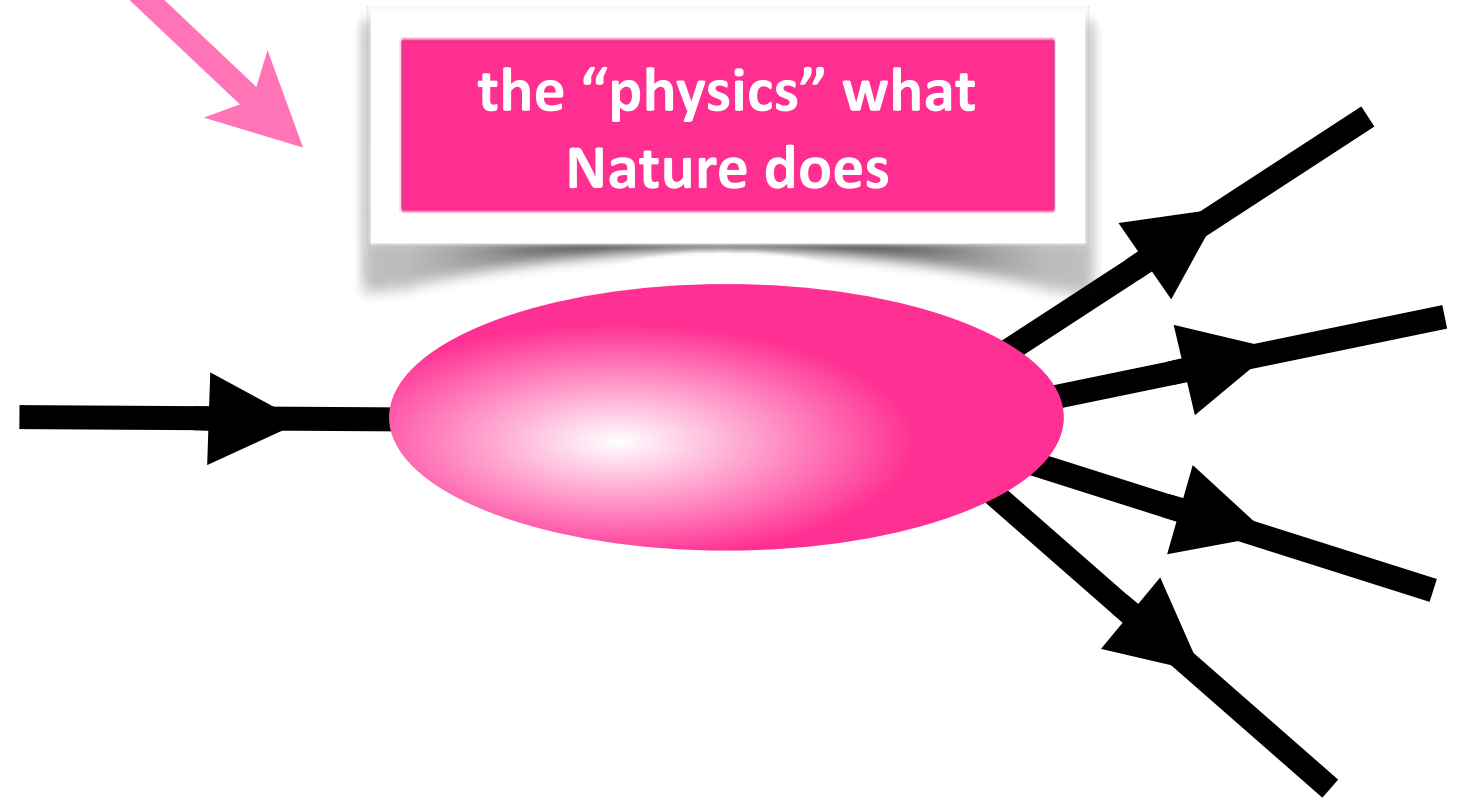
A collision
or
scattering event.



the "initial state"

the "final state"

A decay.



the "initial state"

the "final state"

always
the same
thing

initial "state"

final "state"

in-between

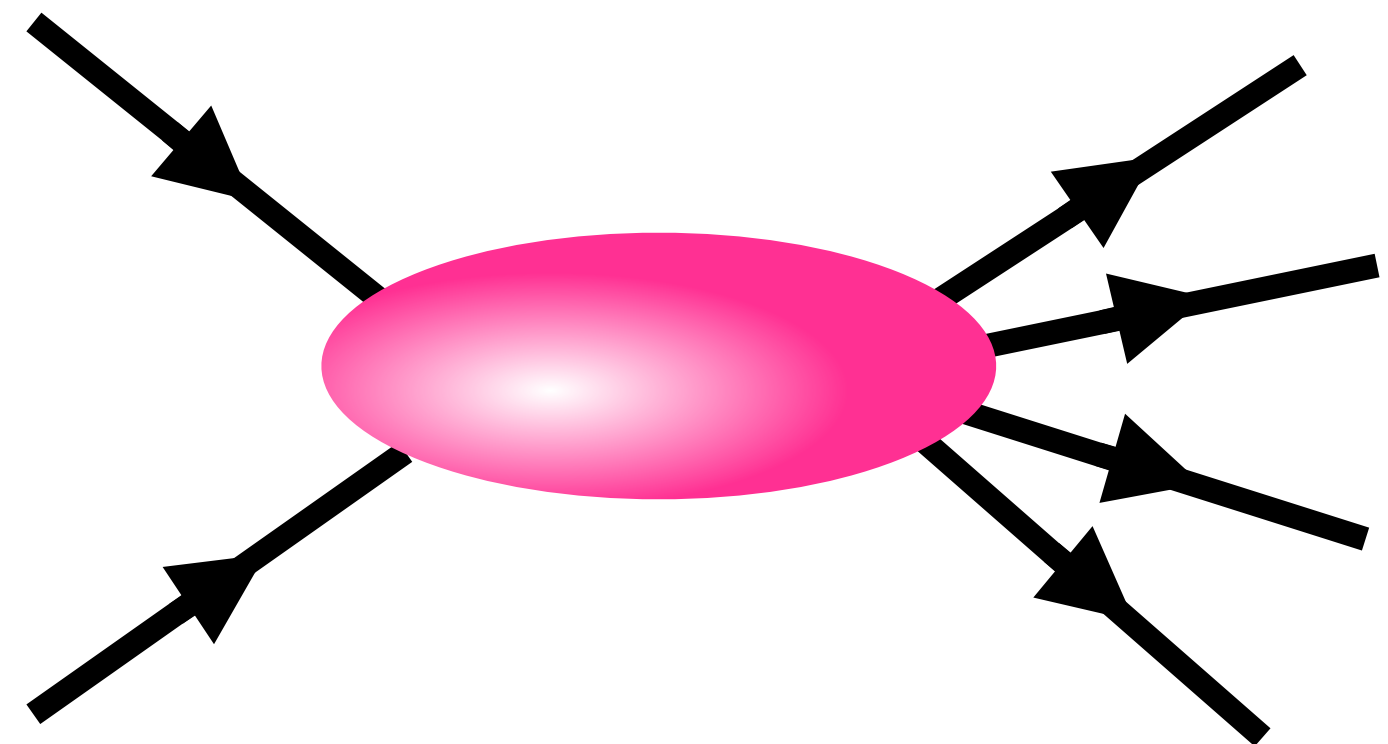
like cooking

probably lousy
metaphor

the "initial state"

the
chemistry of
cooking

the "final state"



the "initial state"

the "final state"

Feynman's approach is really sneaky and really cute

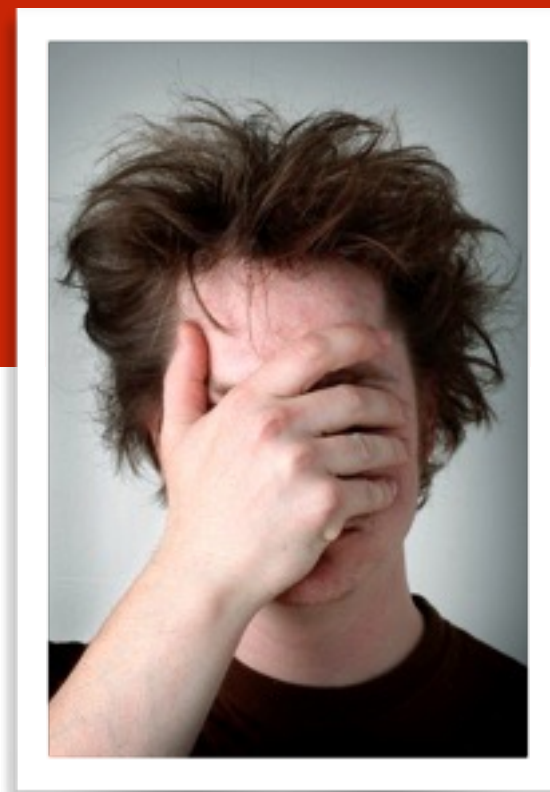
energy and time appear together in the equations:

In essence, this:

either energy solution: $(\pm E)(t)$

just the -E solution: $(-E)(t)$

move the - sign: $(E)(-t)$



Get a whole new interpretation of antimatter

antiparticles

can be interpreted as particles moving backwards in time.

that's it.

and visa versa

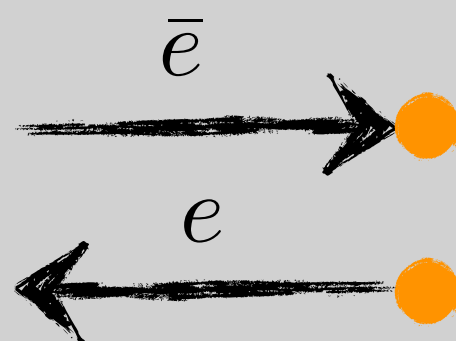
we'll do
this in

two steps

1. I'll show you how spacetime can be manipulated to predict new physical processes out of old ones

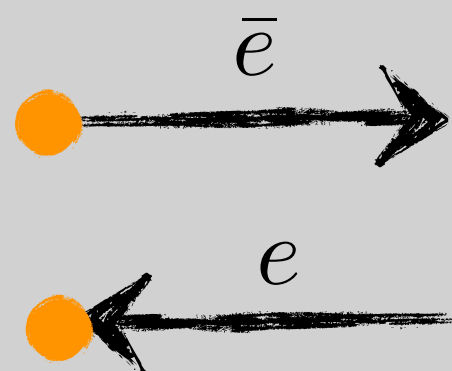
making use of the Feynman idea that antiparticles moving forward in time are the same as particles moving backwards in time

An anti-electron...coming **forwards into** an **initial** state:



is the same thing as
An electron coming
backwards out of an **initial** state

An anti-electron...coming **forwards out** of a **final** state:



is the same thing as
An electron coming
backwards into a **final** state

2. But *the vast majority of our use* will be to develop the handful (11) of “Primitive Diagrams” that we’ll put together like a puzzle

to predict all possible physical processes in the “Standard Model” of particle physics

jargon alert:

fermion

refers to:

any particle with half-integer spin

entomology:

from Fermi's theoretical work on the behavior of large numbers of Fermions

example:

electron, proton, neutron

jargon alert:

bosons

refers to:

any quantum object with integer spin

etymology:

from Satyendra Nath Bose, who worked on the effects of multiple boson aggregates

example:

photon, pion, Higgs Boson

the key

the different kinds
of lines

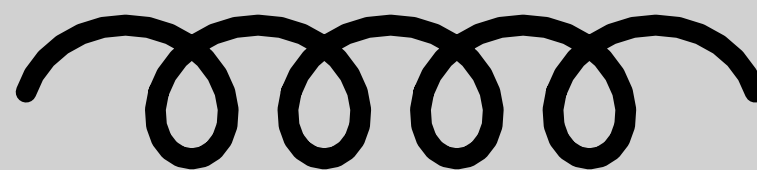
look at your Primitive
Diagram Sheet



fermion, spin $1/2$, e.g., electron



Vector Boson, spin 1, e.g., photon



gluon, spin 1



scalar Boson, spin 0, e.g., Higgs Boson

the first theory of Feynman's

"Quantum Electrodynamics" or "QED"

the full theory of the physics of photons and electrons

strap in

with pencil in hand

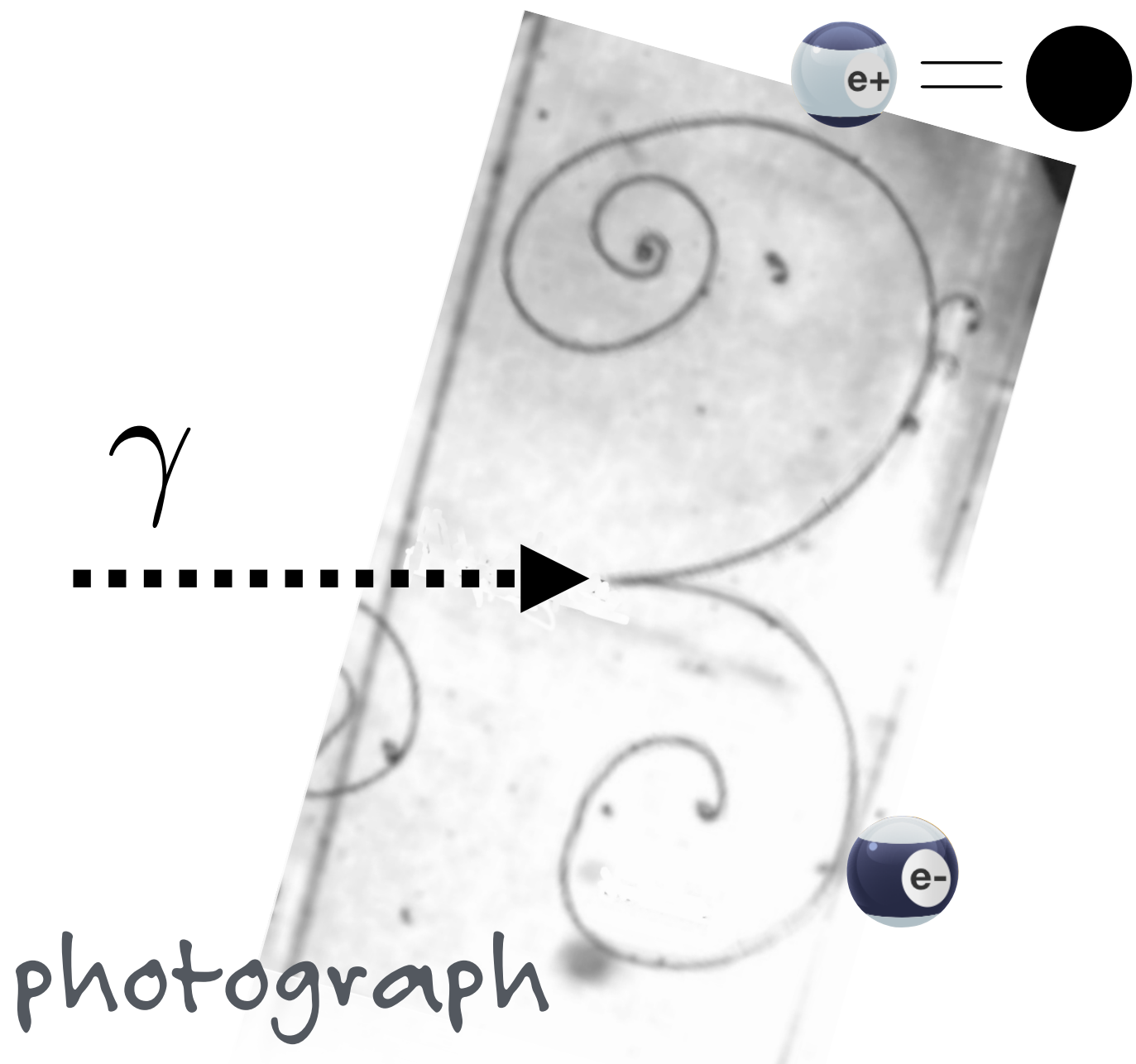
first idea

one can take a single Feynman Diagram that describes a
process

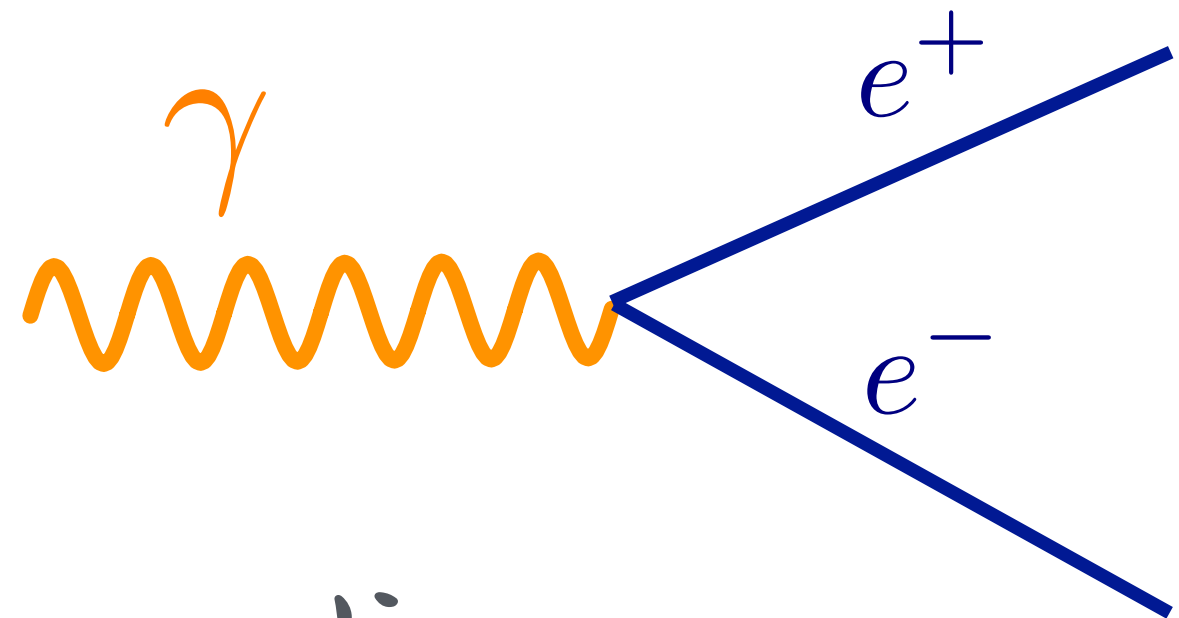
and by rearranging it in spacetime, "predict" additional physical
processes

Dirac's story

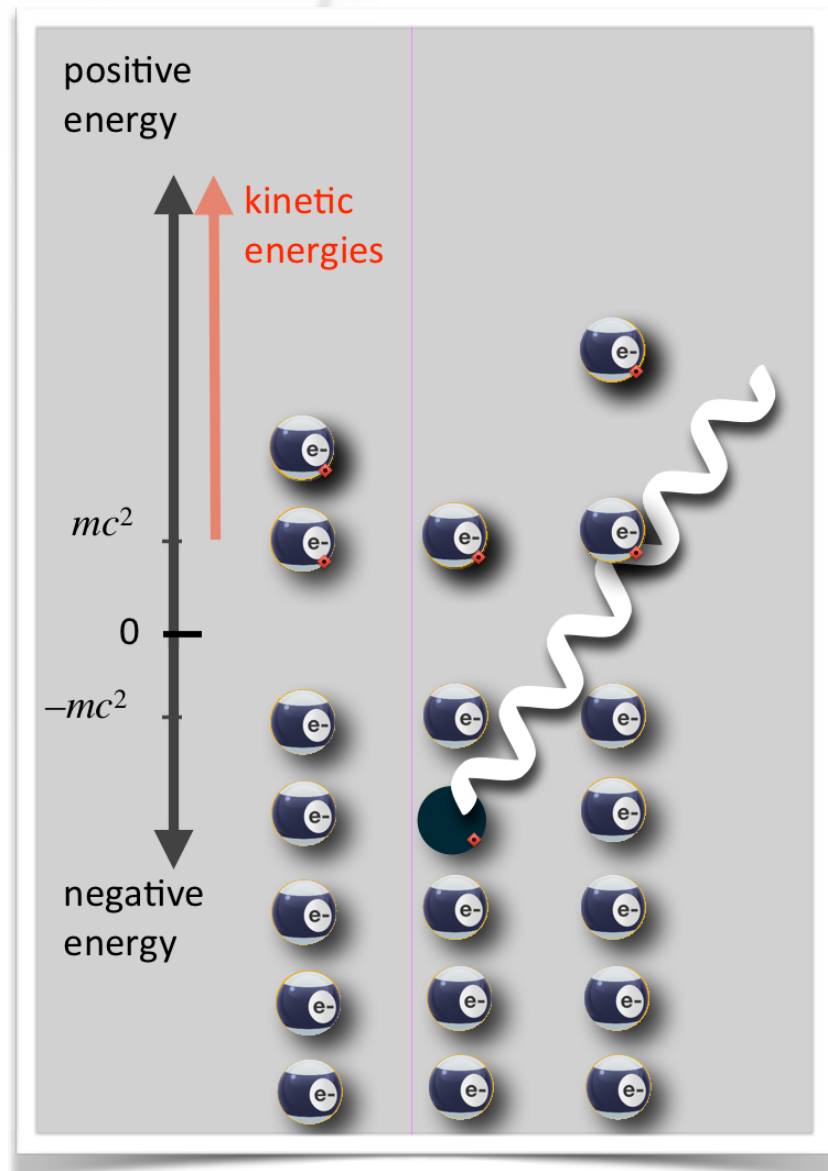
& Feynman's picture



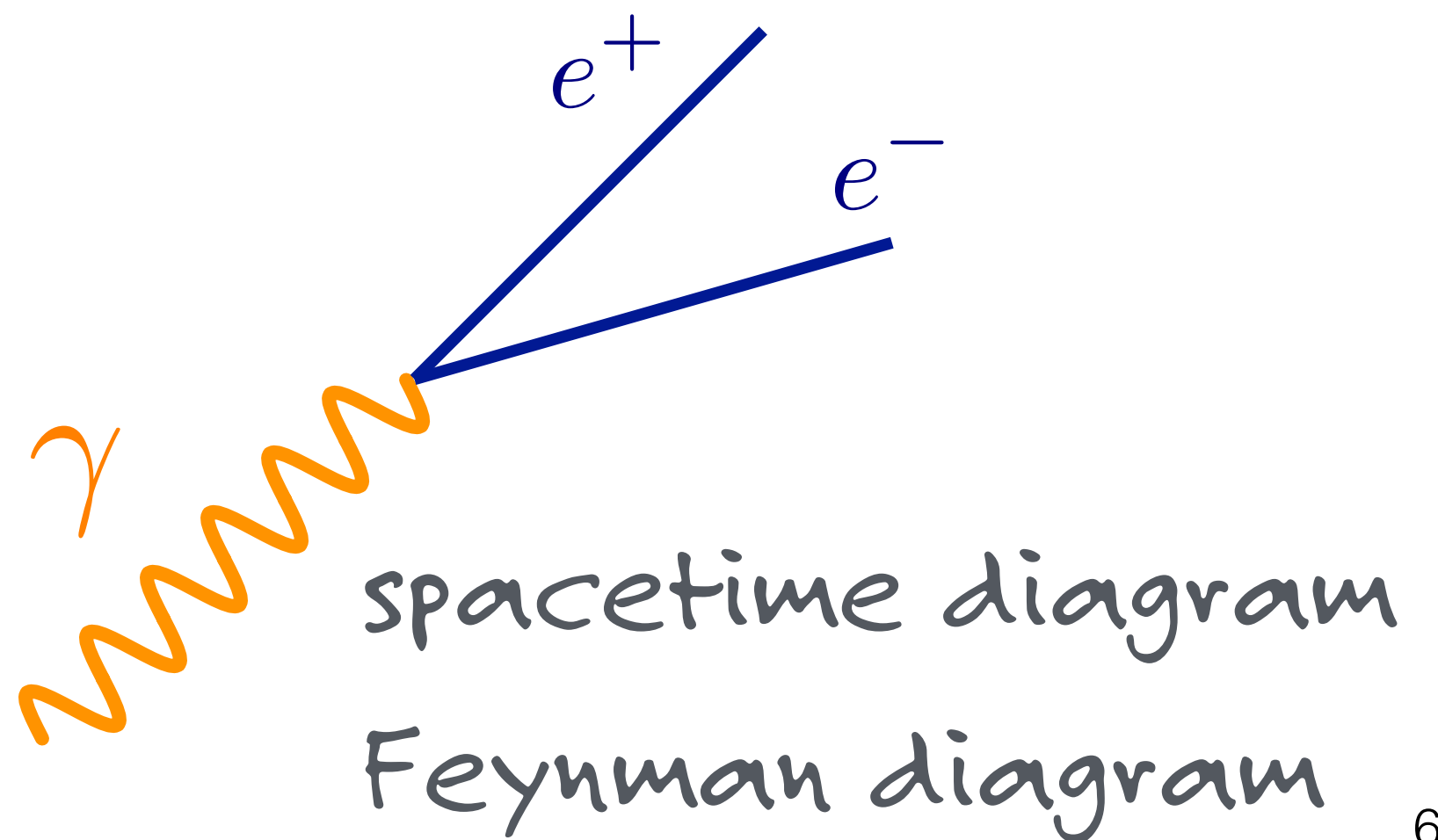
photograph



space diagram



cartoon

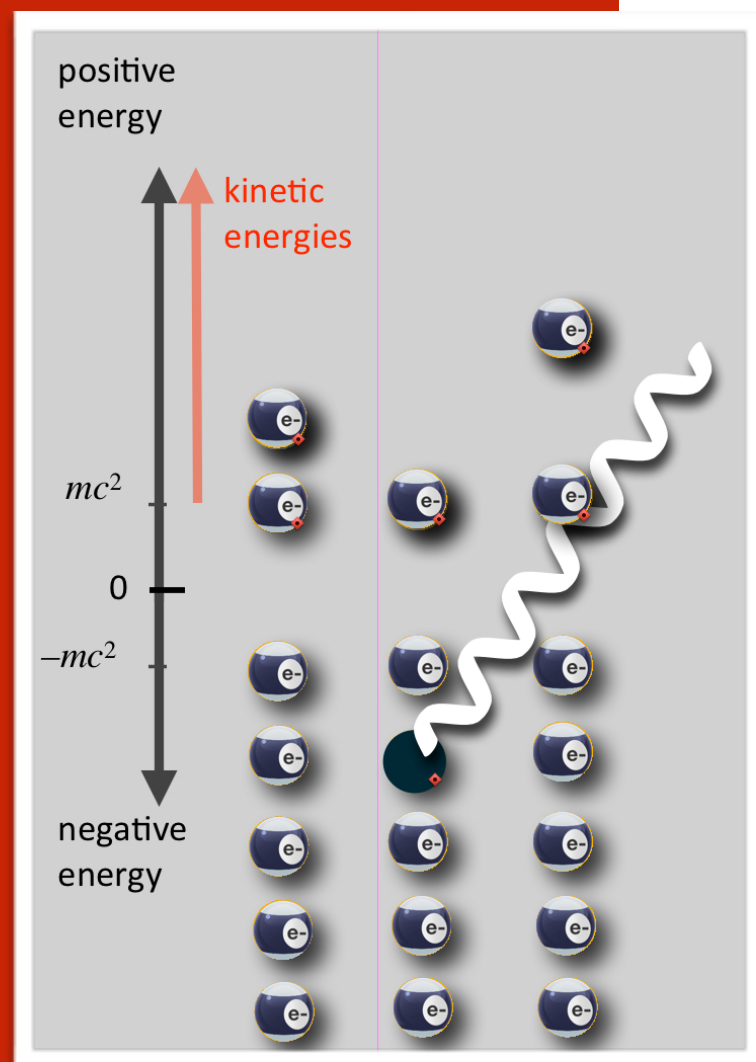


spacetime diagram
Feynman diagram

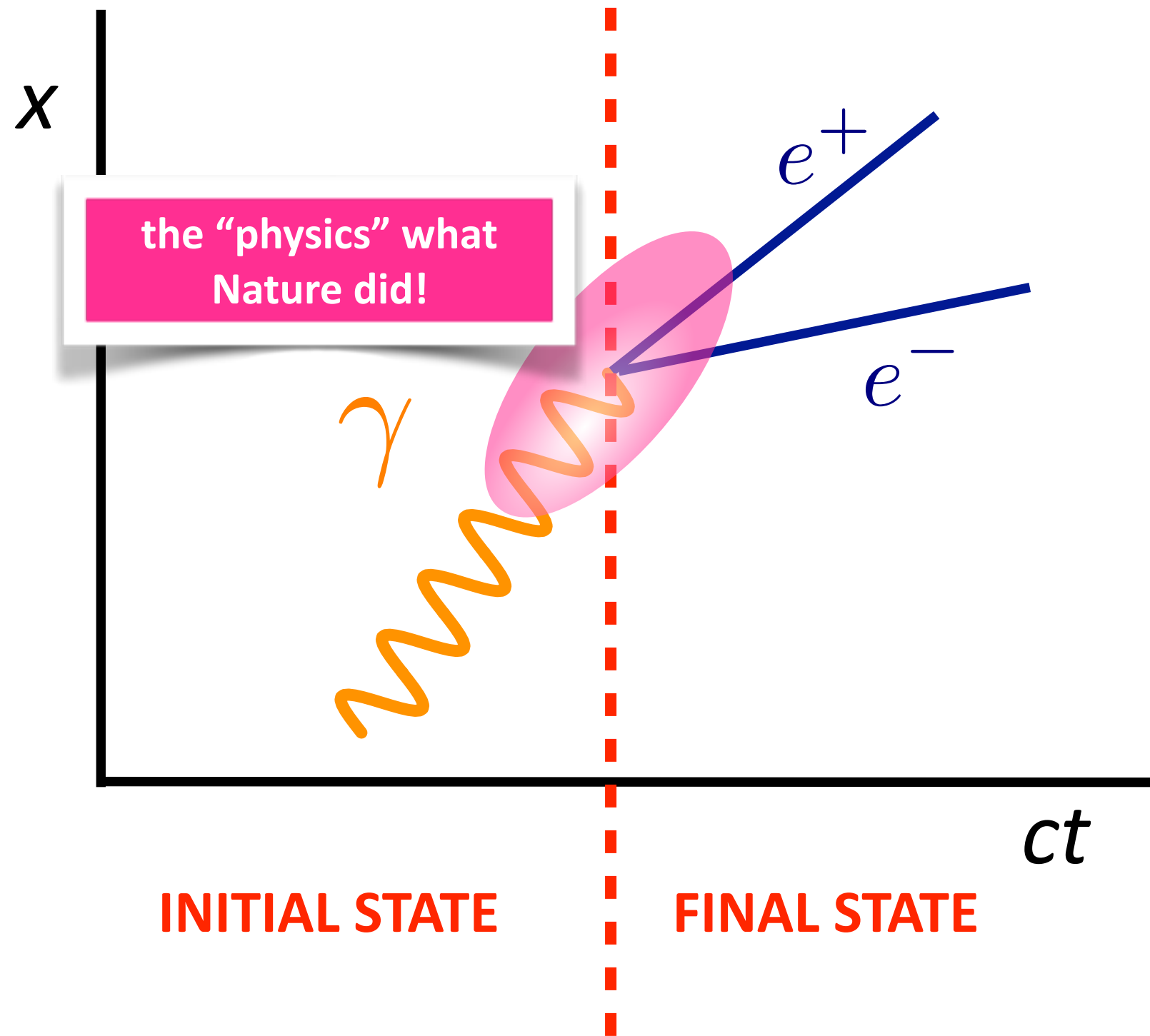
Dirac had photons creating an electron

Feynman's calculus allows that

and more



The Dirac hypothesis is called "Pair Production": photon in, electron & positron out



the "physics" what Nature did!

Now, remember that we treat ct and x identically...

The physics does not care which orientation is which.

note:

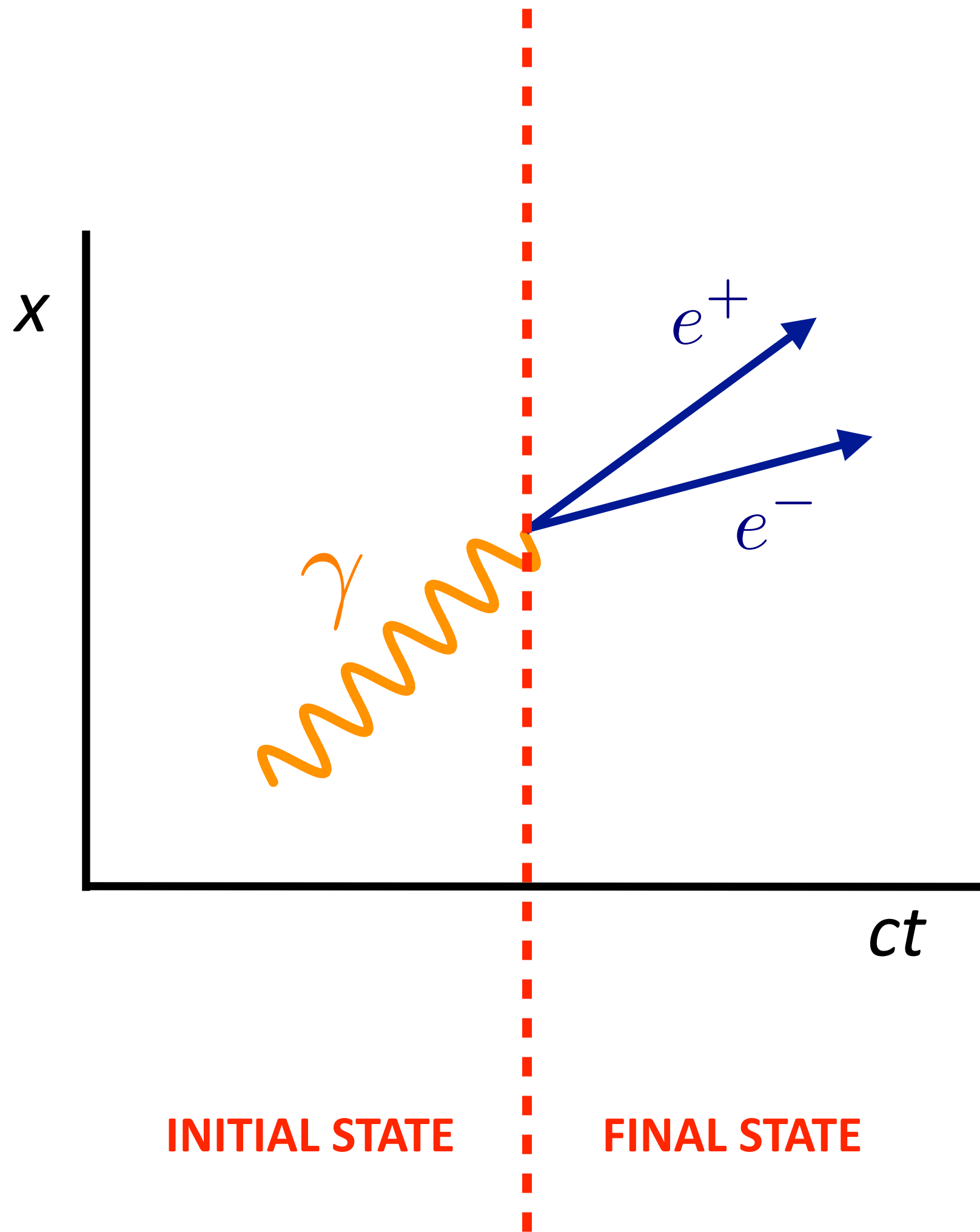
I've been banging on you to keep the slopes right

you know, photons have slope associated with c

We'll relax that now.

can always
rotate any
Feynman
Graph

and get a new one



BUT

We don't deal with particles moving backwards in time
when it happens...we fix it!

Feynman's trick

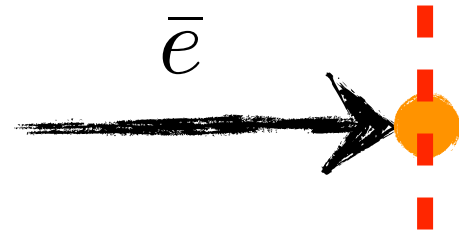
depends on the in and out states.

if some manipulation leaves you with particles going the "wrong" direction?

fix it.

particles in time

An anti-electron...coming **into** an **initial** state to a node:



is the same thing as

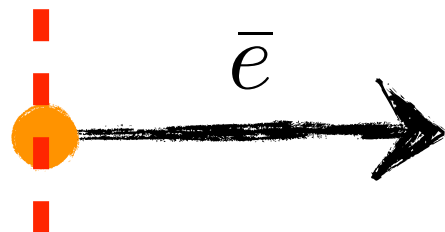
An electron coming **out** of an **initial** state (?)



Yes, this makes sense

Nope, this makes no sense...time-backwards

An anti-electron...coming **out** of a **final** state:



is the same thing as

An electron coming **into** a **final** state (?)

Yes, this makes sense

Nope, this makes no sense...time-backwards

Feynman had rules

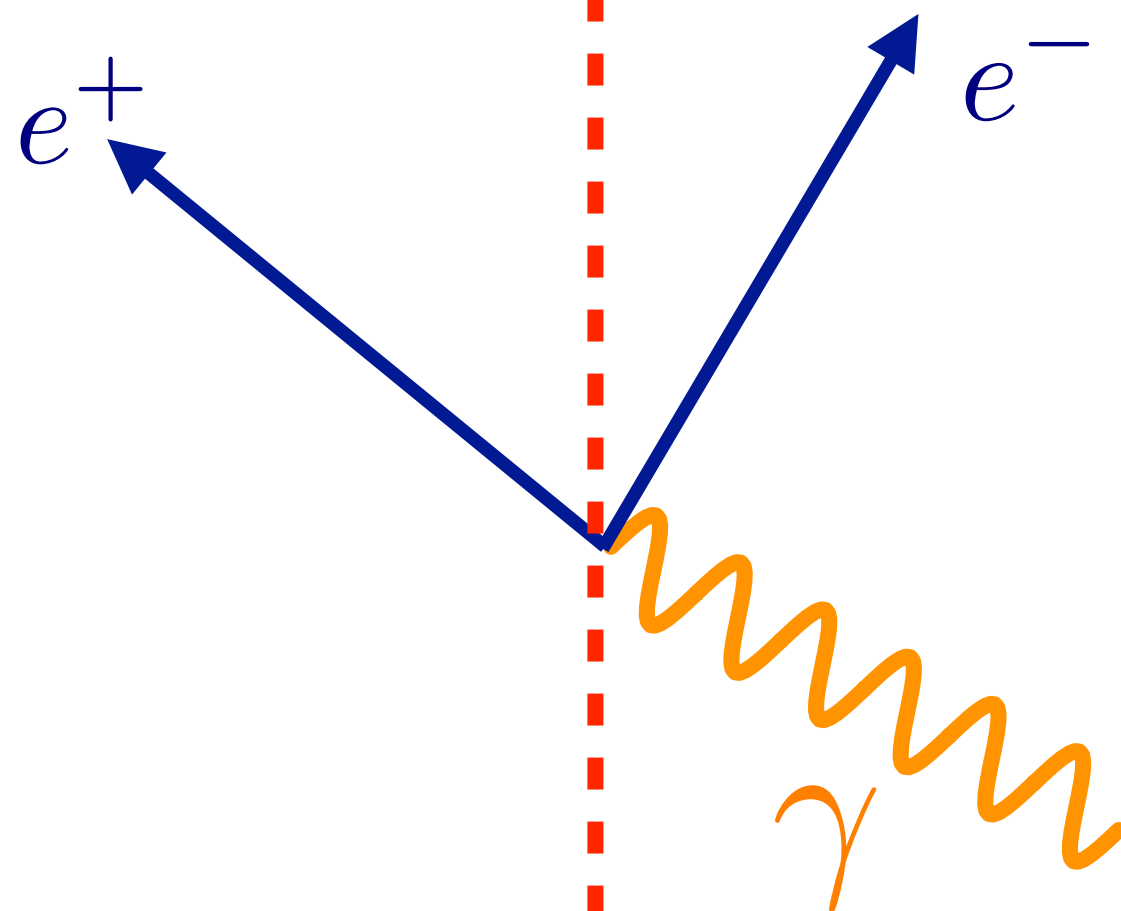
We'll have slightly different rules

but similar in spirit

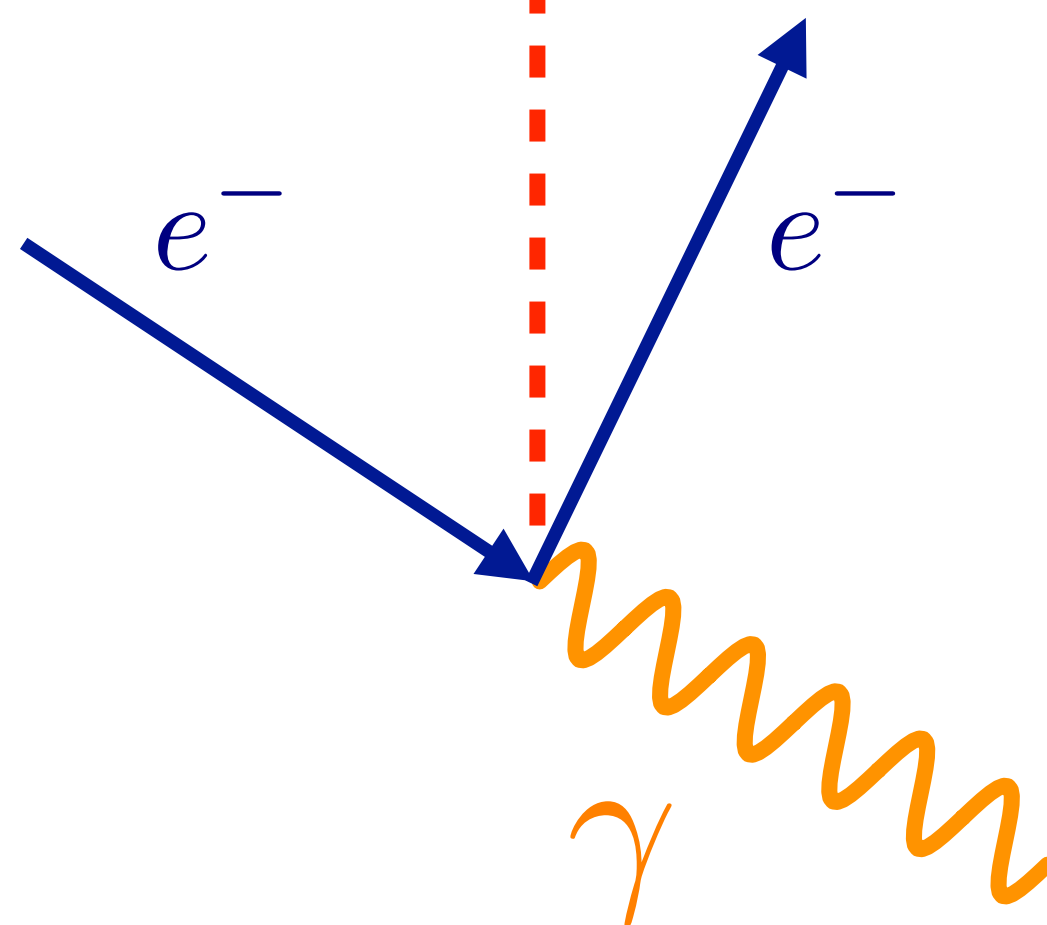
Rule 1.

If you flip a line's arrow forward or backward in time, you change the particle to antiparticle or antiparticle to particle

my rotated diagram...
spread out:



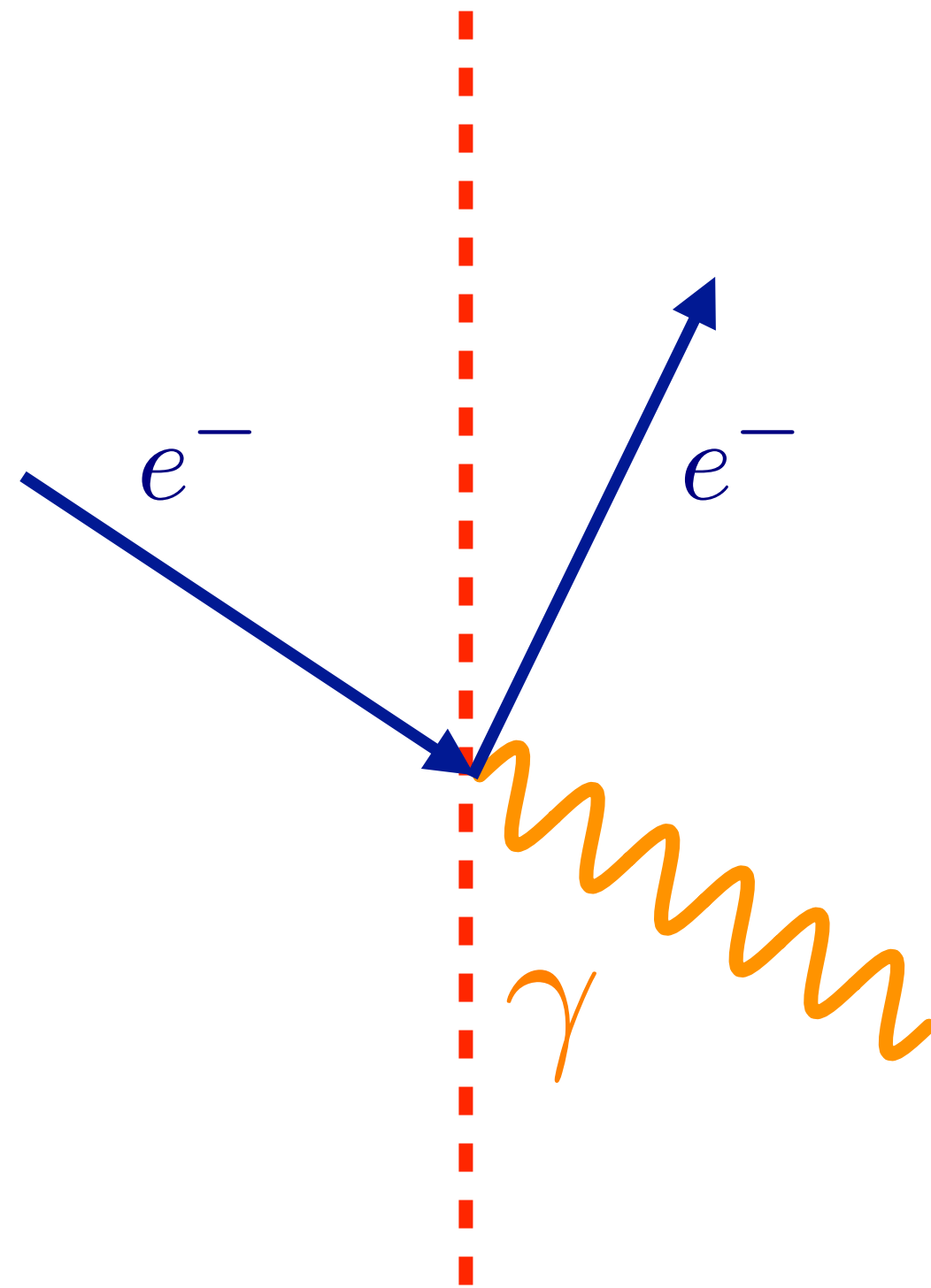
this is the same thing as:



look at
this

you know this.

familiar:



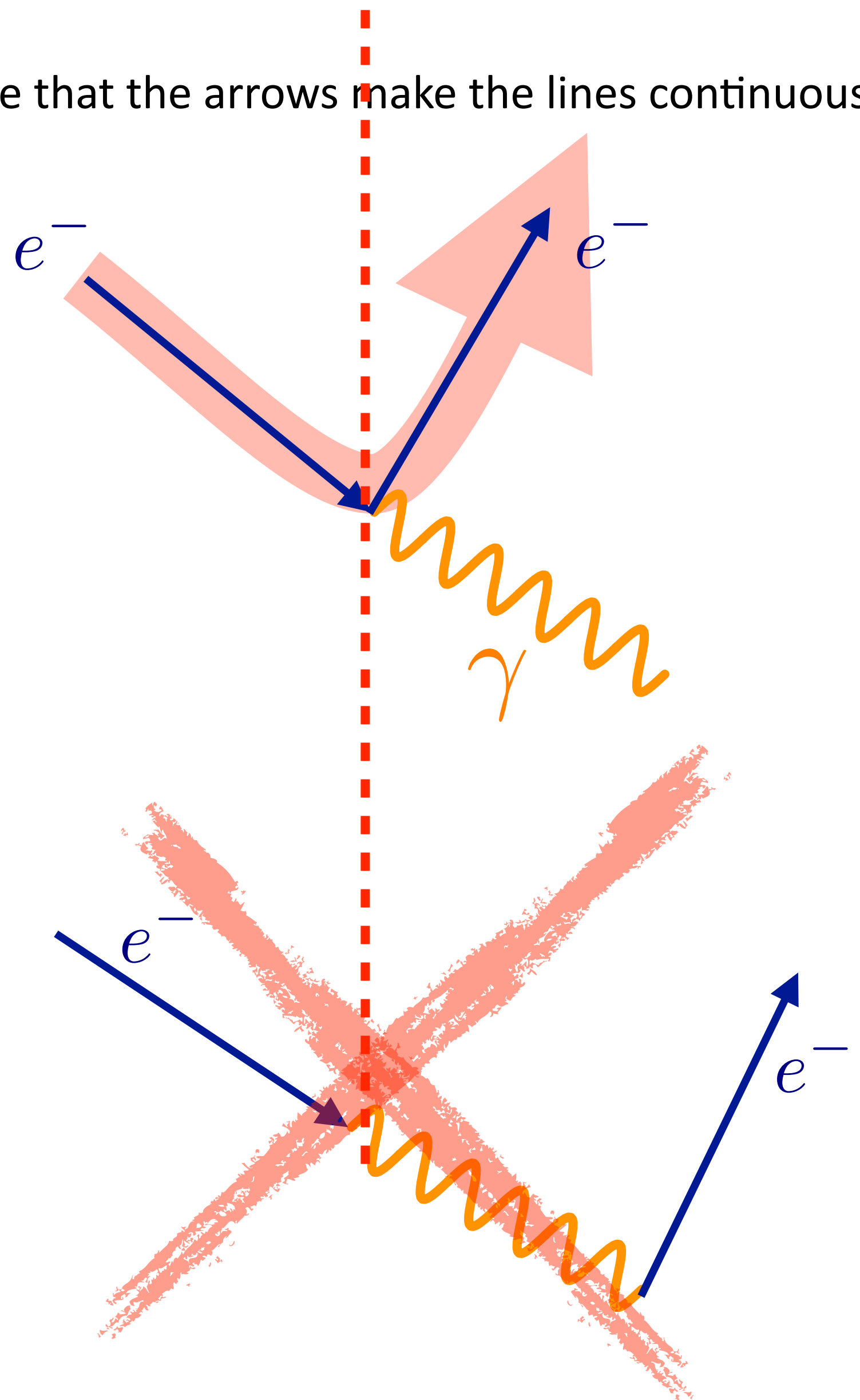
electron comes along and spits out a photon, recoils
and goes on its way

regular old radiation

Rule 2.

fermion lines
must be
continuous

notice that the arrows make the lines continuous



now, let's do it in real-time

repeating a bit, so you can write

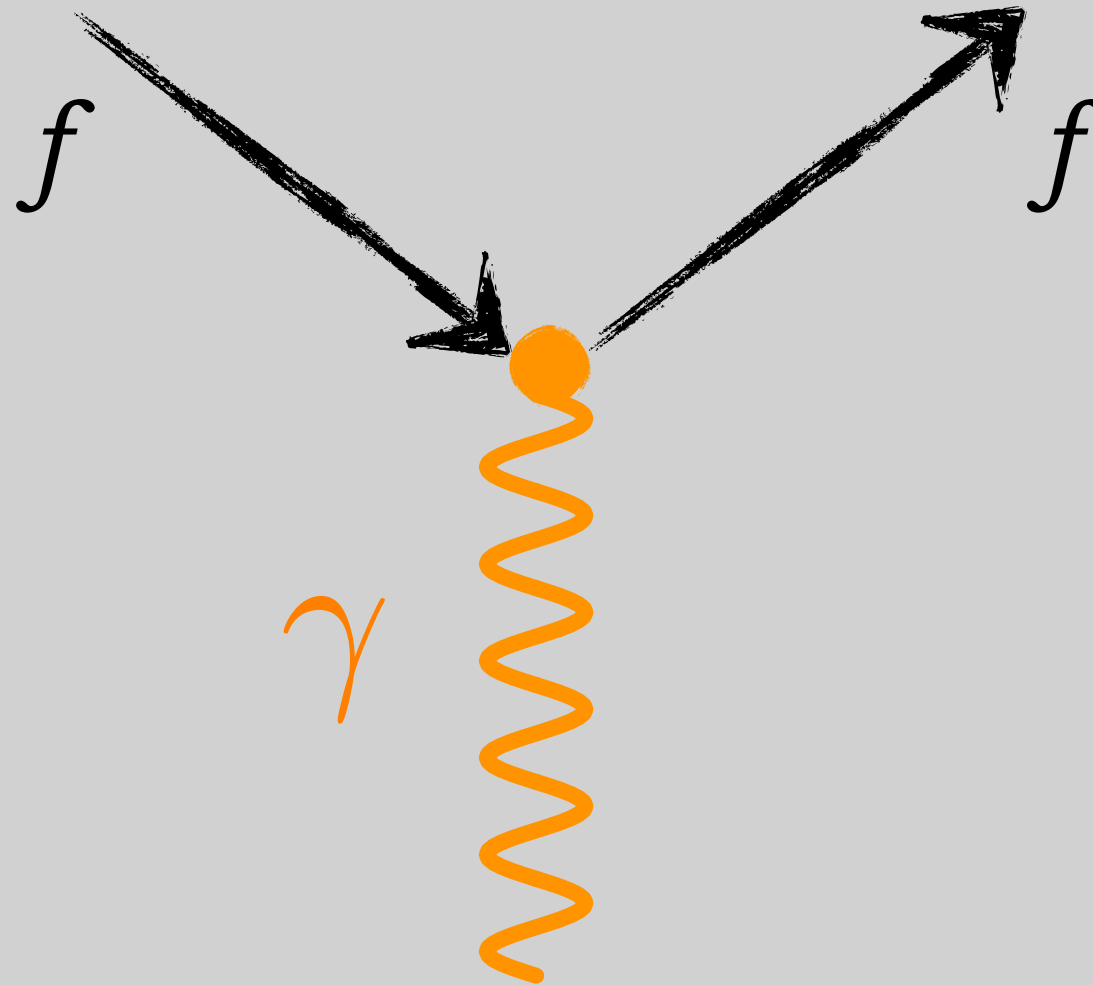


(first)

primitive diagrams

are general

but this is completely general...for any charged fermion:




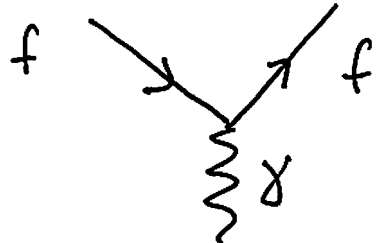
f could be electron, positron, proton, antiproton...and more – any electrically charged **f**ermion.

Their diagrams are identical.

Primitive Diagram Scorecard

your first entry

Primitive Diagrams TIME always: 

1			QED
2		3	Weak Interactions
6		7	
4		5	Strong Interactions
8		9	Higgs Interactions
10		11	

fermion, spin 1/2, e.g., electron Vector Boson, spin 1, e.g., photon gluon, spin 1 scalar Boson, spin 0, e.g., Higgs Boson

