Chapter 7

Diagrammatica: Collisions

Space, Spactime, and Momentum Diagrams

We're all about collisions, and we'll need a vocabulary and a working relationship with our Space, Spacetime, and Momentum diagrams for a few different types of collisions in QS&BB. We started this discussion in Chapter 5.5 with the simplest process of all, the dead ball billiards shot and nursed it through the discussion of Kinetic Energy. There are more.

In this Diagrammatica, I'll add to that list of one and order them all so that we have a readily available inventory of the kinds of collisions that we'll encounter. The emphasis will be on the diagrams, not on the algebra.

Nonetheless, you'll need your pencil. I'll wait…

Apart from some examples that you can work near the end, the presentation will all be about four different objects: A, B, D, and F.

A and B have the same masses, in our fake momentumunits, $m_A = m_B \equiv m = 5$; D is a big guy with $m_D \equiv M = 10$; and F is really well-fed with $m_F \equiv \mathrm{M} = 15$

In the examples that follow, A, B, D , and F will be crashing into one another at varying speeds and we'll fill in tables and draw the diagrams. The organization is intentionally very structured—the arrangement is by type of collision and I want you to return when we come across reactions of these types.

Each of the following sections will have the following features:

- There will be a table with the masses, velocities, and momenta for a particular examples of the collision featured in that section.
- There will be a cartoon of that collision for hopefully easy recognition and remembering later.
- The three diagrams will be shown for each:
- In the **Space Diagram**, imagine that the trajectories in the top frame are taken during the same time window as the bottom. So a longer line means that a particular trajectory is fast. These are sketches, and not to scale. I'll indicate the relative masses with little shaded balls on the arrows to give you a feel.
- In the **Spacetime Diagram**, the speeds are of course represented by the slopes and so pay attention to the size of the slope (steeper means faster) and the sign (positive means to the right and negative means to the left).
- In the **Momentum Diagrams**, the momentum vector's lengths correspond to the scale indicated above the diagrams.

By the way, we now know that both momentum and energy conservation are required in order to solve for most of these situations, but I'll tell you answers in the tables rather than work out the algebra in detail. It's the least I can do. No, seriously. I don't think I could do less than that.

7.1 Elastic, Two Body Scattering Event, $A_0 + B_0 \rightarrow A + B$

Two body, elastic scattering comes in many forms: the masses of the two objects matters and the relative motion of each in the laboratory results in different consequences. We'll look at four different kinds of elastic scattering involving two objects. Let's reprise our original "Simplest Collision" so that we will

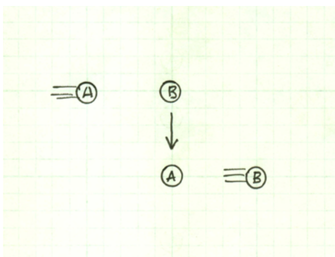
$A_0 + B_0 \rightarrow A + B$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	A:	(5)(2)	= 10	(5)(0)	= 0
	B:	(5)(0)	= 0	(5)(2)	= 10
	total sum:		= 10		= 10

Table 7.1: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

have all of our diagrams in one place. The dead ball billiard shot means that the target is sitting still, so the designation "fixed target." This collision in in one dimension and the three diagrams are show in Fig. 7.1 where now I'm working according to our generic A, B, and D. The example in Table 7.1 is representative of this kind of collision which is very much a standard in many kinds of physics experiments. One prepares a target which could be solid, liquid, or gas, and shoots a beam of particles into it. Simple.

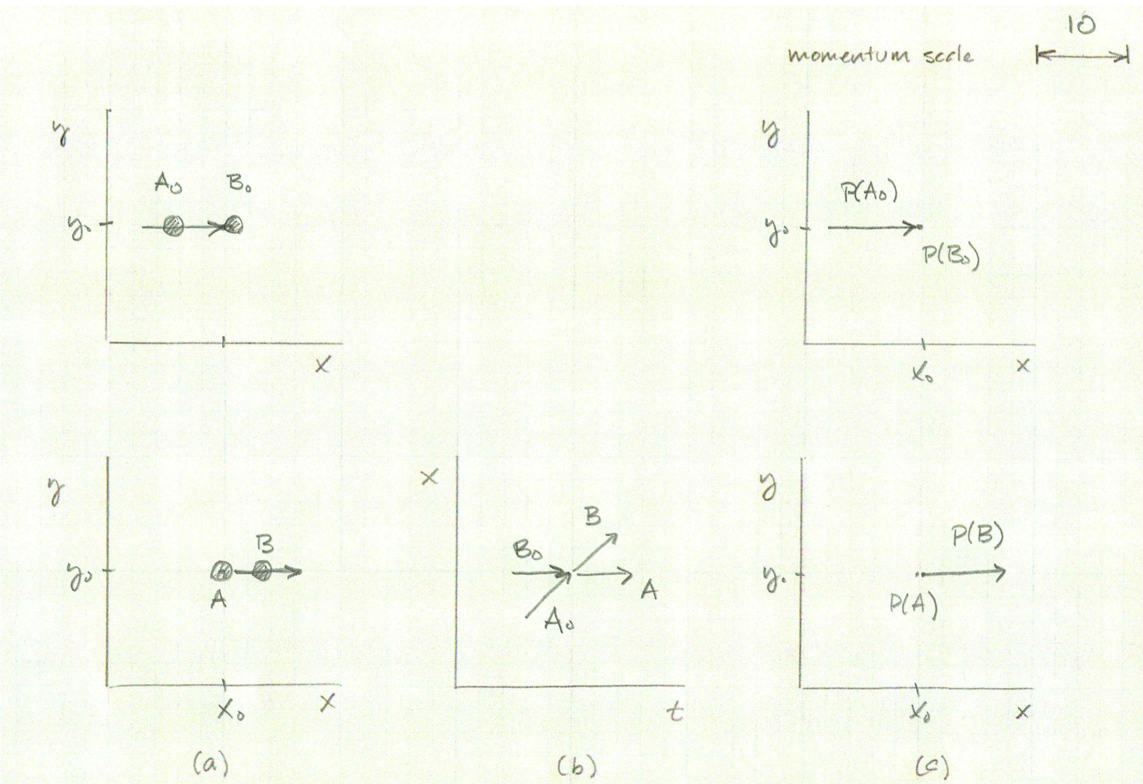


Figure 7.1: FTdiagrams2

7.1.1 Precisely Identical "Colliding Beam" Events: $A_0 + A_0 \rightarrow A + A$

Here's one more elastic collision of a particularly simple kind. Most of the particle physics laboratories of the last 30 years involve the head-on collisions of precisely identical particles (hence, the all A's above). Our relativity and quantum mechanics discussions will help to explain why this is advantageous. So I've separated these kinds of collisions for special treatment here. What follows is really a more general case of the elastic collisions that we started with in Section 7.1. It's always a characteristic of these collisions

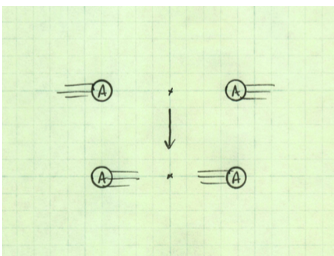
$A_0 + A_0 \rightarrow A + A$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	A:	(5)(2)	= 10	(5)(-2)	$= -10$
	A:	(5)(-2)	$= -10$	(5)(2)	= 10
	total sum:		= 0		= 0

Table 7.2: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

that the total momentum in the initial and final states is zero. We'll see why that's useful as well. By now, you're not surprised to see that the recoil of the As from such a collision is such that they have the same velocities, but oppositely directed.

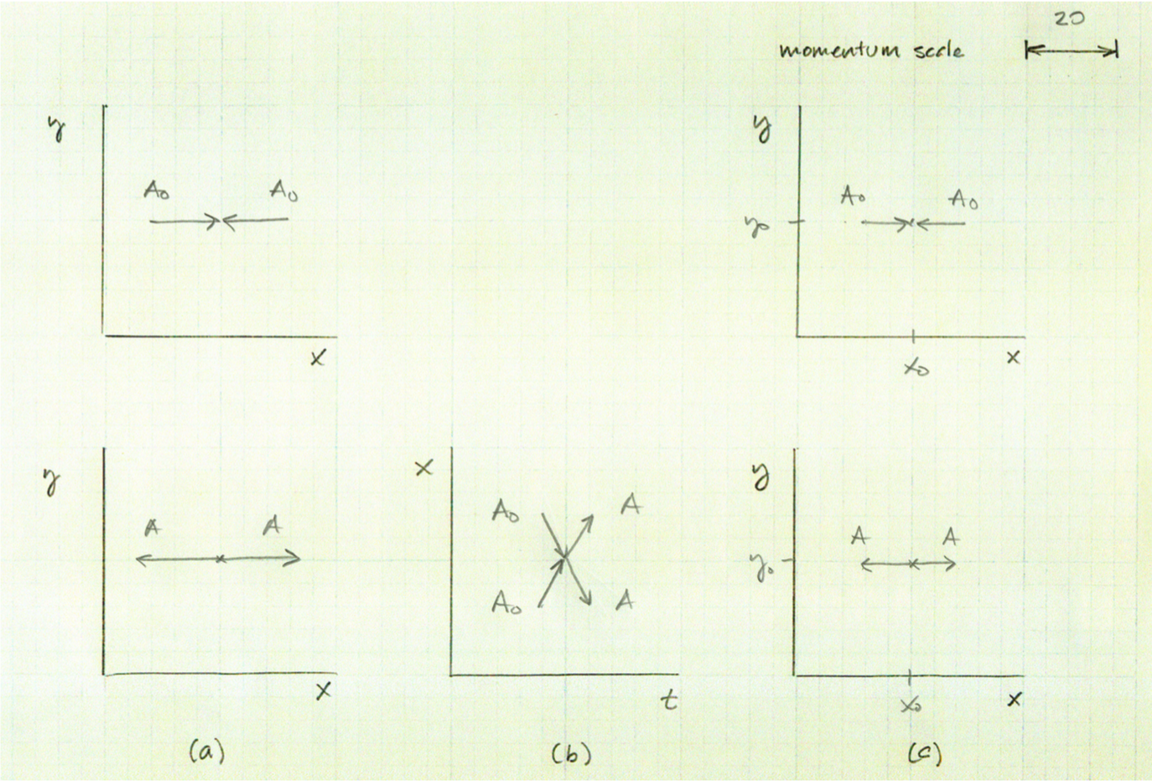


Figure 7.2: cm1Diagram

7.1.2 Moving, Bigger Target, Elastic Scattering Event, $A_0 + D_0 \to A + D$

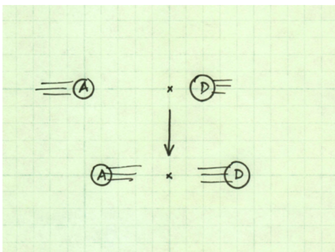
$A_0 + D_0 \to A + d$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	A:	(5)(4)	= 20	(5)(-2 2/3)	$= -40/3$
	D:	(10)(-1)	$= -10$	(10)(2 1/3)	= 70/3
	total sum:		= 10		= 10

Table 7.3: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

Elastic scattering comes in many guises. More generally, if the target is moving, features change slightly. Here, let's involve the heavier D moving to the left slowly at -1. Then (with my back-door calculation) I've calculated the particular reaction parameters in Table 7.3. Notice how the little guy bounces off the big guy. The three diagrams for this reaction follow.

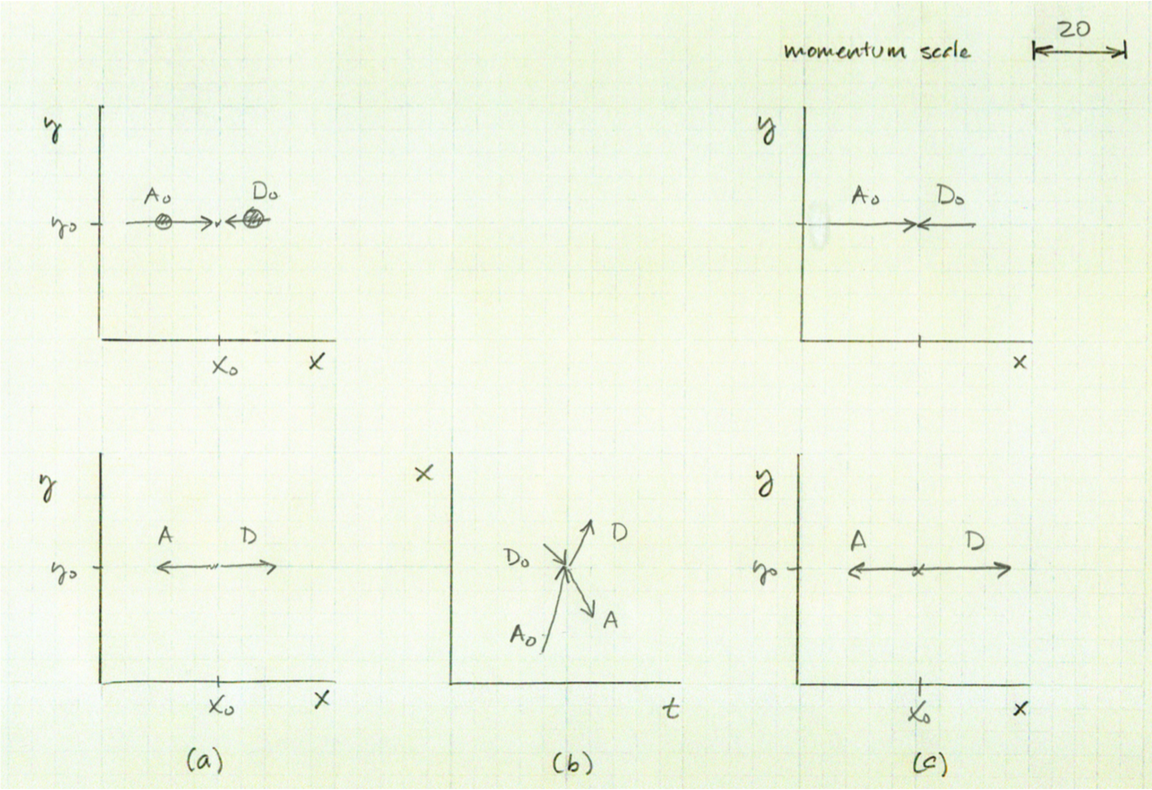


Figure 7.3: elastic2Diagram

7.1.3 Moving, Even Bigger Target, Elastic Scattering, $A_0 + F_0 \rightarrow A + F$

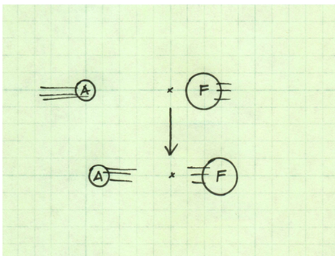
$A_0 + F_0 \rightarrow A + F$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	A:	(5)(5)	$= 25$	(5)(-2.5)	$= -25/2$
	F:	(15)(-1)	$= -15$	(15)(1.5)	$= 45/2$
	total sum:		$= 10$		$= 10$

Table 7.4: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

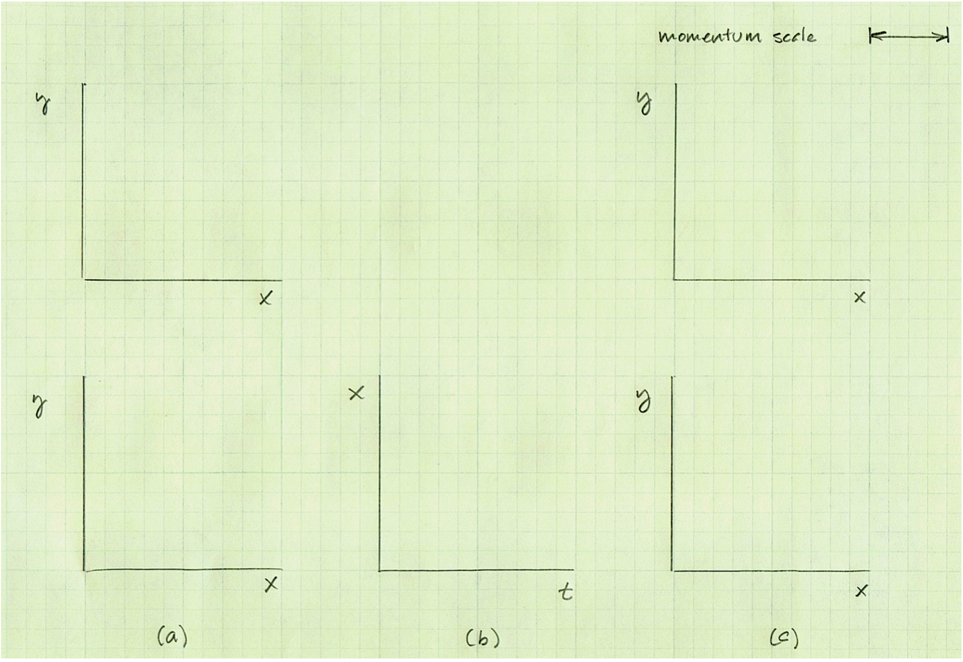
Now, let's involve the larger F moving to the left at only -1 but the smaller guy is moving much faster, and with more momentum. Then (with my back-door calculation) I calculate the reaction in Table 7.4. Notice how the big F bounces off the little guy, who has transferred a lot of momentum to it.

You should amuse yourself by drawing the diagram for this collision represented in Table 7.4.

You Do It 7.1. Spacetime

Draw the Spacetime Diagram for the $A_0 + C_0$ scattering in Table 7.4

or copy the solution



7.2 A "Decay," $D \to A + B$ Event

It's no secret that nuclei, atoms, and subatomic particles are sometimes unstable and decay into other particles. Momentum conservation is a critical part of how physicists understand these situations. Here we'll talk about the decay of one object into two objects, generically:

$$D \to A + B \tag{7.1}$$

where a big thing "decays" into two smaller, but here, identical things. Here, we'll assume that D is decaying at rest...it's sitting still. So this requires some thought!

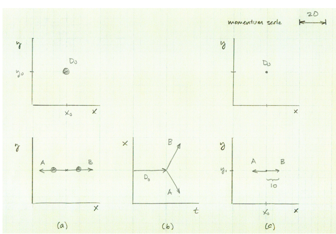
$D_0 \to A + B$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	D:	(10)(0)	$= 0$	gone!	-
	A:	doesn't exist yet!	-	(5)(2)	$= 10$
	B:	doesn't exist yet!	-	(5)(2)	$= -10$
	total sum:		$= 0$		$= 0$

Table 7.5: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

Notice that momentum is happily conserved—always. But kinetic energy is not. There's kinetic energy in each of A and B in the final state, but there's no kinetic energy in the initial state. *Overall energy* is conserved...but Kinetic Energy is not. How does this happen?

Well, think about your experiences with things blowing up. If it's a firecracker or a cannon and cannonball or a person throwing something—these are all "decay" like events where one thing turns into more than one thing. In the first two, the kinetic energy of the final products comes entirely from the chemical energy that created the explosion. In the latter example, if you throw a ball the kinetic energy of the ball is entirely due to your ability to throw hard, which in turn is a feature of the elastic capabilities of your arm and the chemical processes in your muscles. So the energies are internal for all such decays...in real life.[1] We'll see that when relativity comes around, that this whole idea gets a whole new dimension added.

[1] So in Table 7.5, the speeds there are chosen at random. Their value would depend on whatever it was that caused D do explode.

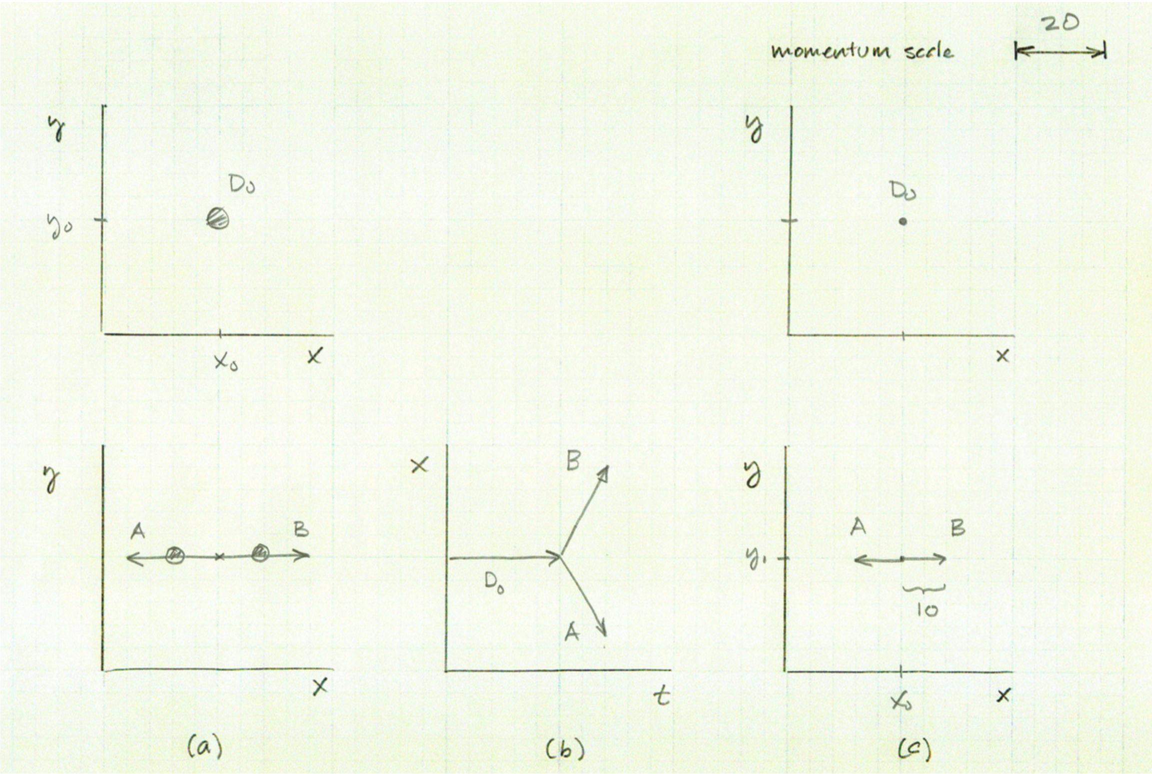


Figure 7.4: decay1DIagram

7.2.1 A “Decay,” $F \to A + D$ Event

If the decay products are not the same mass, then that will affect their momenta, in order to make them balance.

$$F \to A + D \tag{7.2}$$

where a big thing “decays” into two different things.

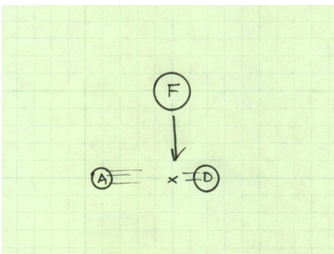
$F_0 \to A + B$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	F:	(15)(0)	= 0	gone!	-
	A:	doesn’t exist yet!	-	(5)(-2)	$= -10$
	D:	doesn’t exist yet!	-	(10)(1)	= 10
	total sum:		= 0		= 0

Table 7.6: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

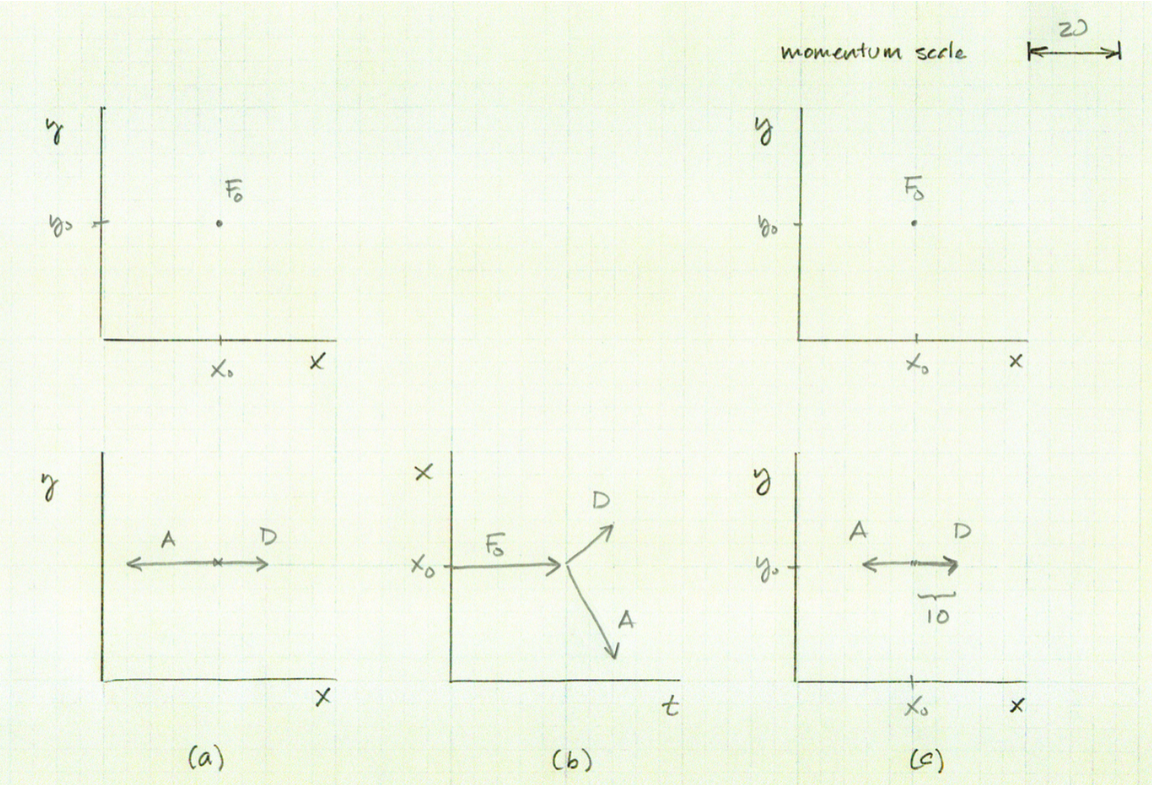


Figure 7.5: decay2Diagram

7.3 Fusion Collisions, $A + D \to F$

Opposite of a decay is a collision in which objects stick together after a collision. This could be a neutron being absorbed by a uranium nucleus in a reactor or an asteroid crashing into a planet, or two cars crashing together or an outfielder catching a fly ball. Generically, we can think of this like:

$$A_0 + D_0 \to F \tag{7.3}$$

Of course, this can be a collision in which the target is stationary, and the two move off together. Or the "target" could be moving toward or away from the beam. Here's a scenario where A and D crash together and make F. This is a completely inelastic collision and so Kinetic Energy is not conserved...but momentum always is. In this case, our objects have the same velocities, but one is twice as massive than the other. Notice that the momentum of the big guy wins, since F lumbers away to the left.

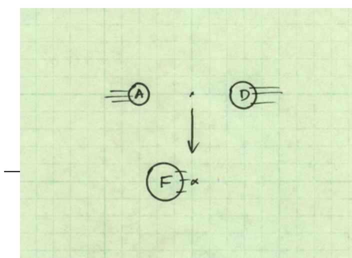
$D_0 \to A + B$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
	A:	(5)(2)	= 10	gone!	–
	D:	(10)(-2)	= −20	gone!	-
	F:	doesn't yet exist!	-	(15)(-2/3)	= −10
	total sum:		= −10		= −10

Table 7.7: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

There"s one for you to do at the end of the chapter. Involving middle linebackers.

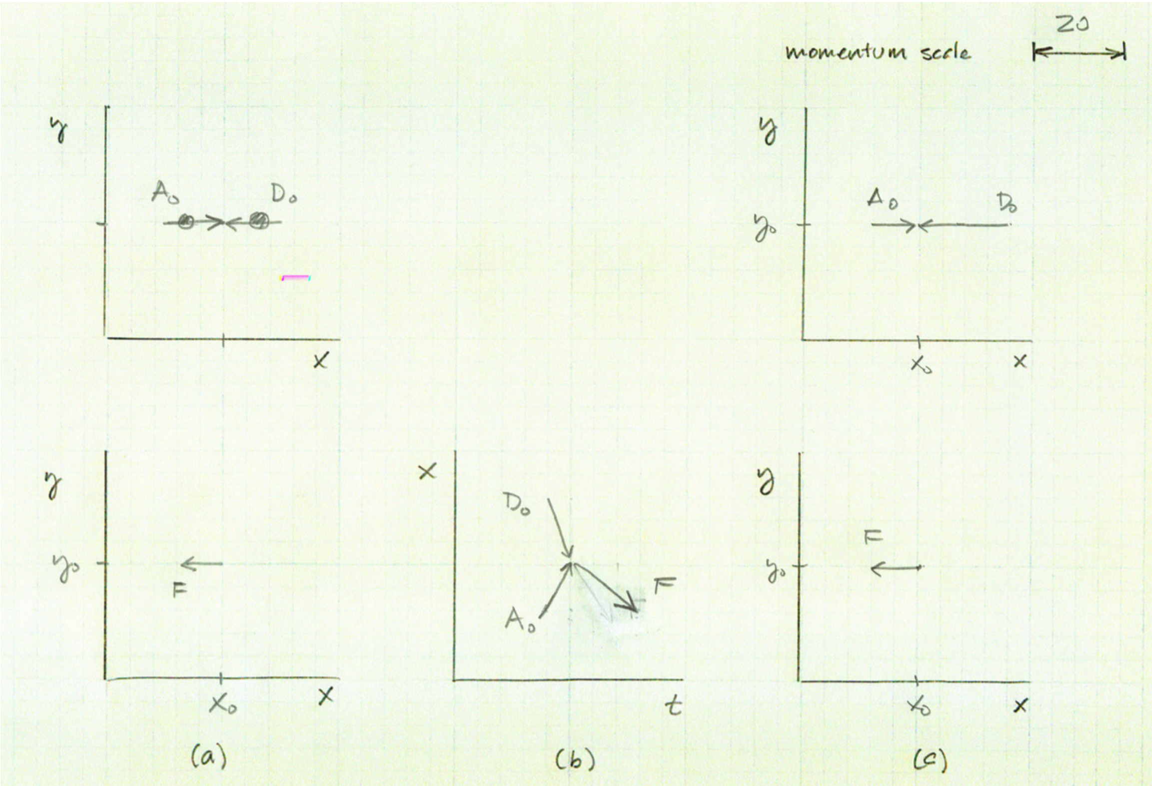


Figure 7.6: fuse1Diagram

7.4 Colliding Beam Events in Two Dimensions: $A_0 + A_0 \rightarrow A + A$

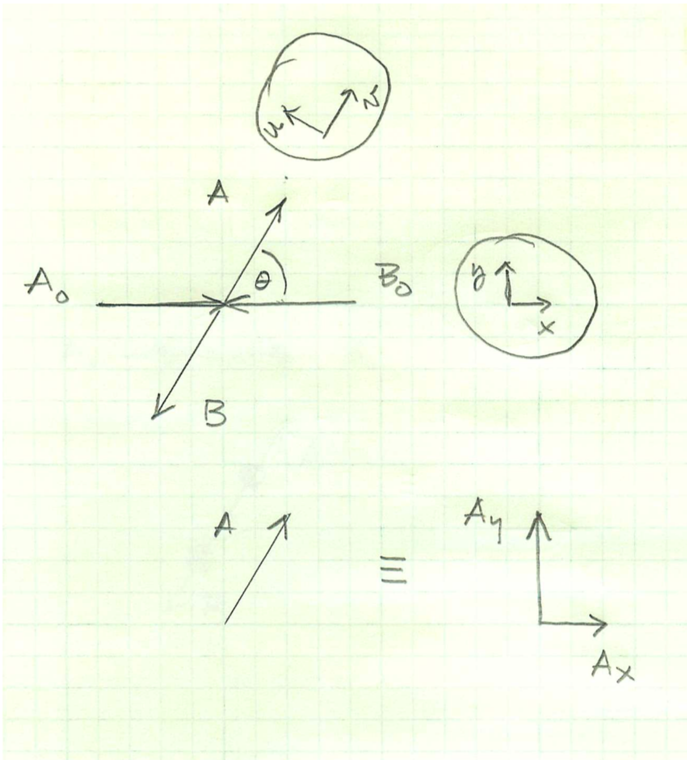


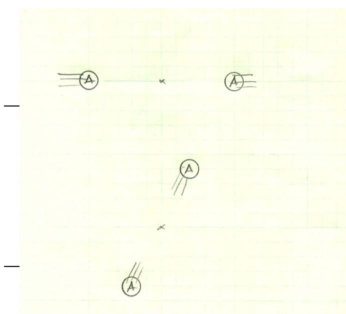
Figure 7.7: cm2dcomp

The only case in which we'll need to think in two dimensions is this: two identical beam objects scatter elastically into two identical outgoing objects. Let's do a little bit of algebra. Let's look at Kinetic Energy conservation, which works since we've defined this to be elastic scattering (no sound emitted, no heat lost, etc). Since the two initial velocities are the same, but oppositely directed and since the masses of all four identical particles are the same, this reduces to a simple relationship between the initial speeds of each and the final speeds of each.

$$\text{KE initial} = \text{KE final}$$
$$1/2mv_0^2 + 1/2mv_0^2 = 1/2mv^2 + 1/2mv^2$$
$$2v_0^2 = 2v^2$$
$$v_0 = v$$

Now let's do an example with our now standard fake momentumunits. Refer to Fig. 7.7. But let's think about those to drawings. The one on the top is just the Space Diagram: A and B come in from the sides, collide, and then go off in another direction. I've indicated two different coordinate systems. The $x-y$ coordinate system is indicated with the circle around little reference axes. Particle A_0 is going in the $+x$ direction and B_0 in the $-x$ direction. Both of their momenta are anti-aligned and the total momentum of the initial state is 0.

Table 7.8: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

$D_0 \to A+B$		BEFORE (mass)(speed)	$= p_0$	AFTER (mass)(speed)	$= p$
				along x and y	
	A:	(5)(2)	$= 10$	(5)(1)	$= 5$
	A:	(5)(-2)	$= -10$	(5)(-1)	$= 5$
	total sum:		$= 0$		$= 0$
				along u and v	
	A:	(5)(2)	$= 10$	(5)(2)	$= 10$
	A:	(10)(-2)	$= -10$	(10)(-2)	$= -10$
	total sum:		$= 0$		$= 0$

7.5 Sort-of Real Life Examples

7.5.1 A Particular Decay Example

A quarterback who is initially holding the football and then throws it...is like a "decay" event. Now, quarterbacks are large and footballs are small (not anchors!), so if you've ever thrown a ball you've probably not experienced the same sort of recoil as with our boat. Let's see how much by supposing that we have the following: using arbitrary units...just numbers:

Here's what we know:

- Mass of football (F) is $m_F = 0.5$
- Mass of the quarterback (Q) is $m_Q = 100$
- Momentum of the football is $p(F) = 10$
- Call the system of Q + F (the quarterback holding the ball), LQ for "Loaded Quarterback."
- The momentum of the LQ state is 0, since the quarterback (with ball) is standing still.

Questions:

1. What is the momentum of the quarterback in the final state?
2. What is the mass of LQ?
3. What is the velocity of the football after being thrown, that is the final state?

Space Diagram

Let's orient our brains by drawing the Space Diagram, which will have before and after pieces, separating the initial and final states. Figure 7.8 shows just that. On the top is the quarterback and ball just standing there. After he throws the ball, the systems are now the Football (F) and the Quarterback (Q) and I've pictured the throw to be only in the x direction...and, I've assumed that the quarterback does indeed recoil (although our mathematics would show this). Roughly, the football goes further (from x_0 to x_f) during the "after" time interval than the quarterback, which is what you'd expect.

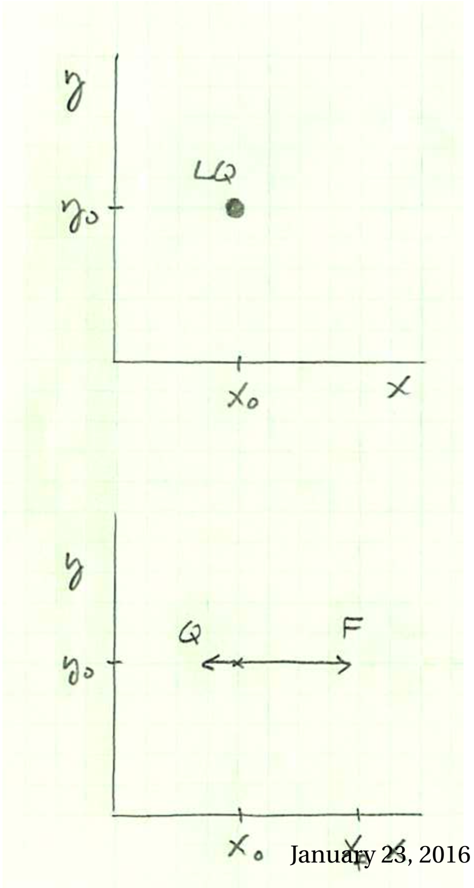


Figure 7.8: quarterSpace

Table 7.9: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

	Before	After
LQ	0	-
Q	-	b
F	-	10
total	0	a

Momentum Conservation

The particular situation is in symbols:

$$A \to b + c$$
$$\text{loaded quarterback} \to \text{ quarterback} + \text{ football}$$
$$LQ \to Q + F$$

Let's turn it into a momentum conservation equation:

$$p(\text{loaded quarterback}) = p(\text{ quarterback}) + p(\text{ football})$$

where I've removed the vector symbols since everything happens along the x axis. So any positive sign means "right" and any negative sign means "left." This is enough information to answer Question #1, what is the momentum of Q in the final state?

Let's analyze the quarterback as in Fig. 7.8. If he and the ball are both stationary, then the initial momentum of the LoadedQuarterback is zero (no velocity, so no momentum). So because of momentum conservation, after the event the final momentum of the resulting (different) system must also be zero. Since the football is thrown in the positive x direction, its momentum is in that direction as well. Think of the momentum analysis as a balancing of the books like in Table 7.9.

Let's unpack the missing entries. How about a? That's pretty easy: Because of momentum conservation, if the initial system's momentum is zero, then the entry for the total final state momentum is $a = 0$. And in order to balance, if the momentum of the football is $p(F) = 10$, then the momentum of the quarterback must be $p(Q) = -10$, so $b = -10$. Remember, while we got rid of the vector symbols for one dimension, directions still matter and the negative sign here means that the momentum direction (and hence the velocity direction) of the quarterback is in the negative x direction.

What's the meaning of this? Remember our boat and anchor? You and the boat move away from the direction you threw it. You recoil, and so does the quarterback from passing the football, precisely as a cannon jumps backward after the cannonball is explosively shot forward.

Wait. *The quarterback is much larger than the football, but they have the same value of momentum. How can that be?*

Glad you asked. *Ah. But the momentum of the quarterback is his mass times his velocity and the total momentum of that flying football can be shared between the quarterback's mass and velocity. His mass is high, so his recoil velocity is tiny. Let's see.*

Using arbitrary units for mass and velocity as well as momentum, the numbers in the list above are not too far from realistic football parameters.[2] So we have this little equation to solve:

[2] These are in the relative fractions associated with a 200 pound quarterback, a 50 mph thrown football, and a 0.5 kg football mass.

$$
\begin{aligned}
p(\text{loaded quarterback}) &= p(\text{ quarterback}) + p(\text{ football}) && (7.4)\\
p(LQ) &= p(Q) + p(F)\\
0 &= m(Q)\,v(Q) + m(F)\,v(F)\\
-m(Q)\,v(Q) &= m(F)\,v(F)\\
v(Q) &= -\frac{m(F)\,v(F)}{m(Q)} && (7.5)\\
v(Q) &= -\frac{(1/2)(20)}{100} = -1/10
\end{aligned}
$$

So the quarterback does recoil, but at a very, very low speed which is a fraction of the football's speed, governed by the ratio of the masses. So with such a large difference here, the recoil is probably not enough for him to even notice. So our more detailed momentum balance sheet would read:

	Before	After	(mass)(speed) = p before	(mass)(speed) = p after
LQ	0	-	(100.5)(0) =0	-
Q	-	-10	-	(100)(-1/10) = -10
F	-	10	-	(1/2)(20) = 10
total sum	0	0	0	0

Table 7.10: The momentum for the objects in the decay of a quarterback-with-the-ball into its constituent parts of quarterback and ball.

7.5.2 Decay: Summary

The best summary of a decay event are the diagrams which are all show in Fig. 7.9

a) We've already seen the Space Diagram with the stationary LoadedQuarterback (LQ) at position (x_0, y_0) so a single dot is the before picture. At time t_0 he throws the ball in the positive x direction—directly

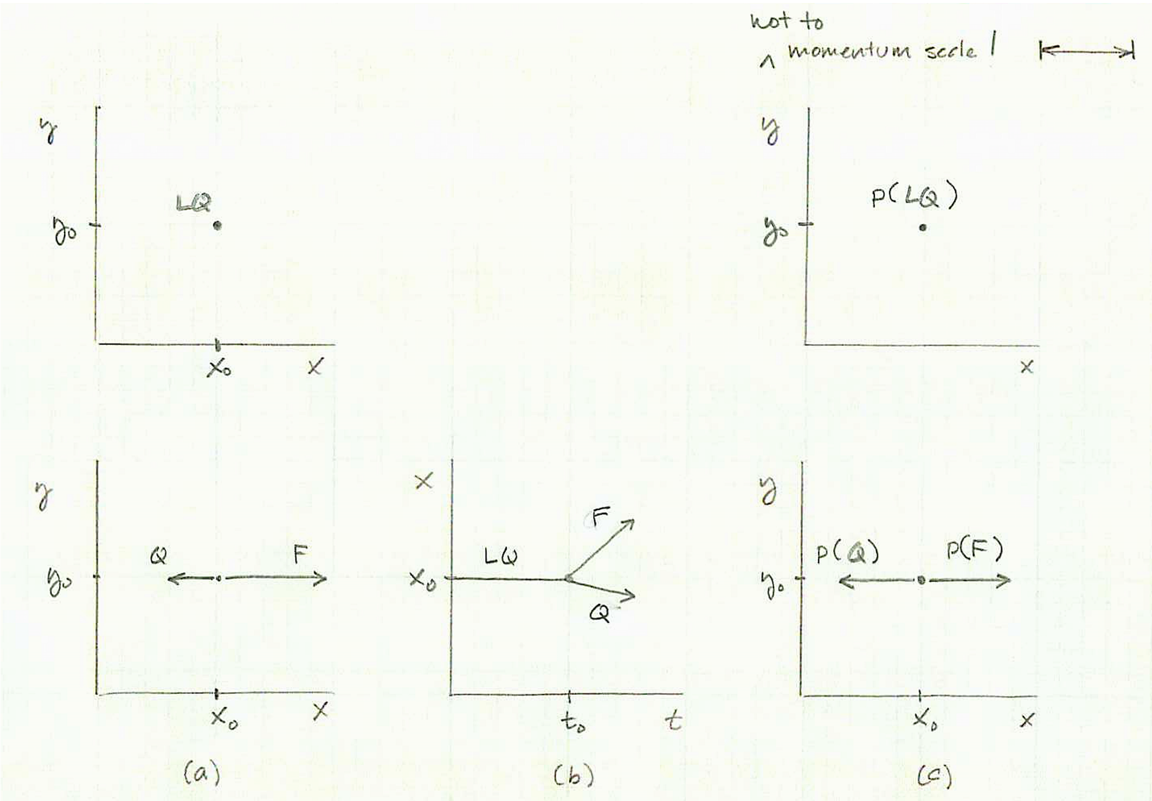


Figure 7.9: This shows all three diagrams for the decay (quarterback) scenario: (a) the Space Diagram, (b) the Spacetime Diagram, and (c) the Momentum Diagram. The top row (for the Space and Momentum Diagrams) shows the before situation and for them, the bottom row shows the after situation. The Spacetime Diagram explicitly calls out time, so before and after are all naturally there.

down-field—without any motion in the y direction (toward either sideline). The bottom picture shows the journey of the football (F) down-field and the recoil motion of the quarterback (Q) to the opposite direction. So, two objects' paths on the same "map."

b) The Spacetime Diagram shows just the x coordinate (as the y coordinate with be very unexciting in this example) as a function of time. The LQ is stationary on the field (and so no variation in x) and moves forward in time (and so the horizontal time journey line) until the event occurs at t_0 when he throws the ball. From that event LQ ceases to exist and the football goes down-field in the positive x direction (and hence the positive slope) and the quarterback (Q) recoils and goes in the negative x direction, away from the ball (and so the negative slope). Notice that the magnitude of the slope of F is much bigger than the magnitude of the slope of Q since the speed of F is much larger than the recoil speed of Q.

c) Finally, the Momentum Diagram is slightly different. First, time is implied and really only distinguished in the before and after intervals. Second, the axes are neither space nor time, but momentum units. Somewhere there could be a scale that told how many inches on the diagram would correspond to how many kg-m/s for any arrow on that diagram. Here, I'm being schematic and not precise. Notice that when LQ is stationary in the top diagram, like in space, the representation is a dot with the value of zero for both the x component of momentum (p_x) and the y component of momentum p_y. In the after representation, the momentum of the football is to the positive direction [which is governed by the positive velocity in (b) which is in turn determined by the direction of positive x] and that the momentum of Q is exactly the same length as that of F's...which is of course, because momentum is conserved. The total momentum of the system (LQ) in the top diagram is zero and so the total momentum of the whole system (now Q and F) must add to zero also. The two equal-length and opposing arrows make that happen.

You Do It 7.2. Lambert

Can we agree that Jack Lambert was the best middle linebacker in the history of the NFL? That's my story and I'm sticking to it. In fact, "sticking" is the very essence of a clean tackle. Lambert ("L") played at 220 pounds (100 kg), not so much bigger than a largish NFL quarterback ("Q" also at 100 kg). If his speed through an offensive line is 20 ft/s (6 m/s) and if a 220 pound quarterback is standing still ($v_0(Q) = 0$), when Lambert tackles him and they stick together as one unit in the final state, what is the speed of the pair of them after they collide? That is, fill in the table with $a, b, \&\ c$::

	Before	After	Your fill-in After
L	$m(L)\,v(L) = (100)(6) = 600$ kg-m/s		-
Q	$m(L)\,v(L) = (100)(0) = 0$ kg-m/s		-
LQ	-	m(LQ)v(LQ) = $(a)(b)$	( )( )
total	600	c	( )

or copy the solution

Now we need some diagrams.