Chapter 17

Antimatter

Paul Dirac’s Second Big Score

Paul Dirac and Werner Heisenberg

Paul Dirac, 1902-1984

“Heisenberg, why do you dance?” A question of Dirac’s to Heisenberg when they were ship-bound for Japan together. Heisenberg liked to dance before the dinners and replied that when there were nice girls he felt like dancing with them. Dirac thought about that and eventually asked Heisenberg, “Heisenberg, how do you know beforehand that the girls are nice?”

When Paul Dirac went to Stockholm to accept his Nobel Prize in 1933 he did all of the standard things—the parties, the banquet, and of course his address to the Royal Court and guests and family. While families of Nobel Laureates always attend the ceremony to see their spouse, parent, or child inducted into history, Paul’s father didn’t attend. He wasn’t invited.

Quantum Mechanics was difficult enough. Relativity was tough too, but somehow a little more accessible, right? Putting them together—which everyone knew had to be done, but nobody could figure out—proved to be the opening of a floodgate that let in all manner of odd realizations of just how Nature works at the deepest level.

17.1 Goals of this chapter:

- Understand:
  - How to calculate distance, time, and speed for uniform and constantly accelerated, linear motion
  - That falling objects all have the same acceleration near the Earth.
  - How to graph simple motion parameters
  - How to read graphs of realistic motion parameters
- Appreciate:
  - The algebraic narratives in the development of the formulas
  - The shape of the trajectory of a projectile
- Be familiar with:
  - Ideas of motion before Galileo
  - Galileo's life
  - Galileo's experiments with motion

key concepts

17.2 A Little Bit of Dirac

Antimatter is the stuff of science fiction —an almost a silly-sounding thing. From blockbuster Hollywood to pulp science fiction, the idea of spooky "stuff" that somehow cancels "normal stuff" has been a part of our cultural imagination for a long time. When you first hear of it, you're skeptical but that's nothing compared with the struggles of the man who invented it—it took him almost three years of frustration before he found a way to interpret what his mathematics was telling him. After very public arguments among many people, he quietly settled in on the most ludicrous interpretation of all, which of course turned out to be the only way Nature would have it.

We're now on the path to Oz that is modern particle physics and the first bricks on that road were laid by Paul Dirac. We're going to live with antimatter throughout the rest of our story. And we'll be puzzled about antimatter right to the present day. And the adventurer who took us there was a very quiet man.

The story of Dirac's young life is legendary, not for its inspiring boy-makes-good theme, but because of the nastiness that he suffered at the hands of his father. Charles Dirac was an expatriate Swiss teacher of French in Bristol at a secondary school affiliated with the University of Bristol.[1]

[1] Cary Grant grew up blocks from Paul Dirac and attended the same primary school, but a year younger!

"Strict" doesn't do justice to the way in which he treated Paul, and in a different way, his older brother and Paul's mother. There was a darkness in the Dirac household which arguably led to tragic suicide of his brother and a discipline that so affected Paul that even in his eighties he could still register deep emotion. He almost never spoke of his childhood, but when he did it was with bitterness.

Paul was commanded from an early age to speak only French with Charles. His mother, brother, and sister ate in the kitchen, while Paul had his wordless dinner at the dining table with his father. Silence was a matter of practicality since any mis-step in his French would result in severe punishment. He also suffered from a delicate digestion throughout his life and there were times when even at the dining table he could not keep his food down. Yet even after such embarrassments, he was still forced to resume eating until his plate was empty. So speaking only when it was absolutely required and eating sparingly were his early choices and habits throughout his life.

Paul was a gifted mathematics student. His brother was as well and wanted to study medicine, but Charles would have nothing but Bristol-educated (free), employable sons so both graduated as engineers. Paul received a bachelor's degree in Electrical Engineering at the age of 19 and after a disastrous industrial internship, went back for a second degree in mathematics. By the time he managed to get admitted to graduate school at Cambridge, he was only 21 years old with two university degrees. At first his classmates were mystified by strange student in the front row correcting the mistakes of lecturers...and then recognized him as a genius.

Years later, Paul's daughter speculated that this was a tragic blow to the uncle she never met as he was miserable in his short life and took his own life when Paul was in graduate school.

The Cambridge Physics Department was associated with The Cavendish Laboratory and Trinity College, which boasted Isaac Newton as an alumnus. By the time Dirac arrived, Rutherford had settled in as Cavendish Director where experimental physics was in the now familiar full-speed-ahead-Rutherford mode. But theoretical physics lagged. Arthur Eddington of the Einstein-eclipse fame, had been named the Director of the Cambridge Observatory a few years before Rutherford arrived and so Relativity was well-represented (but not very popular outside of Eddington's circle). But Quantum Mechanics was less well practiced than General Relativity.

Paul wanted to continue to study Relativity, but didn't get his first choice of faculty advisor and was instead admitted to St John's College, where mathematics and mathematical physics was studied. The man who took him under his wing was Ralph Fowler, exactly the right mentor. Fowler was one of the few experts on Quantum Mechanics at Cambridge and so Paul quickly became expert in that field—so much so that he'd transform it before he graduated: Yes, before Dirac even gained his Ph.D., he was well on his way to revolutionary discoveries greatly impressing the mighty Eddington, as well as everyone on the campus and the continent.

The British would say, "where maths was studied."

Cambridge University is organized in a set of colleges, of which there are 31 now. This is a part of its 500 year history.

Paul was an odd companion—silent at the common St Johns College meals to the point of exasperation for those around him—he did eventually establish friendships with students and faculty and participated fully in the seminars and journal clubs. Yet, he seemed to relate to people in a highly mechanical

His working habits were to completely isolate himself in a spare room for calculation six days a week. Then he always reserved Sunday for walking, and would take off in his only suit and tie for nearly all-day treks in the countryside, a habit he maintained his whole life.

Figure 17.1: Dirac and his family.

Unlike his Cambridge regimen, at FSU, he was much more sociable and found the American campus and department life agreeable and genial. After a lifetime of working one way, he completely switched!

way, seemingly incapable of being insulted or embarrassed and accordingly was often unconsciously the source of discomfort for others. He wasn't intentionally unkind, he was simply unemotional— matter-of-fact to a maddening degree. Not unfriendly, he just wasn't...anything. Eventually people learned to accept him, even affectionately, and he, them. But Dirac Stories abounded.

Paul's letters home were regular but were typically only a few lines, almost never referring to his work. Even when his fame was growing, the Bristol Dirac household had to read of his successes in the newspaper or learn from a neighbor. It just didn't occur to Paul to tell them. Or he didn't want to.

This story does end happily. Paul eventually married the sister of a colleague whom he met at Princeton on one of his scientific stays in the U.S., officially adopting her two children and then together having two of their own. He was appointed the Lucasian Chair (remember, that's Newton's old position) which he maintained following his regular 6+1 weekly schedule (see the side-note above) for almost 30 years, retiring from it at 65 year's of age, as required.

But he wasn't ready to quit and accepted a position at Florida State University where he enjoyed the more personable nature of the American faculty life. So out of a difficult beginning, a happy life evolved. Sufficient reward perhaps for his intellectually lonely work. Because he was often way ahead of everyone.

It is difficult to imagine modern physics that doesn't hold a debt of gratitude to Paul Dirac. To many, he was the second most important theoretical physicist of the 20th century, but because of his low-key manner and the deeply complicated topics that he mastered, he's not as well known today as perhaps he should be. (Indeed, it's hard to imagine his picture on the cover of *Time* magazine!) His contributions were in Quantum Mechanics, General Relativity, Cosmology, materials science, and even some dabbling in experimental physics with a patient colleague.

In 1925 while Dirac was a senior graduate student, Heisenberg visited Cambridge and talked on his still-forming ideas of Quantum Mechanics. They struck up an odd friendship (the athletic and highly social Heisenberg and the quiet, anemic-looking Dirac) and soon after Heisenberg sent Dirac a draft of his famous paper. On one of his Sunday walks he was stunned to realize that a particular mathematical tool that Heisenberg used was identical in form to an old, formal description of "regular" classical mechanics. From this inspiration, Dirac was able to show a connection between the "regular" mechanics of Newton and its subsequent 300 years of development and the new, seemingly foreign Quantum Mechanics. The two descriptions were connected by the Planck Constant, h. Even though it's a tiny, tiny number, were it to become zero on in Dirac's mathematical repackaging, the quantum description would pass over into that old classical description. This was the first time that anyone had succeeded in making that connection. So all he did in his first professional publication was fix the apparent disconnect between the Quantum World and the previous centuries of physics. This shocked the whole of the Revolutionary European Physics Crew of those struggling with the new subject.

In fact, by the time Dirac received his Ph.D. in 1926 (at the age of 24) he published a remarkable 11 papers in which he also showed that the Schroedinger and Heisenberg descriptions of Quantum Mechanics—so different on their face—were mathematically identical. As a student, he tied everything up in to a single picture!

From his degree he took what was becoming the standard trip around Europe: he spent time working in Neils Bohr's institute in Copenhagen, establishing a life-long relationship with the revered Quantum Patriarch; a stint at Gottingen Germany where he worked with Max Born, and met the unusual Robert Oppenheimer;[2] and then to Leiden where he worked with the tragic Paul Ehrenfest.[3]

Cambridge University worked hard to bring him back to the fold and in 1927 he returned as a Fellow and began his career as a teacher and researcher. It was in his capacity as an instructor of Quantum Mechanics in 1930 that he wrote his famous *The Principles of Quantum Mechanics* which is on every physicist's shelf and is as readable today as then. It's impossible to understate the importance of that book. It set the intellectual stage for all of us as the first clear textbook on Quantum Mechanics on which most subsequent texts were based.[4] Every word in that book seems absolutely necessary and no extra words are used—it's quite a pretty piece of scientific literature. Rather an unusual legacy for a 28 year old. (The photograph at the beginning of this chapter is of Dirac taken at just about this time.)

But his most legendary contributions came before 1930: in particular the problems he tackled between 1926–28. He fixed an apparent incompatibility between Quantum Mechanics and Special Relativity, but right before that, he put photons and electrons on the same mathematical footing. Not bad for someone still in his 20's.

Paul Dirac passed away in Tallahassee and is buried there. He was unabashedly content with his new U.S. life, and even blossomed socially in Florida with many friends and the more relaxed atmosphere of the American university campus.

17.3 The Dirac Equation

You know that when someone has an equation named after them...well, there's a story behind that. We saw in the last chapter that in order to make sense of the spectra of even the simplest atoms, something else was required of Schroedinger's original wavefunction—in Wolfgang Pauli's hands it seemed to be made of two pieces according to the ad-hoc idea of spin—the regular wavefunction with the uncalled-for spin addition sort of duct-taped onto it. In this way, the wavefunction for an electron would be represented as

[2] Robert Oppenheimer was a theoretical physicist who was educated in Cambridge and worked for Max Born where he was on the front lines in applying Quantum Mechanics to problems in atomic, nuclear, and even astrophysical problems. He was eccentric and a ferocious worker, often going without food when engaged on a problem and sometimes enthusiastically taking over seminars to the point of irritation by other attendees. He came back to the U.S. and joined the faculty at Berkeley where he essentially founded modern theoretical physics in the U.S. A passionate advocate for social change, he had collaborators and friends that got him into terrible trouble in the 1950's when he lost his security clearance in a very public and humiliating set of Congressional Hearings. This was a big deal since it was Oppenhemier who was tapped to lead the Manhattan Project that organized an enormous collection of physicists, chemists, and engineers to build the first nuclear bombs that ended World War II in the Pacific. The FBI never trusted him and by the time of his triumph as the leader of the project that ended the war, they had much ammunition to use against him—and they did. A colleague noted sarcastically that he he been an Englishman, he would have been knighted for his contributions. But in the McCarthy era in the U.S. he was branded a traitor. He died at the age of 62 from lung cancer.

[3] Ehrenfest was a Dutch theoretical physicist who tragically killed himself when in a depression over his self-perceived inability to keep up with the pace of Quantum Mechanics. Paul was one of the last to see him alive and berated himself for not recognizing that Ehrenfest was troubled to that degree.

[4] In that sense, we all learned Quantum Mechanics from Paul.

Two Component Wavefunction

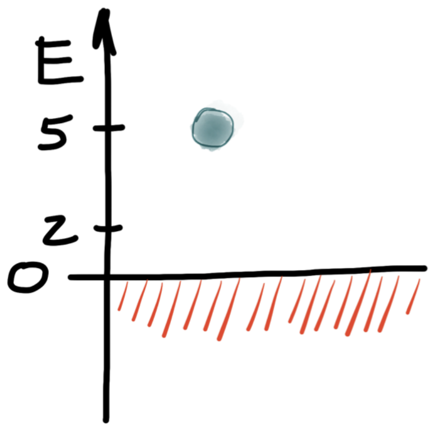

Figure 17.2: The energy of a classical particle can be anything down to zero.

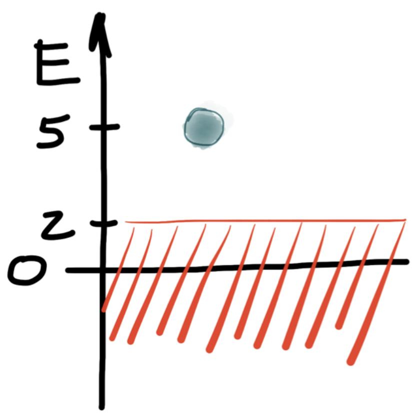

Figure 17.3: The relativistic energy of a particle is made up of the kinetic part (here $E = 5$ and the energy due to mass, here $E = mc^2 = 2$. Energies cannot be less than that due to mass.

either $\psi_\uparrow$ or $\psi_\downarrow$ which represent the spin up (+1/2) or spin down (–1/2) parts. We say that the state of an electron is then represented by a "two component" wavefunction, a double- valued quantity.

Along with the idea of spin came the the Pauli Exclusion Principle which was just indefensibly asserted: No two electrons can occupy the same "state." It was bold and it worked. Everyone was puzzled.

Completely independent of the lack of formal reason for the messy spin solution, Quantum Mechanics also suffered from a more serious embarrassment. The Special Theory of Relativity was by the 1920s essentially undisputed. Yet when one mixed Relativity with Quantum Mechanics of Schroedinger, absurdity was the result. Electromagnetism had been pliable enough to accommodate Relativity but there was no such nicety when Relativity was mixed in with Quantum Mechanics.

There were basically two reasons for this. First, in Schroedinger's picture a wavefunction seemed to be dependent on one's rest frame and that can't work. The chemistry of an atom can't be different for one observer over another. This was a kind of practical problem.

But the second reason is more serious and went to the heart of the quantum idea. Here's how to see it. When we compare the energies of a non-relativistic particle with those of a relativistic one we can see why. The "regular" Kinetic Energy is

$$E = 1/2mv^2,$$

which we can slightly rewrite by using the definition of momentum $p = mv$ to eliminate the v in favor of p. We get:

$$E = \frac{p^2}{2m}.$$

The energy is proportional to the square of speed or momentum. As the momentum goes to zero, the energy does also.

Now take a walk on the wild side: It's obvious that I can walk slower and slower and slower...all the way to a energy of zero in classical physics. Once I reach zero speed, I stop. My kinetic energy is spent and there's certainly no kind of walking that I can do to nudge me into a negative kinetic energy! So far, so good. We can see this in Fig. 17.2 where the energy of a particle in arbitrary units is shown and the disallowed energies are hatched in red. For a classical particle, the energy can go all the way to 0, but not below. Here, "energy" means kinetic energy.

Relativity mixed with Quantum Mechanics messed with this obvious sounding idea of zero as the "lowest you can go."

$$E^2 = p^2c^2 + m^2c^4 \tag{17.1}$$

This is a lot different from the classical kinetic energy in two subtle ways. First, as the momentum goes to zero, the energy settles in to just the rest mass energy, a finite value. As long as a classical object has a mass it will always have a finite amount of energy. We can see this in the cartoon of Fig. 17.3 where the energy now is both kinetic and due to mass and while the particle can slow to zero kinetic energy, there's still always the mass energy remaining. But in Schroedinger's Quantum Mechanics it's a different story.

Remember quantum behavior is unusual since it's *not* continuous. It's jerky. An electron can go slower and slower, but there is nothing to prevent it from quantum-jumping anywhere. And when Relativity was mixed in and negative energy states seemed a consequence, then that was exactly what Schroedinger's Equation predicted: an electron could change its energy from something finite and positive, to quantum-jumping right past zero, all the way to a negative value! Figure 17.4 shows that: a particle with energy of 5 units jumping past 0 into negative oblivion!

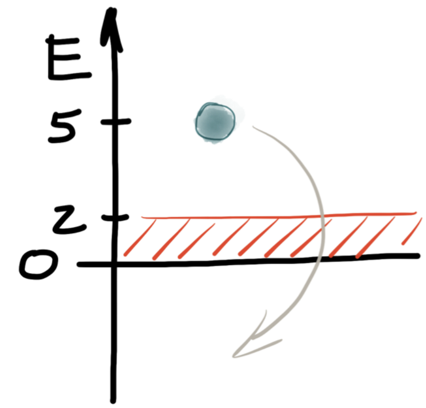

Figure 17.4: In Quantum Mechanics there is nothing to prevent a positive energy electron jumping into a negative energy state, especially one predicted by Relativity!

We can see this from what we learned for the relativistic form of energy:

That's the second way that Relativity messes with Quantum Mechanics. In Schroedinger's equation, what matters is the energy itself, but the relativistic form is a *square*, E^2. So because of that obnoxious "2," the energy by itself has to be:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}.$$

Since both the positive and the negative roots when squared give the primary equation above, both are solutions. So as in Fig. 17.4 a particle with positive energy can quantum-jump to any negative energy!

Forgetting units for a minute: If the relativistic energy squared of an electron were 25, then both +5 and –5 would be solutions. If the rest mass energy of the electron were 2 in these units, then the kinetic energy of the positive electron would be 3 and the kinetic energy of the negative electron would be –3. Should an electron quantum-jump from total energy +5 to –5, then presumably it would be accompanied by a photon of energy 6. Nothing like that had been observed.

It's these two-values of the square root that permit a quantum particle to jump to a negative value for the − sign as well as the more sane positive value of $+E$. If such a thing happened in a classical theory, we'd just throw out the negative solution since it's not reachable. But we have to keep them both in a Relativistic description since quantum-jumping particle could get there!

But it gets worse. When Relativity is added, you also end up with the possibility of a negative probability! *Now that's just insulting.* Negative probabilities don't make any sense at all! Indeed Schroedinger's first attempt at his Quantum Mechanics was just such a relativistic description, but when he ran into the negative probabilities he gave up on Relativity and ran for the hills (the Alps with his girlfriend) settling for his non-relativistic version.

You Do It 17.1. Dirac's Energy

or copy the solution

From the relativistic energy equation, show that if the total energy in Fig. 17.3 is 5 arbitrary energy-units and that if the mass energy in those units is 3, then the energy of motion of that particle is 4 units. Likewise, what happens when the p goes to zero, what total energy results?

There was something incomplete about either Quantum Mechanics or Special Relativity. It was too good to toss out entirely, but at the same time it had to be a part of the truth. Everyone knew this but nobody had any idea how to fix it.

17.3.1 Relativity and Quantum Mechanics

Paul Dirac always had his own approach to theoretical physics. He said that he "played" with equations without regard to their apparent connection to what was in vogue. To him the beauty of the mathematics was most important and we would have to say that this approach worked out pretty well for him. But normal people can't work this way. Somehow hidden in his personal scientific method was a canny physical insight—something that can't be taught.

Wait. *I've never heard of Paul Dirac.*

Glad you asked. *The most famous physicist whom you've never heard of (unless you read the last chapter...then, you're an expert). Ask any pro, and she'll tell you that next to Einstein, that Paul Dirac was probably the most imaginative and influential physicist of the 20th century. You're about to see why with antimatter in a second, and then in the next chapter, our whole notion of particles. All seeds sown by this unusual man.*

Dirac broke the Schroedinger-Relativity problem down and started over, while keeping the squared energy problem squarely in mind. After trial and error, he constructed an equation that was linear in energy (no square root), but he paid a price: instead of the single wavefunction of Schroedinger or the dual wavefunction of Pauli's, what Dirac found that he had to contend with were *four* different wavefunctions. This was good news and bad news.

Definition: Dirac Equation.
The model that Paul Dirac created that merged Quantum Mechanics and Special Relativity.

The good news was that his formulation was invariant with respect to any rest frame. Check. He no longer had the negative probability problem. Check. And he found a surprise: Two of his four wavefunctions had all of the features of Pauli's two spin wavefunction![5] So spin just popped out of the equations for free, without being introduced by hand as Pauli had done! This was quite remarkable. Quantum Mechanical Spin went from being a wild guess to an actual requirement of Relativity. Spin is a purely relativistic quantity.

[5] He found this by subjecting them to a hypothetical magnetic field interaction.

But he still had the negative energy problem—it didn't go away with his new equation. And he had to figure out what the other two wavefunctions meant. He thought pretty hard about about the consequences of these extra solutions over a couple of years, frustrating everyone who had to listen to him as he worked this out. Eventually he came up with a scenario that was controversial, to say the least.

17.3.2 The Vacuum, Spruced Up

We're about to enter a subject that's going to also plague us to the present day: Nothing. Can Emptiness happen? As I hinted when we talked about Newton's Cosmology this subject is an ancient one and most associated with Aristotle who claimed that the very idea of space was defined by objects themselves. Take away objects and there's no way to speak of the space between things, if there are no things to be between. So emptiness didn't exist for him. But we saw that Newton was one of the first to flip that idea in favor of Emptiness as a vessel into which "stuff" in the universe is added. Then, Einstein seemed to bring Emptiness back by ridding science of Newton's Absolute Space. Clearly, much ado about Nothing!

But the quantum realm complicates the subject. Before Dirac, Empty Space was either declared impossible on logical grounds (Aristotle and Einstein), or deemed necessary by definition (Newton). But

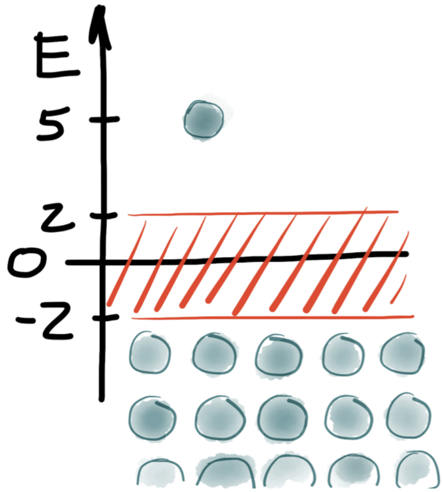

Figure 17.5: In Dirac's picture, positive energy electrons co-occupy the world with negative energy electrons, where the latter fill the vacuum, each occupying each available energy and not overlapping according to the Pauli Exclusion Principle.

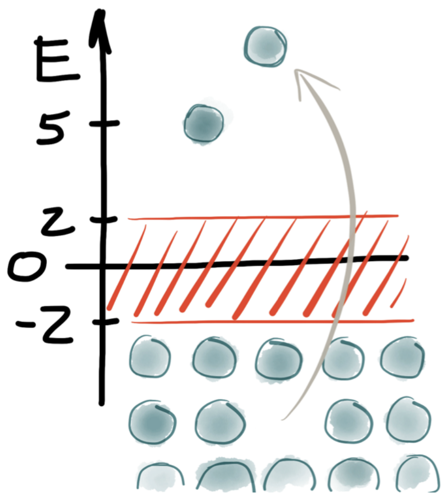

Figure 17.6: In some way, one of Dirac's negative energy electrons has been kicked free, to live a long, happy life among the positive energy world.

nobody questioned that Nothing meant anything but *Absolutely Empty*. Nobody suggested that Nothing could actually be full. That's that path that Dirac tentatively went down in his somewhat desperate attempt to understand the second pair of wavefunctions that arrived, unbidden out of his equation. The Vacuum is a strange place in the quantum world and Paul Dirac first taught us that.

How this gets mixed up in the Quantum Story is the following. The idea that the negative energies couldn't be ignored didn't seem to correspond to the fact that we're here. Everything seeks the lowest energy state, and quantum entities are no different. So if negative energies are available, why haven't all objects quantum mechanically plunged into the lowest, negative energy state...of negative infinity? Here is where his imagery went to work.

Dirac always claimed that the geometry from his mathematical training and isometric drafting from his engineering training guided his physics. He needed a mental picture and what he visualized in this most difficult of all problems was a whole new world that we're not familiar with. In this new world he needed to allow for negative energies, but somehow stop particles from jumping into them...so he took Pauli's Exclusion Principle at its word and simply asserted that all possible negative energies must already be filled with electrons. Since no two electrons could occupy any state, then if the negative energy slots were all unavailable—no positive energy electron could quantum-jump to them. So Dirac's Vacuum is empty on the one side—no particles with positive energies (empty, like a traditional Vacuum) and full on the other side—all negative energy "slots" are filled with negative energy electrons as suggested in Fig. 17.5.

But the real world is active. And he knew that something else must happen besides the empty positive energies and full negative ones. Suppose that some energy is deposited into his dual world on one of those negative energy electrons. If the photon has enough energy, it could liberate it from the confined negative energy sea to the free world of positive energy matter— where we're aware. The picture in your head should be something like that in Fig. 17.6. Think about what has to happen in order for this newly juiced-up negative electron to become "real" as a positive energy, regular electron. The energy has to overcome the negative value to get it from -4 units of energy just to get it to zero. But it's not real yet, since it doesn't have enough mass energy to qualify as a real electron, so that deposit has to have at least 2 more units. So 6 units of energy would get it to just enough to make the positive energy electron...at rest. Any more than 6? Well that just means that the electron has kinetic energy once it's liberated, here more than the 5 units of the spectator electron in Fig. 17.6.

But that's easy to picture. What's left behind where the negative energy electron was before its gift of energy? Dirac called this defect in the sea...a "hole." The characteristics of that hole are very interesting.

17.3.3 The Hole

Not quite Alice's Rabbit Hole, but Dirac's hole idea had bizarre consequences just the same. What would the characteristics be of a hole? This negative energy world, in Dirac's imagination, is all around us but just like a fish is unaware of the water its immersed in, we don't notice it. So let's take each of the primary qualities that define "electron" one at a time.

How about the about the electric charge of a hole: in its full state, the sea has an infinite number of electrons and so it's electric charge is...infinite. We'll call it Q_{sea}.

Infinite. That's just terrific.

Likewise the energy of all of the individual electrons similarly adds to infinity. Let's call it E_{sea}. What Dirac showed was that relative to the sea of the negative charge, negative energy electrons, that a hole—thought of as an absence of a negative charge, negative energy electron—had defined properties which are opposites of regular electrons.

✎

For example, when an electron is removed from the negative sea, what's left over? Well, it would be $[Q_{\text{sea}} - (-e)]$. But *relative to the "sea"* you'd remove the big background charge of that sea and the charge left over of the hole would be:

$$Q_{\text{hole}} = [Q_{\text{sea}} - (-e)] - Q_{\text{sea}} = +e$$

a positive electric charge. Likewise the energy of the hole, also *relative to the sea* would be:

$$E_{\text{hole}} = [E_{\text{sea}} - (-E_e)] - E_{\text{sea}} = +E_e$$

a positive energy.

Didn't like that? Let's count the charges in a different way. Let's suppose that instead of infinity, the total negative electric charge of our story-vacuum is -10. Let's write it as

$$-1-1-1-1-1-1-1-1-1-1=-10.$$

Now let's take out one of our electrons:

$$-1-1-1-1-1-0-1-1-1-1=-10+1.$$

Notice that the left side includes a "hole" where there was a negative electron and that the right side describes that as the addition of $+1$, something with a positive charge. So in the language of the previous paragraph,

$$(-10-(-1))-(-10)=+1$$

since here $Q_{sea} = -10$ rather than negative infinity. Got it? Relative to the sea of negative charge, the removal of one of them acts like a positive charge.

So this hole that's left behind would *behave* as if it were a relatively positively charged object with a positive energy.

Now this confused him, but he pressed on with an interpretation. Remember, Dirac's Equation—which is what we call it now—did an amazing thing: it accounted for spin, it reduced to the Schroedinger Equation for electron velocities small relative to *c*, it also accounted for a slightly different but experimentally correct atomic spectrum. It had a lot going for it. Except: what are those negative energy solutions about? They seemed to point to a positively charged particle when a photon comes along and promotes one into the positive state.

At first Dirac thought that maybe his equation had revealed a reason for the proton to exist—that it was maybe the positively charged hole.[6] But it was pointed out by Herman Weyl and Robert Oppenheimer that whatever the hole is, it had to have the same mass as an electron, so it couldn't be an explanation for the proton. Back to the drawing board.

[6] This is a new way of thinking in and of itself: that an equation might be an actual reason for a particle. It's modern reasoning, which we employ all the time and Dirac was the first to do so.

People became tired of Dirac's trying to find an interpretation for his equation: "The saddest chapter of modern physics is and remains the Dirac theory." A letter from Heisenberg to Pauli. "I find the present situation quite absurd and on that account, almost out of despair, I have taken up another field..." A letter from Heisenberg to Bohr.

Definition: Antiparticle.
An antiparticle is a particle of the same mass, but opposite electrical charge. All particles have antiparticle counterparts, but some of them are their own antiparticle (like the photon).

Definition: Positron.
The antiparticle of the electron got its own special name.

After a couple of years in 1930 Dirac took the huge leap of concluding that the hole behaved like a separate, distinct particle. A new particle. A partner of the electron, but the *anti-partner* of the electron—the "anti-electron," (his words) or as a later journal editor would later suggest, the "positron."

The Dirac Equation would then account for four wavefuctions which uncovered the full electron-family: two spins of each the electron and its anti-particle cousin, the positron. It was an audacious move. He even went so far as to predict that the proton also should have a negatively charged anti-particle cousin, an anti-proton. (That prediction came was confirmed in 1955.) Plus he imagined how it might be unlocked from the negative sea—by the addition of some sort of energy, as we pictured in Fig. 17.6.

Where would energy of that sort come from? It has to be neutral and luckily, at this time the neutron hadn't yet been discovered and so the only candidate was a photon and that's what Dirac suggested: a photon with enough energy could strike a negative energy electron, promote it to a positive energy electron and leave behind a positive energy, antiproton.

We need a nomenclature for antimatter and for reactions. You can probably sense Feynman diagrams are coming! We'll write a process with an arrow connecting what happens at the beginning (the ingredients) with what happens when the dust settles (the cake). So like a chemical reaction. We'll call all particles by their names, which will be a Latin or Greek letter and usually write an antiparticle with the particle's name and a bar over the top of it.[7] We'd write this particular transition of a photon becoming an electron and an anti electron as:

$$\gamma \to e\bar{e}. \tag{17.2}$$

Where did the photon go? For now, we'll say it "converted" into the electron- positron pair and tell you how we deal with its mysterious disappearance in the next chapter. So with lots of photons flying around the universe and presumably lots and lots of negative energy electrons in the sea, you'd maybe expect that electron-positron pairs would be popping up all over. And you'd be right.

As if on schedule, soon after it was proposed by the young Paul, the positron showed up in dramatic fashion in the hands of a young Carl.

[7] Sometimes, I'll be explicit and perhaps write e^+ to indicate the positive election. Context will rule.

As we will see, the hole idea ceased to be an acceptable explanation for antimatter in favor of a more general description. But the hole idea was found to be fruitful in another area of physics, now called Condensed Matter Physics, or Solid State Physics. These are the fields that study materials and their behavior. One of the more interesting materials is responsible for the electronic device on which you might be reading this right now: the semiconductor, which is most precisely described as a material that under the right conditions of voltage behaves at an interface as if conductors (electrons) and holes set up currents which can be switched on and off. The holes have all of the features of electrons, but move in the opposite direction in the presence of an electric field. In an important way, Dirac was the instigator of the most wide-sweeping technological advance in the history of the 20th century!

17.4 Following the Mathematics

We've just crossed a line. To this point we've been messing with common sense in Relativity and Quantum Mechanics, but leaving things like what it means to travel near the speed of light, or how to manipulate a wavefunction as technical recipes. This antimatter business, and how it came about is a different story. Now we're talking about reality. What must be the case. And what Dirac suggests about what must be the case should be testable, and it appears to make little sense. But the mathematics made us do it!

Wait. *You mean that we're going to start to believe in mathematics as if we're forced into it? Don't we have a choice to just say "no"?*

Glad you asked. *Sorry, but being pulled by the mathematics into uncomfortable interpretations of just how Nature seems to function marks the beginning of the modern approach to physics. It wasn't everyone's cup of tea. This is different from everything that came before: the ground starts to shift continually from this point. What made sense yesterday, is cast aside today. What is actually real yesterday is now uncertain today. The Quantum Heroes had to metaphorically close their eyes and walk forward, guided only by their penciled scribblings. They had to cast aside common sense and begin to imagine that our brains were not evolved to actually understand Nature in her most fundamental ways. Sure, we can make predictions and test those predictions, but we can't sensibly describe what's actually happening!*

This was hard for the pioneers. They were inventing modern physics and you can identify how different people had different reactions:

1. Some ignored any interpretation of Quantum Mechanics altogether, letting the mathematics speak and not worrying beyond that. Many (or most?) physicists work in that mode today.
2. Some worked incredibly hard to find a way to describe in words (human language!) what it all means.
3. Some concluded that the whole enterprise was at best incomplete, if not wrong! That was Einstein's position as he became older.

As troublesome as the first painful lessons were, it was only going to get worse!

To remove the suspense, let me say that most of us have learned to live with this state of affairs. We're prepared to "follow the mathematics" and live with the consequences, content with the notion that our brains and our communication skills need not to have evolved beyond simple appreciation for macroscopic objects and everyday velocities. There's no reason why what goes on in the realm of atoms should conform to our notion of common sense. It's strange and beautiful to be able to probe the inner structure of the Universe. It's fascinating that humans are able to find ways to understand things at this level. At least that's my story and I'm sticking to it! So prepare yourself for an unusual ride from this point.

17.5 Just In Time

The drama of discovery that sometimes goes like this: scientist Moe *over there* predicts something that nobody ever dreamed of before while scientist Larry, *over here* has been puzzling over just that phenomenon without knowing of Moe's idea. As if by chance, one hears of the other and both get trips to Stockholm.

That's what happened between Dirac (Moe) and a young Caltech researcher by the name of Carl Anderson (Larry)...but it's even more intriguing since right under Dirac's nose, a group within Rutherford's Cambridge (Curly?) laboratory was puzzling over the same thing without knowledge of even their neighbor's prediction! In fact many people had a surprise in their data that they missed or rather, *dis*missed.

17.5.1 Cosmic Rays

Actually, we're protected here on Earth by the capturing of many of these particles by the magnetic field that surrounds the Earth. One of the unsolved problems for interplanetary space travel is the medical danger from these particles for astronauts who would be outside of our protective magnetic belt for months.

As you read this, you're under attack. Bombarding you from above are hundreds of elementary particles going through your body in a few minutes. Every once in a while, thousands blast through you like a torrential rain of electricity. Feel it? No, you don't although the errant particle can interact with your DNA and perhaps induce some mutation. But as the bag of water that you mostly are, you're mostly unaffected by their intrusion.

"Cosmic Rays" have been a puzzle since about 1910 when they were first taken seriously. Remember how Rutherford used ionization to detect and measure the amount of charged radiations in decays?

Charged alpha, beta, and gamma rays would discharge, electrically charged-up electroscopes. It was puzzling to find that if one was patient enough that even when an electroscope was not near any obvious decaying nuclei, that they would still discharge and it was correctly determined that there must be radiation from the soil on Earth that contributed equaling things out.

The clever Jesuit Theodor Wulf tested this idea. In 1910 he climbed to the top of the Eiffel Tower with an electroscope of his own design and showed that even when you got away from the earthy stuff, the discharge continued. That something *above* the ground was responsible for much of that discharge. By 1914 crazy people like Rudolf Hess were risking their lives in balloons, going nearly 30,000 feet into the atmosphere and finding that their electrscopes' loss of charge not only continued, but was more quicker the higher they went!


Figure 17.7: Next time you're flying across country, imagine looking out your window at cruising altitude and seeing a tiny balloon with a man in full suit and tie manipulating a crude scientific instrument. The Captain had just announced that we're at 30,000 when I took this picture. This is the height that Rudolf Hess climbed in his balloon to measure Cosmic Rays.

It became clear that Cosmic Rays were, well, Cosmic! They come from outer space.

Since those early days studies of this large bombardment by charged particles have revealed remarkable data about them. First, by noticing that they spiraled around the Earth's puny magnetic field in a particular direction—East to West— their origins must be positive particles (we think now, mostly protons and not higher mass nuclei). Next by building detectors over large distances, the size of whole counties, it's clear that a few times an hour that the surface is blasted with huge "air showers" covering large land area with millions of particles. This led to the correct idea that whatever was hitting the upper atmosphere was causing a gigantic cascade of particle production before the Earth (and scientists' instruments) gets in the way. This was the state of the art around 1932. By now we know that the energies of these strangely energetic protons are enormous, as much as 8 orders of magnitude higher than the highest energy accelerators — many orders of magnitude higher than any conceivable man- made accelerator could ever achieve on Earth.

Their origins might be from a variety of sources, most likely shock waves from exploding stars if they're local (from within the Milky Way) or the spittle from terribly angry whole galaxies called Active Galactic Nuclei that shoot out streams of particles from their cores. In any case, much of the birth pangs of Particle Physics came from studies of Cosmic Rays in the 1930's and 1940's as clever experimenters improved on old instrumentation techniques and invented whole new ways of detecting particles. That's where the Dirac story gets interesting. In California and unbeknownst to him, across town at Rutherford's lab.

17.5.2 Anderson's First Experiment

Carl Anderson wasn't in the habit of disregarding his boss. Robert Millikan was legendary, authoritative, and pretty sure of himself. For good reason, since experiments that he'd done had contributed mightily to the burgeoning story of quantum physics—it was he who demonstrated that the electron had a fixed

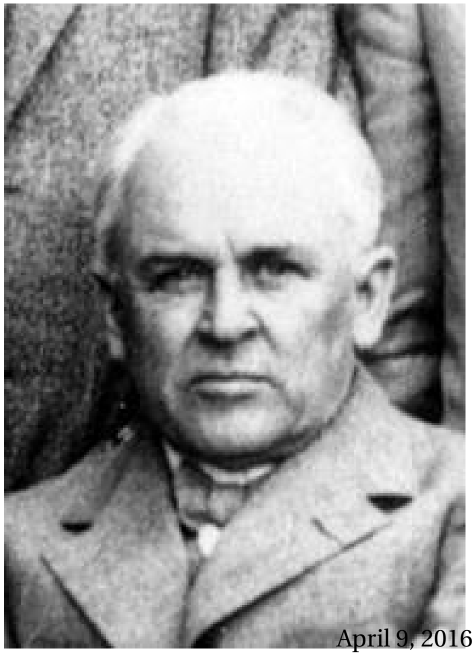
Figure 17.8: Robert Millikan in 1935.

charge and that it couldn't apparently be divided any more than that fixed quantity which we now call *e*. Never shy, he loudly proclaimed that Einstein's photon idea was wrong and then proceeded to demonstrate the opposite in pioneering measurements of the Photoelectric Effect (which hardly tempered his blistering criticism of the photon idea).

Millikan invented the term "Cosmic Rays."

He also had taken up study of Cosmic Rays at the California Institute of Technology—aka Cal Tech—which was becoming the most sophisticated center of experimental physics and observational astronomy in the U.S. He had a theory that Cosmic Rays were high energy photons—the left-over "birth-pangs" of creation— and was in a feud with his former University of Chicago colleague Arthur Compton who suggested that they were protons. Eventually, again, Millikan was found to be wrong when it was shown that the cosmic rays were bent in the Earth's feeble magnetic field. No matter. Millikan was quick to come to a conclusion and would defend it to the end, verbally steamrolling anyone in the way. He certainly wasn't much interested in backing down in measurements of his own devising, in a field he largely pioneered, especially from his former Ph.D. student, Carl.

Anderson had been a student of Millikan's at Cal Tech and then stayed as a young researcher after his doctorate and eventually spent his whole career there as a professor. He recalled that there was a three year stretch when he never saw his advisor one time, perhaps confirming the horror stories of graduate student life in some pockets of academic science (not at our institution, of course). So he became pretty self-reliant and an expert at constructing large Wilson Cloud Chambers and building scary high-current magnets in which to put them.[8] This was the preferred mode of investigating Cosmic Rays. Taking their portraits. Randomly.

[8] The whole laboratory's lights would dim when Carl would turn on his magnets.

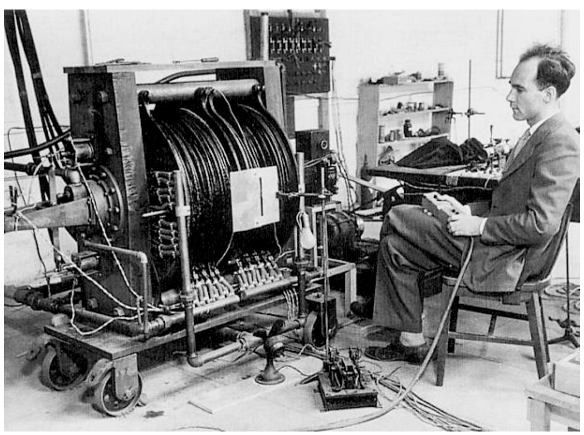

Figure 17.9: Carl Anderson working in front of his magnetized coud chamber. We all dress in wool suits in our laboratories.

Cloud Chambers

By the time that Carl Anderson was studying Cosmic Rays, the Cloud Chamber method was well-established, and very human-intensive. A person would just blindly take hundreds of photographs of the chamber, sufficiently illuminated in order to show the tracks as white dots. By analyzing the density of the dots on a track, researchers had determined that the heaviest particles—protons and nuclear fragments—left dense, heavily ionizing tracks. Electrons left much less dense tracks and were uniform, time after time. Remember, people thought that electrons and protons were all that existed, as the neutron had not yet been discovered (showing up in a cloud chamber in 1932) and so they became adept at picking out their familiar proton and electron patterns.

What Anderson did was build that very large magnet around his cloud chamber, allowing him to bend the protons one way and electrons in the opposite way. By knowing the curvature and the strength of the magnetic field, he could measure the energies of the particles and guess at the Cosmic Ray energies themselves. At least, that was the plan. He fired up his magnet with a uniform 24 kGauss (a small bar magnet has a field strength of about 100 Gauss) solenoidal field then started taking pictures. Lots of them. 1500 of them of which almost all were blank (it was a chancy thing, this picture-taking) but out that bunch a dozen or so were problematic.

You Do It 17.2. title

Draw a plate and a track of cosmic ray electron coming from the top. Assume that the magnetic field is oriented so that positively charged particles bend to the right.

or copy the solution

Right away he began to see ambiguous results. Remember, he could distinguish between a positive proton and a negative electron. What he saw appeared to be positive tracks (because of how they were bent) but with the track densities of electrons, which made no sense. But how could he tell which direction they were going? If a track was a negative electron, but came from above, it would bend one way and appear to be normal, but it could logically have been a positive electron coming from below and be abnormal.

That the track densities of these anomalies were not proton-like and since he was not so Millikan-stubborn, he pursued it and did something clever: he inserted a plate of lead inside his chamber so that when a particle would go through it, energy would be lost and the bending when it emerged would be

"In the spirit of scientific conservatism we tended at first toward the former interpretation, i.e., that these particles were upward-moving, negative electrons. This led to frequent and at times somewhat heated discussions between Professor Millikan and myself in which he repeatedly pointed out that everyone knows that cosmic-ray particles travel downward, and not upward, except in extremely rare instances, and that therefore, these particles must be downward-moving protons. This point of view was very difficult to accept, however, since in nearly all cases the [thickness of the track] was too low for particles of proton mass." Anderson, C. D., American Journal of Physics 29, no. 12 (December 1961): 825.

tighter, a smaller radius. That way he could tell which direction a track was going: if the a track came from the top, it would bend little before the plate, and then more after passing through it. If from the bottom, the opposite. Therein lay the surprise.

On August 2nd, 1932 (18 years to the day before I was born), he captured an iconic event. This was the clearest example of a particle that was obviously an electron (since the track density was electron-like-lite), obviously coming from below (since the curvature greatly became tighter above the plate of Lead), obviously not a proton (since they had learned that a low energy proton would barely have struggled to get out of the Lead at all), and obviously positive (because of the direction that it bent). Staring at him directly was the first definitive evidence of an anti-electron, which he called the "positron" in the paper that he quickly wrote and sent to press, against the wishes of his boss. By September, the rest of the world would know that antimatter existed.

Well, not quite.

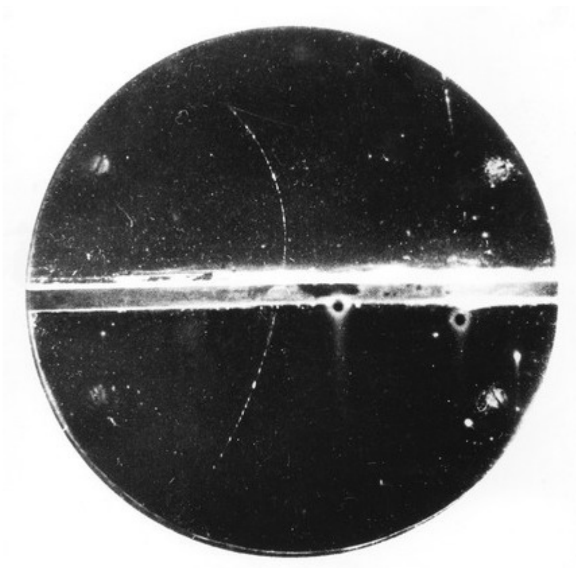

Figure 17.10: The famous photograph capturing the first acknowledged antimatter particle ever discovered. The bend of the particle is according to a positive electrical charge and the loss of energy (tighter spiral) shows that it comes up from the bottom.