Chapter 18

Quantum Mechanics, Grown Up

Paul Dirac's First Big Score

Paul Dirac as a young man.

Paul Dirac, 1902-1984

"In science one tries to tell people, in such a way as to be understood by everyone, something that no one ever knew before. But in the case of poetry, it's the exact opposite!" Paul Dirac as quoted in *Brighter Than a Thousand Suns : A Personal History of the Atomic Scientists (1958)* Robert Jungk

When Paul Dirac went to Stockholm to accept his Nobel Prize in 1933 he did all of the standard things—the parties, the banquet, and of course his address to the Royal Court and guests and family. While families of Nobel Laureates always attend the ceremony to see their spouse, parent, or child inducted into history, Paul's father didn't attend. He wasn't invited.

Quantum Mechanics was difficult enough. Relativity was tough too, but somehow a little more accessible, right? Putting them together—which everyone knew had to be done, but nobody could figure out—proved to be the opening of a floodgate that let in all manner of odd realizations of just how Nature works at the deepest level.

18.1 Goals of this chapter:

- Understand:
  - How to calculate distance, time, and speed for uniform and constantly accelerated, linear motion
  - That falling objects all have the same acceleration near the Earth.
  - How to graph simple motion parameters
  - How to read graphs of realistic motion parameters
- Appreciate:
  - The algebraic narratives in the development of the formulas
  - The shape of the trajectory of a projectile
- Be familiar with:
  - Ideas of motion before Galileo
  - Galileo's life
  - Galileo's experiments with motion

key concepts

18.2 A Little Bit of...haven't decided

In order to dig into modern theories of EPP we want to be able to describe how elementary particles interact with one another and how different theories specify those interactions. In practice we'll be able to concoct a straightforward method[1] to do this but under the hood, these tricks are based on one of the more complicated bits of mathematical physics that's ever been devised. This *tour de force* has a fancy name: "Relativistic Quantum Field Theory" (RQFT) and while a mouthful, it's a catchy phrase and sure to impress at parties. Since you're about to understand what it means, feel free to use it. If you value your privacy.

[1] Easy to say, right?

Definition: RQFT.
Relativistic Quantum Field Theory is the model of how elementary particles behave both quantum mechanically and relativistically.

Wait. *Antimatter was pretty hard to grasp. Can't we come to earth?*

Glad you asked. *Well, you'd better fasten your seatbelt. I've worried out loud a little about "what an equation means" but here it's going to be a critical consideration. You'll see that while we're really talented at turning the crank, we're a little hamstrung when it comes to thinking really deeply about what's actually attached to the crank. Like any group of people, there are physicists who just want the answers and don't worry about the conceptual stuff. And there are others who stay awake at night worrying about exactly what the symbols in the equations all mean!*

What I'm going to describe is likely to frustrate you at first. That's partly because it's a tough subject taught to second year graduate physics students and the details are pretty technical and we just can't go there. It's also a subject which we understand almost solely through mathematics and not words or even reliable mental images. But of course I'm going to use words and pictures, so you'll have to accept metaphor and analogy as substitute for six years of university mathematics.

That's okay with you, right?

Perhaps surprisingly while RQFT is a sophisticated model and well-defined set of rules, it's not entirely understood in each and every solemn detail and some people devote themselves to tying up a few formal, mathematical lose ends.[2] So do we wait around until it's perfect? No: welcome to practical science at its best. The RQFT recipes make predictions which are incredibly precise and experiments confirm those predictions with exquisite accuracy. And that's good enough for most of us and so we don't worry too much about the niceties. All of the progress in most areas of Quantum Mechanics in the last 50 years is due to this theory.[3] So stick with me and I'll highlight the concepts and few formulae that we'll need to make progress. I promise that by using a bit of it you'll grow to tolerate, if not enjoy RQFT. Like us!

[2] This means they try to be sure that every single step be logically consistent with every other and that there are no unwarranted mathematical assumptions.

[3] When it's all tied up in a neat bow, we'll take a couple of hours off and have a party.

We're going to walk a mental tightrope together without a net here. You and I both have this natural desire to put a mental (here, read "mathematical") concept into words, and when we can't do it, it's frustrating. The wavefunction falls into that category and now we're going to go even further into the realm of abstraction. But Quantum Mechanics is different and I feel your pain: living with what a successful equation "means" is a burden that takes some getting used to.

Even in Relativity when we were faced with some odd mechanical circumstances like length contraction or time dilation we didn't have to wonder what a meter stick was or struggle with the "concept" of a clock. So the meaning of the objects affected by the theory was not at issue and while the behavior of these objects was unusual, we had a feel for it.

Sometimes I'll have to say something like, "If we take literally what the equation says, then we have to assume that X is the case." Then I'll describe X which will cause you to shake your head and say, "No, the World can't be like that!" Trust me: that's the best we can do. We have to take the position that if RQFT works so well and yet is a little unnerving when its mathematics is deconstructed into words, well, who's problem is that? Nature continues to function just fine and I suspect she's not staying up nights worrying about our inability to fathom to our satisfaction what She's doing. Again, RQFT works to many decimal places of prediction. Obviously there's something very right about it.

As Feynman put it: "It is not a question of whether a theory is philosophically delightful, or easy to understand, or perfectly reasonable from the point of view of common sense. The theory of quantum electrodynamics describes nature as absurd from the point of view of commonsense. And it agrees fully with experiment. So I hope you can accept nature as she is—absurd." Oerter, Robert (2006-09-26). The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics (p. 131). Penguin Group. Kindle Edition.

With that apologetic introduction, let's take stock of what we know up to this point in our story, which corresponds to the situation in about 1926 when—guess who—Dirac wrote a seminal paper that changed everything. What we knew by then was that electrons were originally found to behave like particles and then grudgingly, were described by deBroglie and then Schroedinger as if they were also waves.

- So electrons: → first interpreted as particles & then also as waves.
- So photons, → first interpreted as waves & then also as particles.

Nature complied:

- particle like behavior of light was confirmed in 1923 in the Compton Experiment: ✓.
- wave-like behavior of electrons was confirmed in 1927 in the Davisson-Germer Experiment: ✓.

So a shaky stand-off was under way when Dirac was in graduate school: particles and waves seem to be opposite concepts, but yet shared by all forms of energy and matter.

A small difficulty: the theories that described both electrons and photons were different! Schroedinger's theory was of non-relativistic electron wavefunctions (and Dirac fixed that in 1928 as we saw). Electromagnetism is clearly a relativistic theory—after all photons move at the speed of light. But the propagation of EM waves and even their interaction with quantum mechanical electrons was still described by Maxwell's theory—it wasn't really quantum mechanical at all! So something was wrong and Paul Dirac quietly set about to fix that in 1926, a year before he wrote down his relativistic electron theory.

I'll try to transcribe his symbolic mathematical narrative that describes this into a story of words and pictures. Again, please don't blame the messenger!

18.3 Uncertainty Principle, Unplugged

You might not have thought of this, but "identity" is a philosophical problem that Dirac inadvertently solved. Of course, he wasn't trying to solve it, he was working on something prosaic, like how atoms absorb and emit light. But let me lead with this because it's so simple a problem that I'll bet you never thought about it before and you'd be in good company. Since Plato worried about it, not much thought had gone into it.

Look around you: what among the things that you experience in everyday life are identical? Sofas are all similar in many respects, but they're really all different in particular features. The big pieces in our living rooms share "sofa-ness" with other such furniture but even if they are the same brand, small differences remain among them rendering them imperfect, a mere shadow of the True Sofa. From those billiard ball examples I'm always using, even the "8 balls" from each set are very similar, but in tiny ways each would be different from one another—slightly different thickness of paint at the micron level, for example. Even mass produced identical objects are not *really* identical.

As I hinted, this is an old subject. Plato surmised that the sofas that we perceive in our lowly human lives aren't actually real, but that only the pure, Idea or Form of a Sofa is the actual, true one and that the apparent sofas that we know then "participate" in the Form of the Sofa. You see, reality for Plato consisted only things that are absolutely true and that was his realm of the Forms, those perfect ideas which are the source of the things we actually perceive. That's why he despised art . It's an imitation of something we see, which itself is an imitation of the Real Form of that thing.

But an electron? That's a different kind of thing. An electron in an iron atom in an arbitrary hemoglobin molecule in my blood stream is absolutely identical to an electron in the atomic hydrogen in the upper atmosphere of a planet in the galaxy Andromeda. Or exactly identical to an electron created in the Big Bang. *Absolutely identical* is a feature that only elementary particles can possess and the ideas seeded by Dirac led to an understanding of how this might be the case and how to write a theory of all of particles

In Plato's language? An electron in our lives is actually its own Platonic Form!

based on that idea. He called the theory that he first published in 1927 **Quantum Electrodynamics**, a quantum theory of Maxwell's and Einstein's electromagnetism. It required some unusual ideas.[4]

[4] These will be some of those "X" things I mentioned before.

Let's think again about the sort of atomic excitation problem from your high school chemistry class, or our description of it in Bohr's and then Schroedinger's Quantum Mechanics. An electron in some state in an atom is exposed to an electromagnetic wave, absorbs a single photon from that wave, and is excited into a higher orbital in the atom. If you wait a bit, that electron will jump down from that excited state and in the process release energy in the form of another photon whose energy is exactly the difference between those two orbitals.

Fine. What actually happened to the photon that was absorbed? Does it wrap itself around the electron? Do they join hands and swing together in that higher state? Does the electron actually get fat by somehow eating the photon? All silly sounding ideas, but something has to happen to it!

The fate of the photon fell out of Dirac's careful quantum mechanical analysis of the problem. Prior to his idea, the quantum mechanical description of atomic transitions treated the photon as if it were a classical, Maxwell wave while treating the atom as a quantum mechanical thing. Shouldn't there be a single Quantum Mechanics that describes the whole system, photon and atom?

Dirac divided the system into three pieces which he wrote explicitly as an equation of the form:

$$\text{Whole System} = \text{Atom alone} + \text{EM field alone} + \text{coupling between the Atom \& EM field.}$$

That last piece is a delicate combination of the relevant pieces of the two systems merged together — the "interaction piece" we call it —with Quantum Mechanics. Following his uncanny instincts and some inspired mathematics he found a single set of equations which made the atom and the field resonate together only if the photons are *individually counted*. He needed a tallying symbolism that started from zero and counted up or down in single photon units. Literally.

"The light-quantum has the **peculiarity that it apparently ceases to exist** when it is in one of its stationary states, namely, the zero state, in which its momentum, and therefore also its energy, are zero. When a light-quantum is absorbed it can be considered to jump into this zero state, and when one is emitted it can be considered to jump from the zero state to one in which it is physically in evidence, **so that it appears to have been created**. Since there is no limit to the number of light-quanta that may be created in this way, we must suppose that there are an infinite number of light-quanta in the zero state..." P. A. M. Dirac, Proc. R. Soc. Lond. A 1927 114, 243-265 His remarkable Photon-quantum paper. The emphases are mine.

Supposed he wanted to describe an electromagnetic wave of a particular frequency with 1000 photons. He wrote $|1,000>$ If one of those photons was absorbed by a nearby atom, he'd then write the wave as $|999>$. The mathematical operations that took one to the other are related to the actual physics of a photon-electron collision, but it would only work if the photons could be individually counted and in this case, decreased by exactly one. That is, the interaction piece above would do something like this:

$$|999> = \mathscr{H}_{int}|1,000> .$$

Here the term $\mathscr{H}_{int}$ is where the mathematical action is. It's a fancy mathematical "operator" that contains the atomic physics.

With these new tools he could explain all interactions of an atom de-exciting from an excited state, an atom exciting to a high state in the presence of a radiation field, to even a new description of a free electron interacting with an electromagnetic field. Notice that this is an inherently particle-like description since we can count photons here, one by one.

Here is the idea and the language: When a photon is emitted from an excited atom, the EM field count goes up by 1 and we say that a photon is **created** in the EM field. If a photon is absorbed by an atom the field count goes down to 1 less and we say that a photon was **annihilated** from the EM field, like in our 1,000 photon example above. In these counting symbols, for the absorption mechanism we have $|1> \rightarrow |0>$.

Wait. *What in the world is: $|0>$???*

Glad you asked. *Oh, it's nothing. In symbols.*

This "zero" part is interesting. Dirac's "picture" was that a perfectly acceptable state for a photon to jump into to is...a "zero state"—a state of there being no photons, the quantum jump to oblivion. Here's another one of those "X" times. The mathematics requires that there be a state of photons in which there are no photons. Say that again:

I was just at a dinner with friends in France outside of the CERN laboratory. Of the eight of us, only two were American. If there's one thing that seems to separate Americans from Europeans, it's ice. Our way of speaking was pointed out to me by a particularly perceptive physicists: she noted that when you order a soda in the US you ask for "ice" or "no ice." Like "no ice" is an actual thing. "I'll have some 'no ice,' please." They all thought that it should be one word: noice. A state of ice, in which there is...no ice.

No particles is a perfectly acceptable state of a particle. *Key Observation 2*

So the photon in an atomic excitation is not absorption into the atom or the transitioning atomic electron—in this formalism, it disappears. Into the Photon-Vacuum state. Let's go to the carnival.

18.3.1 Whac-A-Mole Quantum Mechanics

You've all done it. So have I. Every carnival has the stress-relieving device in which you beat a toy mole on the head with a mallet. Every time you whack it, it disappears...and somewhere else up pops an identical mole which stays there until you whack it. The process goes on and on and there seems to be this unseen, boundless source of identical moles. Before you paid your token there weren't any visible moles—a Mole-Vacuum state. Once you begin, the Mole-Vacuum spontaneously creates a mole until you annihilate it and this goes on over and over.

Back to Dirac's Photons...When one is absorbed, where does it go to? And where does it come from when one is emitted? RQFT demands that there is this invisible, boundless source of photons which we say resides in a field...I'll call it briefly the "Ur-Electromagnetic Field." [5]

[5] Ur was the ancient city in Sumaria that was presumed to be the source of all of Abrahamic tradition. The Source! Also an Ur-text is an original text or language from which other languages originate. "Ur" is an Old German word meaning "primitive" which is a good analog here to what we'll want to think of as the primitive source of all matter.

We'll use the word "field" (lower case) as that of Maxwell, but this Field is an entirely different...shall I say, animal—a primordial *Field* with a capital "F." The Maxwell field was a continuous disturbance in Electricity and Magnetism which has a particular value and direction at every point in space. The thing that oscillates and makes the electrons in the radio antenna in your car oscillate along. Our more modern Ur-EM Field is the source of all photons which has the ability to oscillate at every point in space and each represents the potential to become a quantum of the EM Field—we say an "excitation" of the Field. I'll drop the silly "Ur" but I want you to think of this stuff as a primordial substance (sort of!) of fundamental importance in our world: the EM Field.

We have to think of it as *everywhere* and *everytime*. When a photon is created, it's squirted out of the EM Field and becomes real. When a photon is annihilated, it's sucked into the EM Field and disappears. This goes on all over the Universe, all the time, for all time. It'll happen tomorrow, a lot.

I hope you can see that this idea of photons appearing and disappearing is a little like how Dirac's positive electrons disappear and appear along with his holes—the theory that he came up with a year after his photon Field work. But while the photon story is the modern interpretation of Dirac's atom-idea, it's also the modern version of his hole idea! His holes were just his first try to attach some meaning to his mathematics.

The modern way to interpret Dirac's hole idea is that each electron is the excitation of an Ur-Electron Field. Now we've got two such Fields, simultaneously comprising our Vacuum.[6]

Figure 18.1: mole

[6] If a copy-cat vendor created a Whac-A-Racoon game, and merged the two, sometimes you'd see a Raccoon and sometimes a Mole...there would be a Raccoon-Vacuum as well as a Mole-Vacuum from which only Moles would be produced from the Mole Vacuum, and likewise for the Raccoon-Vacuum. They're separate and when they spit out their particular varmint, you see them and can count them...and can interact with them.

18.3.2 What Particles Are

In fact our modern interpretation is that all elementary particles are excitations of their Fields: an Ur-Field for every one. Were there a state in which there are no excitations among the collection of all of these Fields, well, that would be our Vacuum. It would be *empty* in the traditional sense that if there are no excitations of any of the Fields, then there are no particles of any kind. But it would also be *full* in the sense that these Ur-Fields are there all the time and they contain energy and at every point in space, the **ability** to create a quantum appropriate to their particular field. This is going to be huge at the beginning of the 21st century in both particle physics and in cosmology. You wait.

This description of photons (excitations of the Ur-EM Field) and electrons (excitations of the Ur-Electron Field) puts all particles on the same footing. The mathematical tools we use to manipulate them are the same. After Dirac, the task from the 1930's and into the early 1950's was to formalize this idea and rid it of some mathematical embarrassments into the a very sophisticated, internally coherent description of RQFT. The rules of how Nature Works. Why Relativity figures so prominently in the title we'll see in a bit.

"Wheeler said, "Feynman, I know why all the electrons have the same charge and the same mass." "Why?" Feynman asked. "Because they are all the same electron!" replied Wheeler." Oerter, Robert (2006-09-26). The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics (p. 103). Penguin Group. Kindle Edition.

18.3.3 Back to Plato

But circle back to how I started this discussion: the identical nature of all electrons falls right out of this idea! Because there is only one Ur-Electron Field, every electron is identical to every other since *each is an excitation of the same primordial Field*. You have to fit into your head—where believe me it's crowded now—the idea that these particle fields exist everywhere and everytime and when we collide particles together or when Nature causes natural reactions and decays, it's the tickling of the Fields by the particles and their propensity to do so that account for all of physics.

RQFT is now entrenched: Elementary particles are individually created or annihilated when their Fields are disturbed. Where they go to and where they come from is this increasingly strange Vacuum.

Before we start drawing the pictures that describe all of this formalism, let's see how Quantum Mechanics and Relativity force our Fields into a strange, jittery dance and then see how some clever people devised an experiment to confirm this whole thing.

18.3.4 Pop

32 degrees Fahrenheit is a special temperature. Above that value Nature behaves one way, and below, another. There are many parameters like that in our world that delineate some qualitative boundary — some are temperatures, some are weights or forces or lengths. In Field Theory there is a special length that separates Dr Jekyll and Mr Hyde-like personalities of Nature. This cross-over point can be stated as a particular value of length (or time, Dr Jekyll) in which the Heisenberg Uncertainty Principle (the quantum side, Mr Hyde) and $E = mc^2$ (the relativity side) reveal a strange truth about our Vacuum. Truly, Nothing turns out to be a wild place and Dirac let it loose.

Let's use the Uncertainty Principle as our guide and see where it leads us. Remember it's

$$\Delta x \Delta p \sim h$$

where the ~ symbol means "almost."[7] Whatever Δx and Δp are, their product has to equal h. If our distance uncertainty is small—we're peering at space very finely and the momentum uncertainty must be large in order that their product is always h. In that situation our ability to know about speed or momentum is greatly decreased. Likewise, if we're casual about our investigation of space so that Δx is relatively large, then our ability to know the speed or momentum of something has sharpened up.

Remember Compton Scattering? This was the reaction of photons with free electrons in which the momentum and energy calculation was done by presuming that the photon behaved like a solid, tiny ball. When I went through that calculation in an off-hand way, I highlighted a particular wavelength,

Symbolically it's

$$e\gamma \to e'\gamma'$$

where the primes on the outgoing particles serve to remind us that the outgoing energies and momenta (wavelength, for the photon) are changed in the scattering.

[7] There's actually a factor of 4π involved, but that won't change our argument.

By the way Compton Scattering was re-calculated using the Dirac Equation with Dirac's 1927 counting-photons idea and it was found to only work *only* if both the negative energy states and the positive energy states were included in the calculation. Otherwise, the wrong answer resulted—another indication that the whole thing was right on the money.

called now the Compton Wavelength, which had the value

$$\lambda_C = \frac{h}{mc}.$$

Any object of mass m would have its own, unique Compton Wavelength. I'm going to show you that λ_C is a special length: for a bit of space greater than λ_C, conditions are one way, and for a bit of space that's less than λ_C something different happens. It's the dividing line in Quantum Mechanics which opens a door to havoc in the Vacuum.

For the electron, $\lambda_C(e) = 2.4263102175 \pm 33 \times 10^{-12}$ m. For the proton, $\lambda_C(p) = 1.3214098446 \pm 19 \times 10^{-15}$ m

Now let's imagine that we trap a single electron in a little box of side Δx (in one dimension). Where is the electron? Well, you don't know for sure, but you do know that it's within the box, and so that Δx is a representation of the electron's uncertainty in position. (This of course implies an uncertainty in momentum or velocity.)

But now let's make the box smaller and smaller to the point where it's *so tiny* that its sides are equal to $\lambda_C(e)$. So let's do that. I'll take the Uncertainty relation, replace the uncertainty in position Δx with the Compton Wavelength for an electron and then multiply the whole thing by 1...written in an equivalent but useful way, $1 = \frac{c}{c}$ in the middle...and turn the crank:

$$\begin{aligned} \Delta x \Delta p &\sim h \\ \frac{h}{mc}\Delta p &\sim h \\ \frac{h}{mc^2}\Delta pc &\sim h \\ \Delta pc &\sim mc^2 \end{aligned} \tag{18.1}$$

Notice in the third line that when the two "c's" are distributed the mc becomes mc^2.. Now that's interesting. Let's look at what this last line says which comes from just manipulating the equation to solve for Δpc.

From the relativistic energy relation $E^2 = (pc)^2 + (mc^2)^2$, the pc piece is related to the motion energy of the electron. We've got this electron boxed in to a size that's so tiny...that its kinetic energy *uncertainty*—not its actual kinetic energy, but just the uncertainty of it— is as large as the rest energy of an electron.

What's not forbidden by any rule of physics, must happen.[8] What this suggests is that if we squeeze an electron into a space smaller than its Compton wavelength that shiny, new particles could be produced *purely from the Uncertainty Principle.* No direct, or actual input of energy...just the inability to be precise about energy is sufficient to have to admit to reality: the mass equivalent of that imprecision in energy. That's wild. *It's like the possibility is the reality.*

[8] Have you ever thought about physics like that before?

Not only can we create matter this way, we lose yet another formerly concrete idea. We put into the box, a single elecron, but by changing *the box* we lose the ability to be sure that there's only one electron. Into the clutches of Uncertainty goes the certainty of counting electrons. In some volumes it might be 1, but in other volumes or other circumstances it may be 3 ...or more. Try to make it 1 by putting a particle in a box that's really tiny? You might create more particles.

In fact, you don't need a particle in that box to create matter this way. Just build a little box with nothing in it that's a factor of 2 smaller than the Compton wavelength of, say electrons—and then an electron and positron could pop out of the vacuum. Unbidden, just by the tininess of space itself. It's as if you look at the horizon of the ocean, it looks flat from horizon to horizon. But if you get closer and closer to the water, it looks more and more agitated and frothy. The Vacuum is indeed frothy, except not with sea water. But with elementary particles.

Again, the Uncertainty Principle is the reason. If we accept it—and you'd better!!—then we have to accept the consequences. We used to think of the Vacuum as a state of Nature in which nothing exists and in which there is no energy. That is, we used to believe that $E_{Vacuum} = 0$, exactly. But the Uncertainty Principle says that energy cannot be precisely anything, unless viewed over an infinite time. So the energy of the Vacuum is actually fluctuating around the value of $0 \pm \Delta E$, and that rippling energy fluctuation can produce particle-antiparticle pairs when the $\Delta E \sim 2mc^2$. And does.

Wait. *Why the "2"?*

Glad you asked. *As reckless as Nature seems to be, there are some things that He's pretty particular about and one of them is that electric charges are always balanced. So any frothy particle production from the Vacuum had better add up to zero electric charge, since the Vacuum itself has zero electric charge. So what is produced in the froth? Equal numbers of particles and antiparticles, two by two.*

Gone in Quantum Mechanics is the luxury of imagining that there is some state of Nature with absolutely zero energy! So Nothing...seems to be constantly idling at a very low value perhaps, but it's not shut off. There are observable consequences to this which we'll talk about in a bit.

So how do we force the Vacuum into play? By forcing particles to come together in a volume so small that particle number becomes a variable and we find that particles then appear to change into other particles—if there's enough energy to make them real. What's actually happening—if we strictly read the mathematics and turn it into words—is that in all reactions particles go into the Vacuum where their particular Ur-Field reservoir lives and other particles come out of of their Ur-Fields. Which ones? How many? Well, that's what the laws of Nature dictate. That's where the forces of Nature come into play.

So now to summarize. What's actually in the Vacuum? The mathematics suggests that for every kind of particle there is a "fundamental" field. The Vacuum is the state of those Fields when they've not actually produced any particles. When is that? Well, never, since the Uncertainty Principle forces the spontaneous production of particle-antiparticle pairs all the time.

This is really quite profound and will become more so when we think about the implications for cosmology. But try to imagine just how beautifully active spacetime now is! Close your eyes and pretend to remove all of the furniture around you. Eliminate the room, the building...the Earth, Sun, stars...even any stray radiation from temperature. Create a space in which there are. no. things. There are, however, the Fields and Heisenberg quietly insisting that Zero is not an option, and so your empty mind-universe is suddenly populated by millions and millions of particles. It won't sit still. There is no such thing as nothing. At least in our neck of the woods.

Dirac started this way of thinking, and then couldn't control the direction it took when other brilliant people followed the mathematics and the physics to the highly confirmed theory of Relativistic Quantum Field Theory that we all know and love today. He grew to dislike some of the consequences.

Now let's learn how we'll create particle reactions for any theory in a strictly rules-based way and develop the set of tools that we'll use in the rest of our story. The development of these techniques was not a pretty sight but was a couple of decades of anguish and confusion, oddly blended with impressive prediction-confirmation successes. Such...is Science.

18.4 Feynman Diagrams for Real

Let's cut to the chase and work out the tools we'll need in order to unravel the forces and particles that feel them. The calculations in RQFT are very involved. For example when I teach the trade to second year graduate students the first full-blown calculation I do is our old friend, Compton Scattering. It has features which are illustrative of other processes and since it involves two particles "in" and two particles "out" I can develop some tools that will become useful later in their bewildering journey. It also can be quickly shown to reduce to a non-relativistic, non quantum mechanical result when proper approximations are used.[9]

In order to go from the beginning to end of the calculation of the probability of a photon scattering from an electron requires about 3 hours of hand-scribbling on the chalkboard and about 25 pages of my own handwritten notes. Every line is a mathematical step and the opportunities for mistakes are frustratingly large for the professor and amusing for the students.

[9] In fact, that non-relativistic, non-quantum mechanical result is a calculation that they all learn in undergraduate school that explains why the sky is blue. Go ask Mr. Google about Thomson Scattering and Raleigh Scattering.

Definition: mathematical.
it's silly

That's why we're all indebted to Richard Feynman. His reformulation of Quantum Mechanics had a particularly pretty abstraction into little cartoons that can each stand for the whole of the "histories" of a particle as I explained before. Of course the pictures are Feynman Diagrams which I've used in a non-standard, classical sense just to get you ready to think in terms of Spacetime. RQFT in Feynman's hands is visually instructive—you can "see" the processes as they unfold— yet they really are a quite sophisticated mathematical tool. We're not going to do the mathematics, but we'll make use of the "Feynman Rules" almost in the same way that he intended them to be used. They take into account all of the surprises that Dirac invented and reduce them to little stories.

18.4.1 The Dance

Back to the atom. Our picture of the excitation of an electron will become successively more sophisticated with the Vacuum actually upsetting the simplest explanation of atomic spectra. But let's stay with the simple idea and keep track of what happens in the excitation and emission cycle: EM wave comes in and excites and electron from its ground state, it spends a bit of time in a larger orbit, and then it de-excites back to the ground state by emitting a photon. Let's do it in a really, really detailed way using Dirac's creation and annihilation explanation in the process. Stay with me.

The dance of the electron and the photon takes us through six steps. The partners approach one another, come together in a resonance, and then separate. But the Vacuum is at work between each step swapping particles in and out, like a persistent former boyfriend who keeps trying to cut in. The steps are these:

1. The electron and the photon have their separate existences: the electron in its orbit and the photon as a part of some external radiation field, like a sharply tuned laser beam.
2. They come together and are each annihilated into their respective Ur-Fields.
3. A new, excited electron is created from the Electron-Field.
4. That electron's now in the high state and executes its own solo moves, fully showing off on the high stage.
5. But the high life doesn't last long and the electron is annihilated. Aw.
6. But not to worry, because no sooner has the glittery, solo electron faded from view, but a new electron is created back in the ground state and a new photon is also created and our pair go on to dance again some day.

Each of these moves are executed according to rules that depend on the forces involved, here those of Quantum Electrodynamics.

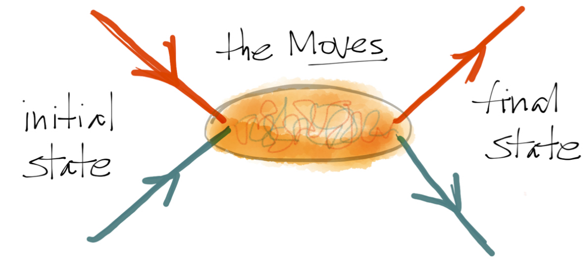

Figure 18.2: The Dancefloor Spacetime Diagram: two dancers come together—bust some moves—and separate.

We need to begin the use of important language, that of the Initial State and the Final State of interactions. It's straightforward, but important since what we can actually measure with our real-world apparatuses.

Let's turn our dance into a liaison. From across the room two partners spy one another and move gracefully together, twirl around executing whatever romantic Latin ballroom technique you like to imagine, and then wistfully move apart at the end. Three stages of this little romance: the Initial State, the Intermediate State, and the Final State. The Intermediate State is where the action is. Before they come together, you're unaware of whether they're going to Fox Trot, Waltz, Tango, or Boogaloo. There's lots of movement and close analysis would reveal their choice. After they're done, their Final State is one of separating and going on to whatever their next project is. We can diagram this three step[10] performance in Spacetime as shown in Fig. 18.2. You see them come in, go out, and do something in the middle. This Dancefloor Spacetime Diagram is generic. Until dancing with three partners becomes standard, this diagram will accommodate all ballroom dancing.

So the Initial State and the Final State we will think of as free dancers (particles). They're far from one another, are not yet interacting, and frankly in the model, untouched by the Vacuum or Quantum Mechanical complications[11]
So with that caveat the lines represent particles, in which the wavelike nature of the Quantum Amplitudes have been buried inside of the nice sketch: Feynman's vision is decidedly a Particle Picture. There's no denying the conceptual import of that and I'm a victim of thinking that way, and so will you too. But deeply imbedded are all of the rules of Quantum Mechanics with the dual nature of particles and waves, but we don't have to worry about that complicating, mind-hurting idea when we use Feynman's Rules. So back to our atom.

The initial state consists of an electron and a photon and the final state's a different electron and photon and what happens in between is governed by the rules of QED (and any other Laws of Nature that allow electrons and photons to interact).

The rules are the following. Figure **??** shows two lines which will be used to represent electrons (and other particles we'll meet). One thing is always true of an electron line and that is that an arrow always points in the direction in time that the electron is headed. For an electron that's somehow managed to violate the laws of RQFT and is just merrily moving along in spacetime without any interactions, we'd just drawn the line and the arrow. It's a "free" electron.

But for an electron that undergoes an interaction with another particle, we draw a dot at the point where they come together as the top line in Fig.~ 18.3. The dot signifies two things. First, it denotes the force strength of whatever model of particles is underway. Second it signals the departure (or arrival) of

[10] No pun intended!

[11] Not quite. In fact, according to Feynman's completely self-sufficient description of Quantum Mechanics—you know, he reinvented Quantum Mechanics for his Ph.D. thesis—these initial states as drawn by a single trajectory are not possible. In fact to him a particle takes *all* possible paths to go from one place to another. The dancers approach one another simultaneously by approaching normally, but also one of them goes out the door, down the hall, and back into the room at the other end, the other also goes to the moon and back...and so on. Each possible path has a probability associated with it. The normal path for people-sized objects? Of course it's the one with the overwhelmingly highest probability. But the other paths have to be included or the answer is wrong. We've already encountered this fact in the description of the two-slit experiment. Our quantum softball's amplitudes go through *both* slits and interfere on the screen. What the lines in the dancer diagram represent is the sum of all possible paths and is therefore a really complicated object.

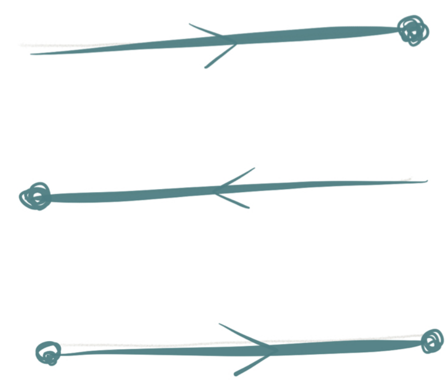

Figure 18.3: The top line represents an electron coming in from the left and interacting at the dot. The second line is an electron coming in from the right (!) and interacting at the dot and the third line represents an electorn that is "born" at the dot (some previous interaction) and lives for a while and then interacts again at the right-hand don.

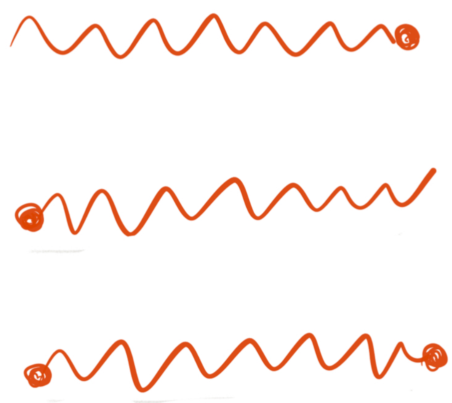

Figure 18.4: Like the electron picture in Fig. 18.3, here the same thing can be represented for photons.

that electron from the stage—it's gone to the Electron-Field and been annihilated if the dot comes after the arrow. If the dot comes before the arrow? Then, happily, a shiny new electron has been created.

Figure 18.4 similarly shows three photon graphs. The top one is just a free photon, a part of a happy EM field, maybe the sunlight, maybe an angry gamma ray from a supernova. (Notice, no arrow.) For a photon that undergoes an interaction with another particle, we do the same thing as with the electrons, namely put a dot at one end or the other (or both) to signify the force "coupling" strength and the annihilation or creation at that point of photons.

Not to be too pedantic, but what does the dance card look like for the catchy Compton Scattering Boogie? The initial state of Compton scattering is and electron and a photon. The final state? A different electron and photon (different, remember because they have different energies from the initial state versions). How does this compare with the dance of an electron and photon in the atomic absorption case? Apart from the energetics (Compton scattering would be X-rays, while the excitation of Hydrogen would be likely UV or visible wavelengths), they are the same physical process and so the same Feynman Diagram!

We literally just add the pieces together following the story: in the beginning, we had an electron and a photon and at the end, likewise. In the middle we had that solo show-off electron which connects the two pairs in the beginning and the end of the dance move. Figure **??** shows the whole thing.

Notice that the electron-ness "flows" in time with the arrows pointing the way. One of the cardinal rules of Feynman Diagrams is that electron (and sister particles we'll come to...I know, I keep saying that) lines are continuous. No breaks from beginning to end. That's not required of photons.

Remember that I've hinted at the fact that we know of four different forces in Nature, three of which are quantum mechanical and can be described in Feynman cartoon-language. In fact we can uniquely characterize every interaction by what I'll call a **Primitive Diagram** (PD). Primitive Diagrams are the starting points for any theoretical physicist who's creating a model for particles—he or she must work in terms of these pieces and then possible scatterings or decays of the particles in their candidate model putting the PDs together in order to create reactions that can be tested in an experiment. So the diagrams that are created include the new ones that he or she are trying out along with whatever "regular ones" that we already know are respected in Nature.

We now have our first Primitive Diagram shown on the largely empty table. As we move along, we'll fill in each box with a new PD for the forces that we know now, and love.

Don't try this at home, as the full implementation of the Feynman calculus is done by Professionals on a closed track. For a physicist, when a diagram is properly pieced together, it represents an algorithm for doing an important calculation: we build an equation that gets a term from each line and each dot.

Sometimes these equations can be quite involved when there are many lines and dots (interactions) and the Feynman Rules allow us to skip many, many steps and use the constructed equation to get to an actual prediction that can be checked in an experiment. For example, in the Compton Scattering marathon that I described above, 15 of those 25 pages of notes are eliminated by starting with the Feynman Rules, drawing the graph of Fig. **??** , and then building the formula and turning the crank. What one gets out of the calculation is the probability that a reaction might occur and even what the momenta of the final state particles would be. A direct prediction that can be tested. So, soup to nuts: Feynman Diagram → Probability → final state particles and their momenta.

18.4.2 Antimatter

But we know more now than just electrons. We've got to take into account its antimatter partner and we'll see that antimatter dances differently from matter. Buckle up your seatbelt because we're about to explore one of the more notorious interpretations of antimatter due to Feynman and his unusual thesis advisor.

Remember that we got into the antimatter game by imagining how Dirac's negative energy electrons might be liberated by being kicked by a photon into a positive energy electron leaving behind a positive energy positron. The process for this is

$$\gamma \to e^+ e^-.$$

We will take on the convention of referring to antimatter with a bar on top of the symbol:

$$\bar{e} \equiv e^+,$$

so that the liberation formula can be written as

$$\gamma \to \bar{e} + e$$

showing that when we do that, we dispense with the electric charge for the matter particle (electron). Given what we said above, we can draw the Feynman Diagram for this process as shown in Fig.**??**. This is a famous physical process called Pair Production, which we'll see motivates particular kinds of particle detection devices.

I've been careful to drawn the photon as going in positive time, left to right, and to label the electron and positron as likewise going in the positive time direction. Here's where it gets interesting because Feynman noticed that the energy and time variables in the equations of Dirac's always arranged themselves in the following ways. For positive energy electrons, they always appeared as products like:

$$(E)(t)$$

while for negative energy electrons they always appeared as

$$(-E)(t).$$

Being a really skilled mathematician [12] he rewrote it as

$$(E)(-t).$$

[12] And a really sneaky one.

Breathtaking, right? This is how Nobel Prizes are made! Moving a negative sign in a way you might have done in 8th grade. What he did was effectively turn a negative energy particle moving forward in time into a positive energy particle moving backwards in time.

Yes, you heard it first here. If we take Dirac seriously and take Feynman seriously—which is sometimes hard to do—we have a brand new interpretation of what antimatter is: it's regular matter moving backwards in time.

Now we don't actually detect backwards-moving electrons...electrons coming at us from the future. Rather we regularly see and manipulate positrons behaving as they should—moving forwards in time. But there's no way to ignore the possibility that antimatter is even stranger than you realized. But if we also take Einstein seriously, then if space and time are on equal footing and if I can walk as easily East as I can West (that move in space in a positive direction or a negative direction), then why can't I also do the same thing in time? Now don't get excited. Macroscopic objects are not coming at us from the future. Why? Well because we have not macroscopic antimatter objects since the annihilation of antimatter and matter happens in the blink of an eye and so macroscopic anti-objects are not possible in our Universe. But it's fun to think about it and frankly if you don't like Feynman's interpretation, you don't need to worry too much about it. But we'll use it and it's necessary in order to actually build a self-consistent RQFT.

So what about our diagram for Pair Production? If we take that positron leg which si going from the past into the future and simply reverse the time direction we turn it into an electron that's coming from the future, into the past (or here, present). The two are equivalent representations of the physical process, but it's important to remember that what we actually measure in the laboratory is always a particle or antiparticle going from the past into the future.

Manipulating Spacetime

Notice that I've drawn our QED PD in a manner that's not consistent with Relativity. Vertical lines in spacetime don't make any sense since they imply motion in space in zero time. But I've done this intentionally as I mean for the Primitive Diagrams to be puzzle pieces, except that instead of there being only one way to use a piece of a jigsaw puzzle, our PDs can be manipulated. Again, since we believe in Einstein and that space and time are on the same footing, any diagram that we draw in one configuration relative to the vertical (space) and horizontal (time) axes...we can rotate around into another process. One diagram actually can represent many different processes, and in fact we can predict physical processes by doing so.

That's how we'll use our PDs. When a new theory is on the table, I'll tell you the PD and fill in the diagram in the table. From them and the Feynman Rules (which I'll strictly enumerate in a bit), we can literally use all of the PDs as puzzle pieces that we can put together to predict reactions that we can expect to detect in the laboratory. If we find them, then great: we've just confirmed the model represented by the PDs. If we don't, then great! We've just disconfirmed a model and that's always fun for an experimenter (unless the theorist is one of your friends).

So let's complete the story by now predicting a number of QED reactions which are all embodied in that one, lowly QED PD.

18.5 The pieces

18.5.1 Spacetime Arrangements

18.5.2 Primitive Diagrams

18.6 How Do We Know?

18.6.1 The Atom Feels the Vacuum

18.6.2 The Vacuum Exerts Pressure!

18.6.3 The Whole Mess Works. Really Well.