Chapter 12

Special Relativity, 1905

space and time aren't what they used to be

Albert Einstein during his time at the Swiss Patent Office.

Albert Einstein (1642-1727)

"Now to the term 'relativity theory.' I admit that it is unfortunate, and has given occasion to philosophical misunderstandings." *To E. Zschimmer, September 30,1921.*

Can you think of a more recognizable face than that of Albert Einstein (1879–1955)? Even in our culture of being famous for being famous, *Time Magazine* named him the *Person of the Century* in its December 31, 1999 issue. The *century*! Einstein's scientific career was as much or more remarkable than Newton's and together, they complete an exclusive club of two.

12.1 A Little Bit of Einstein

In one year Albert Einstein had three breakthroughs of pure thought and simple mathematics, any one of which would put him in textbooks forever.[1] Before 1920 he had at least four more theoretical discoveries that were again, all Prize-worthy. He basically invented five different fields of physics and changed the way humans look at themselves and our universe forever.

Einstein was a complicated man. He was not plagued with the sort of insecurities that blinded Newton, or the self-destructive combativeness of Galileo. He sometimes showed a shocking inability to empathize with individuals, while simultaneously demonstrating great feeling for mankind and professing a highly principled view of world affairs. He had great, life-long friendships and a childlike sense of humor that showed through in photographs and interviews. People around the world loved him and, while he was not prepared for the notoriety that he would achieve—bursting on him overnight on November 7th, 1919—he clearly grew to enjoy the spotlight. The myths surrounding him are, as we've seen, an inevitable consequence of being larger than life.

He died in 1955 (when I was five years old) in Princeton, New Jersey where he and his second wife escaped from the Nazis two decades previously. I think he looked older than his 76 years, perhaps a reflection of a stressful life following his scientific discoveries. Their Mercer Street home is still a private residence and he requested that it not become a museum (as his apartment in Bern, Switzerland is now). Those wishes were respected, although it is listed on the National Register of Historic Places.

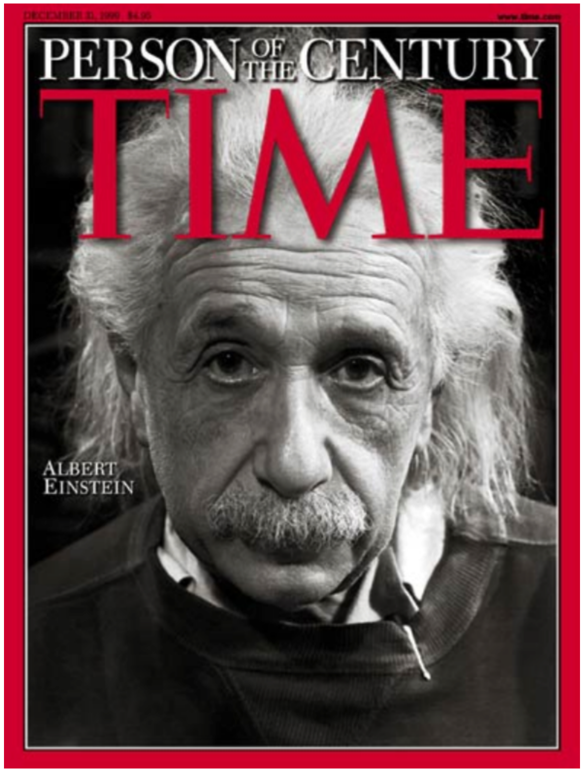

Figure 12.1: The cover of *Time Magazine*, December 31, 1999.

12.1.1 Education

Einstein was so smart that he was born at a very early age. :) He grew up in Munich in a comfortable household with a younger sister he adored. His father and uncle had a successful business of electrifying German municipalities. But it failed when he was a teenager and the family moved to Italy to start over. But Einstein was famous for his independent streak and even at 15 years of age he remained in Munich by himself. The Catholic school in which his Jewish parents had enrolled him was not a good fit, but then most schools weren't.[2]

Wait. *Didn't Einstein flunk math?*

Glad you asked. *That's one of those things that "everyone knows" about him. But it's not true, in fact it's the opposite of true. He mastered differential and integral calculus by the time he was 15 and precociously learned algebra and geometry as a very young child. He was mathematically gifted.*

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[1] Yet when it was time to award a Nobel prize–he received only one–it was held up for a year because of antisemitism in the Nobel Committee, and among many of his then-German colleagues. There was no Nobel Physics Prize in 1920 and then two were awarded in 1921, including his.

[2] This wasn't a match made in heaven and when he got fed up with the regimental manner of education and eventually followed his family south. After he left one of his teachers publicly expressed a sigh of relief. Albert was not shy to challenge teachers as a youngster and certainly as a college student.

Einstein was famously lazy as a student in spite of being brilliant. After he left high school, he took entrance exams for the Swiss Federal Polytechnic[3] in Zürich two years early, but while his scores were exceptional in mathematics and physics, they were unacceptable in other subjects and so he lived with a family near Zurich and attended a private school for a year in order improve his chances. That worked and he entered the Swiss Polytechnic in 1896 to study physics when he was 17 years old.[4] Also enrolled was the only woman in his class of six students, Mileva Marić. They became inseparable and a love-affair built around their joint studies bloomed. Much of their correspondence still exists.

In 1900 he graduated with top marks[5] (and a bad reputation) receiving a high school physics teaching degree. Mileva did not graduate, because of low mathematics scores—she tried a second time and again failed. For the next two years, Einstein unsuccessfully searched for a permanent teaching position and during that period Mileva went back to Serbia to have their out-of-wedlock child, a daughter. The baby's eventual fate is unknown and Einstein never saw her. They were married in a small civil ceremony a year later[6] and eventually had two sons.[7] Einstein had left such a bad taste in the mouths of the Polytechnic faculty that he was the only one of the graduating class to not be offered a continuing research position. He even had reason to believe that the professor who first supported him was by his graduation actively disparaging his former student to other prospective employers.

Marcel Grossmann was a college friend and talented mathematician who figured in Einstein's life multiple times. He was often a source of lecture-notes for when Albert skipped class—which was frequently.[8] And he was from an influential family and prevailed upon his father to help his friend, by securing a job for Albert at the Swiss Patent Office as an examiner in 1902.[9] So he and Mileva moved to the capital city of Bern where he settled in as a middle class clerk.[10] He managed to begin a research project with a professor at the University of Zurich simultaneously and in 1906 received his doctor's degree.
The year before he graduated? He changed the world.

12.1.2 Bern Years

Einstein's work at the Patent Office was not demanding and he could not only pursue his graduate degree part time, but also work on his own, outside of an academic environment.[11] He, Mileva, and Hans Albert lived in a second-floor apartment[12] a few blocks away from work and he would pass through the famous Bern Clock Tower (the *Zytglogge*) twice a day. That iconic, medieval structure and its famous performing clock figures into his later descriptions of how he came to Special Relativity. Let's go boating.

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[3] Now called *Eidgenössische Technische Hochschule*, ETH.

[4] In the process, he also renounced his German citizenship in order to avoid his obligation to go into the military. From a very early age he was a pacifist and ardently opposed to the militarism that was Germany at the turn of the century.

[5] "Oh, that Einstein, always skipping lectures. . . " A remark by one of his professors, Hermann Minkowski, who later put Special Relativity on a firm mathematical foundation.

[6] Einstein's mother was especially disapproving of this marriage.

[7] One, Hans Albert, became a professor of Chemical Engineering at the University of California, Berkeley. The other, Eduard, had a breakdown at the age of 20 and spend the rest of his life in and out of mental health wards with schizophrenia. Einstein expressed affection for a former lover while married to Meliva and eventually married his cousin, Elsa Löwenthal. The divorce from Meliva was very unpleasant and included strict instructions on her handling of their children and behavior towards him and the proceeds from his expected Nobel Prize. Elsa died three years after they moved to Princeton in 1933.

[8] He taught himself Maxwell's theory of electromagnetism out of class, disapproving of the German theory that was taught at the time. The Electromagnetic course was taught by that professor who became an enemy and Einstein had mocked him for teaching only old material rather than the more modern Maxwell.

[9] We'll see Grossmann return a decade later as Einstein's tutor in the advanced mathematics he would need for his General Theory of Relativity.

[10] "That secular cloister, where I hatched my most beautiful ideas and where we had such good times together." From correspondence to his Patent Office, and Olympia Academy pal, Michele Besso.

[11] "Whenever anybody would come by, I would cram my notes into my desk drawer and pretend to work on my office work."

12.2 Frames of Reference

You've all had the sensation. You're on a train and next to your car is another one that suddenly begins to move. Or do you move?... you can't quite tell the difference. Galileo asked whether there was any way to distinguish moving from not moving, and said "no" and of course, Aristotle had said "yes." In order to figure out who was right, let's set up a thought experiment.

Figure 12.3 depicts Galileo (G) and Aristotle (A) each standing on the deck of a boat (B) that's moving to the left at a constant speed relative to the shore (S), where Newton (N) watches them go by. Kepler (K) is at the top of the mast holding an apple[13] and loses his grip and it falls. Being good scientists, G, A, and K each predict the path that the apple will make on the way to the deck... where will it land?

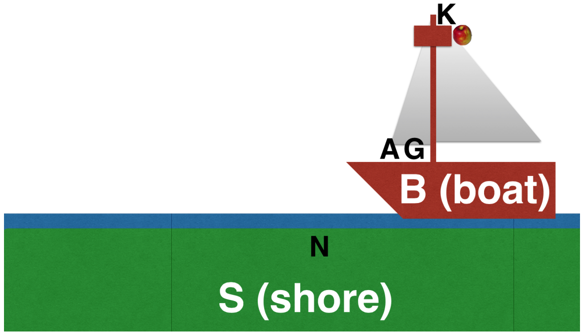

Figure 12.3: boat1

- Of course, A answers that the ship will have moved out from under the apple as it falls and so it would land near the stern, away from the mast. With that, he's repeating one of the classical arguments against a moving Earth: that the atmosphere would be left behind, that birds would fail to reach their destinations if they were flying in the same direction as the Earth, and so on. So the Earth must be still: Earth at the center of Aristotle's universe is absolutely stationary.
- G would tell A, "No!" The apple would land at the foot of the mast, falling straight down.
- K would look at the apple's fall from up above and also agree that it would fall straight down, directly below him.
- Newton (N), on the shore would watch the whole spectacle and when asked where the apple would land, he'd say, "with respect to what?"

Newton's question sets up an important idea that we'll use repeatedly: the notion of a **Frame of Reference**. In this story, there are two main frames of reference: The **shore, S**, and the **boat, B**. Newton would say that B is moving relative to S and that S, with him, is stationary.[14] What would Newton actually see relative to his fixed-Earth frame, S?

- While still in Kepler's hand, the apple would have the motion of the boat, to the left. When he drops it, the apple *still* has that horizontal motion, but now it starts to acquire the downward acceleration due to gravity as shown in Fig. 12.4. What N would observe is the same parabolic trajectory that Galileo discovered when he rolled a ball off the table edge. Here the boat's motion is providing that horizontal speed.

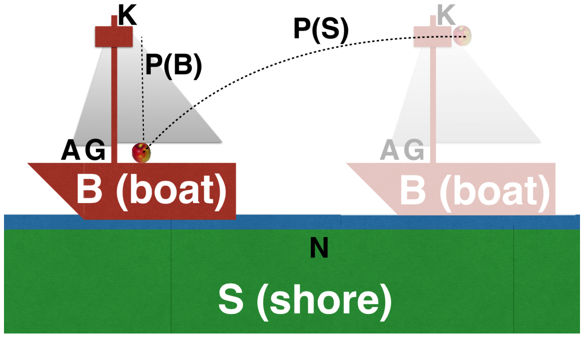

Figure 12.4: boat2

For Galileo and Newton a moving Earth is no problem. Just like the apple, birds and the atmosphere all share the Earth's speed (albeit a rotational speed. which we can pretend to be linear because of the large radius of the Earth) and are dragged along with it.[15]

What would G, A, and K say about the shore, S?

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[13] Fruit, again, play important roles in the history of physics! Not.

[14] He'd go further. Remember our discussion of Newton's Absolute Space. This unique, definitely stationary reference would be the

[15] When you fly across the ocean East or West, you're not helped or hindered by the Earth's rotational speed and direction. You're flying through and relative to the atmosphere which is being dragged along with the Earth.

- A would state that S is stationary, as it's part of the unmoving Earth, and that he and B are moving with respect to it.
- G and K would be more sophisticated, and Galileo in particular worried about this situation. They would say that B (on which they are passengers) appears to be stationary, and that S seems to be moving at a constant speed the other direction from N's assertion.

Frame of Reference. A fixed coordinate system in space and time. *Key Concept 1*

12.3 Galilean Relativity

Let's get technical and instrument our frames with their own space and time measuring devices: Figure 12.5 is a little spacetime measuring kit consisting of a ruler[16] and a clock. We can imagine that every moving object can be so equipped and that events anywhere can be measured in space and time from within any frame.

Let's recap what we've learned so far:

- Every observer is in his own Frame of Reference and at rest relative to it.
- If an observer moves by you at a constant speed, you'd say she's moving and you're not.
- Likewise, she would report that you're moving and that she's not.

Now, let's kick it up a notch.

Let's imagine that Newton and Galileo are playing catch[17] on the shore.[18] Each learns how to throw a given distance and since they're tired and they know the rules of projectiles and force, they rig pitching and catching machines to play their game for them. With a few trials, they can calibrate their machines to accurately pitch and catch, back and forth.

Now they take their machines to the deck of the moving boat.[19] Without making any adjustments at all, the machines pick right up where they left off and continue the game of catch with the same repetitive success as on the shore. Does this surprise you? I'll bet not.

Galileo's and Newton's rules about motion and force work exactly the same in constant speed, co-moving reference frames. Instead of setting up the machines by trial and error, suppose they'd first solved the algebraic equations of motion according to Newton's force and Galileo's motion rules, they would predict the same settings for their machines as they guessed in the trial and error approach. But the exact same equations would work in both the shore and the boat frames. That is, the same rules of physics would be active in both B and S.

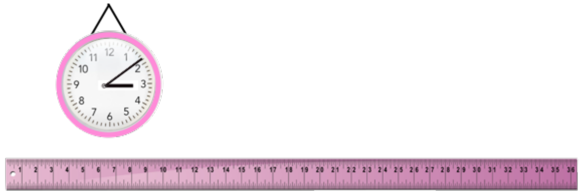

Figure 12.5: A little spacetime tool-kit, suitable for any frame of reference. Blue is a part of the AF equipment and pink is for the HF.

[16] We'll be in no more than two space dimensions and our relative motions will all be in one dimensions.

[17] Maybe with an apple.

[18] In these physics stories, you don't ask why the characters do what they do.

[19] In this chapter, "moving" always means at a constant speed.

Galileo wrote about this in 1632 and it's worth reading. Remember, this is the early days of figuring things out (emphasis, mine):

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), *have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that.* You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.

What Galileo means in his wordy way is that there would be **no** measurement that you could perform that would tell you either that you were moving or stationary. If there were no windows, you would have to conclude that you are at rest.

Definition: Galilean Relativity..
For relatively moving, constant velocity (inertial) frames of reference, that mechanical rules (only) can be transformed between frames by assuming that time is independent of the frame is called Galilean Relativity.

Einstein called this realization Galilean Relativity: **there is no mechanical measurement that can detect that a frame of reference is moving at a constant speed or at rest with respect to any other frame of reference.** A frame of reference that's at rest with respect to itself is called the **Rest Frame**. This makes sense.

Wait. *Does this always work this way?*

Glad you asked. *What if instead of the linearly, and constantly moving boat, they take their machines to a big playground merry-go-round where they set up on opposite rims across the center. Now they try to play catch with the same settings as the previous situations, and they would find that if the merry-go-round is rotating that every throw would be way off target. A throw straight ahead in the merry-go-round frame, would appear to be curved, rather than straight and a good student of mechanics would recognize that as a demonstration that there was a force at work and that the frame was accelerating.*

Frames of reference that are not accelerating, such as the boat (as view from the ground) and the shore (as viewed from the boat) are called **Intertial Frames of Reference**. An accelerating frame of reference, like the merry-go-round is non-inertial. Let's go to the airport.

Definition: Rest Frame..
A frame of reference that has no relative motion to another is a "rest frame." Every object is in its own rest frame.

Definition: Inertial Frame of Reference.
A frame moving at a constant velocity relative to others is called an Inertial Frame. If accelerating, they are non-inertial.

12.3.1 Coordinate Transformations

We'll usually be concerned with two different frames of reference which move side by side (so one dimension) with respect to one another. In this chapter they will be Inertial Frames and transportation industry analogies will be impossible to resist. Figure 12.6 is our tool-kit (rulers and clocks) which we'll pretend is standard equipment in all frames of reference. The overriding puzzle that we must solve is this: using our tool-kit, how can we describe events in an adjacent, co-moving frame from measurements made from our rest frame, or visa versa. An "event" is something that happens at a particular place (with x, y,and z coordinates and time, t). Events could be "happenings" (like an explosion or a light turning on) or the location of the edges of an extended object (like the coordinates—location—of the two ends of a stick at a particular time).

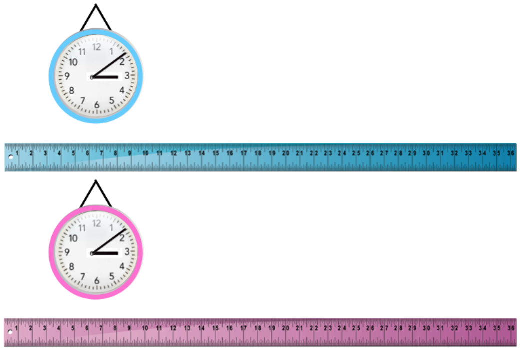

Figure 12.6: rulersclocks

The Moving Sidewalk

You've all been there: the big airport with miles of walking and little time to get from one gate to another. A long time ago airport planners found the solution to your limitations: the moving sidewalk.

Let's define some terms. Instead of calling one frame the "moving frame" and another the "rest frame"[20] we'll refer to the **Home Frame** and the **Away Frame**, or **HF** and **AF**. In this way we avoid any mistakes of language (or physics!). So for our story:

- For G, K, and A, the boat, B, is the Home Frame and the shore, S, is the Away Frame.
- For N, B (the boat) is the AF and S (the shore) is the HF.
- There's no Home Advantage.

Figure **??** shows two resting travelers in the airport watching the weary road warrior just standing there on the moving sidewalk that's going at a constant speed with respect to the airport, and hence, the Couch-People.[21] Another bit of terminology: I'll always use **u** to be the velocity of the Away Frame so u is to be the speed of the moving sidewalk.

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[20] Since Galileo, we all agree that "moving" and "rest" don't make any sense.

[21] If we remember our boat, Newton on the shore is like the Couch-People at the airport.

Pencil 12.1.

Solution

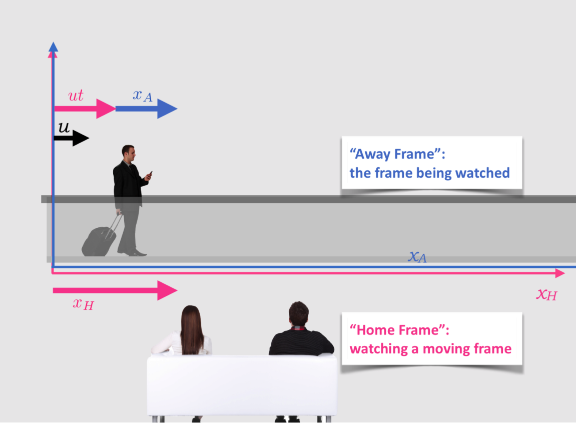

Figure 12.7: From the perspective of the airport (HF) the moving sidewalk (HF) is moving to the right with speed u. The traveler is stationary in the AF. Let's assume that the sidewalk (the AF) moves at a speed of $u = 2$ m/s relative to the airport (the HF). WearyTraveler's stationary foot is 2 meter from the origin of the moving sidewalk's frame, so $x_A = 2$ m. Trick question: after 2 seconds, how far has WearyTraveler's stationary foot moved within the AF? (He's standing still.) Next, the real question: what is x_H after 2 seconds, the position of his foot relative to the airport?

Did you get that $x_H = 6$ m?

What we've just done is called a "coordinate transformation." We've expressed an event that happens in one frame in terms of coordinates in another frame. In order to solve this simple airport problem, you invented a formula in your head:

Pencil 12.2. ✎

$$x_H = x_A + ut \tag{12.1}$$

$$x_H = 2 + (2)(2)$$

$$x_H = 6 \text{ m}$$

Equation 12.1 is the mathematical expression of the **Galilean Transformation**. Actually, there' s one more piece to the Galilean Transformation, and that's the following equation:

$$t_H = t_A \tag{12.2}$$

👍

and therein lies a subtle problem for later.

Definition: **Coordinate Transformation.** The conversion of the coordinates of an "event" in one inertial frame expressed in terms of another inertial frame.

Wait. *If I'm on a moving sidewalk, I know that I'm moving and that the airport is not.*

Glad you asked. *This is a difficult tool to conceptualize, but it's at the heart of Galileo's original notion, just brought up to modern transportation machines. People are not the crucial factors. Just the laws of mechanics. But you have all been in a moving rest frame and forgotten about it. How about a long airplane trip. Hours at cruising altitude, at night, over the ocean, and everyone's got their windows closed. You're simply in a room and cannot tell that you're moving at hundreds of miles an hour. Drop your pillow? It falls directly to the floor, not behind you. There's nothing you can do to show your motion. In fact, in your room, you're a part of a solar system that's moving more than 500,000 mph and don't know it! But don't pick up that pillow and use it because it's dirty down there.*

Now we get to the nub of it. Let's pretend that there were two Newtons... one who lived in the rooms at Cambridge University and the other who lived in a spaceship traveling at a constant speed relative to the British university's campus. They both do the same pendulum experiments and both reach the same conclusions about forces. And the form of the equations that they invent look exactly the same. The Cambridge-Newton uses F to mean force, a to mean acceleration, and m to mean mass, so his Second Law looks like

$$F = ma.$$

The spaceship-Newton uses $\clubsuit$ for force, $\diamondsuit$ for acceleration and $\heartsuit$ for mass and his Second Law looks like

$$\clubsuit = \heartsuit\diamondsuit$$

See that the *form* of the equations are the same? This is an important mathematical idea and the word **Invariance** is used to describe a formula that doesn't change form after some change in coordinates has been made on it. This is another way to say that every experiment performed in either frame would give the same results, and so no experiment could determine a state of motion or of rest. Our two Newtons would agree on the physics because their equations have the same form with the same definitions for the terms.

We say that **Newtons Laws are Invariant with respect to a Galilean Transformation**. Hold that thought, let's chase a beam of light.

Definition: Invariance.
Something is invariant with respect to a change of its coordinates if the coordinates are modified and the form of the formula stays the same as before.

12.4 The Paradoxes of Electromagnetism

When Einstein was a teenager he wondered what it would be like to look at a clock as you move away from it near the speed of light. He knew that light traveled at a large, but finite speed and so presumably as you moved away you'd see the time that the clock *was*. . . when the reflected light from its face bounced off on its way to your eye. What if you were traveling at the speed of light? What would happen then? This strange question stayed with him for a decade.

While he was working at the Patent Office, he and some friends would regularly meet in evenings and discuss matters of interest to them. They were a book-club of sorts[22] and with tongue-in-cheek they dubbed themselves the Olympia Academy. Later he credited the "Academy's" late-night, deep-dives into philosophy, mathematics, and physics as helping him to work through puzzles that bothered him for many years. It was during this time, he changed the world with three pieces of work in one amazing year:

[22] Meliva stayed home with the baby, while Albert played with his friends.

1905, Einstein's *Annus Mirabilis*

Remember that he has no doctorate in 1905. . . he's got a physics hobby, a regular "9-5" job, and an unused teaching diploma.

One. After a paper of his early doctoral thesis work, he published his second academic paper in the prestigious German journal, *Annalen der Physik*. It was about light and how it interacts with matter. We'll talk about that later when we get to the quantum theory, but suffice to note that this *March 1905 paper creates Quantum Mechanics.*

Two. In May of that year, he sent another paper to the same journal, this time related more closely to his slowly forming PhD thesis. All this *May 1905 paper does is convincingly demonstrate that atoms exist.* 2000 years of dispute, essentially settled in this one simple calculation. That's all.

Three. In June of that same year, he sent yet a third 1905 paper to *Annalen der Physik*. This was on one of his favorite topics, electromagnetism, and it's entitled *On the Electrodynamics of Moving Bodies*. In this *June 1905 paper he shows that Galilean Relativity is not correct*—the birth of Special Relativity.

The contents of this last paper are our focus here. He had never gotten his adolescent image of traveling at the speed of light out of his mind and it motivated him to look more closely at the equations of Maxwell once he had enough experience to understand them thoroughly. Electromagnetism got under his skin as he realized that has mathematical subtleties that reveal actual logical paradoxes.

1905 has ever since been called Einstein's *Annus mirabilis*. . . his miraculous year.

Racing Light

The speed of light was well-known by 1900 or so. Remember, we reserve the symbol c to represent this very special number, and in scientific units, c just about $c = 3 \times 10^8$ m/s or $c = 671,000,000$ mph, big in any units![23] Every experimental physicist learns that light travels about a foot in 1 nanosecond (1×10^{-9} s), so how long it takes for an electrical signal to go from one spot an electronics rack to another can be estimated by eye.

If you look in the mirror a foot away, then the light that reflected from your face to the mirror and back into your eyes is stale—you're seeing at what your face looked like 2 nanoseconds ago.

This finite light speed is a lot more dramatic when we think about astronomical objects. For example, the Earth is so far from the Sun, that it takes 8.3 minutes for the light from its surface to reach us. If the Sun suddenly turned off, we'd have 8.3 minutes of sunlight before everything went black. Astronomers refer to this distance as an "Astronomical Unit" or AU. The Earth is 1 AU from the Sun, while Jupiter is about

[23] Actually, it's $c = 299,792,458$ m/s or 670,616,629 mph.

5.2 AU away. They have also invented a self-explantory distance as how far light would travel in a year: a "light-year," which is 9.4607×10^{15} m. So a single AU is $1.58128451 \times 10^{-5}$ ly.[24]

The nearest star to our Sun, Proxima Centauri, is in the constellation Centaurus, the Bull and about 4.2 light years from us. I'm writing this in 2015, so just about now any Proxima Centurions with their TVs tuned towards Earth are just seeing the last of Oprah Winfrey's long-running TV show and the first episode of *Game of Thrones*.

Andromeda is nearest galaxy to ours, and one that probably looks most like the Milky Way. When you look at it,[25] you're seeing the image of what it looked like 2.5 million years ago, so it's 2.5 Mly from the Milky Way. The object that holds the record for being furthest from us is affectionately known as z8_GND_5296 which is 13.1 billion light years away and was only recently discovered. The Universe itself has been determined to be 13.7 billion years old, so this is an image from the universe's adolescence. You'll agree that the finite speed of light leads to interesting, if not completely awesome ideas. But light is tricky.

Tricky Light

Let's list some common sense ideas on the highway:[26]

- Common Sense #1. Suppose we're on the highway traveling at 50 mph and another car goes by us on our left traveling at 60 mph. If we treat our car as at being at rest (like inside the hold of Galileo's ship), we would conclude that other car is not going 60 mph, but 10 mph relative to us.[27]
- Common Sense #2. Suppose your crazy passenger stands up and throws a 90 mph fastball forward through the sun roof of your car. Relative to the ground, the ball is seen then to be moving at 140 mph—superpowered major league material. But relative to the other car? That pitch is just junior varsity, only 80 mph.
- Common Sense #3. Suppose the speeding car beside you is actually a beam of light. Then you would expect that it would be moving at 671,000,000 minus 50 mph, or 670,999,950 mph relative to you.
- Common Sense #4. Suppose your athletic passenger whips out his laser pointer rather than a baseball and points it straight ahead. You would expect that someone on the ground would measure it to be moving at 671,000,000 plus 50, or 671,000,050 mph.

Einstein found hints that something was strange about light. Here are two:

- If you were moving faster and faster away from the Bern tower clock its hands would appear to stop and freeze at the moment you reached light-speed. Its reflected light could not catch up with you and forever you'd see the time that it *was* when you passed that boundary. Time would appear to stop, which is kind of strange. This is Hint #1 that the speed of light is special.

[24] We could also say that the Earth is 8.3 light-minutes from the Sun.

[25] In a dark sky, the Andromeda galaxy is a binoculars object.

[26] Still...with the transportation analogies.

[27] This is so obvious as to be tedious.

- Remember that in an electromagnetic wave, it's the changing **E** field that creates a **B** field, and the changing **B** field that creates an **E** field. They mutually produce one another and mutually depend on one another and the propagating wave motion perpendicular to the **E** and **B** vectors is the result. Well, if like our cars you're speeding up alongside of a light wave—and reach the speed of light so you're now alongside of the light beam—the **E** and **B** fields would appear to go up and down in place, but not appear to propagate forward as a wave any more! That is a direct contradiction to the very clear mathematics in Maxwell's Equations. This is Hint #2 that the speed of light threshold is special.

Common Sense items 1 and 2 above are obvious. But these two hints suggest that *moving relative to light beams is strange*. And since the speed of light is somehow "hard-wired" into Maxwell's Equations, there's simply no good way to deal with situations like these. Before working our way out of this, let's make it even worse:

Definition: Lightyear..
The distance that would be traversed by a beam of light in one year. It is 9.5×10^{15} meters, which is approximately 6 trillion moles.

It Gets Worse

If that's not bad enough, let's think about two situations that Einstein actually writes about. Let's suppose that we have two lines of electric charge, one of positive charges and one of negative charges as in “Situation #1” at the top of Fig. 12.8 and place a positive charge, Q next to them. Since there are as many positive as negative charges, Q will feel no electrostatic force. Now let's have both Q and the negative line of charge move to the right with respect to the page (still no electrostatic force) with a constant velocity, v. In its rest frame, Q would see that the positive line of charge is moving to the *left* (along with the book), which is a current in Q's frame. Since a current produces a magnetic field, **B** into the paper at Q's position, and since Q has a velocity to the right, it would feel a force, up.[28]

Now, let's reverse the situation as in Situation #2 in Fig. 12.8, without seeming to change anything but the relative motion. Leave Q stationary with respect to the page, so it has no velocity in that frame and hence even if there were a magnetic field nearby it would not experience a force. So start the *positive* line of charges moving with speed v to the *left* relative to the page. That creates the same **B** field as before, but since the charge has no velocity, it would not feel anything.

[28] Remember, that the force of a charge in a magnetic field is $F = QvB$. And your right hand.

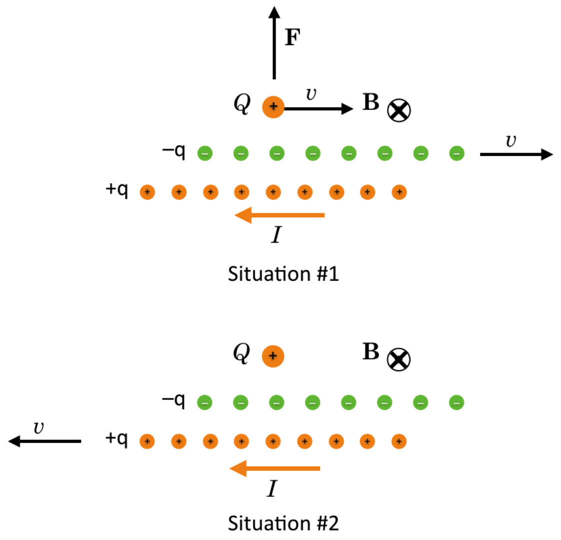

Figure 12.8: chargeparadox

Wait. *The situations aren't really different. In the first one, Q is moving and the positive line is stationary. In the second, Q is stationary and the positive line is moving. They only differ in their reference frame interpretation.*

Glad you asked. *You got it. The two situations only differ by the who is considered moving and who is considered stationary. Such co-moving frames in our Galileo-Newton discussion of mechanical systems didn't lead to any physical differences. But when a magnetic field is involved, Q experiences different outcomes just by interpreting who's Home and who's Away. Yes! It doesn't make any sense!*

Uh Oh.
These are two different physical outcomes for situations that differ only by their relative motion.

Here's another. Remember the magnet and the coil? When we push the magnet through the coil, a current flows in the wire. No battery, just Faraday discovering the generator. Now, suppose we leave the magnet still and move the coil over it. What happens? The same thing happens. The two relatively moving frames give the same physical result. So what's odd about this?

Let's ask what happens *physically* in each situation. When the magnet moves through the coil, the circles of wire capture the changing magnetic field. The changing **B** field creates an **E** field in the wires and that **E** field in the wires causes a force on the charged electrons in the wires, and so they move. . . which is the current that we see.

In the second situation, the wires—including their electrons—are moving toward the field of the magnet. *Now* the electrons in the wires have a velocity *and* they see a magnetic field, so from $F = Q_e vB$ the electrons experience a force. . . but this time it's a force due to **v** and **B**, *not an* **E**.

Uh Oh.
These are two identical physical outcomes for situations which have entirely different causes arising simply from the interpretation of who's Home and who's Away.

12.5 The Postulates of Relativity

Newton's Laws of mechanics gracefully flit about among co-moving inertial frames without any modification. That made sense. But comparing the consequences between co-moving frames when light is involved either leads to logical absurdities or an abandonment of Maxwell's beautiful formalism since there is no messing with c in Maxwell's Theory. Einstein was having none of that. Others also worried about these matters, but they were unwilling to go as far as our young clerk who was offended that the two best theories that explained all known phenomena should act so differently when viewed from co-moving frames. So he resolved to make them both behave.

He thought that Newton's Laws and Maxwell's Equations should not behave differently when viewed from co-moving inertial frames of reference and he resolved to fix this ugly discrepancy. This had been rumbling around in his mind for years. What he wanted was a way to expand on Galileo's notion that mechanical phenomena are blind to steady motion and promote optical phenemena to that level of consistency as well. In his 1905 paper he enunciated this idea and called it a postulate, the "Relativity Postulate": "...the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good."

It says: we require *all phenomena*, mechanical *and* electromagnetic, to work the same in all co-moving intertial frames. So in our example of Galileo and Newton's pitch-and-catch machine above, we could reasonably add that the microwave oven that they use to prepare popcorn for their spectators works the same on the shore is it does on the boat and that the rules that describe its manufacture would be the same as well.

Before we work out the physical consequences of this, let me make a bigger point about the philosophical consequence of the First Postulate, one that has guided all of science ever since Relativity became accepted. If the only tools you've got are mechanical and electromagnetic, then you can **never** tell whether a frame is stationary or moving, relative to any other frame. Never.

Wait. *So, if there's a fixed frame of reference that has no motion, we could never know it?*

Glad you asked. *Yes, that's true, but we need to take that idea and make it into a principle of knowledge—a statement about Reality, itself!*

The importance of the need to make a measurement is now a criterion of science. If you can't make a measurement of some idea, Einstein's work has convinced us all, then you cannot treat that idea as real. What's real in 20th and 21st century science are only those things that can be measured. Say that again:

If you can't measure it, it can't be real. Period.

Let's call this the **Reality Postulate**. The advent of Quantum Mechanics only reinforced this idea and since he was also instrumental in ushering in that most strange theory, Albert Einstein has to be considered as among the foremost *philosophers* of all time, as well as scientists.

The consequences of the Reality Postulate turn into a criterion for what is and what isn't a scientific statement. If your theory contains statements about how things are, but those things cannot be measured, then you're not doing science. Obviously, this hardens the separation between religion and science, but we'll not go any further there.

Now comes the mathematical consequence. Maxwell's Equations say that the speed of light, c, is a single number related to the very properties of empty space which everyone believed was full of the ether. But if you could move relative to the ether and then make a measurement of the speed of light and find it to be different from that predicted by Maxwell, then you'd have detected that you're moving. That would be contrary to the Relativity Postulate, and in fact be responsible for all of the paradoxes that we discussed above.

So this forced him to a second postulate: "...light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. In our language the two postulates are:

1. All laws of physics—mechanical and electromagnetic—are identical in co-moving, inertial frames of reference.
2. The speed of light is the same for all inertial observers.

The import of this is that the Common Sense items #3 and #4 above need to be modified into *un*-Common sense situations:

- *un*-Common Sense #3. Suppose the speeding car beside you is actually a beam of light, and so moving at $671,000,000$ mph relative to the ground. Then you would find that it is also moving at $671,000,000$ mph relative to you also.
- *un*-Common Sense #4. Suppose your athletic passenger whips out his laser pointer rather than a baseball and points it straight ahead so that it would be moving at $671,000,000$ mph relative to your car. Someone on the ground would also measure it to also be moving at $671,000,000$ mph relative to the ground.

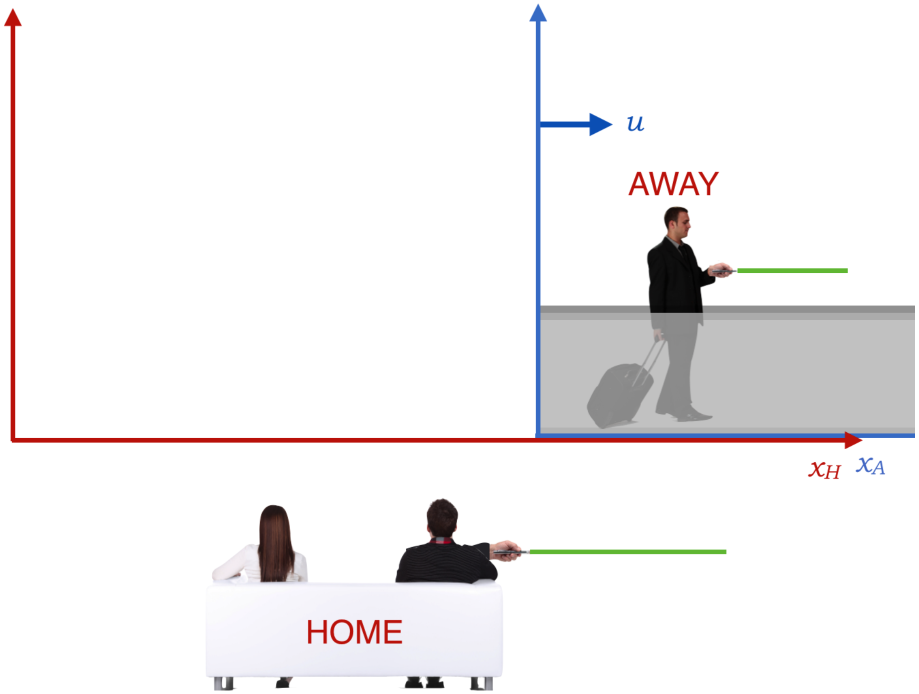

Figure 12.9: lasercar

This is exceedingly strange! What the second postulate requires we'll illustrate in Fig. 12.9 : this shows WearyTraveler playing with his laser pointer, aiming it ahead of him and CouchGuy in the airport with an identical laser pointer. Without bothering to get up, CouchPeople measure the speed of the light from their laser pointer to be c. Meanwhile, WearyTraveler measures his light beam to have the same speed of c, relative to the sidewalk frame. . . . But CouchPeople can also measure the speed of the light generated by WearyTraveler, moving past them at the sidewalk speed of u. They would *not* measure that WearyTraveler's light as moving at $u + c$...they would determine that WearyTraveler's light is *also traveling at that same value of c* as theirs and that he determines for his!

The value of c is the same for all inertial observers, regardless of the frame in which the light is emitted.

Let's take Einstein's leap into brand new territory. In the next chapter we'll unwrap the presents that the two Postulates of Relativity delivered to us and in the chapters after that, dramatically end World War II with the most famous equation ever.