Chapter 6

Energy

A Long Time Coming

James Prescott Joule, Photogravure after G. Patten

James Prescott Joule, 1818-1889

"...wherever mechanical force is expended, an exact equivalent of heat is always obtained." *Joule, August (1843)*

The University of Manchester in that industrial city has been the home of to-be illustrious physicists as well as already-in-the-textbooks physicists for more than 150 years. Ironically, the Manchester scientist credited with one of the most fundamental statements about the word had nothing to do with the university. He made beer. James Prescott Joule was the son of a brewer who joined the management of the family business in his early 20's where he launched intensive research into how to increase the efficiency of or replace its large-scale steam engines. This led to a lifetime of largely private research into the nature of energy.

Figure 6.1: brewery

6.1 Goals of this chapter:

- Understand:
  - How to calculate kinetic energies of moving objects.
  - How to calculate potential energies of objects.
  - How to use the conservation of energy to calculate speeds. parameters
- Appreciate:
  - The importance of the conservation of momentum and energy.
- Be familiar with:
  - The importance of James Joule's work.
  - The importance of Emmy Noether's work.

6.2 A Little Bit of Joule

James Prescott Joule grew up in a wealthy family and was educated by private tutors and by the age of 16, the illustrious John Dalton,[1] also a resident of Manchester. Joule had an adolescent fascination with electricity, probably influenced by the famous work of Michael Faraday in London. When he and his brother were not (literally!) shocking their family and the household staff, James was beginning to conduct research on what causes heat. Motors were beginning to be conceived of and he built several and compared the amount of coal in a steam engine required to perform a fixed mechanical task with the amount of zinc to power a battery-driven motor towards that same task. All the better to figure out what was the best technology for the brewery. Coal won and they didn't adopt the new-fangled electric motor.

As a young man in the family business, Joule would go to the brewery by day and perform his management tasks, and then when he could find time, he would perform his private experiments in his homemade laboratory. From his 20's he carefully charted a course to unraveling three different phenomena, all of which caused objects to heat up, but none of which corresponded to the accepted picture of just what heat was supposed to be. He was suspicious of the commonly held theory that heat is a fluid, "caloric," that was neither created nor destroyed and moved (flowed) from a hot object to a cold one.

His first demonstration in 1841 was to show that when an electrical current flows through a wire, that it heats it. He could explain this by the heat being generated in the wire, and not having been transported from the source of the current. Caloric would have flowed to the point of heating. Today we call this Joule Heating and the formula for the amount of power associated with this heating is due to him: $P = I^2R$,

[1] John Dalton (1766–1844) is considered the father—or at least the favorite uncle—of chemistry. He worked out much of the picture of substances as made of atoms and that chemical compounds are made of atomic constituents. He lived in Manchester his entire adult life where he taught privately and at the university. As a Quaker, he was ineligible for education or employment at many British universities.

where R is the value of the resistance (which was a new idea when he was experimenting) and I is the current. This is the principle behind an electric stove or heater...and the villain to be defeated in the long-distance transmission of electrical power. His second demonstration was to show that when a gas is compressed, that the amount of force required translates directly into the temperature rise in the gas. It's the principle behind an internal combustion engine and the beginning of his notion of a "mechanical equivalent of heat" which led him to his next experiment, for which he's best remembered.

Heat and motion are both forms of energy which can be converted back and forth. Key Observation 4

If you mechanically stir a fluid, it gets warmer. Not a lot. But Joule had inherited from his tutor, Dalton, the idea that a gas was made of atoms (and developed his own theory of gases and the energy of molecules) and that making them move faster was to increase their temperature. He also applied this idea to water. He created a little system with paddles in a beaker of water that could be made to stir the water a specified amount because they were attached to a falling weight. The weight falls a given amount and the paddles reliably turn a specific number of rotations. Joule became skilled at making thermometers[2] and he found that a finite amount of stirring could raise the temperature of water by a single degree Centigrade. He reported this result to the British Association in 1845 and published a paper describing his results in the *Philosophical Magazine*.

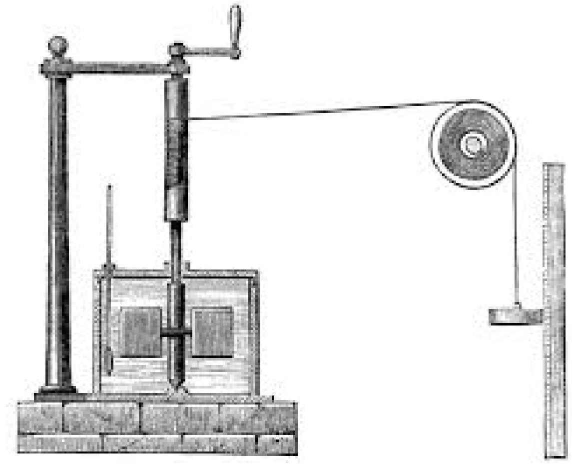

Figure 6.2: joulemechanical

[2] He once made a thermometer so precise that he could measure the temperature of moon-light. That is the temperature rise in air lit only by the moon.

He married Amelia Grimes in 1847 (who tragically died seven years later after they had two sons and a daughter). Their honeymoon was in Chamonix, France (near CERN, actually) where together they tried to measure the difference in temperature between water at the top of a waterfall and the bottom. You gotta love that as a scientist's honeymoon.

Joule was a little isolated while he did much of his work, but increasingly as a result of fortuitous speeches with just the right people in the audience, he became more and more well known and well regarded in Europe. Without any formal education, this recognition came slowly but eventually he was elected to Fellowship in the Royal Society in 1850 and received honorary degrees from Dublin, Oxford, and Glasgow. Finally, in 1872, he served as the President of the British Association. Not bad for a brewery lad.

William Thomson (later Lord Kelvin) wrote about his friendship with Joule and his surprise to discover that James was conducting experiments in waterfalls on his honeymoon."After that I had a long talk over the whole matter at one of the 'conversaziones' of the Association, and we became fast friends from thenceforward. However, he did not tell me he was to be married in a week or so; but about a fortnight later I was walking down from Chamounix to commence the tour of Mont Blanc, and whom should I meet walking up but Joule, with a long thermometer in his hand, and a carriage with a lady in it not far off. He told me he had been married since we had parted at Oxford! and he was going to try for elevation of temperature in waterfalls. We trysted to meet a few days later at Martigny, and look at the Cascade de Sallanches, to see if it might answer. We found it too much broken into spray. His young wife, as long as she lived, took complete interest in his scientific work, and both she and he showed me the greatest kindness during my visits to them in Manchester for our experiments on the thermal effects of fluid in motion, which we commenced a few years later."

Joule convinced everyone that heat and work (we'll see what the formal definition of work is below) are two sides of the same coin: *energy*. That "energy" can be transferred back and forth between heat and work is basically the First Law of Thermodynamics and the basis of the world's industrial economy and many of our household conveniences. It led to the notion of the conservation of energy and guides our thinking to this day.

Joule died in 1889 and is honored forever with his name used as the universal unit of energy: 1 Joule (J) is the equivalent of 1 kg-m^2/s^2. We pay our electricity bills according to Watts used, and its the only everyday metric unit in the U.S.: 1 Watt is 1 Joule per second.

6.3 Ability to Do Damage

Okay. "Ability to Do Damage" isn't a scientific phrase...but I'll bet you'll remember it better than our very specific use of a regular word: "work." If you want to do damage to something, you initiate some sort of contact with it and speed often figures into that process. Want to demolish something with a hammer? Gently pat it? or swing the hammer at high speed? Want to smash a teapot by dropping a rock onto it? Drop it from high up so it's moving really fast when it hits. So you need some speed to do damage. But mass figures in too: a hammer made out of balloons is not a damage-maker and neither is a pebble. So a question is: what's more important, mass or speed in inflicting damage?[3] Let's go back to High School and think about this.

[3] This comes up all the time with major league sluggers. Some will swear that in order to drive a baseball a heavy bat is better than a lighter one. Not true.

A regulation softball has a mass of about 0.22 kg while a regulation baseball has a mass of just about half of that, 0.145 kg. Now here's the question: A decent high school softball pitch is about 50 mph—faster than that, and you've got a college pitcher on your hands. But a 50 mph baseball is not so impressive, less than batting practice quality. Consider these two trade-offs, and think about being hit by each:

- Replace a baseball thrown of 50 mph with a softball of the same speed—a factor of 2 increase in mass, but same speed?
- Replace a baseball thrown at 50 mph with a baseball thrown at 100 mph—a factor of 2 increase in speed but the same mass?

Which replacement would do proportionally more damage? I'd take the first item any day.

Speed matters in this image more than mass, in fact it matters by a lot more. Since mass and velocity contribute to momentum in equal proportions, so this discussion of "damage" is referring to some other quality of motion. That additional quality we call Kinetic Energy. We'll use the symbol K to stand for it and in modern terms, it's written as

Definition: kinetic energy.
Kinetic Energy is the energy possessed by any object in motion.

$$K = \frac{1}{2}mv^2 \tag{6.1}$$

the ability to do damage is related to the square of speed and only linearly with mass.

Equation: Kinetic Energy.
$K = \frac{1}{2}mv^2$

The fancy way to speak about this is in terms of "work" which means something very specific in physics. Work is the product of force × the distance through which the force acts. This is similar to the way that

Impulse is the product of force × the time through which the force acts. Work is then equal to the change in kinetic energy, in the same way that Impulse is equal to the change in momentum.

So while the pitcher increases the momentum of the ball by applying a force to it through a full windup and follow-through (longer time), he also increases the kinetic energy of the ball by applying that force through a long distance. So a long-armed pitcher with a big arc has an advantage. The formal statement of this is:

$$\text{work} = W = F\Delta x = \Delta\left(\frac{1}{2}mv^2\right) \tag{6.2}$$

which looks a lot like

$$\text{impulse} = J = F\Delta t = \Delta mv. \tag{6.3}$$

The partnership between time and space is related to the partnership between energy and momentum, as we'll see a bit later.

6.3.1 Vis Viva

One of the remarkable achievements of Huygens, totally unanticipated by Newton, was the discovery of a second conserved quantity. In this, Huygens had a partner: Gottfried Leibnitz—Newton's bitter rival for the priority of the Calculus—had the same idea. They both found by experiment that if you add up all of the quantities: mv^2 for all of the objects in a collision that the total amount of that quantity before is equal to the total amount afterwards. . . *without regard to direction.* That is, since the velocity is squared these are not vector quantities, but scalar ones. Just numbers.

This was incorrectly given the name of "force" by Leibnitz, in particular the "Life Force" or *"vis viva."* Today (actually, about mid-18th century), a factor of 1/2 is added to create the quantity we call Kinetic Energy, KE= $\frac{1}{2}mv^2$. So, to summarize what's conserved in collisions, we separately conserve:

$$\mathbf{p}_{1,0} + \mathbf{p}_{2,0} = \mathbf{p}_1 + \mathbf{p}_2 \tag{6.4}$$

$$\frac{1}{2}m_1(v_{1,0})^2 + \frac{1}{2}m_2(v_{2,0})^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2. \tag{6.5}$$

The first equation is the Conservation of Momentum, a vector equation and the second is the Conservation of Kinetic Energy.

Now, we can go back to the incomplete Example 5.9 where we were left hanging. Had we also added the Conservation of Kinetic Energy.

Definition: Kinetic Energy.
KE= $\frac{1}{2}mv^2$
Kinetic Energy is proportional to speed squared and mass.

Example 6.1 One Dimensional Collision... continued

Pencil 6.1.

Where we left off was Equation 5.9:

$$v_{1,0} = v_1 + v_2. \tag{6.6}$$

Now, let's include the Kinetic Energy relationship for this particular situation:

$$\frac{1}{2}m_1(v_{1,0})^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2. \tag{6.7}$$
$$(v_{1,0})^2 = v_1^2 + v_2^2,$$

where in order to get the second line I canceled out the equal masses and the common factor of $\frac{1}{2}$.

Now we have two equations and two unknowns to solve, which can be done in a variety of ways (remember?). You always have to keep track of what you're looking for. Here, it's the final velocities. So, let's square the Eq. 6.6 and subtract it from the second one and you get the result:

$$0 = 2v_1 v_2. \tag{6.8}$$

Here are the few lines that lead to that simple conclusion:

$$v_{1,0} = v_1 + v_2$$
$$(v_{1,0})^2 = (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1 v_2$$

set equal to the RHS of Eq. 6.7

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2v_1 v_2$$
$$0 = 2v_1 v_2$$

So, either one or the other of the final velocities must be zero. One of these solutions doesn't make any physical sense. For example, if the target ball (2) is solid, then the target ball can't just fly right through it as if it were not there, so v_2 cannot be zero, it must be something else. That means, that $v_1 = 0$ and going back to Equation 6.6, we see that:

$$v_2 = v_{1,0},$$

which is what we expected.

We need **both** momentum and kinetic energy conservation to describe even the simplest of collisions!

6.4 Energy

The idea of Kinetic Energy was eventually appreciated as a part of a much broader concept. We use the term freely, but it's a subtle thing and the 17th, 18th, 19th and 20th centuries saw repeated recalibration of the idea. It was not until nearly the middle of the 1800s that heat was carefully studied by many, culminating when James Prescott Joule (1818-1889) carefully measured the amount of kinetic energy he put into a beaker of water by stirring it. He found that the water's temperature went up the same amount for the same input of energy—he suggested that heat was a equivalent to energy. Soon after Joule's death, it was decided internationally to honor Joule's memory by naming the basic unit of energy: 1 J = 1 N m after him.

Heat, then is a form of energy, adding to kinetic energy and potential energy as the classical trio of energy forms (nuclear, chemical, and elastic energies are additional kinds). Potential energy is just what the name implies. . . "the potential" for causing damage! Hold a barbell above your foot and let it go, it will change the shape of your foot when it lands, and maybe the floor as well. That suspended weight possess the *potential* for doing "Work," which is a technical term different from the everyday usage. If a force acts on an object through a distance x, then the work is defined as:

$$W = Fx.$$

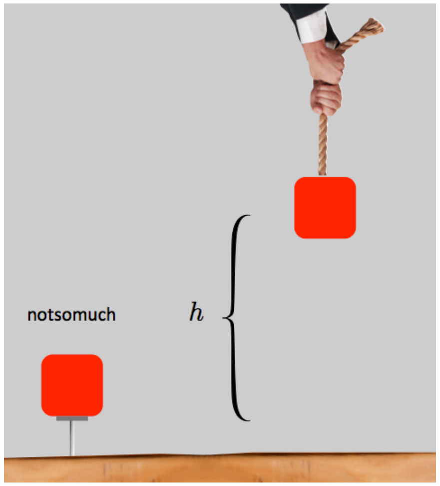

Figure 6.3: (left) Setting a block on a nail does not do much work against the fibers of the wood. (right) Dropping the block from a height onto a nail, drives it into the wood.

6.4.1 Potential Energy

The subtle point about Work is that the force must have a part of its direction along the path through which it's acting. So, if I carry a heavy weight still, but walk across the room, I may be tired and think that I've worked hard, I've done no (technical) Work, since the direction I walked is perpendicular to the force that I exerted (up) in holding the weight. Work figures into the statement of an important theorem in mechanics, the Work-Energy Theorem: The change in kinetic energy in a collision is equal to the Work that's performed. In fact, the exchange of almost all sorts of energy involves doing Work.

For dropping things in a gravitational field, the Potential Energy is:

$$P = mgh \tag{6.9}$$

where h is the vertical distance above the point defined to be the zero value of potential energy. That's sensible since mg is the weight of the object, the force pulled on it by the Earth. So this too is a force times a distance, Wh. The typical example of potential energy at work (no pun intended...or is there?) is driving a nail into a block of wood by dropping a weight from some height as shown in Fig. 6.3. Potential energy is a funny concept and I'll have more to say about it when we talk about Einstein. But, it does have a slippery feature that's sometimes complicated to appreciate:

Definition: Potential Energy.
Potential Energy is possessed by an object by virtue of its "configuration"...height, distance away from a force center, located in a compressed or expanded spring, etc.

Equation: Gravitational Potential Energy, near the Earth.
$P = mgh$ at a distance h above an arbitrarily defined $P = 0$ location.

There is no *absolute* measure of potential energy. Only differences, before and after some change of configuration matter.

If I suspend the ball above the surface of a table, and if I assign the “zero” of potential energy to be that at the surface of the table, then when it falls to the tabletop, it has no potential energy left. But, if I take the zero of potential energy as that at the floor, then when it is done with its motion, still on the table, it still has potential energy left over relative to the floor—that associated with the height of the table. But, that doesn’t change the tabletop result. The difference between before and after is still the same. Again, looking at Fig. 6.3 h is the same whether it’s measured from the surface of the table or from the floor.

This also leads to the notion of a negative potential energy which is the standard idea in chemistry. When an electron is bound to a nucleus, we say that it has negative potential energy. When it’s liberated (ionized), we say that it’s free and has a positive energy and a positive energy must be supplied to the electron in order to free it from its bound state in the atom. Again, that’s just the fact that the zero of the energy scale is defined for ease of use to be zero at the point of ionization.

Joule also studied friction and it became apparent that there was a conservation law at work that was broader than just that of motion alone. If one slides a real object down a plane, for example, it gains speed as it goes (increasing its kinetic energy) and it heats up the plane and the body through friction (heating as a loss of energy) and that adding up all of the energy at the end—kinetic energy gained, heat energy dissipated through the trip—it will all be equal to the potential energy that it had before it was let go.

Joule also pondered the Model that I mentioned earlier about a gas. You’ll recall that picturing a gas as a collection of small, solid spheres colliding with the walls results in Boyle’s Law: $PV =$ constant. Well, the constant can be shown to be the average kinetic energy of all of the little points in the gas. If each one has mass m, then $PV = C\frac{1}{2}mv^2_{\text{ave}}$. (Here C is a constant.) But, the Ideal Gas Law says that $PV = C'T$ (where C' is another constant). The really satisfying thing about this is that T is the temperature of the gas and is therefore simply a measure of the average kinetic energy of the molecules in the gas. So, heat *is* indeed a measure of energy and specifically an account of the motions of the individual molecules of any object with temperature. That’s neat and had been hinted at since Newton’s time, but it took 150 years for the idea to be fleshed out and understood during Joule’s time (although not quantitatively by him).

6.4.2 What Comes In Must Come Out

That these energies add up is the statement of the Conservation of Energy—not just kinetic, not just mechanical, but *all forms of energy*. The idea was hinted at by the German physician, Julius Robert von Mayer (who always felt that he had been ignored by the physics community) and explicitly proposed by the formidable Hermann Helmholtz in 1847, who credited both Joule and Mayer. The statement of the conservation of mechanical energy is:

Definition: Energy Conservation.
The total energy at the beginning of any process is equal to the total energy at the end of that process when losses due to friction and other dissipative processes are included.

$$\left(\text{kinetic energy}\right)_0 + \left(\text{potential energy}\right)_0 = \left(\text{kinetic energy}\right) + \left(\text{potential energy}\right) + (\text{heat lost})$$
$$\text{KE}_0 + \text{PE}_0 = \text{KE} + \text{PE} + \Delta Q \tag{6.10}$$

Total energy is always conserved. *Key Concept 13*

In order to make the point, let’s consider the air-hockey example again in a wordy, rather than formal way in order to account for most of the energy transfers. We’ll start with one puck already moving towards another:

- If you hear it sliding along the polished surface, then two things are going on: first, the rubbing of the surfaces together are heating up the surface of the table and the puck—a tiny bit. That energy loss reduces the kinetic energy.
- The rubbing sound is then propagating a compression of the air between the puck and your ears—heating it along the way. That heat dissipates throughout the room heating everything that is in contact with it. That energy loss reduces the kinetic energy.
- Once the compressed air oscillations reach your ear drum, they set it into vibration—and yes, heating it—which in turn triggers the electrochemical processes in your nervous system which your brain interprets as sound.
- Meanwhile, the puck has struck its neighbor and for a brief time more sound is emitted (more heat) and the two pucks distort slightly as the renaming kinetic energy is converted into potential energy of the lattice structure of the pucks which acts as a spring.
- The potential energy in the (slightly) springy lattice is released pushing the target puck away with the kinetic energy that's left over.

We could follow the energy all the way back to the source of the work that was done on the puck to get it started which would probably have an origin in chemical energy, either within the body of the person who shoved it or some electrical device getting its energy from an electrical grid (which could also have been a nuclear energy source). If it's a person, then the chemical energy in the food that was eaten was partially used to create the muscle action. And of course, if it's from food, then ultimately the energy of the Sun's radiation would have been responsible for the photosynthesis in plants as a direct source, or as food for an animal that was eaten.

But ultimately in any macroscopic mechanical event, what happens when everything has settled down? Everything has become...Heat. This realization, along with sophisticated thermodynamic notions like entropy (which we will not cover in this account) led physicists at the end of the 19th century to begin to speculate about the ultimate "heat death" of the universe as all energy eventually becomes aimless heat. The "death" part would happen when there are no differences among any sources of energy which are large enough to support life. We have a much different view of energy now and this will unfold as we follow Albert Einstein and his colleagues as they redefine the arguments in unexpected ways.

Wait. *You're telling me what energy does, not what energy is... What is energy?*

Glad you asked. *Well, a little uncomfortable that you asked. Let me try to explain with analogy. It's slippery.*

6.5 Okay, But What Is It...Really?

Energy is a sophisticated and abstract thing in physics. In fact, it's not a "thing" at all. It's not a substance. It's a concept that behaves mathematically in particular ways...and manifests itself physically in different guises. It's not surprising that it took more than three centuries to sort all of this out. We now know how to measure energy-guises. But, boy, what a mess for a long time.

Example 6.2 Diamonds are Forever

Energy as an abstraction is "just there." About the best analogy (but not a perfect one) is with the idea of economic value. Is the value of an object, or currency, a "thing"? No, it's a numerical concept which takes different guises and amounts which can at any point in a transaction be assigned a "value." Value is an economic energy.

Take a rough diamond. By itself, it has a *value* (unfortunately a value which often leads to violence and brutality) which is inherent: it can be traded with other objects which also have an equivalent value. . . like cash. In such a trade—a transaction—the total value of the two has not changed, just exchanged hands and in the process, changed kind. If you had diamonds, now you have cash. But you possess the same value.

But, suppose the diamond is cut and polished. Labor—which has a value—has been added and in turn the value of the diamond has increased and an exchange for cash would require more. But the total value of the labor, the raw diamond, and the cash has not changed. . . just shifted. The total value-amount at the beginning (the raw diamond plus the potential value of the labor before it's actually expended) is the same as at the end (the cash) but the potential value of the labor has been expended on transforming the diamond and adding to its value. All the while, this abstract quantity "value" has moved back and forth among the objects—exchanged hands, manifesting itself in various guises, but never actually standing alone as a substance.

Keep that in mind as we think about energy. We physicists tend to stop worrying about these sorts of things as we do calculations and measurements using the concept and so the delicate nature of the "what" gets pushed into the background in favor of the "how." The next example can show you how different energies are "exchanged" in a particularly useful "transaction," that of driving a construction pile into the ground.

Example 6.3 Pile Driver

Are you aware of how supports for bridges and large buildings are anchored into the ground? By brute-force! "Pile drivers" have been in use for centuries, to the present day and are impressive beasts. Even in the 1800s weights of nearly 5000 pounds would be pulled tens of feet into the air above the "pile" (an enormous nail—a beam or steel plate)—and then dropped. And then hoisted again. . . and dropped. Some pile drivers are still functioning after a century. Let's think about the effort and consequence of this machine.

From the point of the maximum height, the weight is just sitting there. It doesn't take any effort to release it but then, it's a different animal. The weight will head to earth, gaining speed as it goes, and eventually crashing into the pile with enormous force—so much that it will drive a very large steel object into the hard ground.[1] Remember that the only thing that can stop something with momentum is a force, corresponding to the total change in that momentum as it stops. Well, the pile driver eventually stops with the pile (and Earth) pushing back and providing that force. A lot has gone on during this transition from suspension to "stop."

[1] A modern pile driver can exert such a tremendous force that it actually heats up the air so much that it is capable of igniting. Diesel fuel is sometimes squirted into the region between the pile and the weight and briefly a one-cylinder diesel piston engine is produced with the fuel exploding and pushing the pile down even more

As we've seen, the trip down increased its speed, an increasing kinetic energy which is enormous since the weights are typically so large. But, free fall eventually ends and the weight begins to drive the pile into the Earth, slowing down considerably in the process as the Earth resists and eventually wins by stopping the pile and the weight. But, through some distance x, the blunt pile has shoved aside, compressed, and made room for itself in soil and rock. During each increment of time that the weight is driving the pile, the momentum of the weight is decreased and the momentum of the pile increased, conserving momentum like any collision. So, since the momentum changes, a force has been exerted on the pile and it's that force that rearranges the soil and rock. The force, created by the changing momentum, acts through a distance and *does work* on the soil.

Figure 6.4: Be the first on your block to own a Sennebogen 683 tp telescopic pile driver which can drop 32 tons over a distance of 34.8 meters.

Now, what are the different piece of energy in the pile driver example? Let's be precise. As the weight falls it is shoving aside and compressing the air which in turn locally heats it. So, potential energy of the suspended pile is going into the kinetic energy of the weight, and the kinetic energy (heat) of the air as it warms. You can probably hear the weight as it falls, and that's again more disturbance in the air that moves until it hits your eardrum. That air heats where the sound waves compresses it, and where it vibrates your eardrum, the air heats it up as well. The amounts of ear-air heating are again provided by the original potential energy. When the weight hits the pile, there's an enormous sound, which is again more air-heating, and it also locally heats the pile. Immediately, the pile (and Earth) push back on the weight which still has lots of momentum. But, that force of resistance slows the pile as it in turn does work on the pile and the earth, this time through friction and compression, heating the soil by—you guessed it—causing the molecules of soil to begin to vibrate, which is heating. Eventually, the rock is moved aside, compressing the surrounding rock and, yes heating it, until everything stops. All of the original potential energy of the weight suspended above the Earth has been converted into: heat.

Let's get a sense of the scale of Joule units of energy.

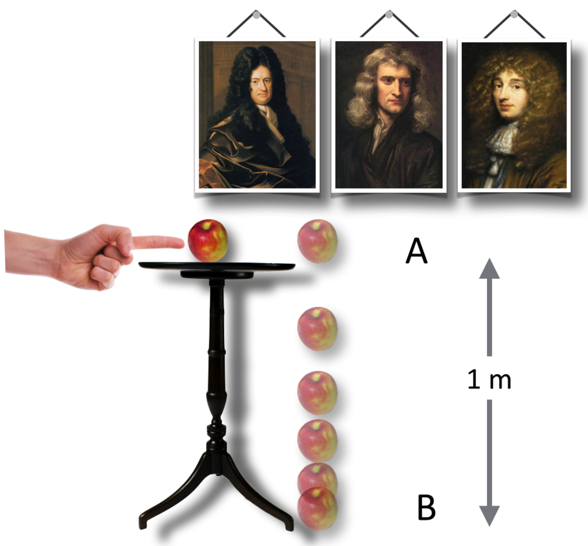

Figure 6.5: apple1m

Example 6.1 Another apple.

Question: What is the kinetic energy of an apple that falls a distance of 1 meter near the Earth?

Solution:

Suppose an apple falls from a table to the floor through a distance of 1 meter. An apple has a mass of 0.1 kg and for simplicity's sake, let's pretend that the acceleration due to gravity is 10 m/s^2 rather than its more precise value of 9.8 m/s^2.

What are the contributions to its energy at point A, point B, and halfway between them?

The contributions ot the energy of the apple would be combinations of potential and kinetic energy. Once we define where the "zero" of potential energy is located, it can be calculated at any height. Obviously, the most sensible thing to do is to define

$$\text{PE}(A) = 0.$$

When the apple is just tipped over the edge of the table, its energy is all potential and would have the value:

$$E = \text{PE}(A) = mgh_A = (0.1)(10)(1) = 1 \text{ J}.$$

That sets the scale of what 1 Joule of energy is like...Dropping an apple a meter above the ground provides it with a potential to do work on whatever it it lands on. When the apple has reached point B, its potential energy is spent, traded for kinetic energy as the apple has sped up from rest at A to the fastest that it will be just before hitting the floor (and deforming into a bruised fruit). So that energy is:

$$E = \text{KE}(B) = 1/2mv^2 = \text{PE}(A) = mgh_A$$

So we could ask how fast the apple is going, and this energy balance gives us the answer:

$$\begin{aligned} mgh_A &= 1/2mv^2 \\ gh_A &= 1/2v^2 \\ v &= \sqrt{2gh} = \sqrt{(2)(10)(1)} = \sqrt{20} = 4.5 \text{ m/s} \end{aligned}$$

But we could have gotten this same answer from Galileo's constant acceleration formula, Eq. 3.12 from Chapter 3. Finally, halfway between A and B, the energy is made up of less potential energy than A and less kinetic energy than at B.

$$\begin{aligned} E &= \text{PE(halfway)} + \text{KE(halfway)} \\ \text{PE(halfway)} &= mgH_{\text{halfway}} = (0.1)(10)(0.5) = 0.5 \text{ J} \\ E &= 0.5 \text{ J} + 0.5 \text{ J} = \text{always} 1.0 \text{ J} \end{aligned}$$

6.5.1 Classification of Collisions

In "regular life" we classify collisions into three kinds depending on how kinetic energy is handled: elastic, completely inelastic, and something in-between.

An **elastic collision** is one in which *kinetic* energy is completely conserved, which means that no energy is lost in any way. So the "normal" kinds of collisions in which the colliding objects make a sound, deform, or experience friction don't qualify. As we saw any of these circumstances take energy away from the motion and eventually all of it eventually becomes heat. We can't gather this energy up and use it efficiently and we say that these phenomena are "irreversible" which is why in part that so-called perpetual motion machines are impossible. Nature always takes energy away and doesn't return it.

Definition: Elastic Collisions.
Perfect collisions which conserve momentum and kinetic energy. Only elementary particles participate in pure elastic collisions.

A completely **inelastic collision** doesn't conserve kinetic energy and it doesn't do so...to the maximum degree possible. This happens when two objects collide and stick together, so a very particular kind of process.

Definition: Inelastic Collisions.
Collisions in which momentum is conserved, but energy is lost to heat so kinetic energy is not conserved.

Finally, in-between collisions are those which are not maximally inelastic, but not quite elastic. They're probably best represented in pool or air-hockey—the stand-in examples that I used to motivate (almost) elastic collisions. They bounce around almost conserving energy, but the fact that we hear them when they collide tells us that they're not quite perfect.

In fact, collisions of elementary particles like electrons and the whole zoo that we'll encounter are the only examples in nature of purely elastic collisions.

Definition: Totally Inelastic Collisions.
Inelastic collisions in which kinetic energy is maximally not conserved. These occur when the target and beam stick together in the final state.

To summarize:

- For Elastic Collisions: momentum is conserved and kinetic energy is conserved.
- For Inelastic Collisions: momentum is conserved, but kinetic energy is not conserved.
- For Totally Inelastic Collisions: momentum is conserved and kinetic energy is maximally not conserved.

Notice that momentum is always conserved. Note too that total energy of all kinds is always conserved. It is the *loss of kinetic energy into heat* through, say friction, that leads to kinetic energy itself being not conserved in the Inelastic Collision cases.

There is only one situation in the Universe in which collisions are perfectly, and precisely elastic: when elementary particles collide.

6.6 Energy and Momentum, From 50,000 Feet

The rules of momentum and energy conservation started as empirical observations. From the 1700s through the 1800s the science of mechanics became more and more mathematically formal. Rather than being a set of rough-and-ready tools at the disposal of engineers, mechanics and its mathematics revealed some neat things about how our universe seems to be put together. In particular, conservation laws went from a nice accounting scheme, to a clever way to solve difficult problems, to arguably the grandest of only a few universal concepts. I'll try to explain some of this later when we delve into symmetry as we understand it today but let's take a stab and meet Emmy.

Amalie Emmy Noether (1882 - 1935) was the daughter of Max Noether, a well-regarded German mathematician from Erlangen University near Munich in the late 19th century. Max Noether was a contributor to algebraic geometry in the highly productive period where algebra was being abstracted as a very broad logical system, in which the puny subject that we learn in high school is only a small part. This particular apple fell very close to the tree and Emmy, as she was always known, turned out to be the most famous member of the Noether mathematical family (she had two brothers who had advanced mathematical training).

Figure 6.6: A photograph of young Emmy Noether , probably around 1907, originally privately owned by family friend Herbert Heisig.

As a woman in Germany, only with an instructor's permission, was she was allowed to sit in on courses at a university—she could not formally enroll as a student. She did this for two years when the rules were changed and she could actually enroll an she steadily advanced to her Ph.D. degree at Erlangen in 1907. She was not able—again, due to German law—to pursue the second Ph.D. that's required in many European universities and so could not be a member of a faculty. So she stayed at Erlangen working with her father and colleagues. She even sponsored two Ph.D. students, formally enrolled under Max's name, but actually working under her. She developed a spectacular reputation and gave talks at international conferences on her work in algebra. Nathan Jacobson, the editor of her papers wrote, "The development of abstract algebra, which is one of the most distinctive innovations of twentieth century mathematics, is largely due to her—in published papers, in lectures, and in personal influence on her contemporaries."

Figure 6.7: Read about David Hilbert (1862-1943) and his 23 Problems. `http://www.famousscientists.org/david-hilbert/`

She was recruited in 1915 to work with the most famous mathematician in Europe, David Hilbert. He was racing Einstein to get to the conclusion of what became the General Relativity Theory of gravity and needed help with the complicated algebra and problems of symmetry, her specialty. Upon arrival at the Mathematics Capital of Europe, Göttingen, she quickly solved two outstanding problems, one of which has come to be known as Noether's Theorem, and which is of fundamental importance in physics today.

Hilbert fought for years for Emmy Noether's inclusion into the Göttingen faculty. He offered courses in his name, for her to teach. He led a raucous (in a early 20th century, gentile German sort of way) discussion in the faculty senate reminding his colleagues that theirs was not a bath house and that the inclusion of a woman was the modern thing to do. She was unpaid and yet still taught and sponsored a dozen Ph.D. students while at Göttingen. Einstein was particularly impressed and wrote to Hilbert, “Yesterday I received from Miss Noether a very interesting paper on invariants. I’m impressed that such things can be understood in such a general way. The old guard at Göttingen should take some lessons from Miss Noether! She seems to know her stuff.”


Figure 6.8: Emmy Noether later in life.

Emmy’s great grandfather was Jewish and had changed his name according to a Bavarian law in the early 1800’s. However, this heritage became a dangerous burden for her and she emigrated to Pennsylvania in 1932 to Bryn Mayr College, outside of Philadelphia. There she resumed lecturing, including weekly lectures at the Advanced Institute at Princeton until she was suddenly and tragically stricken with virulent cancer that took her live in 1935. After her death, which was acknowledged around the world, Einstein wrote in the New York Times, “In the judgment of the most competent living mathematicians, Fräulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians.” But the most moving and personal obituary came from another eminent mathematician, Herman Weyl:

> “You did not believe in evil, indeed it never occurred to you that it could play a role in the affairs of man. This was never brought home to me more clearly than in the last summer we spent together in Göttingen, the stormy summer of 1933. In the midst of the terrible struggle, destruction and upheaval that was going on around us in all factions, in a sea of hate and violence, of fear and desperation and dejection—you went your own way, pondering the challenges of mathematics with the same industriousness as before. When you were not allowed to use the institute’s lecture halls you gathered your students in your own home. Even those in their brown shirts were welcome; never for a second did you doubt their integrity. Without regard for your own fate, openhearted and without fear, always conciliatory, you went your own way. Many of us believed that an enmity had been unleashed in which there could be no pardon; but you remained untouched by it all.”

An amazing person, all the more so for her gender at time when the path for women scientists was non-existent. We’ll see a few more as we go along. In any case, a crater on the Moon is named for her, a street and her childhood school are named for her, as are numerous prizes and scholarships around the world.

6.6.1 Noether's Theorem, In A Nutshell

As I mentioned, mechanics evolved into a formal mathematical framework that exposed a number of fussy, but important details. Encoded in this formalism is the regular Newton's Second law and also momentum conservation, but the wrapper is elegant and accidentally identically important in quantum mechanics and relativity. What Noether found was that this formalism included a hidden surprise. That surprise was how it would react if some of the terms were modified in particular ways.

If we were to take Newton's Second law, good old $F = ma$ and remember that the a term includes space and time coordinates, x's and t's, we can modify their appearance in the equation in particular ways. Suppose I were to take the appearance of every coordinate variable, x and change every one of them to $x + a$ where a is a constant distance, like an inch or a mile. In effect, shifting every space coordinate by a specific amount. What would you expect to happen? Should the rules of Newton change? This is in essence saying that Newton's Second law works fine here, but what if I'm not here, but I'm 20 miles away? Then I should be able to take the x and shift it by $x + 20$ and the rule should still work. My lawnmower works on the east side of my lawn as well as the west side of my lawn. And, the structure of the equation $F = ma$ is such that the 20 would go away. (Calculus is required to see this specifically.)

Definition: Noether's Theorem.
Requiring that equations of physics be invariant under symmetries in variables will insure conservation laws. A remarkable connection between mathematics and physics.

What Noether's theorem says is that this shifting of space coordinates actually speaks to an "invariance" that Newton's Second law respects...its form is not altered—and so my lawnmower works all over the yard—no matter where I am in space. This is a symmetry of nature. Nature's rules hold every*where* the same. And this symmetry has consequences that tumble out of her mathematical description of this symmetry in the hands of the fussy formalism that mechanics had become: momentum conservation falls right out.

Symmetries in physics equations imply conservation laws. *Key Concept 14*

But wait, there's more. My lawnmower works the same today as it did yesterday. And the same at the beginning of the job as at the end of the job. That means that if I take Newton's Second law...and everywhere that time, t appears, I replace it with $t + b$, where b is some constant, like 20 minutes or 24 hours. What tumbles out is another symmetry of nature and another conservation law: Energy conservation.

The remarkable consequence of these observations, is that we now can interpret our conservation laws as not an algebraic accident, or even because of an experimental result. No. Our conservation laws come about because nature requires that our mathematical rules are unchanged whether we use them today or tomorrow, or over there or over here. They hold every**where** and every**when**.

Boy, is this important! Using Noether's Theorem as a recipe, we can pick a symmetry as a test and then ask what our formal mathematical description of nature implies about physical conservation laws. If the laws work out, then we've found a symmetry of nature. If the laws are not observed in experiment, then we can discard that symmetry as not one that works in our universe.

We'll exploit this, but I've used the word "universe" many times. Let's go there.