This is a rougher draft than the other chapters…and incomplete at the end.

Chapter 9

Newtonian Gravitation

the lion roars*

Galileo Galilei, by Justus Sustermans, 1637.

∗ In 1697, a calculus-based mathematical competition was held throughout Europe. Newton was long out of physics, but entered anonymously. Noted the sponsor of the challenge, "we recognize the lion by his claw."

Galileo Galilei, 1564 -1642

""They know that as to the arrangement of the parts of the universe, I hold the sun to be situated motionless in the center of the revolution of the celestial orbs while the earth rotates on its axis and revolves about the sun. They know also that I support this position not only by refuting the arguments of Ptolemy and Aristotle, but by producing many counter-arguments; in particular, some which relate to physical effects whose causes can perhaps be assigned in no other way."" *Letter to Grand Dutchess Christina, 1615.*

Physics got real with Galileo's telescopic discoveries. Everyone knows the highlights of the Galileo story and his embarrassment at the hands of Pope Urban VII. The real story is perhaps different from the urban legends. As in his experiments on terrestrial motion, his conclusions on the moon's and planets' motions were more descriptive than causal. They "why"—the dynamics—of the cosmos was left to Isaac Newton to figure out. His Gravitational Model was so successful, that in the space of his lifetime, Europe went from ignorant of how Nature worked, to believing that everything can be known. The Enlightenment itself owes much of its origins to Newton's work.

9.0.1 Goals of this chapter:

- Understand
  - How to calculate the gravitational force between two masses.
  - How to calculate the weight of any object on any planet.
  - How to calculate gravitational potential energy.
- Appreciate
  - How Galileo's astronomical discoveries charted new ground in astronomy.
  - How Galileo's approach to science laid groundwork for the modern version.
  - Newton's argument regarding the Moon and the Apple.
  - What being in orbit implies about "falling."
  - Escape velocity.
- Be familiar with
  - The later lives of Galileo and Newton.
  - The importance of Galileo's Letter to the Dutchess Catherine.

9.1 A Little Bit More of Galileo

Figure 9.1: One of Galileo's original telescopes. *Museo Galileo*

When we last left Galileo, he was in Padua working out the correct understanding of falling bodies and projectiles. He didn't publish that work until he was under house arrest in his villa outside of Florence and had to smuggle it out of Italy to the Netherlands. How his arrest-story came about is legendary, and not necessarily how most people imagine it. First, some more science, then some of the back-story to his troubles with the Inquisition.

Galileo had been in Padua for 16 years when in May of 1609 he heard of a novelty that was being sold in France, Germany, England, and the Netherlands where it was invented. This was of course the telescope. Remember that he was good with with his hands and eventually employed an instrument maker. Together, within a month, from only a word of mouth description, he was able to grind and polish lenses and construct his own telescope. It was just 3x magnification, not as good as what was "out there." But he persisted in his technique and built 8x, 15x, and 30x versions Figure 9.1 shows one of his first prototypes from the science museum in Florence, Italy. He used it to look across the land and water and then...he looked up.

In August of 1609 he took his then 8x version to Venice and demonstrated it to the intellectual community, and also politicians. Remember, Galileo always had his family's debts on his mind and he gave an exhibition from the top of the St. Marco tower and showed that one could see ships much further away than with the naked eye.[1] Venice, was a maritime power and sometimes the target of naval attack from the East and so the Venetian Senate had a vested interest in this new early warning system. They were impressed, doubled his salary, and awarded him lifetime tenure at the university. . . but also froze his salary at the new level. Although he was now one of the most highly paid professors in the Venetian Republic, the prohibition of any raises for the rest of his life didn't sit well with the ambitious 45 year old.

[1] Famously, an Aristotelian philosopher, Guilio Libri refused to look through the telescope. We'll learn that Galileo's mouth often got him in trouble and he suffered fools badly. Libri died soon after the incident and Galileo remarked that now he could see Jupiter's moons as he passed by it on his way to heaven.

9.1.1 What Galileo Saw!

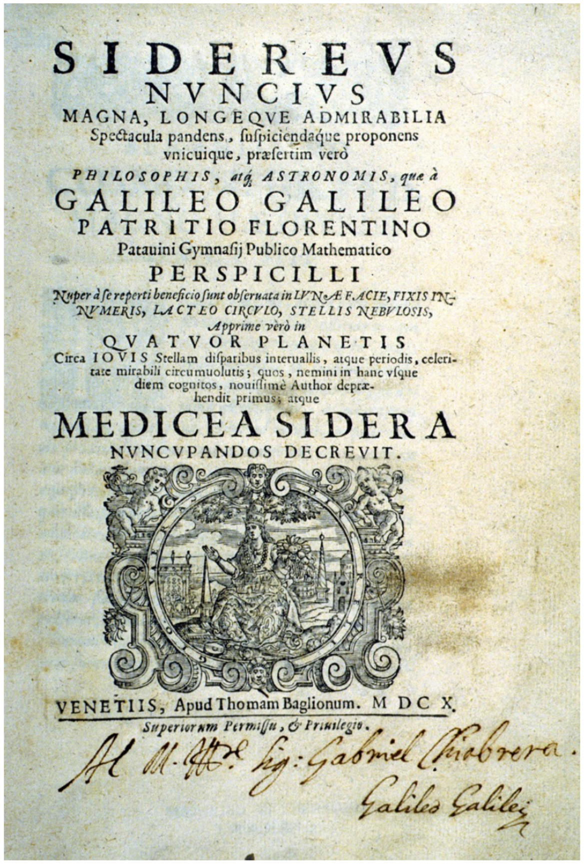
Through the next year Galileo observed things that nobody had previously imagined. In November and December of 1609 he carefully studied the Moon and with his excellent artistic abilities, drew detailed images showing the mountains and craters. This was revolutionary because the Aristotelian model required the celestial bodies to be perfectly unblemished spheres. By carefully mapping the shadows of the Moon, Galileo estimated the height of crater edges and found them to be Earth-like in size. Then a month later he studied Jupiter and found what looked like three bright stars, all in a line. He continued looking in successive nights and saw a fourth "star" peek out from behind the planet and found all four of them to be moving together! Subsequent observations convinced him that they were bound to Jupiter, and not stars at all: Jupiter has moons which today we call the Galilean Moons. And, when he looked into deep space the stars multiplied. He found hundreds of stars that nobody had ever seen before.

It's important to realize that Galileo did not invent the telescope (one of those persistent myths) and he was not the first to use it to discover things in the sky. A British natural philosopher, Thomas Harriot, was first to observe many of the things that are credited to Galileo. Harriot was not as self-promoting nor did he publish as quickly as Galileo, so he lost his historical moment.

In 1611 he published *Sidereus Nuncius*, or *Starry Messenger*, reporting these and other revolutionary observations and interpretations. Figure 9.2 shows the elaborately constructed title page and Fig. 9.3 shows a few pages from the text. When he was at Pisa, he had become friendly with the Medici family, especially the the Grand Dutches Christina. A few summers while he was in Pisa he was brought back by her to Florence in order to tutor her young son, the young Cosimo d'Medici. His bold naming of the moons after his former student—by then the reigning Duke of Tuscany—and his dedication of *Sidereus Nuncius* to him was an obvious ploy to again improve his circumstances and to get a new job without teaching responsibilities.[2]

[2] "...scarcely have the immortal graces of your soul begun to shine forth on earth than bright stars offer themselves in the heavens, which, like tongues [longer lived than poets] will speak of and celebrate your most excellent virtues for all time." A tad syrupy perhaps?

Figure 9.3 shows his sketches of multiple nights' viewing of the Medician Moons. What he had found was a miniature Copernican system within the bounds of our own solar system. Further, *Siderius* described his discovery that Venus had phases like the Moon which explained why it appeared to change brightness periodically, just as Copernicus had predicted. And finally, the number of stars visible with the telescope dwarfed what everyone believed was the full compliment of stars that had been carefully tallied by the Babylonians, Greeks, and Tycho. The universe appeared to be a much more interesting place than anyone had imagined.


SIDEREVS
NVNCIVS
MAGNA, LONGEQVE ADMIRABILIA
Spectacula pandens, suspiciendaque proponens
vnicuique, præsertim verò
PHILOSOPHIS, atq; ASTRONOMIS, quæ à
GALILEO GALILEO
PATRITIO FLORENTINO
Patauini Gymnasij Publico Mathematico
PERSPICILLI
Nuper à se reperti beneficio sunt obseruata in LVNÆ FACIE, FIXIS IN-
NVMERIS, LACTEO CIRCVLO, STELLIS NEBVLOSIS,
Apprime verò in
QVATVOR PLANETIS
Circa IOVIS Stellam disparibus interuallis, atque periodis, celeri-
tate mirabili circumuolutis; quos, nemini in hanc vsque
diem cognitos, nouissimè Author depræ-
hendit primus; atque
MEDICEA SIDERA
NVNCVPANDOS DECREVIT.

VENETIIS, Apud Thomam Baglionum. M DC X.
Superiorum Permissu, & Priuilegio.

THE HERALD
OF THE STARS
unfolding
GREAT, and HIGHLY ADMIRABLE
Sights, and presenting to the gaze
of everyone, but especially
PHILOSOPHERS, and ASTRONOMERS,
those things observed by
GALILEO GALILEI
PATRICIAN OF FLORENCE
Public Mathematician of the University of Padua
with the aid of a
TELESCOPE
which he has recently devised on THE FACE OF THE MOON, IN-
NUMERABLE FIXED STARS, THE MILKY WAY, CLOUDLIKE STARS,
and especially concerning
FOUR PLANETS
revolving around the star of JUPITER with unequal intervals and periods,
with wonderful swiftness, which, known to no-one up to this
day, the Author most recently dis-
covered for the first time; and
DETERMINED TO NAME
THE MEDICEAN STARS

Figure 9.2: The title page of *Sidereus* from a copy in the University of Oklahoma science library. The translation is from a 19th century translation. Notice that Galileo signed this copy.

The effect of all of this news electrified Europe and overnight, Galileo became famous, and remained so for the rest of his life. The good news? He got the job back in Florence. And the bad news: Florence was within the sphere of influence of Rome and the Pope. In fact, there had been a number of Medici popes in the family. Venice was much more liberal and was often at odds with the Vatican over one or another issue.[3] Galileo's. . . unusual views. . . were safe in Venice, but dangerous in Florence.

He negotiated the position that not only paid well, but also importantly, raised his stature: he was The Chief Mathematician of the University of Pisa and Philosopher and Mathematician to the Grand Duke. The last title was important, for it was Philosophers who ruled the academic roost and mathematicians

[3] Pope Clement V excommunicated the entire population of Venice in 1309! Interdicts—forbidding any ecclesiastical functions were instituted against Venice in 1202, 1284, 1480, 1509, and again in 1609.

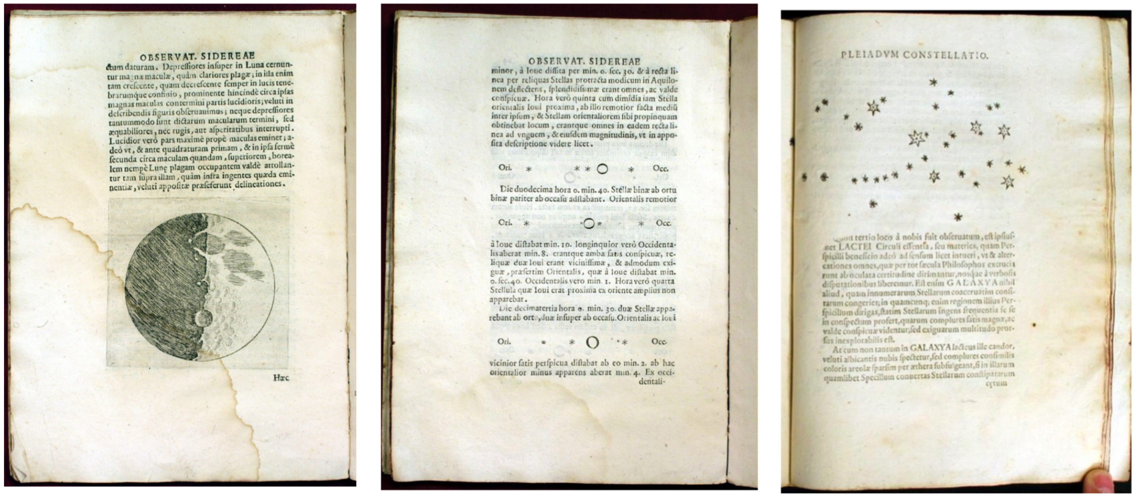


Figure 9.3: Three of the many sketches in Sidereus Nuncius. The Moon picture is famous and meticulous. The middle drawing is one of many documenting the motion of Jupiter's four moons orbiting the planet. The right figure is his sketch of the Pleiades constellation with its seven ("seven sisters") stars and then all of the new ones visible through his telescope.

were the least respected. Galileo insisted on this dual, contradictory title. He took multiple victory laps in Rome where he was celebrated by the College of Jesuits and where the then Cardinal Barberini took great pleasure in Galileo's friendship. The 47 year old was riding high.

The Moon has a rough surface with mountains and valleys. *Key Observation 7*

Other planets in our solar system have moons that according to Kepler's model. *Key Observation 8*

Figure 9.4: Dominican friar, Tommaso Caccini, raised the first public attack on Galile from this pulpit in Santa Maria, Novella in Florence in 1614.

In years to come, Galileo studied many things and wrote books on Sunspots (He learned to train his telescope on the Sun, but a student taught him to project the image onto a piece of paper so that he would not damage his eyes. The result was another kind of blemish in a heretofore perfect celestial sphere: sunspots.) and buoyancy. Here he began to get himself in trouble as a respected Jesuit competitor disagreed with him on the origin of sunspots (were they just another set of planets?) and buoyancy…what caused things to float. In both cases, Galileo was over the top and *ad hominem* in his nasty criticisms of his scientific adversaries. Ths cost him support among some of his Jesuit colleagues.

The Sun has blemishes on its surface that change in time. *Key Observation 9*

9.1.2 The Most Famous Letter in the History of Science

In 1614 Galileo was denounced by name from the pulpit (Fig. 9.4)of Santa Maria Novella—in Florence—by a conservative Dominican priest. He had begun to be suspected of heresy—was formally denounced to the Inquisition in 1615—and there was a growing unhappiness with him within the most doctrinaire of the Church's hierarchy. He reacted in what was to become the Galileo-way: a strong defense is always a strong offense.

In 1615 Galileo circulated a long, open letter[4] to the Grand Dutches Christina purporting to explain a debate at a meal that he was not at, but where his views were the topic of discussion.[5] It's worth quoting in length, for it forms the rallying cry of the new approach of Natural Philosophy as it morphs into a real scientific attitude:

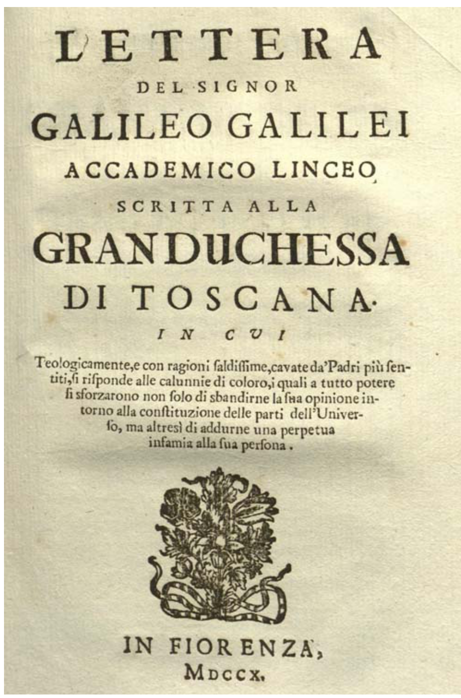
[4] http://inters.org/galilei-madame-christina-Lorraine

[5] He admits that he was influenced by the bumper-sticker comment of Cardinal Baronies: "The intention of the Holy Ghost is to teach us how one goes to heaven, not how heaven goes."

> Some years ago as Your Serene Highness well knows, I discovered in the heavens many things that had not been seen before our own age. The novelty of these things, as well as some consequences which followed from them in contradiction to the physical notions commonly held among academic philosophers, stirred up against me no small number of professors–**as if I had placed these things in the sky with my own hands in order to upset nature** and overturn the sciences. . .

> Showing a greater fondness for their own opinions than for truth, they sought to deny and disprove the new things which, if they had cared to look for themselves, their own senses would have demonstrated to them. To this end they hurled various charges and published numerous writings filled with vain arguments, and they **made the grave mistake of sprinkling these with passages taken from places in the Bible which they had failed to understand properly**.

He's just getting warmed up:



LETTERA
DEL SIGNOR
GALILEO GALILEI
ACCADEMICO LINCEO
SCRITTA ALLA
GRANDUCHESSA
DI TOSCANA.
IN CUI
Teologicamente,e con ragioni ſaldiſſime,cavate da'Padri più ſentiti,ſi riſponde alle calunnie di coloro,i quali a tutto potere ſi ſforzarono non ſolo di sbandirne la ſua opinione intorno alla conſtituzione delle parti dell'Univerſo, ma altresì di addurne una perpetua infamia alla ſua perſona.
IN FIORENZA,
MDCCX.

Figure 9.5: Title page of the Letter to the Grand Dutches. Galileo wrote it formally in 1616 and it was eventually printed in Latin in 1636, three years after he'd been incarcerated.

> Again, to command that the very professors of astronomy that they must not see what they see and must not understand what they know, and that in searching they must find the opposite of what they actually encounter is beyond any possibility of accomplishment.

And the punch-line:

> Now, if truly demonstrated **physical conclusions need not be subordinated to biblical passages, but the latter must rather be shown not to interfere with the former**, then **before a physical proposition is condemned it must be shown to be not rigorously demonstrated**... and this is to be done not by those who hold the proposition to be true, but by those who judge it to be false.

Finally:

> Inasmuch as the Bible calls for an interpretation differing from the immediate sense of the words, it seems to me that as an authority in mathematical controversy it has very little standing... **I believe that natural processes which we perceive by careful observation or deduce by cogent demonstration cannot be refuted by passages from the Bible**. . . .The primary purpose of the Holy Writ is to worship God and save souls...

The nub of the argument was:

> They know that as to the arrangement of the parts of the universe, I hold the sun to be situated motionless in the center of the revolution of the celestial orbs while the earth rotates on its axis and revolves about the sun. They know also that I support this position not only by refuting the arguments of Ptolemy and Aristotle, but by producing many counter-arguments; in particular, some which relate to physical effects whose causes can perhaps be assigned in no other way.

Figure 9.6: christina

His war with theology has begun. He's saying: the Bible does not determine what is the case in the physical world. What's observed does. In order to overturn a fact in the world, only another observation can condemn it, not scripture. His critics accused him of re-interpreting the Bible, which smacked of protestantism and was against Church law according to the defensive Council of Trent.

Only measurements can challenge observations about the physical world. *Key Concept 17*

Galileo had become a Copernican and there's some evidence that this evolution in his belief happened early, but it was first enunciated in a letter to Kepler in 1597 (".…Like you, I accepted the Copernican position several years ago") but the letter to Catherine was his coming-out.

> "All our Fathers of the devout Convent of St. Mark feel that the letter contains many statements which seem presumptuous or suspect, as when it states that the words of Holy Scripture do not mean what they say; that in discussions about natural phenomena the authority of Scripture should rank last… [the followers of Galileo] were taking it upon themselves to expound the Holy Scripture according to their private lights and in a manner different from the common interpretation of the Fathers of the Church…"
> Letter to a member of the Inquisition.

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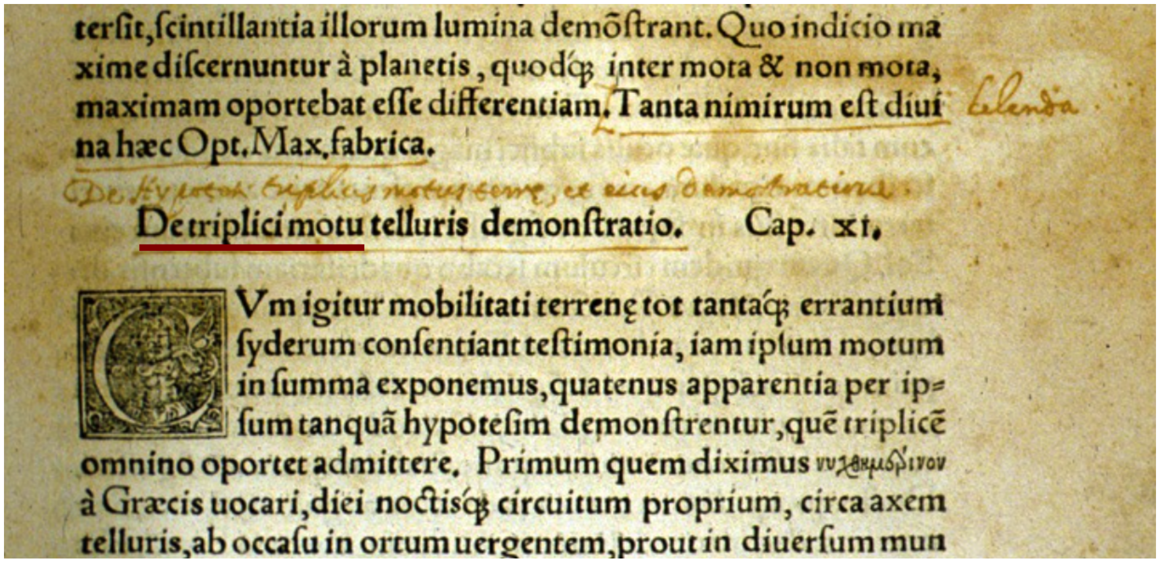
> Letter to a member of the Inquisition.

A council of advisors was established by Pope Paul V to review the theological aspects of Copernicanism. They reached conclusion on two issues: Does the Sun sit immobile? Does the Earth move?

- On the first, they concluded that to hold that the Sun was immobile and at the center of the solar system: "...foolish and absurd in philosophy, and formally heretical since it explicitly contradicts in many places the sense of Holy Scripture."
- On the second, that the Earth moves: "...receives the same judgment in philosophy and... in regard to theological truth it is at least erroneous in faith."


terſit,ſcintillantia illorum lumina demõſtrant. Quo indicio ma
xime diſcernuntur à planetis, quodq; inter mota & non mota,
maximam oportebat eſſe differentiam. Tanta nimirum eſt diui
na hæc Opt. Max. fabrica.

De triplici motu telluris demonſtratio. Cap. xi.

CVm igitur mobilitati terrenę tot tantaq; errantium
ſyderum conſentiant teſtimonia, iam ipſum motum
in ſumma exponemus, quatenus apparentia per ip-
ſum tanquã hypoteſim demonſtrentur, quẽ triplicẽ
omnino oportet admittere. Primum quem diximus νυχθημερινόν
à Græcis uocari, diei noctiſq; circuitum proprium, circa axem
telluris, ab occaſu in ortum uergentem, prout in diuerſum mun

Figure 9.7: At the end of the paragraph, Copernicus wrote: "Tanta nimirum est divina haec Opt. Max. Fabrica." (*"So vast, without any question, is the divine handiwork of the most excellent Almighty."*. That was to be eliminated. The beginning of the next chapter entitled, "*De hypothesi triplicis motus telluris eiusque demonstratione*" ("On the explication of the three-fold Motion of the Earth"was too much for the Inquisition and they suggested instead, "On the Hypothesis of the Three-fold Motion of the Earth and its Explication" which is written in above.)

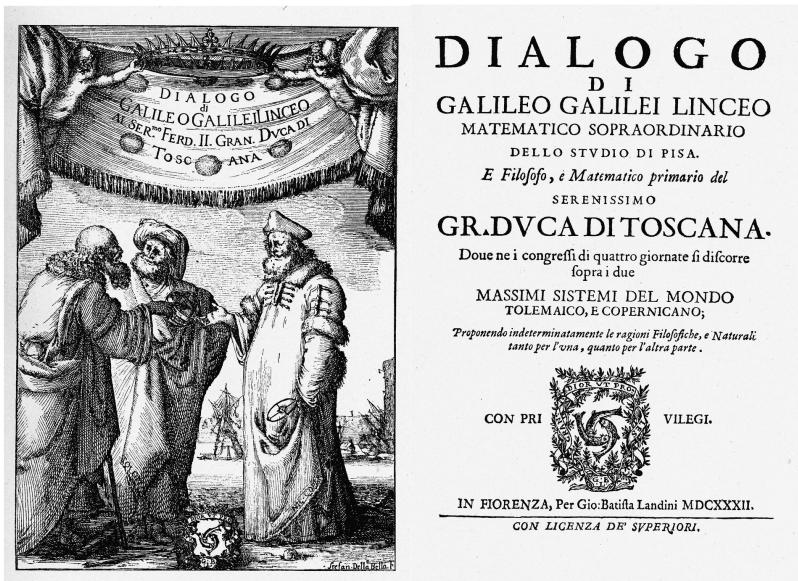
This led to a banning of Copernicus' book until corrections were made. Figure 9.7 is a page of Revolutiononibus showing Inquisitor's corrections.

The consequences were not terribly significant. The Pope's advisors reported to him on February 24, 1616 and Paul asked the respected Robert Cardinal Bellarmine (who had previously defended Galileo's letter to the Grand Dutches to the Pope) to advise Galileo to not claim that Copernicanism as fact. This was in a written document signed by both that hypothetical discussions were okay. Galileo said, "okay." He had a nice meeting with the Pope and went home. Some years later a letter surfaced that suggested that Galileo had been admonished to not speak of Copernicanism even in hypothetical terms, but there's ample reason to suspect that this letter was fraudulent and created in order to create a legal case to silence or imprison Galileo.

It wasn't necessary. Galileo was perfectly capable of creating his own problems, all by himself.

9.1.3 Unforced Errors

Paul died in fall of 1616 and was replaced by Cardinal Scipione Caffarelli-Borghese as Pope Greggory XV, who then was succeeded in 1623 by Maffeo Barberini who became Pope Urban VIII. This was good, thought Galileo. Urban was a personal friend! Barbarini had supported him with poems and a stipend. . . even supporting Galileo's son.



DIALOGO
DI
GALILEO GALILEI LINCEO
MATEMATICO SOPRAORDINARIO
DELLO STVDIO DI PISA.
E Filoſofo, e Matematico primario del
SERENISSIMO
GR.DVCA DI TOSCANA.
Doue ne i congreſſi di quattro giornate ſi diſcorre ſopra i due
MASSIMI SISTEMI DEL MONDO TOLEMAICO, E COPERNICANO;
Proponendo indeterminatamente le ragioni Filoſofiche, e Naturali tanto per l'vna, quanto per l'altra parte.
CON PRIVILEGI.
IN FIORENZA, Per Gio:Batiſta Landini MDCXXXII.
CON LICENZA DE' SVPERIORI.

Figure 9.8: The cover shows from the left, Salviati (actually, Copernicus, who looks more like Galileo...in the next editions a more Copernicus-looking young person is depicted), Sagredo (actually, Ptolemy, hence the turban), and Simplicio (actually, Aristotle).

SIxteen years after his meeting with Bellarmine and Paul XV, Galileo finally went to print with his definitive publication on Cosmology, *Dialogue Concerning the Two Chief World Systems.* He chose to write it as a "dialog" among three people: Salviati (an actual friend of Galileo's) is the enlightened modern thinker who defends Copernicanism, Sagredo is an intelligent layperson who's slowly convinced by Salviati, and Simplicio who is an Aristotelian, whose name says it all about how he's portrayed. Figure 9.8 is the cover of Dialogo.

For some reason, Galileo puts Urban's own words to him in the mouth of Simplicio and that was his undoing. Within two months, the *Dialogue* was removed from all shops (it was in Italian, so laypeople could read it) and Galileo was summoned to Rome to stand trial for heresy.

Figure 9.9: ghome

Wait. *So Galileo's problems weren't necessarily because of his views of Copernicus?*

Glad you asked. *No. Galileo could surely have survived if he'd managed his "mouth" better than he did. Remember, after the original Inquisitional investigation, he was free to work for 16 more years with the friendly backing of multiple Popes. It was only when he ridiculed his former friend and patron that he was arrested. He brought it on himself.*

9.1.4 The End

Galileo was sentenced to house arrest for the balance of his life. Eventually, he was allowed to live in his villa outside of Florence where he was tended to by his son and other supporters. He slowly went blind and suffered many physical ailments, but was forbidden by the Pope Urban to be allowed to see a doctor in Florence. He still managed to put his Paduan work on motion into a new book which is also a dialog among the same three characters, *Discourses and Mathematical Demonstrations Relating to Two New Sciences*. But now the characters are representative of Galileo himself at different stages of his intellectual life. So no Pope is in the cast of that story.

Galileo died in 1642 within a few months of the birth of Isaac Newton. His burial was a mess, as he was not originally allowed to be buried in consecrated ground. Urban refused to allow it and was buried just outside of the famous Basilica of Santa Croce which includes many Renaissance heroes like Michelangelo and Machiavelli (and Galileo's famous namesake relative). Finally in 1737 he was reburied in the main room of the Basilica, but during the transfer three fingers and a tooth were taken from his body. One of those fingers is on display at the Museo Galileo in Florence.[6] Figure 9.10 represents Galileo still editorializing to the world from beyond it.

[6] Actually, after writing this I visited the museum again and found that they'd recovered the missing pieces and now all fingers are on grissly display. Terrific.

Figure 9.10: finger

The damage had been done to the Aristotelean picture of the solar system in a steady stream of observational blows from Tycho through Galileo. Our favorite Italian was famous and persuasive. His book on mechanics was the basis for further work in motion and with the adoption of algebra, Descartes' analytic geometry, and the decimal place...mathematics was brought to bear in Britain. But also damage to Italian science was serious and stifled for nearly 200 years after the Galileo embarrassment.

The Inquisition lifted the ban on Galileo's books in 1718! In 1741, the Pope authorized a publication of his works, somewhat edited. Not until 1758 was heliocentrism allowed in other publications, although Copernicus' books remained banned until 1835. Finally in 1992 Pope John Paul II publicly regretted the Galileo affair and the Church's handling of it.

9.2 The Apple Moment

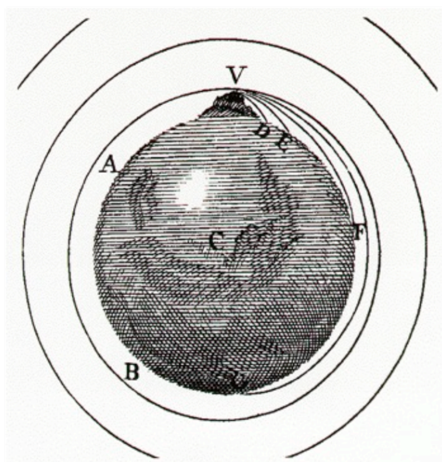


Figure 9.11: From the *Principia*. Perhaps the most wihimsical thing that Isaac Newton might ever have done!

Suppose you could go to a mountain and shoot a cannonball horizontally, like Galileo's table-top. If you were to increase the charge so that the cannonball is given more and more horizontal velocity. . . it would go further and further and eventually—it misses the ground. Figure 9.11 from the *Principia* is perhaps the most fanciful thing that Newton ever depicted. [7] He surmised that if given enough velocity the cannonball would continue to “fall" around and around: it would go into orbit. [8] This is a part of the famous idea that transformed physics forever. Yes, the Apple.

Box 9.1 The Apple Changed Everything

What I’m about to describe arguably changed not only natural science, but was the catalyist for the creation of the Enlightenment itself. What came from the Enlightenment, you ask? Everything we know today as how to think, how to govern, and the role of rationality in deciphering how the world works.

There’s no way to minimize the importance of Newton’s Gravitational law. It made precise predictions about a number of physical phenomena, which were tested and shown to be confirmed. The very idea that a model of the universe would be quantiative and that predictions would be worth testing was itself a new idea. Before Newton there was superstition. After Newton, there was science. Gravitation theory was the reason. The Aristotelian view disappeared. Ptolemey’s model disappeared. The solar system and the Sun’s rightful place was established, not to be unseated again.

During the 17th century the rules governing celestial objects were supposed to be different from those on Earth. Copernicus didn’t question this. Kepler hinted at a common set of rules. Galileo didn’t go there. And so Newton would have been taught in Cambridge eduction with still a strong hint of Aristotle. But somehow he–unlike anyone before him–imagined that there was only one set of rules that governed Earth-bound and celestial objects. And he, first among all, figured out how to make a model and test it.

When he was at the farm during the plague he had a number of remarkable ideas, among which one presumably came from the apple story. There was (and still is) an apple tree at his childhood home. He might indeed have watched one fall. The only account of an apple in this story comes from his first biographer, William Stukeley, about a dinner he and the great (now old) man enjoyed in 1726:

> "After dinner, the weather being warm, we went into the garden & drank tea under the shade of some apple tree; only he and myself...

[7] ...an adult. As a child he built intricate little kites on which he mounted burning candles. Then he launched them all one evening terrifying the townspeople. A Newton-prank.

[8] So this is important: “weightlessness” as a description of life in the International Space Station is a misnomer. Everything has weight as everything is still attracted to the Earth by its gravity, albeit at a slightly smaller value than the g that we experience on the ground. But if everything in the space station is falling together, it looks like nothing has weight. You would have to go much further than the Station’s orbit to be virtually free of a gravitational attraction.

> "Amid other discourse, he told me, he was just in the same situation, as when formerly the notion of gravitation came into his mind. Why should that apple always descend perpendicularly to the ground, thought he to himself; occasioned by the fall of an apple, as he sat in contemplative mood.

> "Why should it not go sideways, or upwards? But constantly to the Earth's centre? Assuredly the reason is, that the Earth draws it. There must be a drawing power in matter. And the sum of the drawing power in the matter of the Earth must be in the Earth's centre, not in any side of the Earth.

> "Therefore does this apple fall perpendicularly or towards the centre? If matter thus draws matter; it must be proportion of its quantity. Therefore the apple draws the Earth, as well as the Earth draws the apple."

There it is. That's the story. Further, if the Earth has "drawing power," maybe whatever it is about the Earth that draws an apple to the ground might also pull on the Moon. How much? Galileo thought that the force of the Earth on objects would be constant everywhere, but Newton guessed that it was not. . . that it should be diluted as one moves away from the Earth. He guessed (and later showed mathematically) that it could be presumed to be the center of the Earth.

He had to assume that the Moon moves in a (near) circle around the Earth. And he had to presume that the same rules governing objects moving in a circle on the Earth would hold for the Moon. He (privately in 1666) and Huygens (publicly in 1659) demonstrated mathematically that a centripetal acceleration must be pulling to the center and have the form v^2/R. Maybe that causal force—the centripetal force—is the one and same force that attracts the apple. Hmm?

Wait. *So did the apple hit his head as we're all taught?*

Glad you asked. *He never made mention of it, so it's another fable told by supporters later. Did Washington cut down the cherry tree? Does fruit always figure into Great Person Myths?*

With this set of ideas, he's way outside of the normal way of thinking. He's violating Aristotle's principles and he's violating his hero, Descartes' principle of reasoning from first principles. He's going to work out the consequences of such a guess, *without first identifying the original cause.*

> **Wait.** *Where did that original motion of the Moon come from? What's the Moon-cannon? Didn't he worry about that?*
>
> **Glad you asked.** *That's the brilliance of Newton. He chose not to start with a set of deductions from a principle, which is like a "why" question. He thought it appropriate to answer the "how" question and if satisfactory explanation resulted, then progress has been made. We understand this primordial velocity of the Moon as related to whatever early spinning was going on 4.5 By ago when the Earth and the Solar System were formed from rotating dust.*

Here's what he wrote of his summer many years later:

> I began to think of gravity extending to the orb of the Moon & (having found out how to estimate the force with which [a] globe revolving within a sphere presses the surface of the sphere) from **Kepler's rule of the periodical times of the Planets being in sesquialterate proportion** of their distances from the center of their Orbs, I deduced that the forces which keep the Planets in their Orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve & thereby compared the Moon in her Orb with the force of gravity at the surface of the Earth & found them answer **pretty nearly.**

What's he saying here?[9]

[9] I'll bet you didn't know that "sesquialterate" means "...in a ratio of one and a half to one." Neither did I.

9.2.1 How to Support a Moon In Its Orbit

What's coming in the next three pages is the longest mathematical story in the whole book. But it's the very definition of "game changer" and you know enough to be able to enjoy it with me! Let's develop the most important physics chapter in the book of western science.

Newton's model asserted the following:

1. The force of gravity that pulls on the Moon is the same force that pulls on an apple. That force is the centripetal force that causes the Moon to move in a circle. (He's using his first law here.)

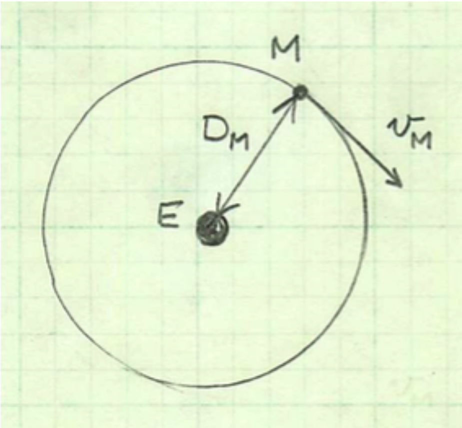


Figure 9.12: earthmoonsetup

2. The Moon's circular trip can be approximated by an infinite number of straight, tangential paths, which are pulled back by gravity to the circle. Over and over.
3. That "pull" back...is "falling" and that's what he models for the Moon and compares to falling near the Earth.
4. He uses Kepler's 3rd Law as a guide, and shows that this implies that the force of gravity on an object decreases by the inverse of the distance away from the source. Kepler introduced it for the planets around the Sun, but Newton extends the idea to also work for the Moon around the Earth.

Pencil 9.1.

The setup is shown in Fig. 9.12. I'll use R_E for the radius of the Earth and D_M to mean the distance from the center of the Earth to the center of the Moon, and v_M to be the speed of the Moon in its orbit.

What's the Moon's Centripetal Acceleration?

He needed to find the centripetal acceleration of the Moon using what he knows about the Moon's motion (it takes a month to go around the Earth) and Kepler's Third Law: $T^2 \propto D_M^3$.

$$a_C(\mathrm{M}) = \frac{v_M^2}{D_M}$$

How does he actually know v_M? Well, that's the easy part. The speed is the distance traveled—the circumference of the Moon's orbit—divided by the time that it takes to go around, its period (1 month), which we'll call T. So the speed is

$$v_M = \frac{\text{circumference}}{\text{period}} = \frac{2\pi D_M}{T}.$$

which we can substitute into the centripetal acceleration relation:

$$v_M = \frac{2\pi D_M}{T}$$

to get

$$a_C(\mathrm{M}) = \frac{4\pi^2 D_M^2}{T^2 D_M}$$
$$a_C(\mathrm{M}) = \frac{4\pi^2 D_M}{T^2} \qquad (9.1)$$

Then he used Kepler's rule from Eq. 8.1

$$T^2 \propto D_M^3$$
$$T^2 = kD_M^3 \qquad (9.2)$$

where I've inserted a constant of proportionality, k.[10]

Substituting the period of the Moon's orbit into Eq. 9.1, we get:

$$a_C(\mathrm{M}) = \frac{4\pi^2 D_M}{kD_M^3}$$
$$a_C(\mathrm{M}) = \frac{4\pi^2}{k}\frac{1}{D_M^2}. \qquad (9.3)$$

This is huge. He's demonstrated that the centripetal acceleration (and hence the force) for the orbiting Moon would vary as the inverse-square of the distance that the Moon is from the Earth. This is something that everyone suspected, but nobody figured out before this moment. It was buried in Kepler's law all the time. Keep it in mind.

[10] This is a big leap from Kepler! The constant k for the planets, that Kepler assumed, would be completely different from that assumed for the Earth-Moon relationship. Newton picks out the idea and applies it in a direction that Kepler never intended.

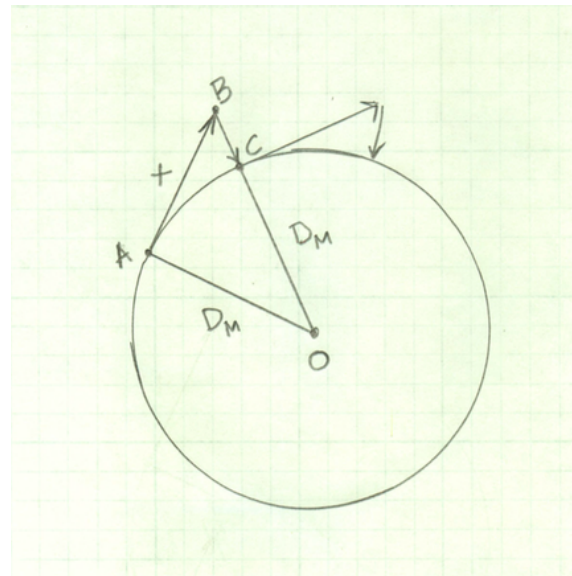


Figure 9.13: moonstraight

How Newton Confirmed His Model of Gravity

The second bold (in both sense of the word "bold"!) phrase refers to a calculation using his model which is embodied in the simple diagram in Fig. 9.13. The Moon is traveling in a circle, but thinking like a calculus-inventor, Newton imagined that this circular orbit is really an infinite number of little tugs across the intervening space and to the center of the Earth. The Moon goes in a straight line from point A to point B at speed v_M and then is pulled to the center—to point C—by the Earth's gravity. This pull happens in some time interval, which we'll take to be 1 second. Then it goes in *another* straight line and is pulled back.

In essence he asked how far is it tugged? It's as if the Moon is at point B and in one second "falls" —FALLS!—to point C...just like the apple falls. How far is that $\overline{BC}$ distance?

Actually, that triangle OAB is a right triangle and the hypotenuse is $\overline{BC} + D_M$.

- He knows D_M to be about $60 \times R_E$ and he can calculate the $\overline{AB}$ leg by knowing how fast the Moon is going (v_M) and using the regular formula for speed: $\overline{AB} = v_M t$.
- We have the form of the Moon's speed and he used an average month to be 27.3 days, which is $T = 2,360,000$ seconds.
- We'll use a modern value for the distance from the Earth to the Moon, where he used an ancient result that D_M is about $60 \times R_E$.[11]

[11] He also made some other mistakes and approximations. For example, he assumed that a mile is 5,000 ft. But hey, when you're inventing a whole discipline, sometimes you gotta cut corners.

Let's put all of this together and calculate that speed:

$$v_M = \frac{\text{circumference of Moon's orbit}}{\text{1 month}} = \frac{2\pi D_M}{T} = \frac{(2\pi) \times 1,031,400,000}{2,360,000} = 2,746 \text{ ft/sec.}$$

We found the speed above, so the distance $\overline{AB} = x(\text{Moon})(1 \text{ second}) = 2,746$ ft along that tangent, for 1 second.

$\overline{ABO}$ is a right triangle and we can find all three legs: we just found $\overline{AB}$ and we know that $\overline{AO}$ is D_M. The leg $\overline{BO}$ is really $D_M + \overline{BC}$. We can use Pythagoras' Theorem to find $\overline{BO}$ and therefore, to isolate $\overline{BC}$. This is the "fall" of the Moon through that 1 second! The result of the calculation is:

$$\overline{BC} = 1/20\text{th of an inch} = 0.004167 \text{ ft.}$$

Let me repeat this, because it's important: Newton calculated that in 1 second, our Moon "falls" to Earth by 0.004167 ft.

Let's put this together with the discovery that the Earth's gravity is diluted by the square of the distance, then the distance that an apple would fall on Earth can be related to the distance that the Moon falls away from the Earth!

$$x(\text{apple}) = x(\text{Moon}) \frac{D_M^2}{R_E^2}$$
$$(\text{apple}) = 0.004167(60)^2 = 15 \text{ feet} \tag{9.4}$$

Now, if an apple falls through a distance of 15 feet in 1 second, what's the acceleration due to gravity for it from what Newton called "Galileo's Theorem"?

$$x = 1/2 g t^2$$
$$g = 2x/t^2 = 2(15)/1^2 = 30 \text{ ft/s}^2 \tag{9.5}$$

This must have been satisfying: 30 ft/$s^2$is pretty close to what he knew little* g* to be, 32 ft/s^2. So indeed, "pretty nearly."

He's (you've!) done an amazing thing!

He's measured the acceleration of gravity on the Earth parameters from the Moon!

Think about that.

As he said in Book III of *Principia* where he summarized this earlier work, "And heavy bodies do actually descend to the earth with this very force." Understated as the most important few lines of work in the history of science!

The same physical theories govern motion on Earth and the cosmos. *Key Concept 18*

Now Think Big!

In looking back to the days on the farm in 1666 where he first tried out this idea, he used some incorrect numbers and was still working out the mathematics but even though he never published his results. . . he worked off-and-on for years. But eventually, he started to think about the actual force that the Earth would exert.

> If we stuck in the Moon's mass, then we'd arrive at a formula for the gravitational force, similarly diluted by that same factor.

That is, with his Second law $F = ma_C$ and the centripetal acceleration we would find that the force of attraction by the Earth on the Moon is:

$$F_{EM} = K\frac{m_M}{D_M^2}$$

where I've rolled all of the constants ($\frac{4\pi^2}{k}$) into one big K for tidiness.

Now we remember Newton's *Third* law that says that if you push on me, I'll push back on you. So what's the force of attraction *for* the Earth, *from* the Moon? The same. We can't just replace the Moon's mass with the Earth's mass, since that would change the force. The only way for to work is that whatever this force is it has to be symmetrical between the Moon and Earth. So it hast to look like this:

$$F_{EM} \propto \frac{m_M m_E}{D_M^2}.$$

The proportionality constant has a name. . . and a long history to this day.

So this is rather remarkable. For the first time the physics of the Moon is convincingly shown to be identical to physics on the Earth. But he's not done.

9.2.2 Universal Gravitation

With the idea of centripetal force and his third law of motion—and confirmation using the Moon's parameters—he's connected the Moon's orbital motion to regular Earth-bound circular motion and connected the Moon's motion to objects falling on the Earth. The nature of these forces seem to not care about the objects that cause and experience them—like Galileo insisted—indeed, they're not different forces, but manifestations of a single force. He's connected the Earth to the Moon: one theory.

Later he analyzed the motions of the moons of Jupiter and Saturn and eventually comets and showed that they obeyed Kepler's Law, like Kepler's planets and now, like the Moon. Suddenly, the whole solar system, including the moons of all planets hung together in a single mathematical system. One set of rules for the whole of our visible universe and our terrestrial home.[12]

At this point, Newton makes a breathtaking leap. He assumes that a gravitational attraction exists between *any two objects with mass*. Right now you are being attracted by the Earth, but also by the Sun, and the Moon, and Jupiter, and by the banana on your desk that you're saving for lunch . All objects in the Universe attract one another according to the following universal rule, which we played with in our mathematics review way back in Chapter 2. The force acting on 1, because of the pull of 2 is:[13]

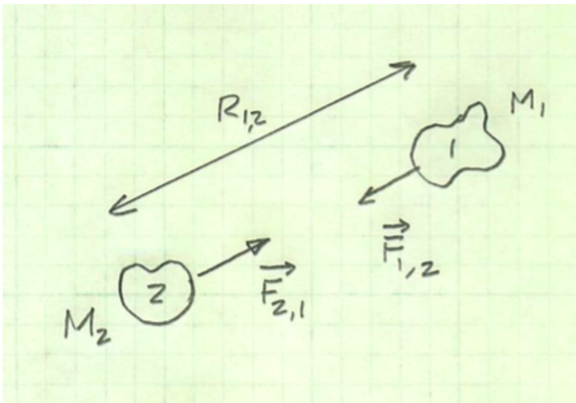


Figure 9.14: force12

Figure 9.14 shows the situation. Some object #1 with a mass M_1 attracts some other object #2, also with a mass M_2. . . and *visa versa*. This attraction is along a line connecting their centers which are $R_{1,2}$ apart. This is called the Universal Law of Gravitation and the constant of proportionality, G is Newton's Constant or the Gravitational Constant. The force of attractoin *on* 1 due to 2 is $F_{1,2}$ while the force of attraction *on* 2 due to 1 is $F_{2,1}$. From Newton's Third law? They're equal.

An interesting fact about this equation is that it can only be solved exactly for two objects. Add a third object—or a fourth, or fifth, etc—and the equation cannot be solved. Rather it is necessary to solve it *approximately* and after Newton people became very skilled at doing very complicated approximation calculations called "perturbations."

Equation: **Newton's Gravitational Law.**

$$F = G\frac{Mm}{R^2}$$

Constant of nature: **Newton's Gravitational Constant.**
$G = 6.67 \times 10^{-11}\ \mathrm{m^3kg^{-1}m^{-2}}$

[12] Aristotle has now left the building. Never to return.

[13] Of course there is the force acting on 2 because of its attraction by 1, $F_{2,1}$. And, they're equal.

The Gravitation Constant

The constant of proportionality, G, is very tiny and not known well.[14] Newton had no estimate for its value, rather he worked in ratios of forces but it was measured in a laboratory by the very odd Henry Cavendish about a century later. It is a fundamental constant of nature. It just is. There's no deriving it. Were it different by a little, our world would be very different.

[14] the uncertainty on that number is 0.021 out of 6.67, or about 0.3%. For a fundamental constant of nature, that's not very precise.

The gravitational force is very weak and characterized by a single constant of nature. *Key Concept 19*

Little *g* Again

Now we can understand Galileo's results from a modern point of view. With the Universal Law of Gravitation and Newton's Second Law, the acceleration due to a gravitating body can be isolated from Newton's rule by finding the "a" and the "m." To see what I mean, look at Fig. 9.15.

Pencil 9.2. ✎

Keep in mind Newton's simple second law:

$$F = ma.$$

Place your apple on the ground—notice that it's distance from the center of the Earth is, R_E. Let's calculate the force on that little apple with mass m due to the big Earth, with mass M_E. Newton taught us that the force between them is

$$F = G\frac{M_E m}{R_E^2}$$

Now isolate the little m outside of the other terms:

$$F = m\left(G\frac{M_E}{R_E^2}\right) = ma$$

and can you see that we've discovered an acceleration buried in the middle term by recognizing $F = ma$ in it:

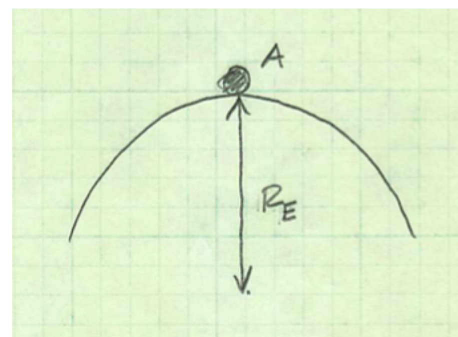
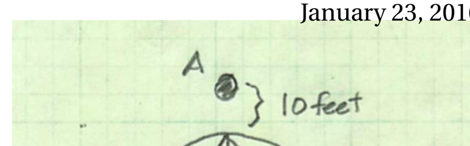


Figure 9.15: An apple sitting on the ground a distance R_E from the center of the Earth.



$$a = G\frac{M_E}{R_E^2}.$$

Since this situation is an apple on the surface of the Earth, what we've really found is a derivation for Galileo's g! So we can just identify:

$$g = G\frac{M_E}{R_E^2}. \tag{9.6}$$

You Do It 9.1. Calcuating g

or copy the solution

Using the following parameters

- $G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}m^{-2}.
- $R_E = 6.37 \times 10^6$ m.
- $M_E = 5.97 \times 10^{24}$ kg.

and using Eq. 9.6, calculate g.

Did you get 9.8 m/s^2? Look familiar? So, that's where our weight comes from. The Earth attracts us with a force that's $F = mg$, which is a constant—on the surface of the Earth. When you step on a scale, it pushes back and is calibrated to read back how much spring-force is required to balance your weight.

Example 9.1

Constant g?

Question : Let's calculate the acceleration due to the Earth's attraction for an apple in a tree, 10 feet above the Earth's surface as shown in Fig. 9.16.

Solution:

$$a(\text{tree}) = G\frac{M}{(R_E + 10 \text{ feet})^2}.$$

Ask Mr. Google and you'll find that that R_E is more than 17 million feet and you can readily see that the little 10 foot addition is minuscule. This calls for the use of the approximations that we listed in Section 2.6, in particular Eq. 2.20 which looks like our function with $10/R_E$ playing the role of x. Here's how this works. Manipulate the equation above so that the denominator is like $1+$ something...and we do that by dividing out R_E. Then "something" is a very small number and we can use Eq. 2.20. So let's do that: Now just use the first two terms of the approximation from Eq. 2.20:

$$\frac{1}{(R_E+10)^2} = \frac{1}{(1+10/R_E)^2}\frac{1}{R_E^2} \text{ now the approximation:}$$

$$\approx \frac{1}{R_E^2}[1-2(10/R_E)] = \frac{1}{R_E^2}(1-20/17,000,000)$$

$$\approx \frac{1}{R_E^2}(1-0.0000012) \text{ So, from this the acceleration at 10 feet:}$$

$$a(\text{tree}) = GM\frac{1}{R_E^2}(0.99999882) = 0.99999882g \tag{9.7}$$

So for all practical purposes $a(\text{tree}) = g$.

Teaching Moment!

Wait. *But we've been saying that Galileo showed that the acceleration due to gravity is a constant. Now you're saying that it depends on how far away one is? Which is it?*

Glad you asked. *Yes. Galileo's g is really not a constant, but it varies very little...even for large distances above the Earth. So for all practical purposes, we can consider it to be a constant. In fact, let's calculate that for the highest (above the Earth) that you've ever been:*

Here we have a situation that's going to repeat itself over and over in the history of physics. Galileo said that the acceleration due to gravity was constant. Then along came Newton who showed that this wasn't right in the strictest sense: that the acceleration due to gravity varies as you move away from the gravitating object. Over and over we'll have to grapple with a question that in this case, looks like:

Was Galileo wrong?

For almost half a century, Galileo's discovery was considered a fact of nature. But then it was shown to be the case only in a restricted domain...in this case, when you're close to the surface of the Earth. As you'll see in the next steps, there's really no circumstance that you or any of us (except for a handful of astronauts) will ever experience in which Galileo was incorrect.

The scenario runs like this: First, Theory A explains a feature of the world and establishes a fact of nature and a mathematical Model that uses it. Then along comes Theory B that shows that the facts and the models of Theory A are not strictly correct. Yet if the facts of Theory A and the models in Theory A are included in the facts and the models of Theory B *within a domain of experience that's smaller than the domain that Theory A describes*, then we'd say two things: First, Theory B is more inclusive than Theory A . It explains more about the universe. And second, Theory A is still the case when applied to the restricted domain of experience that's a subset of the domain of Theory B.

In this case, Newton's theory (B) explains gravitation everywhere. Galileo's theory explains gravitation only in the region near the surface of the Earth (A). We still happily—and reliably—use a constant g in the design of any structure or vehicle, for example. Keep this notion in mind, since Newton's gravitation law...will become a "Theory A" in a few hundred years at the hand of Albert Einstein!

You Do It 9.2. title

or copy the solution

Let's say that an airplane is 5 miles above the surface of the Earth. If I drop my delicious snack on the floor of the cabin, what acceleration due to the Earth's gravity would it experience compared with if I had dropped it on the ground? The radius of the Earth, which is $R_E = 3960$ miles.

9.3 Three Problems for Newton

The successes of Newton's model for gravitation were many and astounding. It's sometimes said that the Enlightenment was a direct result of the success of the naturalistic approach to explaining the world. Here are some of what his theory of gravitation demonstrated:

- He showed that the inverse-square rule for gravitation explained Kepler's Laws, that they would accommodate circular, elliptical, and parabolic orbits. Famously, Hayley's Comet was discovered and the predictions that Newton's friend made were based on Newton's rules. He simply assumed that the comet's path was elliptical (but squashed) around the Sun as its focus and could then use Newtons' Gravitation law. He was right...Halley's Comet's path takes it all the way past Neptune before it starts coming back towards the Sun. It's a 76 year round trip.
- The Earth's axis wobbles a tiny bit and Newton explained that, the precision of the equinoxes.
- He explained the tides as a feature of the Moon's attraction for the ocean water closest to it as opposed to the water on the other side of the Earth from the Moon.
- The Earth should not be a perfect sphere since it's not an absolutely rigid mass. Because it rotates material closest to the axis through the poles (near the poles) would feel a different gravitational force due to the material inside of its radius from material furthest away from the axis of rotation (equator). So there should be a measurable difference in the gravitational attraction at different longitudes and this stimulated heroic teams of explorers who traveled very far north with pendulums to make measurements of g everywhere they could. Newton's explanation worked.
- And of course his model explained all of the observed orbital motions of the known planets, a concept that was not even thought possible, or even desirable while Newton was a child. He determined the relative masses of the planets and the Sun.

  Of course in addition, he had other unparalleled (including to this day) achievements:

- He properly conceived of the idea of momentum and completely describe motion and dynamics.
- He correctly conceived of the theory of colors as mixing together to make white, in contradiction to the prevailing views led by Descartes.
- He invented and the pioneered the use of calculus.

By the time he died in 1726, magic was gone. Subservience to Aristotle was gone. Everyone believed that...everything could be known. The very essence of the Enlightenment period in western history.

But there were issues that were more philosophical that required his attention.

9.3.1 Action At A Distance

Two distinct camps developed in physics. While a dominant belief in naturalism now reigned in Europe, the French followed the lead of Descartes while the British remained loyal to Newton. But everyone agreed that the actual mechanism of gravity was problematic. In a letter he wrote:

"It is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter without mutual contact...that one body may act upon another at a distance through a vacuum, without the mediation of anything else...is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it."

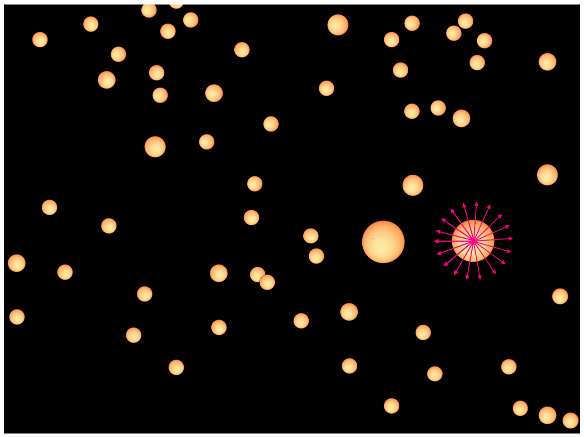


Figure 9.17: newtoninfiniteU

Everyone agreed but two camps developed on proper process. The Continental view was that until you can enunciate the mechanism of gravity and then reason deductively from that, you're not doing acceptable science. By contrast, there was the British view—and the one that we all follow today—that what's important is that if it works, that's good enough. In many ways, Newton differentiates the contrast between **why** a phenomenon occurs and **how** it occurs...and an answer to *how* can be a mathematical model. He famously said: "I feign no hypotheses." which even is its own Latin catch-phrase, "*hypotheses non fingo*" (Google it!).

The bar to making progress that Descartes set up (the Continental view) is too high. One should "hypothesize" (I'd say model-build) and deduce empirical observables, test them and then refine your model. Then you've turned science into a Process that improves on its conclusions. Eventually—and gravity is a good example—one might find an acceptable why...but until that, how is good enough and makes progress possible.

9.3.2 Stability of the Universe—Cosmology

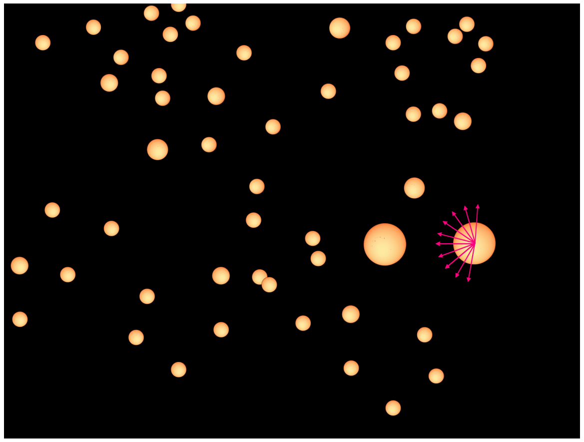


Figure 9.18: newtonfiniteU

Newton wasn't shy about how to apply his model. As described above, there were plenty of terrestrial and astronomical applications that were predicted and tested positive. But what about the whole enchilada? What about the whole Universe?

He recognized quickly that he faced a puzzle even more prickly than Action at a Distance. He couldn't explain the improbability of why we're here at all. Here's the problem, which I can form as a question:

Is the Universe finite or infinite? His theory seems to suggest that the Universe must be infinite, with an infinite number of stars (all anyone knew about were planets and stars...no galaxies). Imagine this enormous space filled with stars, each of which is attracting every other object in it, and is in turn being attracted by every other object. Figure 9.17 is a cartoon of such a situation. That one star is being pulled on by everyone...and the fact that the Gravitation law varies like $1/R^2$ means that there is an influence from all objects, all the way to infinity.

If the Universe had an edge, then Fig. 9.18 would crudely be the story. Notice now that our target star is being pulled to the left and there's no balancing set of forces to the right. That should start our star accelerating which would then pull on other stars differently as it moves and they'd start to accelerate—the end result would be a huge collapse of everything on top of itself. Since we're here, this hasn't happened and so the Universe is infinite. That's the argument, but it's flawed...or at least highly improbable.

Suppose the Universe is infinite and this incredibly delicate balance is at work. A butterfly could cause the whole thing to collapse, much less Jupiter orbiting the Sun. That is, the nature of his Gravitational law is such that the delicate balance that holds everything just right...has to be absolutely perfect. That seems improbable.

Newton had a famous correspondence with the leading theologian in Britain, Richard Bentley in 1692. Bentley was erudite and familiar with science and Newton took him seriously. He wrote to the reverend:

> As to your first query, it seems to me that if the matter of our sun and planets and all the matter in the universe were evenly scattered throughout all the heavens, and every particle had an innate gravity toward all the rest, and the whole space throughout which this matter was scattered was but finite, the matter on the outside of the space would, by its gravity, tend toward all the matter on the inside, and by consequence, fall down into the middle of the whole space and there compose one great spherical mass. But if the matter was evenly disposed throughout an infinite space, it could never convene into one mass; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered at great distances from one to another throughout all that infinite space. And thus might the sun and fixed stars be formed, supposing the matter were of a lucid nature. But how the matter should divide itself into two sorts, and that part of it which is fit to compose a shining body should fall down into one mass and make a sun and the rest which is fit to compose an opaque body should coalesce, not into one great body, like the shining matter, but into many little ones; or if the sun at first were an opaque body like the planets, or the planets lucid bodies like the sun, how he alone would be changed into a shining body whilst all they continue opaque, or all they be changed into opaque ones whilst he remains unchanged, I do not think explicable by mere natural causes, but am forced to ascribe it to the counsel and contrivance of a voluntary Agent.

And again,

> The reason why matter evenly scattered through a finite space would convene in the midst you conceive the same with me, but that there should be a central particle so accurately placed in the middle as to be always equally attracted on all sides, and thereby continue without motion, seems to me a supposition as fully as hard as to make the sharpest needle stand upright on its point upon a looking glass. For if the very mathematical center of the central particle be not accurately in the very mathematical center of the attractive power of the whole mass, the particle will not be attracted equally on both sides. And much harder it is to suppose all the particles in an infinite space should be so accurately poised one among another as to stand still in a perfect equilibrium. For I reckon this as hard as to make, not one needle only, but an infinite number of them (so many as there are particles in an infinite space) stand accurately poised upon their points. Yet I grant it possible, at least by a divine power; and if they were once to be placed, I agree with you that they would continue in that posture without motion forever, unless put into new motion by the same power. When, therefore. I said that matter evenly spread through all space would convene by its gravity into one or more great masses, I understand it of matter not resting in an accurate poise.

> … a mathematician will tell you that if a body stood in equilibrio between any two equal and contrary attracting infinite forces, and if to either of these forces you add any new finite attracting force, that new force, howsoever little, will destroy their equilibrium and put the body into the same motion into which it would put it were those two contrary equal forces but finite or even none at all; so that in this case the two equal infinities, by the addition of a finite to either of them, become unequal in our ways of reckoning; and after these ways we must reckon, if from the considerations of infinities we would always draw true conclusions.

Newton's solution to this delicate balance was an appeal to God. God's job is holding everything in place. While today we don't do that in science, we'll see this particular problem come back for our other full-blown Scientific Hero and we'll find that Einstein provided a different explanation.

Cosmology

This is an important point in the history of physics. It's the beginning of quantitative, predictive science of the universe. This subfield of physics and astrophysics is called Cosmology: the study of the whole universe.[15]

[15] Cosmogony is another similar word that describes the origin of the universe. Formerly, thought to be outside of the provence of science. Now...a regular part of physics. We tend to have expanded the word Cosmology to include origins.

definition Cosmology

definition Cosmogony

While Kepler came close, after all, he provided a formula that was descriptive of how the planets move. We understand Kepler's law now as a logical (meaning: algebraic) consequence of Newton's Gravitation...s0 it was eventually appreciated to be derivative.

Here we have Newton using an abstract (meaning: using a mathematical formula) explanation to describe the entire universe. His equation's form insists that a gravitational force only goes to zero at infinity, so no matter how far away two objects with mass are situated, they will still attract one another. This presented a problem that needed explanation: his model was predictive, but not complete. His approach to this level of incompletion was to give up and require a deity. Our approach is to leave it open as a problem remaining to be solved. In that sense, we're more Newtonian ("no hypothesis") than he was!

9.3.3 Absolute Space and Time

Newton's mechanics led to big questions that required speculation about space and time...that is, **S**pace and **T**ime! He asked himself questions like this (although not this particular one). Suppose the universe consists of only four objects: you, your friend, a rope, and a knife. You and your friend are connected by a rope. Are you stationary or are you *rotating* around the center point of the rope? Remember, the universe is empty but for you two. How could you tell?

This is sticky matter of relative motion. If I were to ask if you were moving linearly with respect to your friend, you could tell me that because you'd see your friend approach, pass you, and recede. Which would be really sad. (By the way, your friend would see exactly the same thing except in the other direction.) So you might not agree about who is moving and who is stationary, but you'd have no trouble believing that relative motion exists between you.

But rotation is a different matter and this question is specifically about an accelerated "frame of reference" since in order to rotate about that center point, a centripetal force through the rope would be required and so an acceleration is at work. Well, one of you has a knife and if you cut the rope one of two things might happen. Nothing! In which case you'd conclude that you were not rotating because the other thing that might happen could be that you'd immediately begin to separate meaning that: you had each been orbiting the center and that when the rope no longer connected you, you'd start straight line motion in accordance with Newton's First law.

The question is...if you are rotating in this situation, *with respect to what* are you rotating? Newton felt that he needed an absolute measure for inertia and acceleration and he chose Space, with a capital S. To Newton, space was a thing. Take everything out of the universe and space will still be there acting as an absolute coordinate system. All motion, constant velocity and accelerated, can be described mathematically with respect to this absolute coordinate system. So he said. Newton also insisted that there is an absolute clock...absolute Time, with a capital T.

Needless to say, there was also the Continental point of view, championed by Leibnitz who said that space was defined by the relative positions of things. Take away the things and there is no space...it's completely relative...to stuff.

This argument is going to come back to haunt us a few more times before we reach the 21st century! But the important thing is that nobody talked like this; nobody theorized scientifically about the universe before Newton.

9.4 Gravitational Energy