ISP220 Homework #13
30 points

New version: figure references fixed.

April 24, 2014!!

Your Name: ____________________

- um...**STAPLE these pages**!!
- This homework set is due on Thursday, April 29, which is Final Exam day.
- I need you to show all of your work—if you have comments about your reasoning, say them.

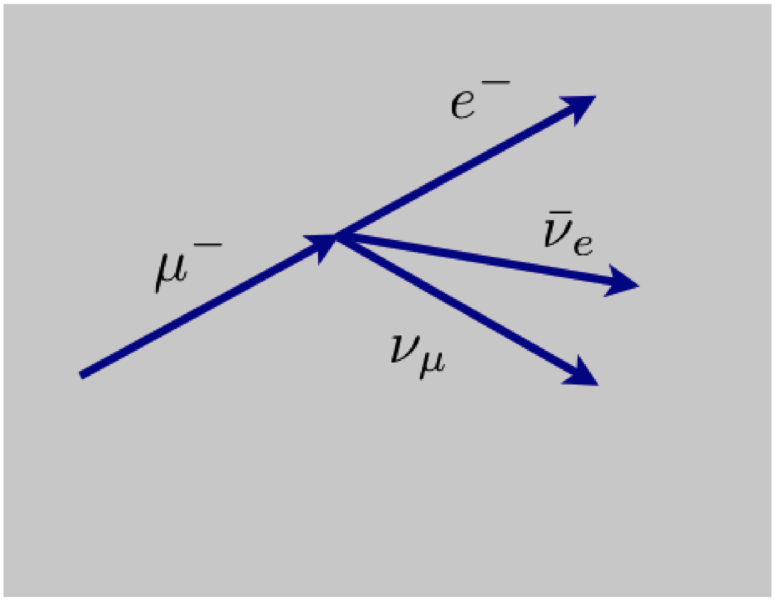

Figure 1: Decay of a muon (particle, not antiparticle) into an electron and two neutrinos.

Manipulating Feynman Diagrams

This problem involves the muon decay Feynman Diagram shown in Figure 1. It's what I showed for class, but with the explicit charges shown. The convention for what's a particle and what's an antiparticle for leptons is different than for nucleons—just a historical quirk (not quark). The electron and muon particles are negatively charged and the antiparticles are positively charged. For this problem, represent them with their charges as I've done, not with the bar over them. For the neutrinos, however, use the bar over the symbol to represent the antiparticle and don't forget that there are two kinds of neutrinos here and they have to be kept separate.

So, here's the plan. The standard neutrino beams produced at accelerators consist overwhelmingly of muon neutrinos (not antineutrinos). These neutrinos can scatter from quarks (which we'll talk about) and from electrons. What I want is to construct the Feynman Diagram for the process:

$$\nu_\mu + e \rightarrow \mu^- + \nu_e \tag{1}$$

Let's do this in the steps that I've outlined and written down in the slides.

1. (2 points) Pull the muon neutrino line from the final state into the initial state, push the initial muon line from the initial state into the final state, and bring the electron line from the final state into the initial state. Draw that diagram keeping track of the arrows. They will now not make sense. That is, you'll now have an electron leaving the initial state going backwards in time, etc.

2. (2 points) So, now fix the lines by using the Feynman trick of reversing the directions of the arrows and doing the appropriate thing to the particle-antiparticle nature of each adjusted leg.

3. (3 points) Notice, that this is not the reaction we want. Why? (Hint: can you think of any positron targets?) So turn it into the reaction we want...use the rule I talked about on Tuesday and do it: Draw the final Feynman Diagram corresponding to Equation 1.

4. (2 points) **There are reasons why the following reaction cannot occur. Tell me three.** You might need to watch the Quantum Numbers video, if you haven't already.

$$\pi^- + p \to \Sigma^0 + \Delta^{++} + \bar{e}$$

5. (2 points) **This is almost a trick question: Why is it impossible for the following decay to occur? The neutron is at rest. (Remember relativity.)**

$$n \to p + p + \bar{p} + e^- + \bar{\nu}_e$$

Let's warm up for the final exam by doing one of the calculations like I did on the video at `https://vimeo.com/64457879` . What collides at Fermilab and the LHC are pairs of quarks within protons (or antiprotons). In particular to produce the following reaction at Fermilab:

$$p\bar{p} \to Wg,$$

If this were the Final, the things that I would provide to you are the following:

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i) **Luminosity**. The Accelerator is the proton-antiproton collider at Fermilab with a luminosity of L = $10^{32}\text{cm}^{-2}\text{s}^{-1}$.

ii) **The production Feynman Diagram** is shown in Fig. 2.

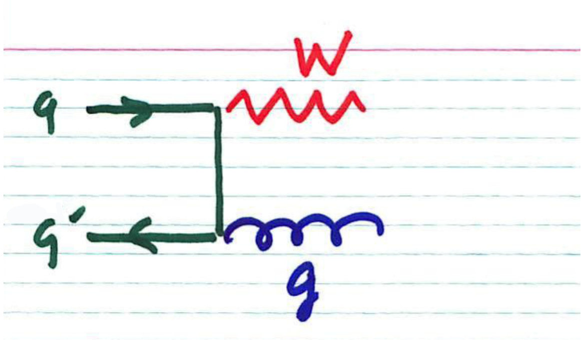

Figure 2: The production Feynman Diagram for the process.

iii) **The cross section** for the process is $\sigma = 4 \times 10^2$ pb. (picobarn)

iv) **The final state** for this problem is demands that the W boson decay into a muon and a muon neutrino, so the whole reaction is:

$$q\bar{q} \to W + g$$
$$\hookrightarrow \mu + \nu_\mu$$

v) **The Branching Ratios** are: $BR(W \to qq') = 0.7$, $BR(W \to e\nu_e) = 0.1$, $BR(W \to \mu\nu_\mu) = 0.1$, and $BR(W \to \tau\nu_\tau) = 0.1$.

From the old lecture, you know two things that you need for the conversion of picobarns to cm^2:

- a "barn" is a unit of area in which 1 barn $= 10^{-24}$ cm^2 and
- 1 pb $= 10^{-12}$ b.

I also will remind you that a year has 3.15×10^7 seconds, which we playfully call "$\pi \times 10^7$ seconds" in order to remember it.

So now the questions:

6. (2 points) Show that the number of square centimeters that corresponds to 1 pb is 10^{-36} by filling in the formula, canceling the dimensions, and getting the answer (see the example on the last page to see what I mean for you to do here):

$$1 \text{ pb, in cm}^2 = 1 \text{ pb}\left(\frac{\qquad \text{b}}{1 \text{ pb}}\right) \times \left(\frac{\qquad \text{cm}^2}{1 \text{ b}}\right) = 10^{(\qquad)} \text{cm}^2$$

7. (3 points) From the familiar formula of $N = L\sigma\Delta t \cdot BR$ number of events in a year for

$$q\bar{q} \to W + g$$
$$\hookrightarrow \mu + \nu_\mu$$

at this accelerator with the stated luminosity for this final state is about (circle your choice):

(a) 12×10^5 events
(b) 12×10^4 events
(c) 12×10^3 events
(d) 12×10^2 events

8. (3 points) Draw the Feynman Diagram for the whole process of

$$\begin{aligned} q\bar{q} &\to W + g \\ &\quad\ \hookrightarrow \mu + \nu_\mu \end{aligned}$$

9. (4 points) On Fig. 3 indicate what the final state would look like. Do NOT draw in any neutrinos, but leave the obvious imbalance of momentum apparent from your picture. (Notice that I'm not specifying the charges, so bend the particles if necessary, arbitrarily.)

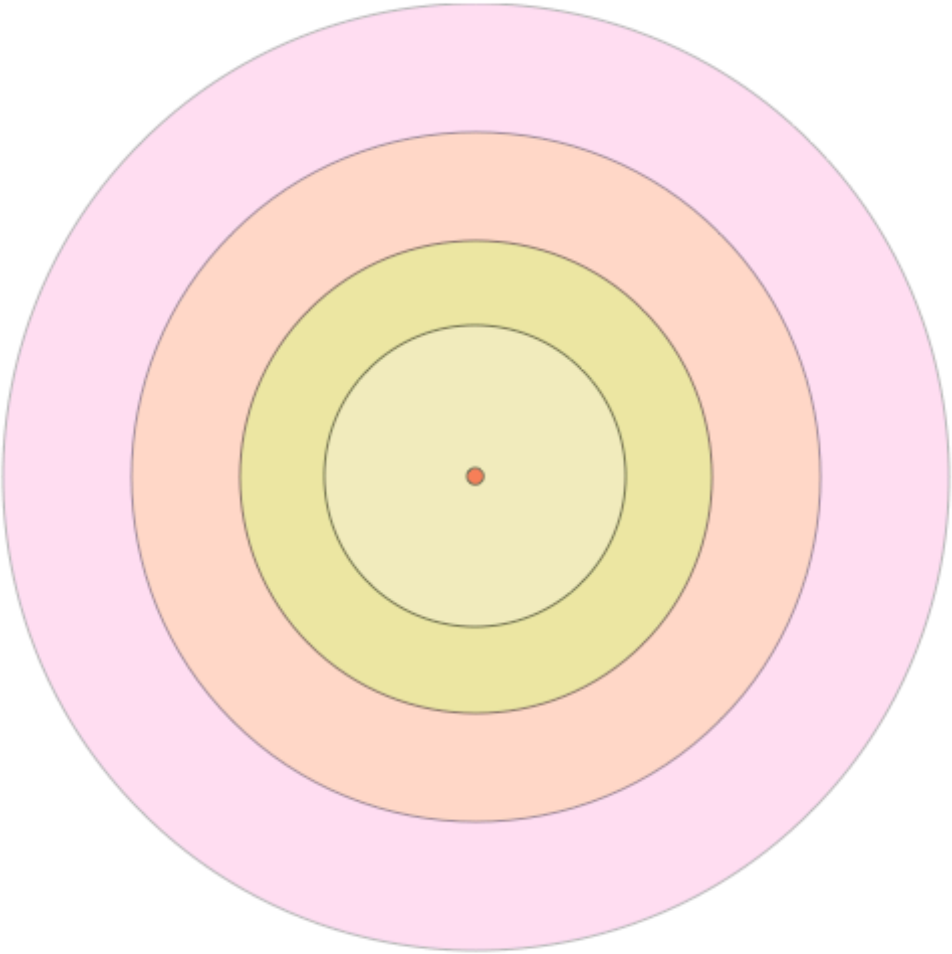

Figure 3: How the reaction $p\bar{p} \to Wg$, with $W \to \mu\nu_\mu$ would look in the cartoon detector.

Bubble Chamber Reactions
What follows refers to photographs at the end of the set. You don't have to print them out and turn them in if you don't want to.

I have three bubble chamber photographs of events which were exposed to a K^- beam in a liquid hydrogen bubble chamber. So, the target consists of protons. I'll work one as an example, and you work the other two. The Example refers to Figure 4 on page 14. See the letters imbedded in the photograph? That's what the questions will refer to in what follows and you need to circle the letter you think corresponds.

1. Which track shows a collision of beam particle with a proton:

   a b c d e f g

2. Which track shows the decay of a neutral particle.

   a b c d e f g

3. Which track shows a collision of a beam particle with an electron.

   a b c d e f g

4. Which track shows a collision of a neutron with a proton.

   a b c d e f g

5. Collision of a photon with an electron.

   a b c d e f g

Here is my analysis:

1. A kaon hitting a proton will result in some strong interaction, so you're looking for a charged kaon coming in from the bottom and at some point multiple charged tracks

emerging from a point on a kaon track. That would be the production of pions and protons and at least some strange particle. It could also include production of neutral pions, which you'll remember decay into two photons, which you'd see convert into e^+e^- pairs. There's nothing like that. But, c does appear to be a beam-proton collision.

2. This will be one of the V's found in cosmic rays. That's apparent at point d.

3. This will be the collision of a kaon and an atomic electron, which will probably be a "glancing" blow and the electron will spiral quickly and the kaon will continue on its way since it's so much more massive than a stationary electron, it's momentum will be hardly affected. Look at a. Here is exactly this circumstance. Notice that this also shows you the direction of the magnetic field: negative particles will bend to the right. The kaon beam is moving pretty fast and it's hard to tell that they are negative.

4. A neutron hitting a proton is obviously not the consequence directly of the incoming kaon beam. But, neutrons are always a background in beams since they can be produced way up stream and then they're really hard to stop.[1] So, the occasional neutron will happen and it does here. The initial state would have a net positive charge of +1 since it's due to the target proton alone—the neutron hasn't any charge. So, that's why the V in the previous question doesn't qualify. That was a net zero charge in those two decaying particles. Point e has three tracks emerging, one negative (bends right) and two positive, as expected.

5. Photons could also come with a charged beam since they might be radiated from a beam particle. So, we expect to see nothing and then an electron suddenly spiraling to the right. Point b is exactly that.

Now it's your turn.

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[1] That's why the so-called "neutron bomb" is such a deadly battlefield weapon. There is no protection for people, even inside of tanks. The water in the body is a large amount of nearly free protons, so the neutrons elastically scatter from them and do terrible damage. Of course, it's indiscriminate and civilians and not just soldiers would be killed. I don't like the neutron bomb.

(Using Figure 5 on page 15), answer the following:

10. (2 points) Which track shows the decay of a neutral particle.

v w x

11. (1 point) Which track shows a collision of a beam particle with an electron.

v w x

12. (1 point) Which track shows a collision of a beam particle with a proton.

v w x

(Using Figure 6 on page 16), answer the following:

13. (1 point) Which track shows the decay of a neutral particle.

    v w x y

14. (1 point) Which track shows a collision of a beam particle with an electron.

    v w x y

15. (1 point) Which track shows a collision of beam particle with a proton:

    v w x y

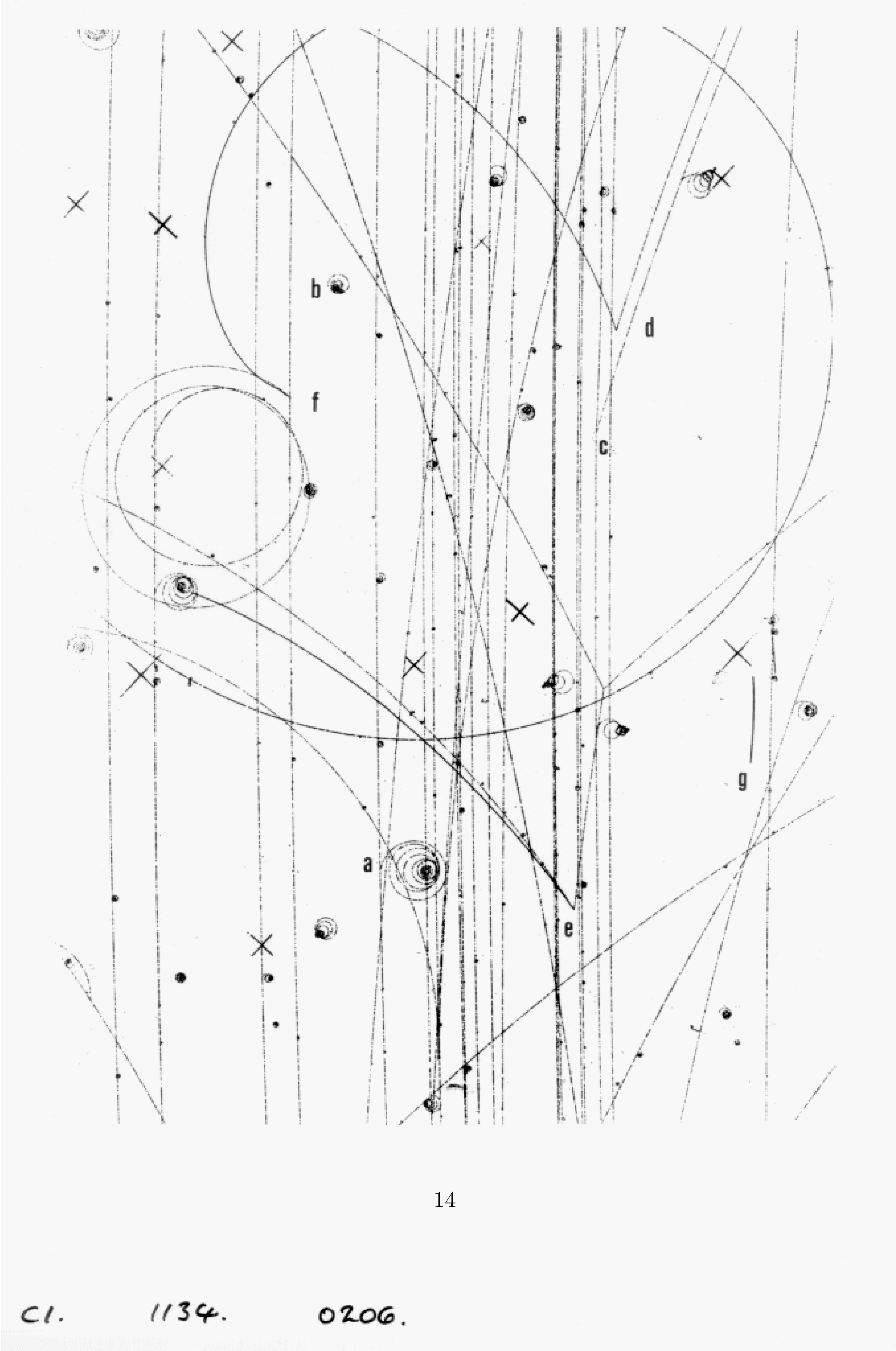


Figure 4: Example events in a negative Kaon beam.

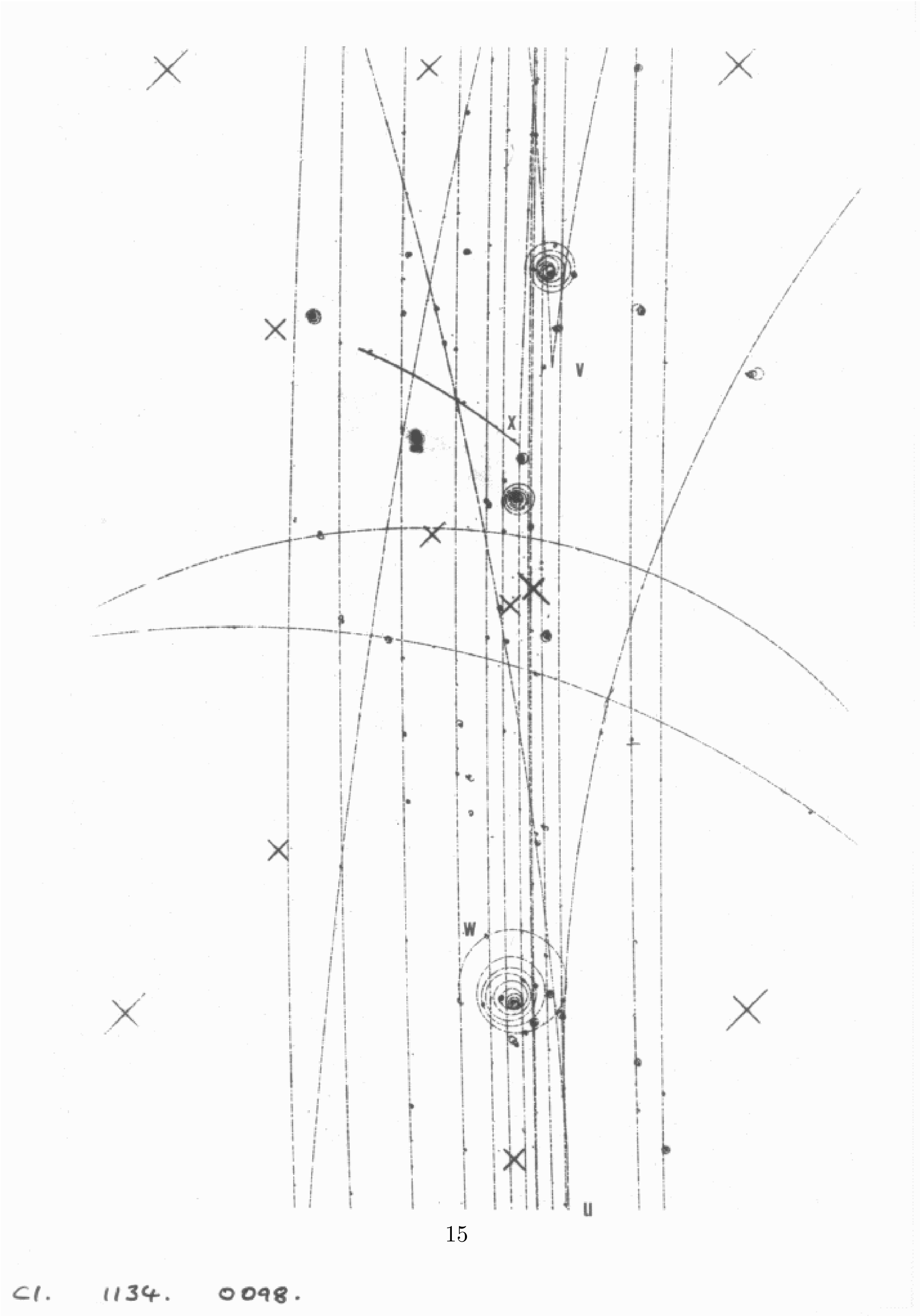
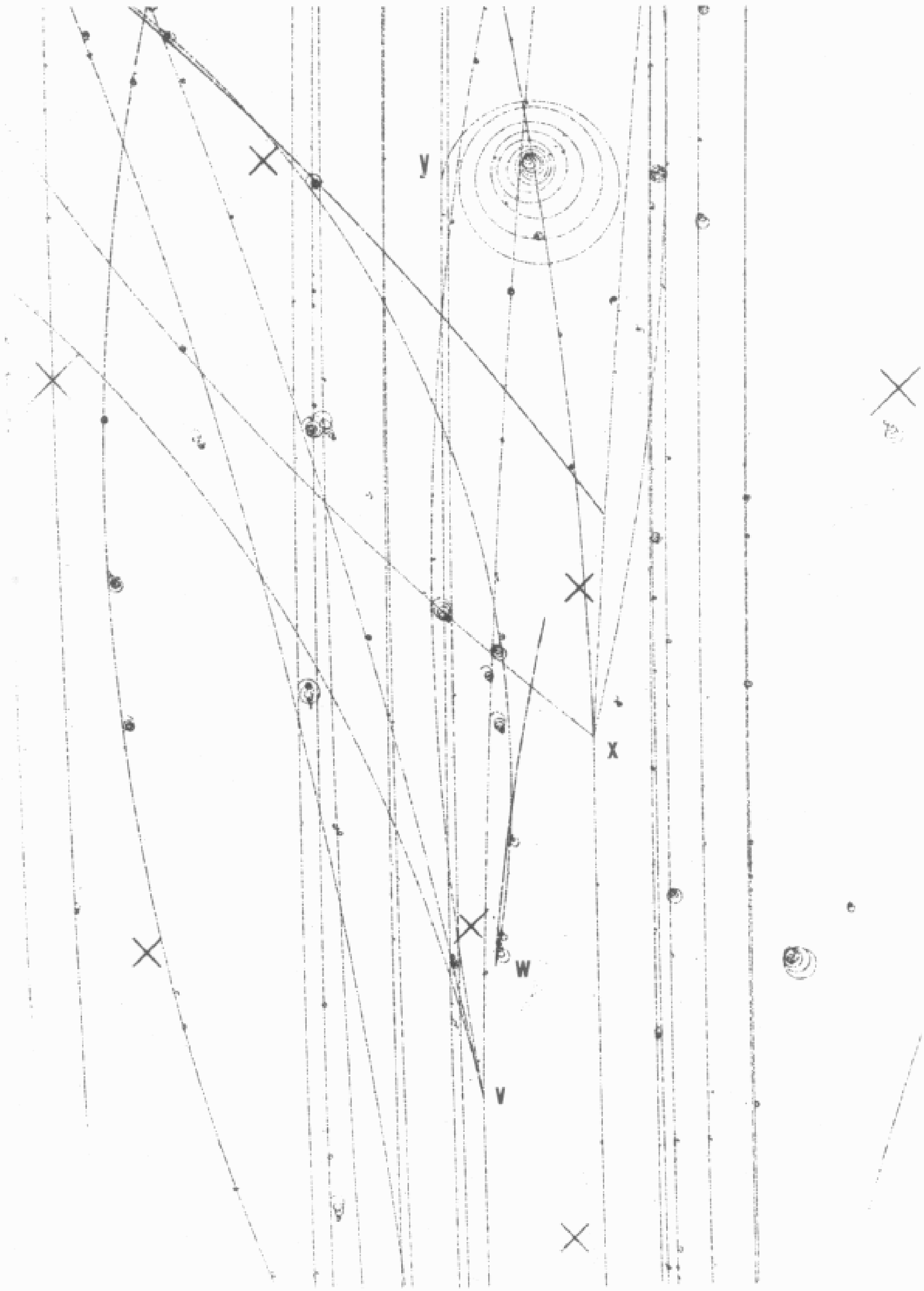


Figure 5: Problem 1 bubble chamber picture.



Cl. 1134. 0275.

Figure 6: Problem 2 bubble chamber picture

I promised an example to go along with the formula in question 5. Suppose we want to calculate 1 millennium in years. You know:

- 100 years in a century (“cen”) and
- 10 centuries in a millennium (“mill”).

Then in the same spirit, I would fill in:

$$1\ \text{mill (in years)} = 1\,\cancel{\text{mill}}\left(\frac{100\ \text{y}}{1\ \cancel{\text{cen}}}\right) \times \left(\frac{10\ \cancel{\text{cen}}}{1\ \cancel{\text{mill}}}\right) = 10^{3}\ \text{y}$$